```
In[50]:= (*************************************************************)
(* 2. Scalar / Vector / Tensor sectors via NDEigensystem    *)
(*************************************************************)

(* --- Geometry & potential parameters (1 mm wormhole) --- *)

ClearAll["Global`*"];



(* Physical scale: take r in meters, with R0 = 1 mm *)
A = 1.;         (* potential strength *)
R0 = 1.*10^-3;      (* 1 mm throat radius *)
w  = 0.5 R0;       (* Gaussian width - adjust as needed *)
ϵ = 0.1 R0;   (* small regulator scale, r ≥ ϵ *)

(* Radial domain for the eigenproblem *)
rmin = ϵ;
rmax = 10 R0;
nModes = 6;

(* --- Scalar potential Φ(r) --- *)

(* 3D definition (for reference, not used directly in 1D problem):
   r[x_, y_, z_] := Sqrt[x^2 + y^2 + z^2];
   Φ3D[x_, y_, z_] := Module[{rr = Max[r[x, y, z], ϵ]},
     -A (1 - R0/rr) Exp[-(rr - R0)^2/w^2]
   ];
*)

(* 1D radial restriction: r ≥ rmin ≥ ϵ, so no explicit Max needed *)
Φ[r_] := -A (1 - R0 / r) Exp[-(r - R0)^2 / w^2];

(* This Φ(r) is the same one that appears in the metric:
   ds^2 = -Exp[2 Φ[r]] dt^2 + Exp[-2 Φ[r]] dr^2 + r^2 dΩ^2
*)

(* --- 2a. Scalar sector --- *)

potentialS[r_] := Φ[r];

Ls = -D[ψs[r], {r, 2}] + potentialS[r] × ψs[r];
bcS = DirichletCondition[ψs[r] == 0, r == rmin || r == rmax];
```

```
{valsS, funsS} =
 NDEigensystem[{Ls, bcS}, ψs, {r, rmin, rmax}, nModes];

Print["\nScalar ω^2 = ", valsS // N];
If[AllTrue[valsS, # > 0 &],
 Print["☑  Scalar sector stable"],
 Print["✕  Scalar instability"]
];

(* --- 3. Vector / Tensor sectors --- *)

(* Numeric first & second derivatives of Φ *)
dΦ =
 Function[{x}, Evaluate[D[Φ[r], r] /. r → x]];

d2Φ =
 Function[{x}, Evaluate[D[Φ[r], {r, 2}] /. r → x]];

(* Illustrative effective potentials for perturbations *)
Vv[r_] := -d2Φ[r] + dΦ[r]/r; (* vector potential *)
Vt[r_] :=  d2Φ[r] + dΦ[r]/r; (* tensor potential *)

Lv = -D[ψv[r], {r, 2}] + Vv[r] × ψv[r];
Lt = -D[ψt[r], {r, 2}] + Vt[r] × ψt[r];

bcV = DirichletCondition[ψv[r] == 0, r == rmin || r == rmax];
bcT = DirichletCondition[ψt[r] == 0, r == rmin || r == rmax];

{valsV, funsV} =
 NDEigensystem[{Lv, bcV}, ψv, {r, rmin, rmax}, nModes];
{valsT, funsT} =
 NDEigensystem[{Lt, bcT}, ψt, {r, rmin, rmax}, nModes];

Print["\nVector ω^2 = ", valsV // N];
Print["Tensor ω^2 = ", valsT // N];

If[AllTrue[valsV, # > 0 &],
 Print["☑  Vector sector stable"],
 Print["✕  Vector instability"]
];
```

```
If[AllTrue[valsT, # > 0 &],
 Print["☑  Tensor sector stable"],
 Print["✕  Tensor instability"]
];

(*********************************************************)
(* 4. Plots: scalar, vector, tensor eigenfunctions        *)
(*********************************************************)

(* --- Scalar eigenfunctions --- *)
Plot[
 Evaluate@Table[funsS[[k]][r], {k, nModes}],
 {r, rmin, rmax},
 PlotLabel → "Scalar eigenfunctions psi_s[n](r)",
 PlotLegends → Table["n = " <> ToString[k], {k, nModes}],
 AxesLabel → {"r", "psi_s[n]"}
]

(* --- Vector eigenfunctions --- *)
Plot[
 Evaluate@Table[funsV[[k]][r], {k, nModes}],
 {r, rmin, rmax},
 PlotLabel → "Vector eigenfunctions psi_v[n](r)",
 PlotLegends → Table["n = " <> ToString[k], {k, nModes}],
 AxesLabel → {"r", "psi_v[n]"}
]

(* --- Tensor eigenfunctions --- *)
Plot[
 Evaluate@Table[funsT[[k]][r], {k, nModes}],
 {r, rmin, rmax},
 PlotLabel → "Tensor eigenfunctions psi_t[n](r)",
 PlotLegends → Table["n = " <> ToString[k], {k, nModes}],
 AxesLabel → {"r", "psi_t[n]"}
]
```

Scalar ω^2 = $\{100700., 402805., 906361., 1.61154\times10^{6}, 2.51879\times10^{6}, 3.629\times10^{6}\}$

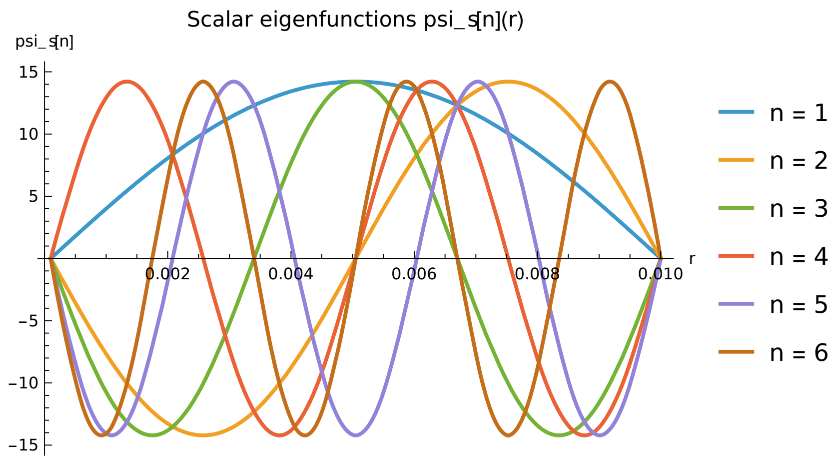
☑ Scalar sector stable

```
Vector ω^2 = {19 339.2, 242 827., 698 933., 1.3982×10^6, 2.34391×10^6, 3.53147×10^6}
Tensor ω^2 = {98 917.4, 402 190., 912 359., 1.60108×10^6, 2.41922×10^6, 3.40355×10^6}
```

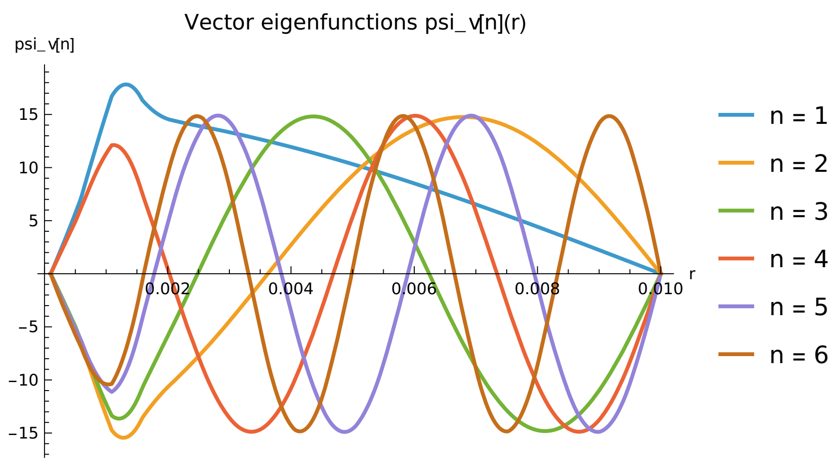
☑ Vector sector stable

☑ Tensor sector stable

Out[79]=



Out[80]=



Out[81]=