```
In[•]:= (* ========================================*)
(*1 mm Static Wormhole:Potential& Metric*)
(* ========================================*)
(*---Clear all previous definitions---*)ClearAll["Global`*"];

(*---Basic parameters (SI units)---*)
R0 = 2.0
w = 0.2
A = 0.01
ε = 0.04

(*--------------------------------------*)
(*1D RADIAL SCALAR POTENTIAL Φ(r)*)
(*--------------------------------------*)

(*Φ(r)=-A (1-R0/r) Exp[-(r-R0)^2/w^2]*)
Φ[r_] := -A (1 - R0 / r) Exp[-((r - R0)^2) / w^2];

(*Flare-out condition:d/dr[Exp[-2 Φ(r)]] at r=R0*)
flareOutExpr = Exp[-2 Φ[r]];
flareOutDerivative = D[flareOutExpr, r] /. r → R0 // FullSimplify;

Print["\n--- Flare-out Condition at r = R0 = ", R0, " m ---"];
Print["d/dr[Exp[-2 Φ(r)]]|_(r=R0) = ", N[flareOutDerivative, 10]];
If[flareOutDerivative > 0, Print["☑ Flare-out condition satisfied at the throat."],
  Print["✗ Flare-out condition violated at the throat."]];

(*Scan over different w values (in units of R0)*)
Print["\n--- Scanning flare-out over w (multiples of R0) ---"];
Table[Module[{wval, φloc, ddr}, wval = fac * R0;
   φloc[r_] := -A (1 - R0 / r) Exp[-((r - R0)^2) / wval^2];
   ddr = D[Exp[-2 φloc[r]], r] /. r → R0 // N;
   {wval, ddr, If[ddr > 0, "☑ OK", "✗ Violated"]}], {fac, 0.5, 3, 0.1}] // TableForm

(*--------------------------------------*)
(*3D SCALAR POTENTIAL Φ(x,y,z)*)
(*--------------------------------------*)

(*Regularized radius:r(x,y,z)=max(Sqrt[x^2+y^2+z^2],eps)*)
rReg[x_, y_, z_] := Max[Sqrt[x^2 + y^2 + z^2], eps];

(*Φ(x,y,z) built from rReg*)
Φ3D[x_, y_, z_] := Module[{r = rReg[x, y, z]}, -A (1 - R0 / r) Exp[-((r - R0)^2) / w^2]];

(*Quick checks on the 3D potential*)
Print["\n--- 3D potential checks ---"];
```

```
Print["Φ3D[0,0,0] = ", N[Φ3D[0, 0, 0], 10]];
Print["Φ3D[R0,0,0] = ", N[Φ3D[R0, 0, 0], 10]];
Print["Φ3D[2 R0,0,0] = ", N[Φ3D[2 R0, 0, 0], 10]];

(*--------------------------------------*)
(*METRIC FROM Φ(r)*)
(*--------------------------------------*)

(*Spherical coordinates (t,r,θ,φ)*)
coords = {t, r, θ, ϕ};

(*Metric components:ds^2=-e^{2Φ(r)} dt^2+e^{-2Φ(r)} dr^2+r^2 dθ^2+r^2 sin^2θ dϕ^2*)
g = {{-Exp[2 Φ[r]], 0, 0, 0},
   {0, Exp[-2 Φ[r]], 0, 0}, {0, 0, r^2, 0}, {0, 0, 0, r^2 Sin[θ]^2}};

Print["\n--- Metric components g_{μν}(r,θ) ---"];
MatrixForm[g] // Print;

(*Check determinant and signature at the throat r=R0*)
gAtThroat[θ0_] := g /. {r → R0, θ → θ0};

detgAtThroat = Det[gAtThroat[Pi / 2]] // FullSimplify;
eigsAtThroat = Eigenvalues[gAtThroat[Pi / 2]] // N;

Print["\nDet[g]|_(r=R0, θ=π/2) = ", detgAtThroat];
Print["Eigenvalues[g]|_(r=R0, θ=π/2) = ", eigsAtThroat];
Print["(Expect one negative, three positive for Lorentzian signature.)"];
```

Out[•]=

```
2.
```

Out[•]=

```
0.2
```

Out[•]=

```
0.01
```

Out[•]=

```
0.04


--- Flare-out Condition at r = R0 = 2. m ---
d/dr[Exp[-2 Φ(r)]]|_(r=R0) = 0.01
☑ Flare-out condition satisfied at the throat.

--- Scanning flare-out over w (multiples of R0) ---
```

Out[•]//TableForm=

1.	0.01	☑ OK
1.2	0.01	☑ OK
1.4	0.01	☑ OK
1.6	0.01	☑ OK
1.8	0.01	☑ OK
2.	0.01	☑ OK
2.2	0.01	☑ OK
2.4	0.01	☑ OK
2.6	0.01	☑ OK
2.8	0.01	☑ OK
3.	0.01	☑ OK
3.2	0.01	☑ OK
3.4	0.01	☑ OK
3.6	0.01	☑ OK
3.8	0.01	☑ OK
4.	0.01	☑ OK
4.2	0.01	☑ OK
4.4	0.01	☑ OK
4.6	0.01	☑ OK
4.8	0.01	☑ OK
5.	0.01	☑ OK
5.2	0.01	☑ OK
5.4	0.01	☑ OK
5.6	0.01	☑ OK
5.8	0.01	☑ OK
6.	0.01	☑ OK

--- 3D potential checks ---

$$\Phi 3D[0,0,0] = -0.01 \times 2.718281828^{-25.\,(-2.+\text{Max}[0.\times 10^{-10},\text{eps}])^2}\left(1.000000000 - \frac{2.}{\text{Max}[0,\text{eps}]}\right)$$

$$\Phi 3D[R0,0,0] = -0.01 \times 2.718281828^{-25.\,(-2.+\text{Max}[2.,\text{eps}])^2}\left(1.000000000 - \frac{2.}{\text{Max}[2.,\text{eps}]}\right)$$

$$\Phi 3D[2\ R0,0,0] = -0.01 \times 2.718281828^{-25.\,(-2.+\text{Max}[4.,\text{eps}])^2}\left(1.000000000 - \frac{2.}{\text{Max}[4.,\text{eps}]}\right)$$

--- Metric components g_{μν}(r,Θ) ---

$$\begin{pmatrix} -e^{-0.02\,e^{-25.\,(-2.+r)^2}\left(1-\frac{2.}{r}\right)} & 0 & 0 & 0 \\ 0 & e^{0.02\,e^{-25.\,(-2.+r)^2}\left(1-\frac{2.}{r}\right)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\,\text{Sin}[\theta]^2 \end{pmatrix}$$

Det[g]|_(r=R0, Θ=π/2) = -16.

Eigenvalues[g]|_(r=R0, Θ=π/2) = {4., 4., -1., 1.}

(Expect one negative, three positive for Lorentzian signature.)