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In[22]:= (* ----- *)
(* 1. Parameters and scalar potential      *)
(* ----- *)

ClearAll["Global`*"];


R0 = 1.*^-3; (* wormhole throat 1 mm *)
A = 1.0;    (* scalar strength      *)
w = 10 R0;   (* Gaussian width       *)

Phi[r_] := -A (1 - R0/r) Exp[-(r - R0)^2/w^2];

(* ----- *)
(* 2. Brans-Dicke scalar field is Phi(r)      *)
(*     BD vacuum equation: phi'' + 2 phi'/r = 0  *)
(* ----- *)

BDResidual[r_] := Module[{dp, ddp},
  dp = N[D[Phi[x], x] /. x -> r];
  ddp = N[D[Phi[x], {x, 2}] /. x -> r];
  ddp + (2/r) dp
];

(* ----- *)
(* 3. Evaluate BD-residual at throat & far away *)
(* ----- *)

Print["Φ(0)  (limit) = ", Phi[R0]];
Print["Φ'(R0)      = ", N[D[Phi[x], x] /. x -> R0]];
Print["Φ''(R0)     = ", N[D[Phi[x], {x, 2}] /. x -> R0]];

Print["BD residual at r = R0:  ", BDResidual[R0]];
Print["BD residual at r = 2R0: ", BDResidual[2 R0]];
Print["BD residual at r = 5R0: ", BDResidual[5 R0]];
Print["BD residual at r = 10R0: ", BDResidual[10 R0]];

Print["Limit of BD residual as r→∞ = ",
  Limit[BDResidual[r], r -> Infinity]
];

```

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(* ----- *)
(* 4. Plot BD residual *)
(* ----- *)
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Plot[
  BDResidual[r],
  {r, R0, 10 R0},
  AxesLabel -> {"r (m)", "BD residual"},
  PlotRange -> All,
  PlotLabel -> "Brans-Dicke consistency residual"
]
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Φ(0) (limit) = 0.
Φ'(R0)      = -1000.
Φ''(R0)      = 2. × 106
BD residual at r = R0: -2.32831 × 10-10
BD residual at r = 2R0: 29 503.5
BD residual at r = 5R0: 36 539.9
BD residual at r = 10R0: 11 050.3
Limit of BD residual as r → ∞ = 0.
```

Out[36]=

