In[644]:=

```
(* ======================================== *)
(* CORRECT FLARE-OUT TEST FOR YOUR EXACT METRIC *)
(* ======================================== *)

ClearAll["Global`*"];



(* --- Your exact parameters --- *)
R0 = 1.*10^-3;    (* 1 mm throat *)
A = 1;
w = 1.5 R0;
ϵ = 0.1 R0;     (* Small regularization *)

(* --- Your exact potential --- *)
rCoord[x_, y_, z_] := Max[Sqrt[x^2 + y^2 + z^2], ϵ];
Φ[r_] := -A (1 - R0 / r) Exp[-((r - R0)^2) / w^2];

(* --- Your exact metric --- *)
(* ds² = -e^{2Φ(r)}dt² + e^{-2Φ(r)}(dr² + r²dΩ²) *)

(* --- Areal radius --- *)
Rareal[r_] := Exp[-Φ[r]] * r;

(* --- Flare-out condition --- *)
dRareal = D[Rareal[r], r] /. r → R0 // N;

Print["\n--- Flare-Out Test for YOUR Exact Metric ---"];
Print["Metric: ds² = -e^{2Φ}dt² + e^{-2Φ}(dr² + r²dΩ²)"];
Print["Potential: Φ(r) = -A(1-R0/r)Exp[-(r-R0)²/w²]"];
Print["Parameters: R0 = ", R0, ", A = ", A, ", w = ", w];
Print["Areal radius R_areal(r) = e^{-Φ(r)} r"];
Print["dR_areal/dr at r = R0 = ", dRareal];

If[dRareal > 0,
 Print["☑ FLARE-OUT SATISFIED at throat."],
 Print["☒ FLARE-OUT VIOLATED at throat."]
];

(* --- Detailed analysis at throat --- *)
Print["\n--- Detailed Throat Analysis ---"];
```

```
Print["Φ(R0) = ", Φ[R0] // N];
Print["Φ'(R0) = ", D[Φ[r], r] /. r → R0 // N];
Print["e^{-Φ(R0)} = ", Exp[-Φ[R0]] // N];
Print["Flare factor = 1 - R0*Φ'(R0) = ", 1 - R0*(D[Φ[r], r] /. r → R0) // N];
```

```
--- Flare-Out Test for YOUR Exact Metric ---
Metric: ds² = -e^{2Φ}dt² + e^{-2Φ}(dr² + r²dΩ²)
Potential: Φ(r) = -A(1-R0/r)Exp[-(r-R0)²/w²]
Parameters: R0 = 0.001, A = 1, w = 0.0015
Areal radius R_areal(r) = e^{-Φ(r)} r
dR_areal/dr at r = R0 = 2.
☑ FLARE-OUT SATISFIED at throat.

--- Detailed Throat Analysis ---
Φ(R0) = 0.
Φ'(R0) = -1000.
e^{-Φ(R0)} = 1.
Flare factor = 1 - R0*Φ'(R0) = 2.
```