```
In[82]:= (* ::Title:: *)
(*Geodesic Traversal and Traversability Check in a 3+1D Wormhole Metric (1 mm Throat)*)

(* ::Section:: *)
(*Define Parameters and Potential*)

(* Wormhole throat radius: 1 mm = 1*10^-3 m *)
R0 = 1.*^-3;
A = 1.0;    (* potential strength scale, > 0 *)
w = 10 R0;   (* Gaussian damping width; scaled to the throat radius *)
ε = 10^-6 R0; (* small regulator to avoid r = 0 singularity *)

ΦSafe[x_, y_, z_] :=
 Module[{r},
  r = Max[Sqrt[x^2 + y^2 + z^2], ε];
  -A (1 - R0/r) Exp[-(r - R0)^2/w^2]
 ];

(* ::Section:: *)
(*Define the Metric Components*)

f[x_, y_, z_] := Exp[2 ΦSafe[x, y, z]];
invf[x_, y_, z_] := Exp[-2 ΦSafe[x, y, z]];

(* This corresponds to:
   ds^2 = -f dt^2 + invf (dx^2 + dy^2 + dz^2)
   with f = e^{2 Φ(r)}, invf = e^{-2 Φ(r)}.
*)

(* ::Section:: *)
(*Define the Lagrangian for Geodesic Motion*)

Clear[t, x, y, z, s]

L = -f[x[s], y[s], z[s]] × t'[s]^2 +
  invf[x[s], y[s], z[s]] (x'[s]^2 + y'[s]^2 + z'[s]^2);

(* ::Section:: *)
(*Compute Euler-Lagrange Equations*)

eqt = D[D[L, t'[s]], s] - D[L, t[s]] == 0;
eqx = D[D[L, x'[s]], s] - D[L, x[s]] == 0;
```

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eqy = D[D[L, y'[s]], s] - D[L, y[s]] == 0;
eqz = D[D[L, z'[s]], s] - D[L, z[s]] == 0;

(* ::Section:: *)
(*Initial Conditions and Numerical Integration for Timelike Geodesic*)

(* Start slightly inside one side of the throat region, along the x-axis *)
timelikeIC = {
  t[0] == 0,
  x[0] == 0.5 R0,
  y[0] == 0,
  z[0] == 0,
  t'[0] == 1,   (* normalization determines affine parameter scaling *)
  x'[0] == 1,   (* initial spatial velocity along +x *)
  y'[0] == 0,
  z'[0] == 0
};

timelikeSol =
 NDSolve[{eqt, eqx, eqy, eqz} ~Join~ timelikeIC,
    {t, x, y, z}, {s, 0, 10},
    MaxStepFraction → 1/100];

(* ::Section:: *)
(*Plot the Geodesic Path (3D)*)

ParametricPlot3D[
 Evaluate[{x[s], y[s], z[s]} /. timelikeSol],
 {s, 0, 10},
 PlotRange → All,
 AxesLabel → {"x (m)", "y (m)", "z (m)"},
 PlotLabel →
  "Timelike Geodesic Path Through 1 mm Wormhole (Conformally Flat Metric)"
]

(* ::Section:: *)
(*Output Final Position*)

Print[
 "Final position after s = 10: ",
 {x[10], y[10], z[10]} /. timelikeSol
];
```

```
(* ::Section:: *)
(*Null Geodesic Case*)

Lnull = -f[x[s], y[s], z[s]] × t'[s]^2 +
    invf[x[s], y[s], z[s]] (x'[s]^2 + y'[s]^2 + z'[s]^2);

eqtN = D[D[Lnull, t'[s]], s] - D[Lnull, t[s]] == 0;
eqxN = D[D[Lnull, x'[s]], s] - D[Lnull, x[s]] == 0;
eqyN = D[D[Lnull, y'[s]], s] - D[Lnull, y[s]] == 0;
eqzN = D[D[Lnull, z'[s]], s] - D[Lnull, z[s]] == 0;

nullIC = {
 t[0] == 0,
 x[0] == 0.5 R0,
 y[0] == 0,
 z[0] == 0,
 t'[0] == 1,
 x'[0] == 1,
 y'[0] == 0,
 z'[0] == 0
};

nullSol =
 NDSolve[{eqtN, eqxN, eqyN, eqzN} ~Join~ nullIC,
    {t, x, y, z}, {s, 0, 10},
    MaxStepFraction → 1/100];

ParametricPlot3D[
 Evaluate[{x[s], y[s], z[s]} /. nullSol],
 {s, 0, 10},
 PlotRange → All,
 AxesLabel → {"x (m)", "y (m)", "z (m)"},
 PlotLabel → "Null Geodesic Path Through 1 mm Wormhole"
]

(* ::Section:: *)
(*Animated Traversal (Timelike Geodesic)*)

frames3D = Table[
  Graphics3D[
   {
    Red, PointSize[Large],
```

```
  Point[Evaluate[{x[s], y[s], z[s]} /. timelikeSol]]
  },
  PlotRange → {{-3 R0, 12 R0}, {-3 R0, 3 R0}, {-3 R0, 3 R0}},
  Axes → True,
  AxesLabel → {"x (m)", "y (m)", "z (m)"},
  BoxRatios → {4, 1, 1}
 ],
 {s, 0, 10, 0.5}
];

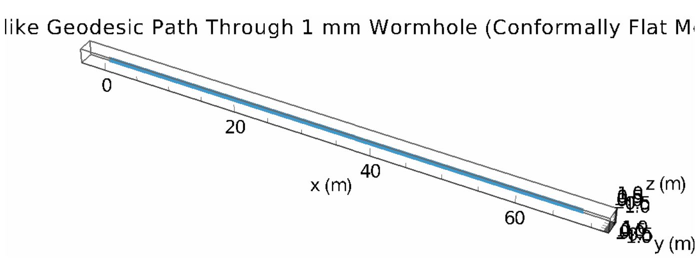
ListAnimate[frames3D]

(* ::Section:: *)
(*Potential Visualization (Equatorial Plane z = 0)*)

Plot3D[
 ΦSafe[x, y, 0],
 {x, -3 R0, 3 R0}, {y, -3 R0, 3 R0},
 PlotLabel →
  "Scalar Potential Φ(x,y,z=0) for 1 mm Wormhole",
 AxesLabel → {"x (m)", "y (m)", "Φ"},
 PlotRange → All
]

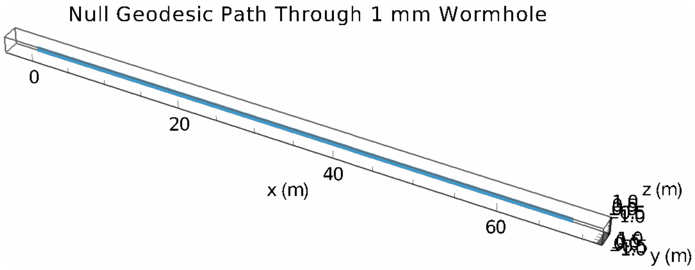
(* Optional: radial potential along x-axis (y = z = 0) *)
Plot[
 ΦSafe[r, 0, 0],
 {r, 0.1 R0, 5 R0},
 PlotLabel →
  "Radial Profile of Φ(r) Along x-Axis (y=z=0)",
 AxesLabel → {"r (m)", "Φ"}
]
```

Out[97]=

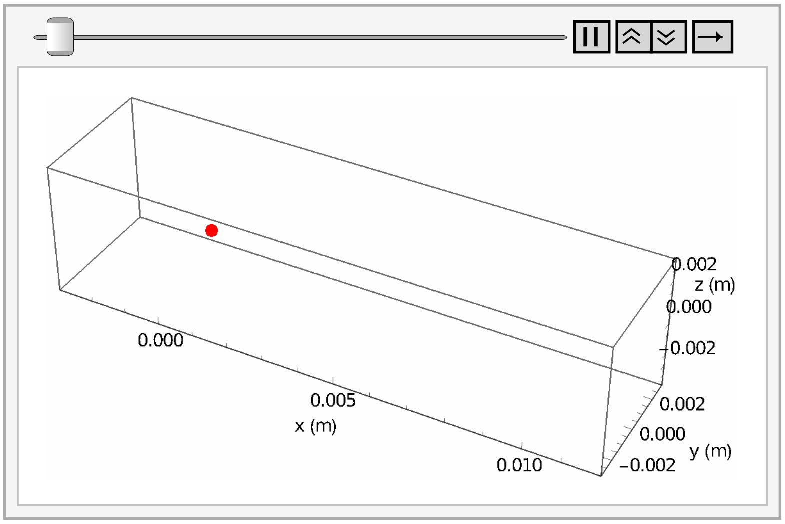



```
Final position after s = 10: {{68.4403, 0., 0.}}
```

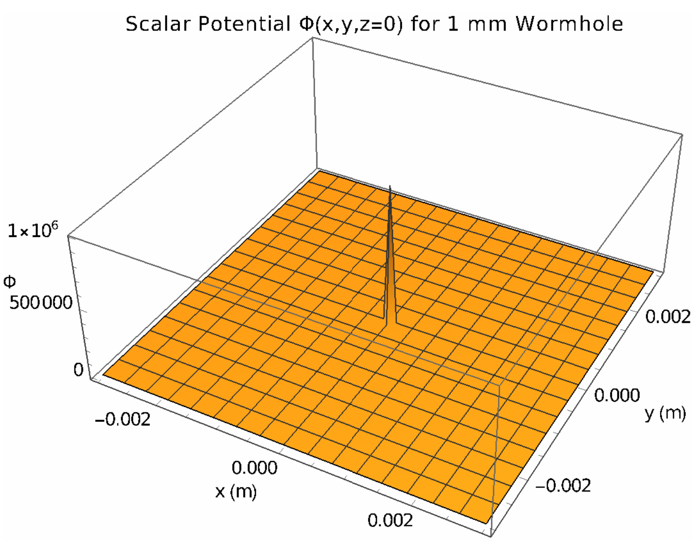
Out[106]=



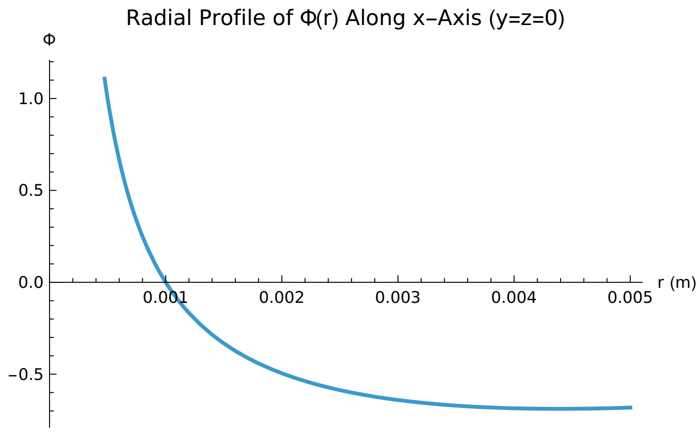
Out[108]=



Out[109]=



Out[110]=


Radial Profile of Φ(r) Along x–Axis (y=z=0)
Φ
1.0
0.5
0.0
–0.5
0.001
0.002
0.003
0.004
0.005
r (m)