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In[*]:= (* =====*)
(*CORRECT FLARE-OUT TEST FOR YOUR EXACT METRIC*)
(* =====*)ClearAll["Global`*"];

(*---Your exact parameters---*)
R0 = 2.0
w = 0.2
A = 0.01
ε = 0.04

(*---Your exact potential---*)
rCoord[x_, y_, z_] := Max[Sqrt[x^2 + y^2 + z^2], ε];
ϕ[r_] := -A (1 - R0 / r) Exp[-((r - R0)^2) / w^2];

(*---Your exact metric---*)
(*ds^2 = -e^{2ϕ(r)} dt^2 + e^{-2ϕ(r)} (dr^2 + r^2 dΩ^2) *)

(*---Areal radius---*)
Rareal[r_] := Exp[-ϕ[r]] * r;

(*---Flare-out condition---*)
dRareal = D[Rareal[r], r] /. r → R0 // N;

Print["\n--- Flare-Out Test for YOUR Exact Metric ---"];
Print["Metric: ds^2 = -e^{2ϕ} dt^2 + e^{-2ϕ} (dr^2 + r^2 dΩ^2)"];
Print["Potential: ϕ(r) = -A(1-R0/r) Exp[-(r-R0)^2/w^2]"];
Print["Parameters: R0 = ", R0, ", A = ", A, ", w = ", w];
Print["Areal radius R_areal(r) = e^{-ϕ(r)} r"];
Print["dR_areal/dr at r = R0 = ", dRareal];

If[dRareal > 0, Print["☑ FLARE-OUT SATISFIED at throat."],
  Print["✗ FLARE-OUT VIOLATED at throat."]];

(*---Detailed analysis at throat---*)
Print["\n--- Detailed Throat Analysis ---"];
Print["ϕ(R0) = ", ϕ[R0] // N];
Print["ϕ'(R0) = ", D[ϕ[r], r] /. r → R0 // N];
Print["e^{-ϕ(R0)} = ", Exp[-ϕ[R0]] // N];
Print["Flare factor = 1 - R0*ϕ'(R0) = ", 1 - R0 * (D[ϕ[r], r] /. r → R0) // N];

Out[*]=
2.

Out[*]=
0.2

Out[*]=
0.01

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Out[8]=

0.04

--- Flare-Out Test for YOUR Exact Metric ---

Metric: $ds^2 = -e^{2\Phi} dt^2 + e^{-2\Phi} (dr^2 + r^2 d\Omega^2)$

Potential: $\Phi(r) = -A(1-R_0/r) \text{Exp}[-(r-R_0)^2/w^2]$

Parameters: $R_0 = 2.$, $A = 0.01$, $w = 0.2$

Areal radius $R_{\text{areal}}(r) = e^{-\Phi(r)} r$

dR_{areal}/dr at $r = R_0 = 1.01$

☒ FLARE-OUT SATISFIED at throat.

--- Detailed Throat Analysis ---

$\Phi(R_0) = 0.$

$\Phi'(R_0) = -0.005$

$e^{-\Phi(R_0)} = 1.$

Flare factor $= 1 - R_0 \Phi'(R_0) = 1.01$