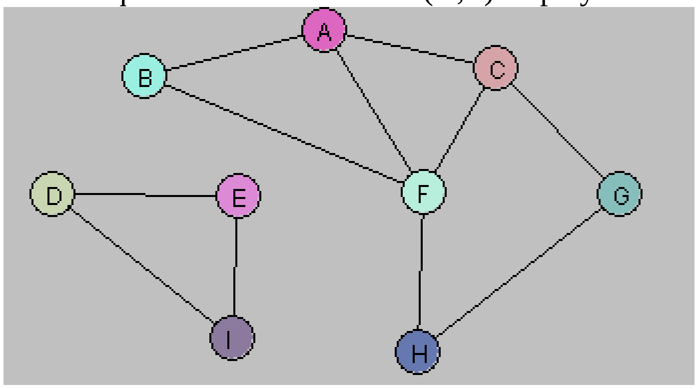
Lab 11 Solutions

1. Answer questions about the G = (V,E) displayed below.



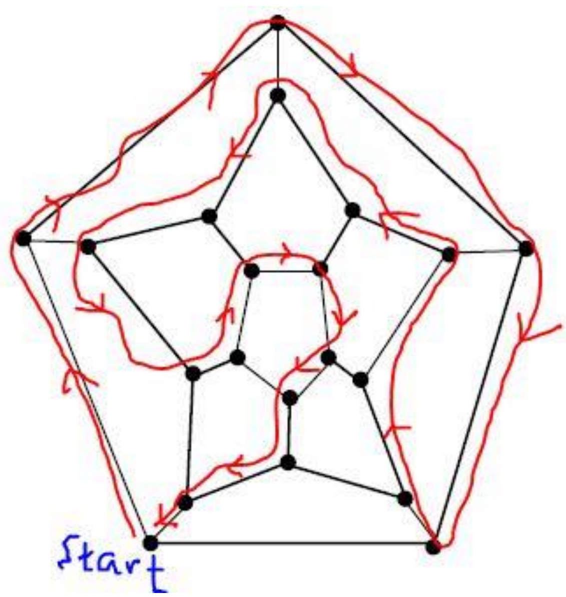
   A. Is the graph G connected? If not, what are the connected components for G?
   Solution: G is not connected. It has two connected components…

   B. Draw a spanning tree/forest for G.
   Solution: T = {DE, EI, FB, FA, FC, FH, GH}

   C. Is G a Hamiltonian graph?
   Solution: No, it has no Hamiltonian Cycle.

   D. Is there a Vertex Cover of size less than or equal to 5 for G? If so, what is the Vertex Cover?
   Solution: Yes. C={D, E, F, A, G}

2. *Hamiltonian Graphs.* The following graph has a Hamiltonian cycle. Find it.



3. Express in pseudo-code an algorithm which accepts as input a graph G and which outputs a vertex cover for G of smallest possible size. You may make use of the PowerSet algorithm without showing any pseudo-code details indicating how it works. Also, you may assume that your algorithm can make use of these operations freely:

   `computeEndpoints(e)` //returns the two endpoints of the edge e

   `belongsTo(x, U)` // returns true if vertex x belongs to set U; false otherwise

   Follow the rules for the pseudo-code language as completely as possible.

Solution:

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Algorithm: SmallestVertexCover
   Input: A graph G whose set of vertices is denoted V and set
      of edges is denoted E
   Output: Smallest size of a vertex cover U for G

   pow ← PowerSet(V)
   minCover ←  V
   minVal ← |V|
   for each U in pow do
      isCover ← true
      //verify U is a vertex cover
      for each e in E do
         (u,v)  ← computeEndpoints(e)
         if( !(belongsTo(u,U) and !belongsTo(v,U))
            isCover ← false

      if(isCover and U.size() < minCover.size()) then
            minCover ←  U
            minVal ←  |U|
   return minVal
```