

Problem1

Determining whether Bubble sort is stable or not?

Given a list $S = \{40, 10, 3, 1, 40^*, 100, 2, 5\}$

1st attempt: swaps

$\{10, 40, 3, 1, 40^*, 100, 2, 5\}$

$\{10, 3, 40, 1, 40^*, 100, 2, 5\}$

$\{10, 3, 1, 40, 40^*, 100, 2, 5\}$ No swap

$\{10, 3, 1, 40, 40^*, 100, 2, 5\}$

$\{10, 3, 1, 40, 40^*, 2, 100, 5\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

$\{10, 3, 1, 40, 40^*, 2, 5, 100\}$

Repeat the process:

Implying that Bubble sort is a Stable Algorithm since the order of duplicates will never be changed.

Determining whether selection sort will be stable?

$\{4, 20, 1, 10, 2, 20^*, 12\}$

Perhaps $\{1, 20, 4, 10, 2, 20^*, 12\}$

$\{1, 2, 4, 10, 20, 20^*, 12\}$ No swap

$\{1, 2, 4, 10, 20, 20^*, 12\}$

$\{1, 2, 4, 10, 12, 20^*, 20\}$ loop will break after one more check

This implies that selection sort is not a stable algorithm

Insertion sort

{ 4, 9, 1, 9*, 2 }

We start at position 1; make a comparison only to realise it's smaller than the element at position 0; meaning we keep traversing.

{ 4, 9, 1, 9*, 2 } No swap, while loop won't be executed.

{ 1, 4, 9, 9*, 2 }, swap detected and element has been put in its position.

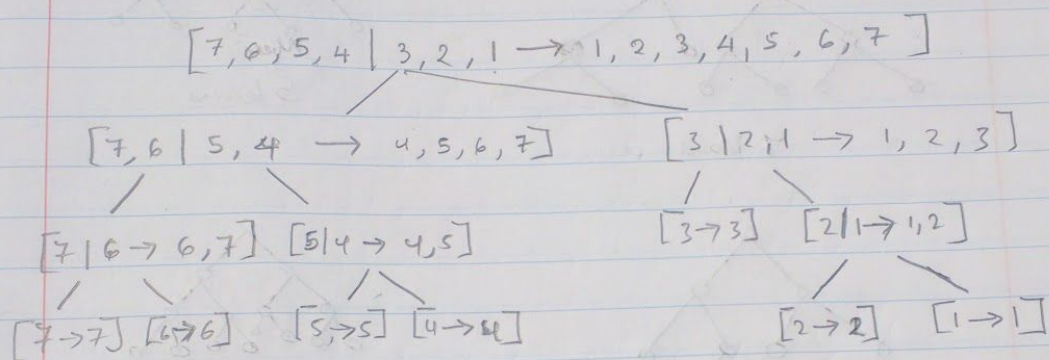
{ 1, 2, 4, 9, 9* }, Swap occurred and now our list is sorted without interchanging the duplicates ordering.

This sorting algorithm is therefore stable.

Problem2

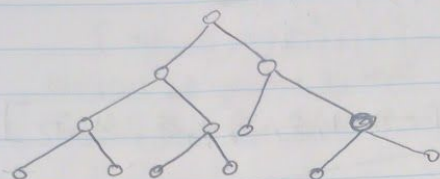
Problem

Merge sort on $[7, 6, 5, 4, 3, 2, 1]$

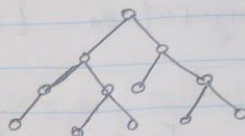


Problem4

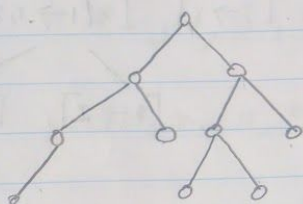
a)



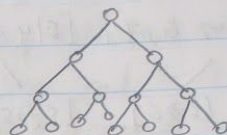
7 leaves



5 leaves



5 leaves



8 leaves

b) The deduction that every binary tree can hold at most 8 leaves is true. ($2^3 = 8$). Basing on the above results;

c) The height of a tree, n and the number of leaves
The maximum number of leaves is 2^n .

Therefore the condition

$$\text{Number of leaves} \leq 2^n.$$

