Problem 1

An NP complete problem belongs to both NP and NP-Hard (NPH). Thus, we must show that TSP belongs to both NP and NPH.

The TPS problem uses a complete graph (G = (V, E)) to find if there is a Hamiltonian cycle (C) that has the sum of weights along the edges less than or equal to a positive integer k.

Proving that TSP is an NP problem (Its solution can be verified in polynomial time)

- 1. Proving that TSP is a Hamiltonian cycle (C)
 - Proving that C is a complete graph can be done by using BFS to create a spanning tree of C (this runs in 0 (m' + n') where m' and n' are the edges and vertices of C) to create a spanning tree T with m_t edges and then comparing if $m' > m_t$ to determine whether it's a cycle.
 - Proving that C contains all vertices of G runs in O (n) time.
 - Proving that all edges of C are also edges in G runs in O (m) time.

This can all be done in linear time.

2. Proving that the sum of the weights is less than or equal to k runs in 0 (m) time in the worst case. Since verifying the solution of TSP runs in polynomial time, then TSP is an NP problem.

Proving that TSP is an NPH problem

Assuming R is an NP problem, we must show that R --poly TSP. It is true that R --poly HC since HC is an NP complete problem. Also, HC--poly TSP. Thus, R --poly HC --poly TSP.

So, R --poly TSP.

Since TSP has passed both the conditions of being NP and NPH, then TSP is an NP-Complete problem.

Problem 2

Let SS be a subset problem of size in consisting of the set S = fs., S., Sa --- Sol of poster lategers and a non regative Integer K. The tack is to transform I into a Knaplack problem (K&) in Polynomial time.

Let set S from SS be the KN set of Hems. for each i, let wi = 50 and let You = Si. let H= K and let Y= K where H is the maximum weight and Y is the minimum Value. These definitions specify KN as an Instance of O(n).

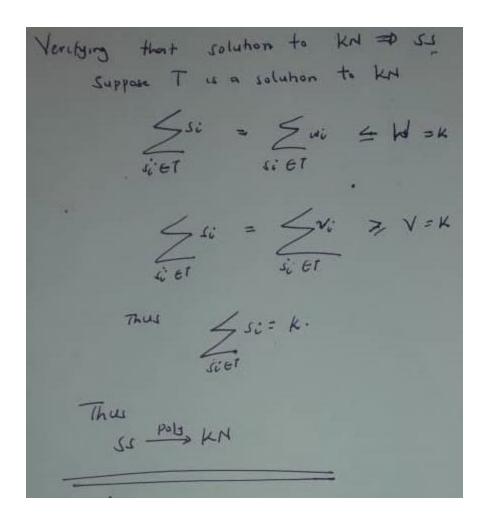
Proving Solutions

Solution that SS => KN suppose T is asolution to SS (Tis asubjet of Swhoose sum 11 K)

$$\leq Si = K$$
 $Siet$

$$\frac{1}{\sum_{s \in T} w_i} = \sum_{s \in G} s_i = k = H$$

That the solution to SS tields the Solution to KN



Problem 3

Minimum vertex cover $s = \{B, D\}$

Vertex cover approximation = $\{A, B, C, D\} = 2*s$

Problem 4

Vertex cover for a graph G=(V, E) is a set C which is a subset of V, such that every edge e which is an element of E has at least one end vertex in C. This can be obtained by getting the power set P of V and checking for the subset with the minimum number of vertices that satisfies the vertex cover definition. This algorithm runs in O (2^n) .

Given a problem with input G and k where is a non-negative integer, to verify the solution of vertex cover, the following steps will be taken

- 1. Check that all vertices in C are elements of V. This runs in O (n) time.
- 2. Check that every edge e of E has at least one vertex in C. This takes 0 (m * n). $m = O(n^2)$ for dense graph. Thus, this step takes $O(n^2)$ time.
- 3. Checking if the number of vertices is less than k, takes 0 (1) time.

Therefore, the verification process takes $O(n^2)$ which is polynomial time. This means that the vertex cover problem is an NP problem.