```
In[•]:= (*---------------------------------------------------------*)(*Coordinates and Metric*)
(*---------------------------------------------------------*)coords = {t, r, θ, ϕ};

(*Scalar potential Φ(r):with regularized radius r=Sqrt[r^2+ε^2]*)
rReg[r_] := Sqrt[r^2 + ε^2];

Φ[r_] := -A (1 - R0 / rReg[r]) Exp[- (rReg[r] - R0) ^2 / w^2];

metric = DiagonalMatrix[{-Exp[2 Φ[r]], Exp[-2 Φ[r]], r^2, r^2 Sin[θ] ^2}];

invMetric = Simplify[Inverse[metric]];

(*Define scalar field for illustration*)
CurlyPhi[r_] := Exp[Φ[r]];

(*---------------------------------------------------------*)
(*Christoffel Symbols*)
(*---------------------------------------------------------*)

Γ = Table[Sum[1 / 2 invMetric⟦i, k⟧
     (D[metric⟦k, j⟧, coords⟦l⟧] + D[metric⟦k, l⟧, coords⟦j⟧] - D[metric⟦j, l⟧, coords⟦k⟧]),
    {k, 1, 4}], {i, 1, 4}, {j, 1, 4}, {l, 1, 4}];

(*---------------------------------------------------------*)
(*Riemann and Ricci Tensors*)
(*---------------------------------------------------------*)

Riemann = Table[D[Γ⟦i, j, k⟧, coords⟦l⟧] - D[Γ⟦i, j, l⟧, coords⟦k⟧] +
    Sum[Γ⟦i, m, k⟧ × Γ⟦m, j, l⟧ - Γ⟦i, m, l⟧ × Γ⟦m, j, k⟧, {m, 1, 4}],
   {i, 1, 4}, {j, 1, 4}, {k, 1, 4}, {l, 1, 4}];

Ricci = Table[Sum[Riemann⟦m, i, m, j⟧, {m, 1, 4}], {i, 1, 4}, {j, 1, 4}];

RicciScalar = Simplify[Sum[invMetric⟦i, j⟧ × Ricci⟦i, j⟧, {i, 1, 4}, {j, 1, 4}]];

(*---------------------------------------------------------*)
(*Kinetic Term*)
(*---------------------------------------------------------*)

dCurlyPhi = Grad[CurlyPhi[r], {r}];
kinetic = Simplify[1 / 2 invMetric⟦2, 2⟧ × dCurlyPhi⟦1⟧ ^2];

(*---------------------------------------------------------*)
(*Scalar Potential V(φ)*)
(*---------------------------------------------------------*)
```

```
V[φ_] := λ φ^4;
potentialTerm = V[CurlyPhi[r]];

(*-------------------------------------------------------*)
(*Full Lagrangian*)
(*-------------------------------------------------------*)

L = Simplify[CurlyPhi[r]^2 RicciScalar - kinetic - potentialTerm];

Print["Ricci scalar R(r) = ", RicciScalar];
Print["Lagrangian ℒ(r) = ", L];

Print["Limit Lagrangian r→R0 = ", Limit[L, r → R0]];
Print["Asymptotic series at infinity: ", Series[L, {r, ∞, 2}]];

(*-------------------------------------------------------*)
(*Example numeric plot for a 2meter wormhole*)
(*-------------------------------------------------------*)

Plot[Evaluate[L /. {A → 0.01, R0 → 2.0, w → 0.2, ε → 0.04, λ → 1}],
 {r, 1.9995, 2.0005}, PlotRange → All]
```

Ricci scalar R(r) =

$$-\frac{1}{r^2\,w^4\,\left(r^2+\varepsilon^2\right)^3}\,2\,e^{-2\,A\,e^{-\frac{\left(R0-\sqrt{r^2+\varepsilon^2}\right)^2}{w^2}}\left(1-\frac{R0}{\sqrt{r^2+\varepsilon^2}}\right)-\frac{2\left(R0-\sqrt{r^2+\varepsilon^2}\right)^2}{w^2}}\left(e^{\frac{2\left(R0-\sqrt{r^2+\varepsilon^2}\right)^2}{w^2}}\left(1+e^{2\,A\,e^{-\frac{\left(R0-\sqrt{r^2+\varepsilon^2}\right)^2}{w^2}}\left(1-\frac{R0}{\sqrt{r^2+\varepsilon^2}}\right)}\right)w^4\left(r^2+\varepsilon^2\right)^3-\right.$$
$$2\,A^2\,r^4\,\Big(4\,r^6+4\,R0^4\,\varepsilon^2+4\,\varepsilon^6-4\,R0^3\,\sqrt{r^2+\varepsilon^2}\,\left(w^2+4\,\varepsilon^2\right)-4\,R0\,\varepsilon^2\,\sqrt{r^2+\varepsilon^2}\,\left(w^2+4\,\varepsilon^2\right)+$$
$$R0^2\,\left(w^4+8\,w^2\,\varepsilon^2+24\,\varepsilon^4\right)-4\,r^4\,\left(-6\,R0^2-3\,\varepsilon^2+4\,R0\,\sqrt{r^2+\varepsilon^2}\right)-$$
$$4\,r^2\,\left(-R0^4-3\,\varepsilon^4+4\,R0^3\,\sqrt{r^2+\varepsilon^2}-2\,R0^2\,\left(w^2+6\,\varepsilon^2\right)+R0\,\sqrt{r^2+\varepsilon^2}\,\left(w^2+8\,\varepsilon^2\right)\right)\Big)+A\,e^{\frac{\left(R0-\sqrt{r^2+\varepsilon^2}\right)^2}{w^2}}\,r^2$$
$$\Big(4\,r^8-2\,r^6\,\left(-6\,R0^2+w^2-6\,\varepsilon^2+6\,R0\,\sqrt{r^2+\varepsilon^2}\right)+w^2\,\varepsilon^2\,\left(-2\,R0^2\,\varepsilon^2-2\,\varepsilon^4+R0\,\sqrt{r^2+\varepsilon^2}\,\left(w^2+4\,\varepsilon^2\right)\right)-$$
$$2\,r^4\,\left(3\,\varepsilon^2\,\left(w^2-2\,\varepsilon^2\right)+2\,R0^3\,\sqrt{r^2+\varepsilon^2}-2\,R0^2\,\left(w^2+6\,\varepsilon^2\right)+R0\,\sqrt{r^2+\varepsilon^2}\,\left(w^2+12\,\varepsilon^2\right)\right)-$$
$$\left.2\,r^2\,\left(3\,w^2\,\varepsilon^4-2\,\varepsilon^6+2\,R0^3\,\varepsilon^2\,\sqrt{r^2+\varepsilon^2}-R0^2\,\varepsilon^2\,\left(w^2+6\,\varepsilon^2\right)+R0\,\sqrt{r^2+\varepsilon^2}\,\left(w^4-w^2\,\varepsilon^2+6\,\varepsilon^4\right)\right)\Big)\right)$$

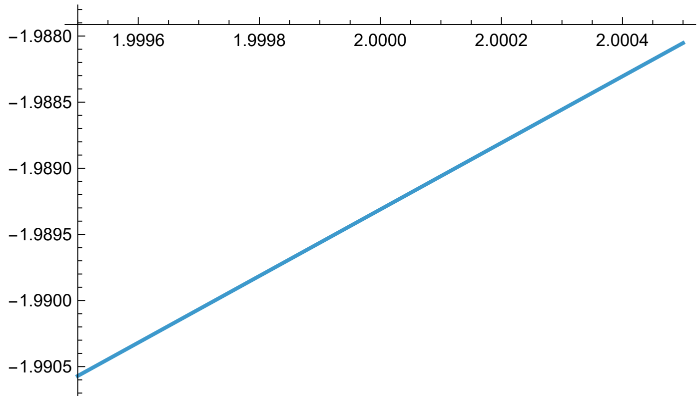
$$
\begin{aligned}
\text{Lagrangian } \mathcal{L}(r) \;=\; & \frac{1}{2}\, e^{-4 A\, e^{-\frac{\left(R0-\sqrt{r^2+\varepsilon^2}\right)^2}{w^2}} \left(1-\frac{R0}{\sqrt{r^2+\varepsilon^2}}\right)} \Bigg(-\frac{4\left(1+e^{2 A\, e^{-\frac{\left(R0-\sqrt{r^2+\varepsilon^2}\right)^2}{w^2}} \left(1-\frac{R0}{\sqrt{r^2+\varepsilon^2}}\right)}\right)}{r^2} - \\
& \frac{A^2\, e^{-\frac{2\left(R0-\sqrt{r^2+\varepsilon^2}\right)^2}{w^2}}\, r^2 \left(2\, R0^2 \sqrt{r^2+\varepsilon^2} + 2\, \varepsilon^2 \sqrt{r^2+\varepsilon^2} - R0\left(w^2+4\,\varepsilon^2\right) + 2\, r^2\left(-2\, R0 + \sqrt{r^2+\varepsilon^2}\right)\right)^2}{w^4\left(r^2+\varepsilon^2\right)^3} + \\
& \frac{1}{w^4\left(r^2+\varepsilon^2\right)^3}\, 8\, A^2\, e^{-\frac{2\left(R0-\sqrt{r^2+\varepsilon^2}\right)^2}{w^2}}\, r^2 \Big(4\, r^6 + 4\, R0^4\, \varepsilon^2 + 4\, \varepsilon^6 - 4\, R0^3 \sqrt{r^2+\varepsilon^2}\left(w^2+4\,\varepsilon^2\right) - \\
& \quad 4\, R0\, \varepsilon^2 \sqrt{r^2+\varepsilon^2}\left(w^2+4\,\varepsilon^2\right) + R0^2\left(w^4+8\, w^2\, \varepsilon^2+24\,\varepsilon^4\right) - 4\, r^4\left(-6\, R0^2 - 3\,\varepsilon^2 + 4\, R0\sqrt{r^2+\varepsilon^2}\right) - \\
& \quad 4\, r^2\left(-R0^4 - 3\,\varepsilon^4 + 4\, R0^3\sqrt{r^2+\varepsilon^2} - 2\, R0^2\left(w^2+6\,\varepsilon^2\right) + R0\sqrt{r^2+\varepsilon^2}\left(w^2+8\,\varepsilon^2\right)\right)\Big) - \\
& \frac{1}{w^4\left(r^2+\varepsilon^2\right)^3}\, 4\, A\, e^{-\frac{\left(R0-\sqrt{r^2+\varepsilon^2}\right)^2}{w^2}} \Big(4\, r^8 - 2\, r^6\left(-6\, R0^2 + w^2 - 6\,\varepsilon^2 + 6\, R0\sqrt{r^2+\varepsilon^2}\right) + \\
& \quad w^2\,\varepsilon^2\left(-2\, R0^2\,\varepsilon^2 - 2\,\varepsilon^4 + R0\sqrt{r^2+\varepsilon^2}\left(w^2+4\,\varepsilon^2\right)\right) - \\
& \quad 2\, r^4\left(3\,\varepsilon^2\left(w^2-2\,\varepsilon^2\right) + 2\, R0^3\sqrt{r^2+\varepsilon^2} - 2\, R0^2\left(w^2+6\,\varepsilon^2\right) + R0\sqrt{r^2+\varepsilon^2}\left(w^2+12\,\varepsilon^2\right)\right) - \\
& \quad 2\, r^2\left(3\, w^2\,\varepsilon^4 - 2\,\varepsilon^6 + 2\, R0^3\,\varepsilon^2\sqrt{r^2+\varepsilon^2} - R0^2\,\varepsilon^2\left(w^2+6\,\varepsilon^2\right) + R0\sqrt{r^2+\varepsilon^2}\left(w^4 - w^2\,\varepsilon^2 + 6\,\varepsilon^4\right)\right)\Big) - 2\,\lambda \Bigg)
\end{aligned}
$$

$$
\begin{aligned}
&\text{Limit Lagrangian } r\to R0 = \frac{1}{2}\, e^{-4 A\, e^{-\frac{\left(R0-\sqrt{R0^2+\varepsilon^2}\right)^2}{w^2}} \left(1-\frac{R0}{\sqrt{R0^2+\varepsilon^2}}\right)} \Bigg(-\frac{4\left(1+e^{2 A\, e^{-\frac{\left(R0-\sqrt{R0^2+\varepsilon^2}\right)^2}{w^2}} \left(1-\frac{R0}{\sqrt{R0^2+\varepsilon^2}}\right)}\right)}{R0^2} - \\
&\frac{A^2 e^{-\frac{2\left(R0-\sqrt{R0^2+\varepsilon^2}\right)^2}{w^2}} R0^2 \left(2 R0^2 \sqrt{R0^2+\varepsilon^2} + 2\varepsilon^2 \sqrt{R0^2+\varepsilon^2} - R0\left(w^2+4\varepsilon^2\right) + 2 R0^2\left(-2 R0+\sqrt{R0^2+\varepsilon^2}\right)\right)^2}{w^4\left(R0^2+\varepsilon^2\right)^3} + \\
&\frac{1}{w^4\left(R0^2+\varepsilon^2\right)^3}\, 8 A^2 e^{-\frac{2\left(R0-\sqrt{R0^2+\varepsilon^2}\right)^2}{w^2}} R0^2 \Big(4 R0^6 + 4 R0^4 \varepsilon^2 + 4\varepsilon^6 - 4 R0^3 \sqrt{R0^2+\varepsilon^2}\left(w^2+4\varepsilon^2\right) - \\
&\quad 4 R0\, \varepsilon^2 \sqrt{R0^2+\varepsilon^2}\left(w^2+4\varepsilon^2\right) + R0^2\left(w^4+8 w^2\varepsilon^2+24\varepsilon^4\right) - 4 R0^4\left(-6 R0^2-3\varepsilon^2+4 R0\sqrt{R0^2+\varepsilon^2}\right) - \\
&\quad 4 R0^2\left(-R0^4-3\varepsilon^4+4 R0^3\sqrt{R0^2+\varepsilon^2}-2 R0^2\left(w^2+6\varepsilon^2\right)+R0\sqrt{R0^2+\varepsilon^2}\left(w^2+8\varepsilon^2\right)\right)\Big) - \\
&\frac{1}{w^4\left(R0^2+\varepsilon^2\right)^3}\, 4 A\, e^{-\frac{\left(R0-\sqrt{R0^2+\varepsilon^2}\right)^2}{w^2}} \Big(4 R0^8 - 2 R0^6\left(-6 R0^2+w^2-6\varepsilon^2+6 R0\sqrt{R0^2+\varepsilon^2}\right) + \\
&\quad w^2\varepsilon^2\left(-2 R0^2\varepsilon^2-2\varepsilon^4+R0\sqrt{R0^2+\varepsilon^2}\left(w^2+4\varepsilon^2\right)\right) - \\
&\quad 2 R0^4\left(3\varepsilon^2\left(w^2-2\varepsilon^2\right)+2 R0^3\sqrt{R0^2+\varepsilon^2}-2 R0^2\left(w^2+6\varepsilon^2\right)+R0\sqrt{R0^2+\varepsilon^2}\left(w^2+12\varepsilon^2\right)\right) - \\
&\quad 2 R0^2\left(3 w^2\varepsilon^4-2\varepsilon^6+2 R0^3\varepsilon^2\sqrt{R0^2+\varepsilon^2}-R0^2\varepsilon^2\left(w^2+6\varepsilon^2\right)+R0\sqrt{R0^2+\varepsilon^2}\left(w^4-w^2\varepsilon^2+6\varepsilon^4\right)\right)\Big) - 2\lambda \Bigg)
\end{aligned}
$$

Asymptotic series at infinity:

$$
\begin{aligned}
&e^{e^{-\frac{r^2}{w^2}+\frac{2 R0\, r}{w^2}+\frac{-R0^2-\varepsilon^2}{w^2}+\frac{R0\,\varepsilon^2}{w^2 r}+O\left[\frac{1}{r}\right]^3}\left(-2 A+\frac{2 A\, R0}{r}+O\left[\frac{1}{r}\right]^3\right)} \left(-\frac{2}{r^2}+O\left[\frac{1}{r}\right]^3\right) + e^{e^{-\frac{r^2}{w^2}+\frac{2 R0\, r}{w^2}+\frac{-R0^2-\varepsilon^2}{w^2}+\frac{R0\,\varepsilon^2}{w^2 r}+O\left[\frac{1}{r}\right]^3}\left(-4 A+\frac{4 A\, R0}{r}+O\left[\frac{1}{r}\right]^3\right)} \left(-\lambda-\frac{2}{r^2}+O\left[\frac{1}{r}\right]^3\right) + \\
&e^{e^{-\frac{r^2}{w^2}+\frac{2 R0\, r}{w^2}+\frac{-R0^2-\varepsilon^2}{w^2}+\frac{R0\,\varepsilon^2}{w^2 r}+O\left[\frac{1}{r}\right]^3}\left(-4 A+\frac{4 A\, R0}{r}+O\left[\frac{1}{r}\right]^3\right)+\left(-\frac{r^2}{w^2}+\frac{2 R0\, r}{w^2}+\frac{-R0^2-\varepsilon^2}{w^2}+\frac{R0\,\varepsilon^2}{w^2 r}+O\left[\frac{1}{r}\right]^3\right)} \\
&\left(-\frac{8 A r^2}{w^4}+\frac{24 A\, R0\, r}{w^4}+\frac{4 A\left(-6 R0^2+w^2\right)}{w^4}+\frac{4 A\, R0\left(2 R0^2+w^2-3\varepsilon^2\right)}{w^4 r}-\frac{8\left(A\, R0^2\left(w^2-3\varepsilon^2\right)\right)}{w^4 r^2}+O\left[\frac{1}{r}\right]^3\right) + \\
&e^{e^{-\frac{r^2}{w^2}+\frac{2 R0\, r}{w^2}+\frac{-R0^2-\varepsilon^2}{w^2}+\frac{R0\,\varepsilon^2}{w^2 r}+O\left[\frac{1}{r}\right]^3}\left(-4 A+\frac{4 A\, R0}{r}+O\left[\frac{1}{r}\right]^3\right)+\left(-\frac{2 r^2}{w^2}+\frac{4 R0\, r}{w^2}-\frac{2\left(R0^2+\varepsilon^2\right)}{w^2}+\frac{2 R0\,\varepsilon^2}{w^2 r}+O\left[\frac{1}{r}\right]^3\right)} \\
&\left(-\frac{2 A^2 r^2}{w^4}+\frac{8 A^2 R0\, r}{w^4}-\frac{12\left(A^2 R0^2\right)}{w^4}+\frac{2 A^2 R0\left(4 R0^2+w^2-2\varepsilon^2\right)}{w^4 r}-\frac{2\left(A^2 R0^2\left(R0^2+2 w^2-6\varepsilon^2\right)\right)}{w^4 r^2}+O\left[\frac{1}{r}\right]^3\right) + \\
&e^{e^{-\frac{r^2}{w^2}+\frac{2 R0\, r}{w^2}+\frac{-R0^2-\varepsilon^2}{w^2}+\frac{R0\,\varepsilon^2}{w^2 r}+O\left[\frac{1}{r}\right]^3}\left(-4 A+\frac{4 A\, R0}{r}+O\left[\frac{1}{r}\right]^3\right)+\left(-\frac{2 r^2}{w^2}+\frac{4 R0\, r}{w^2}-\frac{2\left(R0^2+\varepsilon^2\right)}{w^2}+\frac{2 R0\,\varepsilon^2}{w^2 r}+O\left[\frac{1}{r}\right]^3\right)} \\
&\left(\frac{16 A^2 r^2}{w^4}-\frac{64\left(A^2 R0\right) r}{w^4}+\frac{96 A^2 R0^2}{w^4}-\frac{16\left(A^2 R0\left(4 R0^2+w^2-2\varepsilon^2\right)\right)}{w^4 r}+\frac{16 A^2 R0^2\left(R0^2+2 w^2-6\varepsilon^2\right)}{w^4 r^2}+O\left[\frac{1}{r}\right]^3\right)
\end{aligned}
$$

Out[#]=


1.9996
1.9998
2.0000
2.0002
2.0004
−1.9880
−1.9885
−1.9890
−1.9895
−1.9900
−1.9905