In[145]:=

```
(* ==================== *)
(* Coordinates          *)
(* ==================== *)

ClearAll[t, r, θ, ϕ, Φ, φ]
coords = {t, r, θ, ϕ};

(* ==================== *)
(* Scalar potential Φ(r) *)
(* ==================== *)

Φ[r_] := -A*(1 - R0/r) * Exp[-(r - R0)^2/w^2];

(* Scalar field φ(r) = exp(Φ) *)
φ[r_] := Exp[Φ[r]];

(* ==================== *)
(* Metric: conformally-flat spatial slice *)
(* ds^2 = -exp(2Φ) dt^2 + exp(-2Φ) dr^2 + r^2 dΩ^2 *)
(* ==================== *)

metric = DiagonalMatrix[{
  -Exp[2 Φ[r]],
   Exp[-2 Φ[r]],
   r^2,
   r^2 Sin[θ]^2
}];

invMetric = Simplify[Inverse[metric]];

(* ==================== *)
(* Christoffel symbols   *)
(* ==================== *)

Γ = Table[
  Sum[
   1/2 invMetric[[i, k]]*
    (D[metric[[k, j]], coords[[l]]] +
     D[metric[[k, l]], coords[[j]]] -
     D[metric[[j, l]], coords[[k]]]),
   {k, 1, 4}],
  {i, 1, 4}, {j, 1, 4}, {l, 1, 4}
```

```
];

(* ==================== *)
(* Ricci tensor & scalar *)
(* ==================== *)

Riemann = Table[
  D[Γ[[i, j, k]], coords[[l]]] -
  D[Γ[[i, j, l]], coords[[k]]] +
  Sum[
   Γ[[i, m, k]] × Γ[[m, j, l]] -
   Γ[[i, m, l]] × Γ[[m, j, k]],
   {m, 1, 4}],
  {i, 1, 4}, {j, 1, 4}, {k, 1, 4}, {l, 1, 4}
];

Ricci = Table[
  Sum[Riemann[[m, i, m, j]], {m, 1, 4}],
  {i, 1, 4}, {j, 1, 4}
];

R = Simplify[
  Sum[invMetric[[i, j]] × Ricci[[i, j]], {i, 1, 4}, {j, 1, 4}]
];

(* ==================== *)
(* Scalar kinetic term    *)
(* ==================== *)

dφ = D[φ[r], r];
kinetic = Simplify[1/2 * invMetric[[2, 2]] * dφ^2];

(* ==================== *)
(* Potential term         *)
(* ==================== *)

V[φ_] := λ φ^4;

potentialTerm = V[φ[r]];

(* ==================== *)
(* Full Lagrangian        *)
```

```
(* ==================== *)

L = Simplify[φ[r]^2 R - kinetic - potentialTerm];

Print["Ricci scalar R(r) = ", R];
Print["Lagrangian L(r) = ", L];

(* ==================== *)
(* Plot for sample wormhole *)
(* ==================== *)

Plot[Evaluate[L /. {A → 1, R0 → 1*10^-3, w → 5*10^-4, λ → 1}],
  {r, 1*10^-3, 5*10^-3},
  PlotRange → All,
  PlotLabel → "Lagrangian for 1 mm Wormhole"]
```

Ricci scalar R(r) =
$$
\begin{aligned}
&\frac{1}{r^4}\, e^{-2\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}+\left(-16000.-1.2\times 10^{7}\, r\right) r} \\
&\left(1.34185\times 10^{-9}\, e^{\frac{0.002\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}}{r}+r\left(32000.+4.\times 10^{6}\, r\right)} - 0.0000214696\, e^{\frac{0.002\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}}{r}+r\left(32000.+4.\times 10^{6}\, r\right)}\, r + \right. \\
&\quad 0.0000732626\, e^{\frac{0.002\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}}{r}+r\left(24000.+8.\times 10^{6}\, r\right)}\, r + \\
&\quad 0.128818\, e^{\frac{0.002\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}}{r}+r\left(32000.+4.\times 10^{6}\, r\right)}\, r^{2} - 0.5861\, e^{\frac{0.002\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}}{r}+r\left(24000.+8.\times 10^{6}\, r\right)}\, r^{2} - \\
&\quad 1.\, e^{2\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}+r\left(16000.+1.2\times 10^{7}\, r\right)}\, r^{2} - 2.\, e^{\frac{0.002\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}}{r}+r\left(16000.+1.2\times 10^{7}\, r\right)}\, r^{2} - \\
&\quad 364.983\, e^{\frac{0.002\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}}{r}+r\left(32000.+4.\times 10^{6}\, r\right)}\, r^{3} + 2637.45\, e^{\frac{0.002\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}}{r}+r\left(24000.+8.\times 10^{6}\, r\right)}\, r^{3} + \\
&\quad 515271.\, e^{\frac{0.002\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}}{r}+r\left(32000.+4.\times 10^{6}\, r\right)}\, r^{4} - 6.74016\times 10^{6}\, e^{\frac{0.002\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}}{r}+r\left(24000.+8.\times 10^{6}\, r\right)}\, r^{4} - \\
&\quad 3.43514\times 10^{8}\, e^{\frac{0.002\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}}{r}+r\left(32000.+4.\times 10^{6}\, r\right)}\, r^{5} + 7.03321\times 10^{9}\, e^{\frac{0.002\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}}{r}+r\left(24000.+8.\times 10^{6}\, r\right)}\, r^{5} + \\
&\quad 8.58784\times 10^{10}\, e^{\frac{0.002\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}}{r}+r\left(32000.+4.\times 10^{6}\, r\right)}\, r^{6} - 2.3444\times 10^{12}\, e^{\frac{0.002\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}}{r}+r\left(24000.+8.\times 10^{6}\, r\right)}\, r^{6} + \\
&\quad \left. 1.\, e^{2\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}+r\left(16000.+1.2\times 10^{7}\, r\right)}\, r^{2}\, \mathrm{Cot}[\theta]^{2} - 1.\, e^{2\, e^{-4.\times 10^{6}(0.001-1.\, r)^{2}}+r\left(16000.+1.2\times 10^{7}\, r\right)}\, r^{2}\, \mathrm{Csc}[\theta]^{2}\right)
\end{aligned}
$$

Lagrangian L(r) = −

$$\frac{1.07348\times 10^{10}\, e^{e^{-4.\times 10^{6}(0.001-1.\,r)^2}\left(-4.+\frac{0.004}{r}\right)+\left(16000.-8.\times 10^{6}\,r\right) r}\left(1.25\times 10^{-10}-1.\times 10^{-6}\,r+0.002\,r^2-1.\,r^3\right)^2}{r^4} -$$

$$e^{-4\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}\left(1-\frac{1}{1000\,r}\right)}\lambda +$$

$$\frac{1}{r^4}\, e^{e^{-4.\times 10^{6}(0.001-1.\,r)^2}\left(-4+\frac{1}{500\,r}\right)+\left(-16000.-1.2\times 10^{7}\,r\right) r}\Bigg(1.34185\times 10^{-9}\, e^{\frac{0.002\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}}{r}+r\left(32000.+4.\times 10^{6}\,r\right)} -$$

$$0.0000214696\, e^{\frac{0.002\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}}{r}+r\left(32000.+4.\times 10^{6}\,r\right)} r + 0.0000732626\, e^{\frac{0.002\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}}{r}+r\left(24000.+8.\times 10^{6}\,r\right)} r +$$

$$0.128818\, e^{\frac{0.002\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}}{r}+r\left(32000.+4.\times 10^{6}\,r\right)} r^2 - 0.5861\, e^{\frac{0.002\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}}{r}+r\left(24000.+8.\times 10^{6}\,r\right)} r^2 -$$

$$1.\, e^{2\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}+r\left(16000.+1.2\times 10^{7}\,r\right)} r^2 - 2.\, e^{\frac{0.002\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}}{r}+r\left(16000.+1.2\times 10^{7}\,r\right)} r^2 -$$

$$364.983\, e^{\frac{0.002\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}}{r}+r\left(32000.+4.\times 10^{6}\,r\right)} r^3 + 2637.45\, e^{\frac{0.002\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}}{r}+r\left(24000.+8.\times 10^{6}\,r\right)} r^3 +$$

$$515271.\, e^{\frac{0.002\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}}{r}+r\left(32000.+4.\times 10^{6}\,r\right)} r^4 - 6.74016\times 10^{6}\, e^{\frac{0.002\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}}{r}+r\left(24000.+8.\times 10^{6}\,r\right)} r^4 -$$

$$3.43514\times 10^{8}\, e^{\frac{0.002\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}}{r}+r\left(32000.+4.\times 10^{6}\,r\right)} r^5 + 7.03321\times 10^{9}\, e^{\frac{0.002\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}}{r}+r\left(24000.+8.\times 10^{6}\,r\right)} r^5 +$$

$$8.58784\times 10^{10}\, e^{\frac{0.002\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}}{r}+r\left(32000.+4.\times 10^{6}\,r\right)} r^6 - 2.3444\times 10^{12}\, e^{\frac{0.002\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}}{r}+r\left(24000.+8.\times 10^{6}\,r\right)} r^6 +$$

$$1.\, e^{2\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}+r\left(16000.+1.2\times 10^{7}\,r\right)} r^2\, \mathrm{Cot}[\theta]^2 - 1.\, e^{2\,e^{-4.\times 10^{6}(0.001-1.\,r)^2}+r\left(16000.+1.2\times 10^{7}\,r\right)} r^2\, \mathrm{Csc}[\theta]^2\Bigg)$$

Out[162]=

Lagrangian for 1 mm Wormhole