APPM 4610 — Project list

Project 1: Adaptive time stepping

In this project, you will investigate adaptive time stepping methods for solving initial value problems. This means that you use a time stepping method with different time step sizes that are adjusted as needed. You will begin this project by deriving and testing you own adaptive Runge-Kutta. Does the step size adjust how you think it should for a collection of problems? Next you will investigate other adaptive time stepping techniques. Can you just take the ideas from adaptive Runge-Kutta and apply them to any time stepping method or does it require some finesse? How do the different time stepping methods you consider perform for a collection of problems? Is there a situation where one is better than others? You could also integrate this into solving a PDE such as the heat equation.

Suggested starting references:

Section 5.7 of textbook

"Adaptive time-stepping and computational stability" by Gustaf Söderlind and Lina Wang. Journal of Computational and Applied Mathematics Volume 185, Issue 2, 15 January 2006, Pages 225-243

additional reference

Project 2: Time stepping for higher order initial value problems

Your homework was the only time you considered a second order initial value problem. The technique we took for solving that problem was to rewrite it as first order system of equations and apply the our first order time stepping methods. In this project you will explore time stepping methods that you apply directly to the higher order initial value problem. Such time stepping methods include the leap frog method and Newmark method. You will investigate the stability of such methods and compare their performance to using the methods from class for the corresponding first order system.

Suggested starting references:

Leapfrog:
John Cook
Python software
another resource
Newmark:
Section 5.11.2 of George Lindfield, John Penny, in Numerical Methods (Fourth Edition), 2019
T.K. SARKAR, T. ROY, M. SALAZAR-PALMA, A.R. DJORDJEVIC, CHAPTER 8 - Finite-Element Time Domain Method, Time Domain Electromagnetics, 1999,Pages 279-305, (Available online through library)
Zhihui Zhou, Ying Wen, Chenzhi Cai, Qingyuan Zeng, Chapter 7 - Step-by-step integration method,

Copyright APPM 2024

Editor(s): Zhihui Zhou, Ying Wen, Chenzhi Cai, Qingyuan Zeng, Fundamentals of Structural Dynamics, 2021, Pages 245-266, (Available online through library)

Project 3: Spectral collocation methods for boundary value problems

In class we talked about two methods for boundary value problems; Finite element and finite difference methods. With both of these methods it is difficult to create high order approximations. In this project you will explore spectral collocation. This method easily obtains high order accurate approximations and easily extends to higher dimensions. Of course there are some challenges. Possible project extensions include exploring the new HPS method and solving time dependent problems with this method.

Suggested starting references:
Spectral Methods in MATLAB by Trefethen

Chebyshev and Fourier Spectral Methods by John Boyd.

A direct solver with O(N) complexity for variable coefficient elliptic PDEs discretized via a high-order composite spectral collocation method by A Gillman, PG Martinsson - SIAM Journal on Scientific Computing, 2014

Project 4: Modal form of the spectral method

In class, we talk about approximating the solution to boundary value problems via the spectral method. This is approximating the solution by a collection of Chebychev polynomials and determining the value the approximation at each of the corresponding discretization points. This can suffer from a loss in accuracy due to conditioning of the the derivative matrices. To avoid this, we can rewrite the approximation in terms of a Chebychev expansion. Taking derivatives via a transformation of the basis results in well-conditioned tridiagonal linear system. In this project, you will explore this technique called the Ultraspherical method. You will derive it and compare its performance with the other methods from class.

Suggested starting references:
A fast and well-conditioned spectral method by Sheehan and Townsend. SIAM Review 2013.

Project 5: Explore other discretizations

For this project, you can explore other discretizations that were not covered in class, such as discontinuous Galerkin method. You will derive the method, investigating the stability, consistency and convergence. Next you will compare its performance to the other methods we explored in class. You are welcome to choose your independent extension.

Suggested starting references:
Discontinuous Galerkin Methods for Solving Elliptic and Parabolic Equations: Theory and Implementation Book by Beatrice Rivière

Project 6: Integral equation techniques

Copyright APPM 2024

All the techniques mentioned previously are based on discretizing space. For constant coefficient problems, it is possible to write the solution as a convolution of a Green's function with an unknown density and/or a forcing term. These techniques are called Integral equation methods. In the case where you have to solve for an unknown density, you have the benefit of it only living on the boundary of the geometry. This is what we call a reduction in dimensionality. In other words, for one dimensional boundary value problems, the unknown is at two points. For two dimensional boundary value problems the unknown is on a one dimensional curve, etc. For the project extension, you can explore adaptive discretization techniques or integral equations for two dimensional boundary value problems.

lsp

Suggested starting references:

Gillman notes

A FAST ADAPTIVE NUMERICAL METHOD FOR STIFF TWO-POINT BOUNDARY VALUE PROBLEMS by Lee and Greengard

An introduction to the study of integral equations / by Maxime Bôcher.

Copyright APPM 2024