Lecture course "Fast Solvers for Large Systems of Equations" (AMCS 343)

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Exercise 6

Exercise 5.1 Use the script convdiff.lua to solve the convection-diffusion problem

$$u_t + \nabla \cdot (\mathbf{v} u) - \nabla \cdot (\epsilon \nabla u) = 0, \qquad x \in (0,1)^2$$

with Dirichlet boundary conditions and a "hill" as the initial condition. The velocity $\mathbf{v}$ rotates the hill around the center. Discretization: FV, full-unwind method for the convection term, implicit Euler method for the time derivative.

(a) [2 point] Compute the 40 time steps with the linear iteration preconditioned with the GMG method (Jacobi smoothing) on grid refinement levels 5, 6 and 7 (option -numRefs). Compare the maximum value of u after the last time step for these grids. How can you explain the difference? Does the numerical solution converge to the analytic one if only the spacial grid is refined?

(b) [1 point] How does the convergence rate of the linear solver vary when the grid is refined.

(c) [1 point] Replace the linear iteration with the conjugate gradient method. Try it on the grid refinement level 5. What do you observe? Why?

(d) [2 point] Set BiCGStab (bicgstab) as the linear solver instead of the linear iteration. Test it on the grid refinement level 7. What can you say about its convergence in the time steps?

Exercise 5.2 [2 points] Use the script ns.lua to solve the Navier-Stokes equation in a 2d domain with a hole. The problem is non-linear, and the linear solver is used to solved the linearized problems in the non-linear iterations. Try two settings for the relative reduction in the linear solver: 10^{-10} and 10^{-5}. Report the numbers of the steps of the non-linear method in both the cases.