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In[1]:= (* =====
        Isotropic wormhole (3+1D)
        ds^2 = -N(r)^2 dt^2 + ψ(r)^4 (dr^2 + r^2 dΩ^2)
        Units G=c=1; T^μ_ν = (1/8π) G^μ_ν (mixed components)
        ===== *)

(* ---- Switch: machine-fast vs high-precision ---- *)
useHighPrecision = False;

(* ---- Parameters ---- *)
If[useHighPrecision,
  a = 1; eps = 1/1000; rmax = 5;,
  a = 1.; eps = 1.*10^-3; rmax = 5.;
];

(* ---- Metric functions ---- *)
ClearAll[Nfun, ψfun];
Nfun[r_?NumericQ] := 1;      (* lapse, simple choice *)
ψfun[r_?NumericQ] := 1 + a/(2 r);  (* isotropic wormhole conformal factor *)

(* ---- Derivatives ---- *)
ψ1[r_?NumericQ] := Evaluate[D[ψfun[x], x] /. x → r];
ψ2[r_?NumericQ] := Evaluate[D[ψfun[x], {x, 2}] /. x → r];
N1[r_?NumericQ] := Evaluate[D[Nfun[x], x] /. x → r];
N2[r_?NumericQ] := Evaluate[D[Nfun[x], {x, 2}] /. x → r];

(* ---- Einstein tensor components (mixed) ---- *)
einsteinMixed[r_?NumericQ] := Module[
  {ψ = ψfun[r], ψp = ψ1[r], ψpp = ψ2[r],
   N = Nfun[r], Np = N1[r], Npp = N2[r]},
  (* formulas derived for isotropic metric in spherical coords *)
  <|
  "tt" → (4 ψpp/ψ^5 + 2 (ψp/ψ^2)^2 + 8 ψp/(ψ^3 r)),
  "rr" → (2 Np/(N ψ^4 r) + 4 ψp/(ψ^5 r) + 2 Npp/(N ψ^4) - 2 Np^2/(N^2 ψ^4)
    + 4 ψpp/ψ^5 - 2 (ψp/ψ^2)^2),
  "thth" → (Np/(N ψ^4 r) + 2 ψp/(ψ^5 r) + Npp/(N ψ^4) - Np^2/(N^2 ψ^4)
    + 2 ψpp/ψ^5),
  "phph" → (Np/(N ψ^4 r) + 2 ψp/(ψ^5 r) + Npp/(N ψ^4) - Np^2/(N^2 ψ^4)

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      + 2  $\psi_{pp}/\psi^5$ )
    |>
  ];

(* ---- Stress-energy ---- *)
tMixed[r_?NumericQ] := (1/(8 Pi)) einsteinMixed[r];

rho[r_?NumericQ] := - tMixed[r]["tt"];
pr[r_?NumericQ] := tMixed[r]["rr"];
pt[r_?NumericQ] := tMixed[r]["thth"];

(* ---- Energy-conditions helpers ---- *)
NECrad[r_?NumericQ] := rho[r] + pr[r];
NECtang[r_?NumericQ] := rho[r] + pt[r];
WECdensity[r_?NumericQ] := rho[r];
DECrad[r_?NumericQ] := rho[r] - Abs[pr[r]];
DECTang[r_?NumericQ] := rho[r] - Abs[pt[r]];

(* ---- Throat areal radius ---- *)
arealR[r_] :=  $\psi_{fun}[r]^2 r$ ;
rGrid = Exp@Subdivide[Log[eps], Log[rmax], 500];
rThroat = First@MinimalBy[rGrid, arealR];
fmt[x_] := NumberForm[x, {Infinity, 6}];

Print["Wormhole Energy Condition Analysis (Isotropic metric)"];
Print["====="];
Print["Parameter: a=", a];
Print["\n--- At the throat ( $r \approx$  ", NumberForm[rThroat, {Infinity, 3}],
      ",  $R =$ ", NumberForm[arealR[rThroat], {Infinity, 3}], ") ---"];
Print["rho          : ", fmt[rho[rThroat]]];
Print["pr           : ", fmt[pr[rThroat]]];
Print["pt           : ", fmt[pt[rThroat]]];
Print["NEC_radial      : ", fmt[NECrad[rThroat]]];
Print["NEC_tangential  : ", fmt[NECtang[rThroat]]];
Print["DEC_radial ( $\rho \geq |p_r|$ ) : ", rho[rThroat]  $\geq$  Abs[pr[rThroat]]];
Print["DEC_tangential ( $\rho \geq |p_t|$ ): ", rho[rThroat]  $\geq$  Abs[pt[rThroat]]];

(* ---- Plots (the four panels) ---- *)

p1 = Plot[{Abs[rho[r]], Abs[pr[r]], Abs[pt[r]]}, {r, eps, rmax},
  ScalingFunctions  $\rightarrow$  {"Log", "Log"},

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PlotLegends → {"| $\rho$ |", "| $p_r$ ", "| $p_t$ "},
Frame → True, GridLines → {rThroat, None},
GridLinesStyle → Directive[GrayLevel[.7], Dashed],
FrameLabel → {"r", "|Energy|", |Pressure|},
PlotLabel → "Absolute Values (Log Scale)";

p2 = Plot[{NECrad[r], NECtang[r]}, {r, eps, rmax},
  ScalingFunctions → {"Log", None},
  PlotLegends → {" $\rho+p_r$  (NEC radial)", " $\rho+p_t$  (NEC tangential)"},
  Frame → True, GridLines → {rThroat, {0}},
  GridLinesStyle → Directive[GrayLevel[.7], Dashed],
  FrameLabel → {"r", "NEC"},
  PlotLabel → "Null Energy Condition";

p3 = Plot[{DECrad[r], DECTang[r]}, {r, eps, rmax},
  ScalingFunctions → {"Log", None},
  PlotLegends → {" $\rho-|p_r|$  (DEC radial)", " $\rho-|p_t|$  (DEC tangential)"},
  Frame → True, GridLines → {rThroat, {0}},
  GridLinesStyle → Directive[GrayLevel[.7], Dashed],
  FrameLabel → {"r", "DEC"},
  PlotLabel → "Dominant Energy Condition";

p4 = Plot[{WECdensity[r]}, {r, eps, rmax},
  ScalingFunctions → {"Log", None},
  PlotLegends → {" $\rho$  (WEC density)"},
  Frame → True, GridLines → {rThroat, {0}},
  GridLinesStyle → Directive[GrayLevel[.7], Dashed],
  FrameLabel → {"r", " $\rho$ "},
  PlotLabel → "Weak Energy Condition (Density)";

GraphicsGrid[{{p1, p2}, {p3, p4}}, Spacings → {0.6, 0.8}]

Wormhole Energy Condition Analysis (Isotropic metric)

=====

Parameter:  a=1.

--- At the throat (r  $\approx$  0.501, R=2.000) ---

rho          : 0.098899
pr           : 0.000134
pt           : 0.009956

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NEC_radial      : 0.099033
NEC_tangential  : 0.108854
DEC_radial ( $\rho \geq |p_r|$ ) : True
DEC_tangential ( $\rho \geq |p_t|$ ): True
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Out[39]=

