# Modeling Stars:
# Two Method Comparison and Analysis

Najla Alahmadi, Michael Cock, Rebecca Halloran, Brady Metherall, Hector Robinson

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## 1 Introduction

In this project, we modelled a spherically symmetric, static star by numerically integrating the four stellar equations. We started by doing this for a star that had the same core conditions as the sun, and then changed the conditions to determine final values for many other stars. We had to be careful with the initial conditions we used to ensure that the temperature and pressure went to zero at the surface. Using the results, we can plot the relation between luminosity and temperature of the stars to create an Hertzpring-Russell diagram (HR diagram). We created code that would be able to take a stars composition, mass and some initial conditions to determine effective temperature, surface luminosity and radius so we can plot the surface luminosity versus the effective temperature on our HR diagram. Before we get there, lets talk about our program that numerically integrates one star.

## 2 Method

To perform the calculations we started from the stellar core and integrated outward using the fourth order Runge-Kutta method. We specified a core pressure, temperature, density, and composition. From there, the outward integration calculates mass, luminosity, pressure, and temperature at every iteration of the radius. The integration stops when the boundary conditions are met, that is, the pressure or temperature goes to zero. Ideally they both go to zero at the same radius, if the initial conditions are valid. Once this point is reached it means we are at the surface, and have our final values. Figure 1 shows our main code that ran the simulations. Once we have this working for one star using the sun's conditions, we can extend the code to work for many stars. The way to do this is simply by changing the initial conditions. All of our stars had the same compositions; what we changed was the initial core temperature and pressure. We multiplied the sun's core temperature and pressure by different values to get different stars.

## 3 Results

Figure 2 shows our results using the initial conditions of the core of the sun. These plots have the general trend we would expect for a 1 solar mass star, but they do not get the exact final values of the sun. One thing that differs from our star to the sun is that it radiative transport only occurs for two radius steps, which is about $2 \times 10^5$ meters, and convective transport occurs for the rest of the star. This is in contrast to the sun, where the radiative zone is about 70% of the whole star. Because of this, the boundary conditions were not met perfectly; the surface temperature should go to zero but our pressure hit zero first, triggering the integration to stop every time. The mass of our star was about 0.78 $M_\odot$, it's radius was only around 0.38 $R_\odot$, which is considerably smaller, the luminosity was about 1.8 $L_\odot$ and there was a surface temperature of 8600 K. If the model was perfect, the final values for mass, luminosity, temperature, and pressure would be $1M_\odot$, $1L_\odot$, 5770 K, and 0 Pa, respectively.

```
#Define all our constants. This is where we pass X, Y, Z
initialize.init(0.7,0.25,0.05)
val = np.logspace(-5., 5., 100)
count = 1
r_end = 10**10 #in units of m

for i in val:

    print i, "solar initial conditions"
    print count, "out of 100"
    count += 1
    # Initial core conditions of the star
    P0 = i*2.477 *10**16. # units of pascal
    M0 = 10**-4 # units of kg
    L0 = 10**-4 # units of J/s or W
    T0 = i*1.571*10**7 # units of K
    rho0 = i*1.622*10**5 #units of kg/m^3

    data = open('stars.dat','a')

    P,M,L,T,r,kappa_array,rho_array = RK(10**-6, r_end, 10.**5., P0, M0, L0,
   T0, rho0, dP, dM, dL, dT, rho, kappa, stop, 0.000001)

    print >> data, r[-1], M[-1], T[-1], L[-1]
    data.close()
```

(a) First *main.py* calls *initialize.init* to define our constants, then the core conditions are specified. We then open our data file, run our Runge-Kutta method, and then print the results to the data file.

Figure 1: main method of the code.

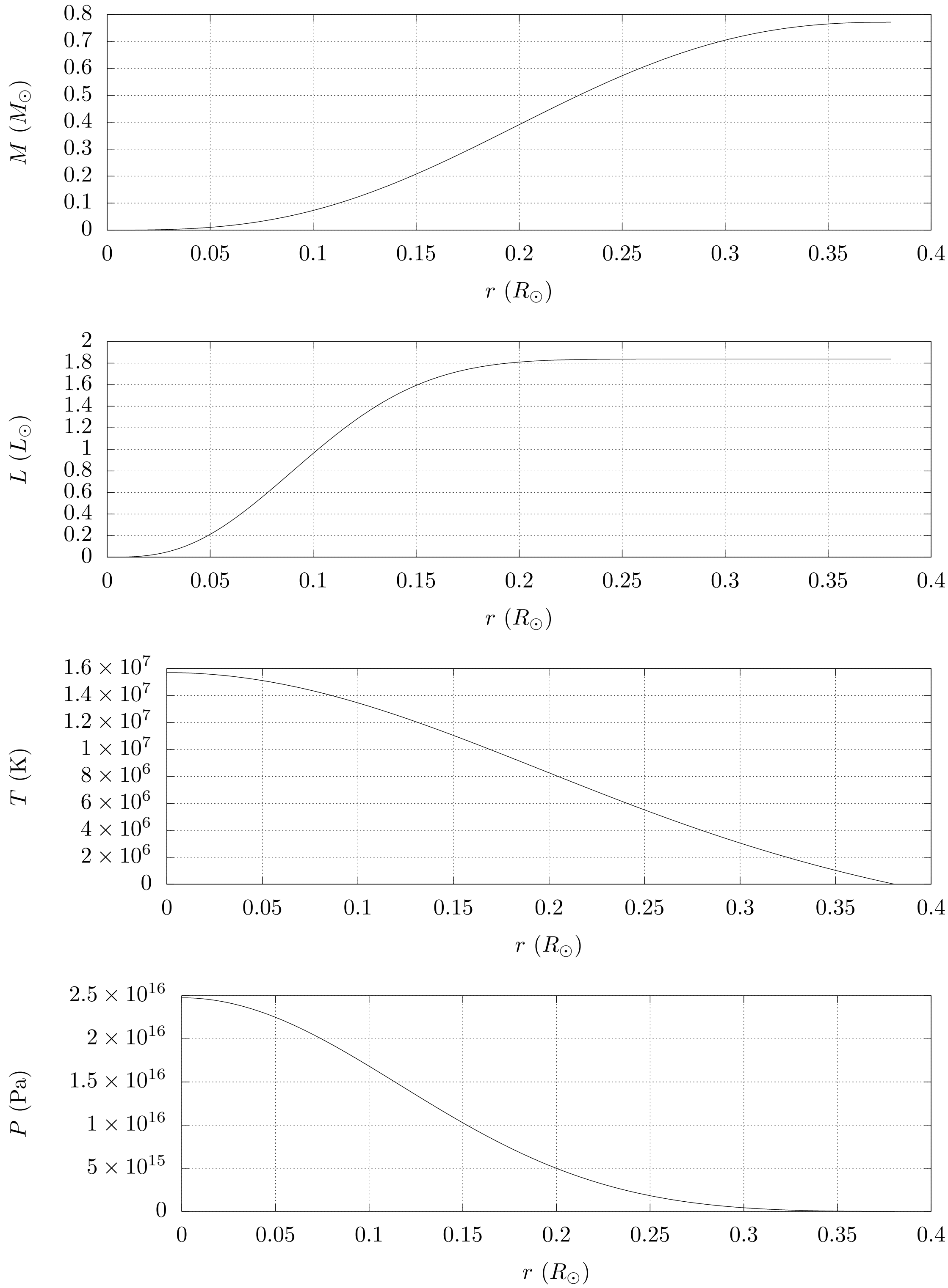



Figure 2: Simulation results of the Sun.

Knowing our model works generally, but not perfectly, we went ahead and used it on other initial conditions, to create other stars. Doing so, should produce a general form of the main sequence on an HR diagram. An HR diagram is a log plot of the luminosity vs. effective temperature. The effective temperature is observationally found by matching the color of a star with the temperature on a blackbody curve that emits that color the most. Due to this, the temperature measured observationally it is not the surface temperature, rather, the temperature from a depth related to the optical depth inside the star. Our attempts to calculate the optical depths of our stars resulted in wildly large values that were thousands of times larger than the star itself. However, during our integrations the pressure was going to zero, and the surface temperature did not. We then decided to use this temperature in place of the effective temperature. This is not perfect, but should still give us a good relationship between different stars' temperatures. Our first attempt at an HR diagram used all of the raw data and is shown in Figure 3a.

The first obvious thing that you'll notice is that the stars seem to have four distinct categories. A small group with high temperatures and luminosities, two groups with low temperatures and luminosities, and a large group somewhere in the middle. To help make some sense of what was going on, we have included the radii of the stars on a new plot, shown in Figure 3b.

In this plot, we see that the two groups at the bottom have either large, or super small radii. The group with small radii are around $0.001R_{\odot}$ and the larger group are on average $10R_{\odot}$. Both of these groups also have very low luminosities. The group near the top has very high surface temperatures, they range between $10^8$ and $10^9$ K. This is about 100 times hotter than the core temperature of the sun. The one group in the middle however, shows promise. The luminosity of this group increases with temperature, and their radii increase gradually as well. Furthermore, this group exists in about the same region we would expect to find stars on an actual HR diagram. What we do is toss out all the data from the three other groups as they seem to be working past the limits of our model and are unphysical. The remaining region is focused in on and we reran the simulations to get more precise data here. We made two separate plots of our final HR diagrams, one comparing it real star data as well as MESA (Modules for Experiments in Stellar Astrophysics) data (Figure 4), and the other showing the mass and radii of our simulated stars (Figure 5).

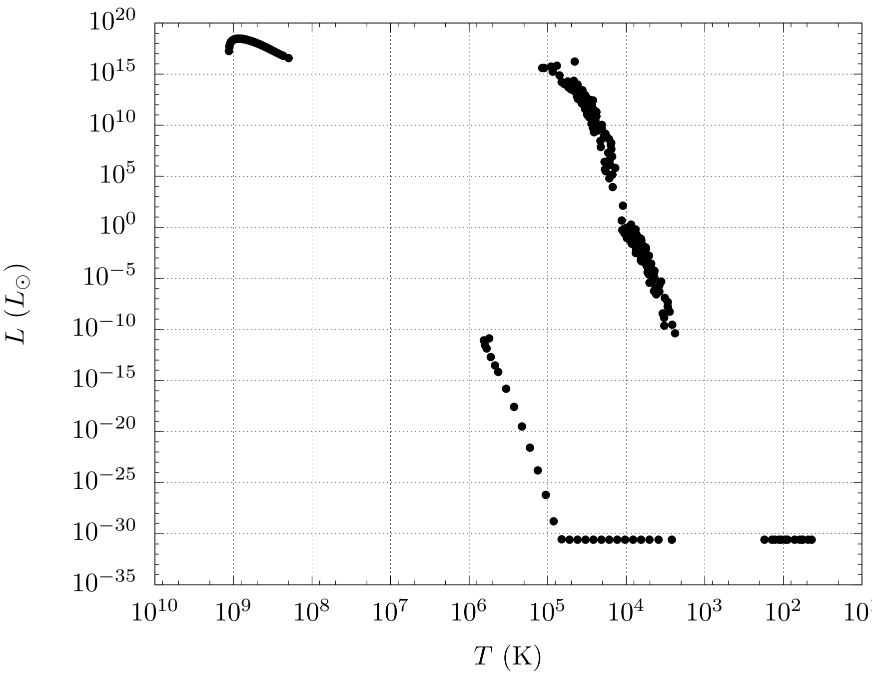
Figure 4 plots the HR diagram for our simulated stars in the region of interest, along with a few stars from a real HR diagram [Zombeck, Martin V. (1990). Handbook of Space Astronomy and Astrophysics, pages 68, 73]. The main difference is that the general slope of our simulated data is steeper. One of the more likely reasons for this is that we used the surface temperature, rather than the effective temperature. This would be due to the temperature not increasing linearly as depth increases. Aside than that, our model accurately predicted the shape and location of actual data.

MESA is a different modelling package, that also models main sequence stars. It differs from our model mainly because it dynamically changes the overall state of the star through time. We ran several MESA simulations for stars along the main sequence. MESA allowed to alter the stars' initial mass and composition. We used masses similar to that of our model, and compositions that are similar to our sun. MESA supplies with a wealth of information for each star (100+ MB each), but we only extracted the data pertaining to their location on an HR diagram. On our plots we can see where each MESA star existed throughout the majority of its life. They exist in the general area of the main sequence and move about it as time progresses.

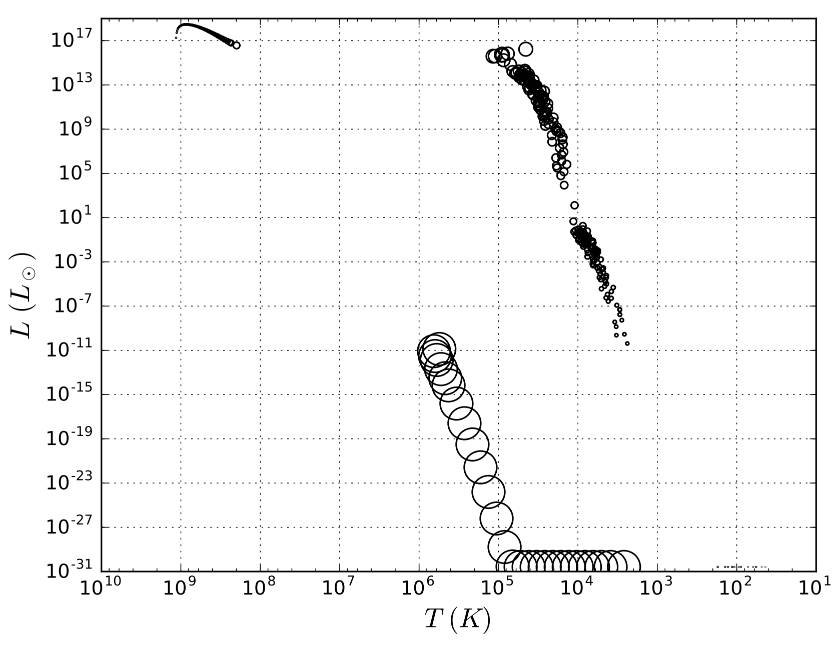
In Figure 5, we plotted only our model's stars but also included their radii and mass. What this shows is that as you travel further up the main sequence, stars get larger and more massive. Physically this makes sense and our model accurately predicts it.

# 4 Discussion

The two main weaknesses of our implementation have already been mentioned; the use of surface temperature, and size of transport zones. Some weaknesses of the model itself are that it only decribes static stars, they do not progress through time at all. Another is that it also only describes low mass stars. The stars are




(a) A crappy version of HR.




(b) A crappy version of HR with radii.

Figure 3: First attempts at an HR diagram.

assumed to be spherically symmetric and non-rotational, which might possibly cause non-trivial differences. The main goal of this project was to create an HR diagram by simulating stars and plotting the data. To do this we needed to complete many smaller tasks along the way. The first objective for our group was to create a profile on GitHub and ensure everyone was able to to push and pull properly. At first, there were a few troubles with ensuring our local versions remained up to date while making changes. Eventually, everyone was able to pull, add, commit then push a change with no problems. Splitting up work in a fair and convenient way was a small challenge, as it was difficult to work around each others schedules. Working in a large group made it easier to write the code because we were all able to bring our ideas together and come up with solutions faster. We learnt to work as a group in an efficient way, even though our ideas would not work sometimes. We were able to listen to each other and everyone was able to contribute. Different people have different styles of coding; when working in a group of five, each person had to learn to read and interpret each others' code. At the same time, we learnt to comment our code in a way which would allow the group members to understand what had been written and why.

We really enjoyed working on this project. It was very satisfying to work with a model where the final results make sense and are tangible (and really cool!) even though they may not be perfect. Overall, great project, had a blast working on it 10/10!

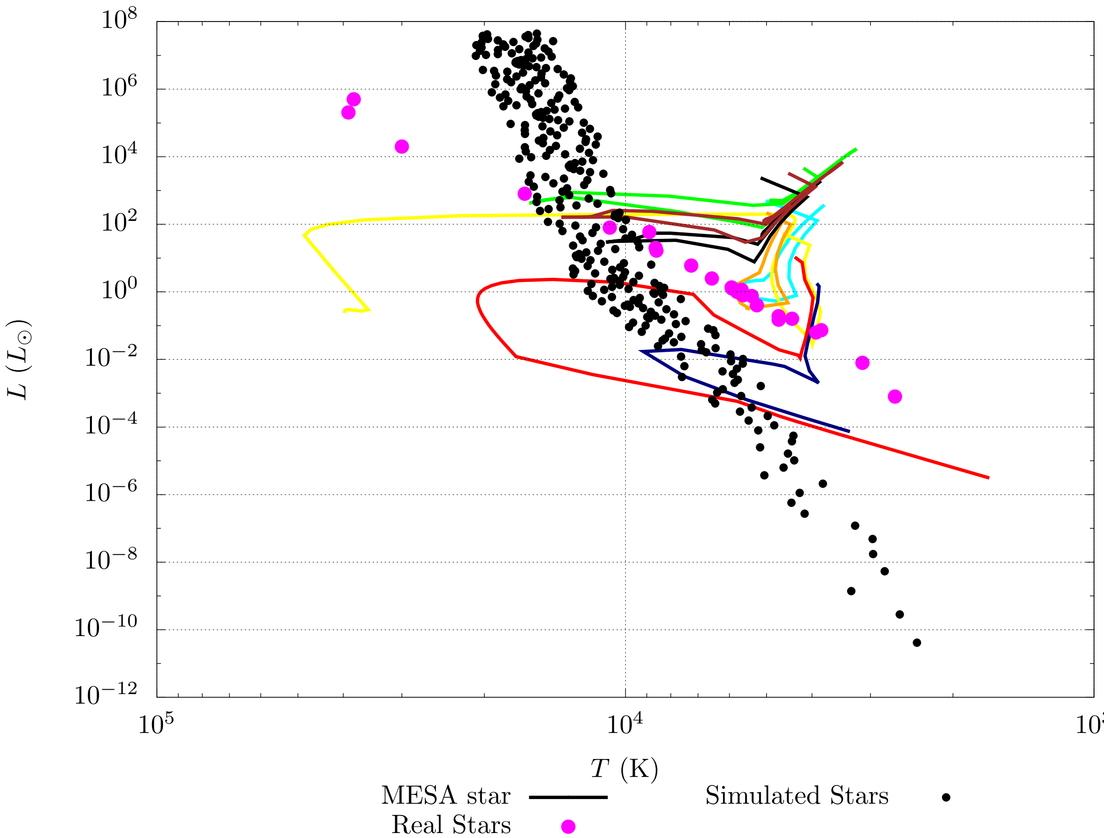



Figure 4: Our reproduction of the HR diagram.

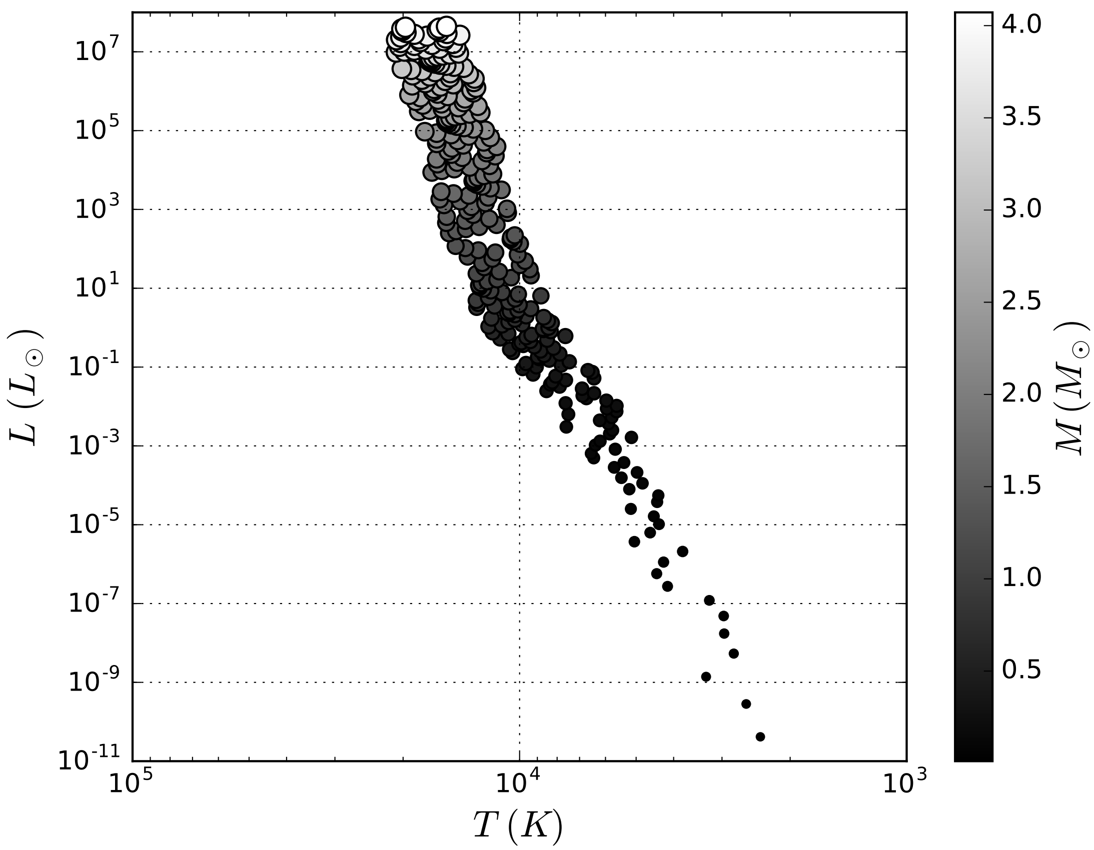



Figure 5: HR diagram from only simulated data