In[187]:=

```
(* ::Title:: *)
(*Two-Sided Wormhole: NEC at Throat + Geodesic Across Two Universes*)

(* ::Section:: *)
(*Parameters and Radial Profile*)

ClearAll["Global`*"];



R0 = 0.001; (* 1 mm throat *)



A = 1.0;
w = 10 R0;

r[l_] := Sqrt[l^2 + R0^2];

ϕ[l_] := -A (1 - R0 / r[l]) Exp[-(r[l] - R0)^2 / w^2];

(* ::Section:: *)
(*Metric (Spherical Symmetry, Proper Radial Coordinate l)*)

(* Coordinates: t, l, θ, ϕ *)
coords = {t, l, th, ph};

metric = DiagonalMatrix[{
  -Exp[2 ϕ[l]],
   Exp[-2 ϕ[l]],
   Exp[-2 ϕ[l]] × r[l]^2,
   Exp[-2 ϕ[l]] × r[l]^2 × Sin[th]^2
}];

(* ::Section:: *)
(*Asymptotic Flatness on Both Sides (Two Universes)*)

limitp = Limit[metric[[1, 1]], l → Infinity];
limitm = Limit[metric[[1, 1]], l → -Infinity];

Print["Asymptotic g_tt at l -> +∞: ", limitp];
Print["Asymptotic g_tt at l -> -∞: ", limitm];
```

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(* Both should be -1 for asymptotically flat regions on both sides *)

(* ::Section:: *)
(*NEC at the Throat (Rigorous Formula)*)

(* We derived:
   NEC_throat = ((-Phi1^2 R0^2 + Phi2 R0^2 - 1) Exp[4 Phi0])/(4 Pi R0^2),
   where Phi0 = ϕ[0], Phi1 = ϕ'[0], Phi2 = ϕ''[0].
*)

Phi0 = ϕ[0];
Phi1 = D[ϕ[l], l] /. l → 0;
Phi2 = D[ϕ[l], {l, 2}] /. l → 0;

NECThroat = ((-Phi1^2 R0^2 + Phi2 R0^2 - 1) Exp[4 Phi0]) / (4 Pi R0^2) // N;

Print["Φ(0)    = ", Phi0 // N];
Print["Φ'(0)   = ", Phi1 // N];
Print["Φ''(0)  = ", Phi2 // N];
Print["NEC at throat (l = 0): ", NECThroat];

(* If NECThroat < 0, the null energy condition is violated at the throat,
   which is the usual GR signature of
 a traversable wormhole requiring exotic matter. *)

(* ::Section:: *)
(*Radial Timelike Geodesic Across the Throat*)

Clear[τ, ell, s];

(* Proper radial coordinate function ell[s], and t-coordinate τ[s] *)

Lgeo = -Exp[2 ϕ[ell[s]]] × τ'[s]^2 +
    Exp[-2 ϕ[ell[s]]] × ell'[s]^2;

eqτ   = D[D[Lgeo, τ'[s]], s] - D[Lgeo, τ[s]] == 0;
eqell = D[D[Lgeo, ell'[s]], s] - D[Lgeo, ell[s]] == 0;

(* Initial conditions: start in l < 0 universe and move towards +l *)

geoIC = {
```

```
 τ[0] == 0,
 ell[0] == -5 R0,  (* start 5 throat radii into the "left" universe *)
 τ'[0] == 1,    (* normalization choice *)
 ell'[0] == 1   (* initial velocity towards the throat *)
};

geoSol = NDSolve[
 {eqτ, eqell} ~Join~ geoIC,
 {τ, ell}, {s, 0, 40},
 MaxStepFraction → 1/100
];

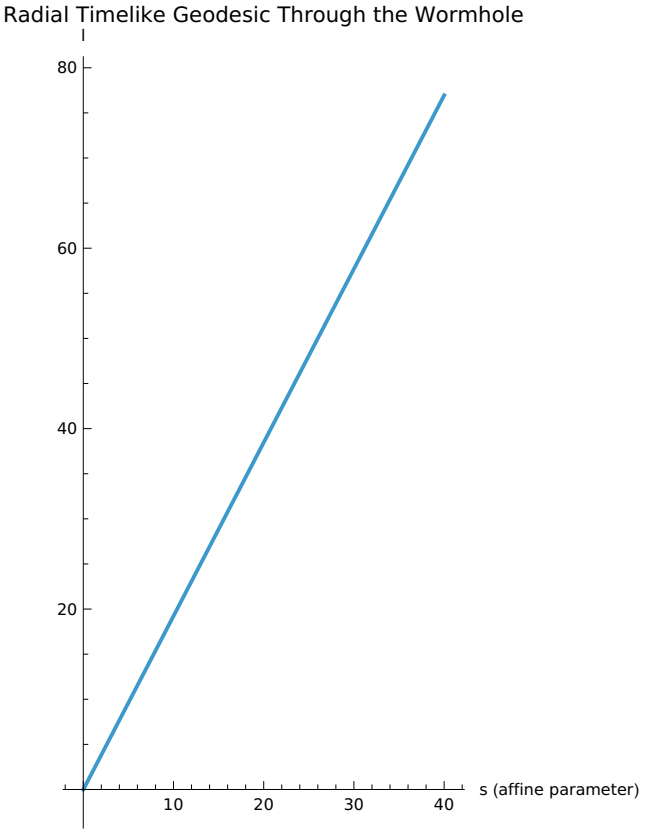
Print["Sample values of l(s) at s = 0, 10, 20, 30, 40:"];

Print[
 Table[
  {ss, ell[ss] /. geoSol},
  {ss, 0, 40, 10}
 ]
];

(* Optional: plot the trajectory in l *)
ParametricPlot[
 Evaluate[{s, ell[s]} /. geoSol],
 {s, 0, 40},
 AxesLabel → {"s (affine parameter)", "l"},
 PlotLabel → "Radial Timelike Geodesic Through the Wormhole"
]
```

```
Asymptotic g_tt at l -> +∞: -1.
Asymptotic g_tt at l -> -∞: -1.
Φ(0)    = 0.
Φ'(0)   = 0.
Φ''(0)  = -1.×10^6
NEC at throat (l = 0): -159155.
Sample values of l(s) at s = 0, 10, 20, 30, 40:
{{0, {-0.005}}, {10, {19.2231}}, {20, {38.4632}}, {30, {57.7033}}, {40, {76.9434}}}
```

Out[215]=


Radial Timelike Geodesic Through the Wormhole
80
60
40
20
10
20
30
40
s (affine parameter)