```
(*-----------------------------------------------------------*)
(* Coordinates and Metric                                    *)
(*-----------------------------------------------------------*)

coords = {t, r, θ, ϕ};



(* Scalar potential Φ(r): with regularized radius r = Sqrt[r^2 + ε^2] *)
rReg[r_] := Sqrt[r^2 + ε^2];



Φ[r_] := -A (1 - R0/rReg[r]) Exp[-(rReg[r] - R0)^2/w^2];



metric = DiagonalMatrix[{
  -Exp[2 Φ[r]],
   Exp[-2 Φ[r]],
   r^2,
   r^2 Sin[θ]^2
}];



invMetric = Simplify[Inverse[metric]];



(* Define scalar field for illustration *)
CurlyPhi[r_] := Exp[Φ[r]];



(*-----------------------------------------------------------*)
(* Christoffel Symbols                                       *)
(*-----------------------------------------------------------*)

Γ = Table[
  Sum[
   1/2 invMetric[[i, k]] (
    D[metric[[k, j]], coords[[l]]] +
```

```
    D[metric[[k, l]], coords[[j]]] -
    D[metric[[j, l]], coords[[k]]]
   ),
  {k, 1, 4}
 ],
 {i, 1, 4}, {j, 1, 4}, {l, 1, 4}
];




(*--------------------------------------------------------*)
(* Riemann and Ricci Tensors                              *)
(*--------------------------------------------------------*)

Riemann = Table[
 D[Γ[[i, j, k]], coords[[l]]] -
 D[Γ[[i, j, l]], coords[[k]]] +
 Sum[
  Γ[[i, m, k]]×Γ[[m, j, l]] -
  Γ[[i, m, l]]×Γ[[m, j, k]],
  {m, 1, 4}
 ],
 {i, 1, 4}, {j, 1, 4}, {k, 1, 4}, {l, 1, 4}
];




Ricci = Table[
 Sum[Riemann[[m, i, m, j]], {m, 1, 4}],
 {i, 1, 4}, {j, 1, 4}
];




RicciScalar = Simplify[
 Sum[invMetric[[i, j]]×Ricci[[i, j]], {i, 1, 4}, {j, 1, 4}]
];




(*--------------------------------------------------------*)
```

```mathematica
(* Kinetic Term                                            *)
(*------------------------------------------------------*)


dCurlyPhi = Grad[CurlyPhi[r], {r}];
kinetic = Simplify[1/2 invMetric[[2, 2]] dCurlyPhi[[1]]^2];


(*------------------------------------------------------*)
(* Scalar Potential V(φ)                                  *)
(*------------------------------------------------------*)


V[φ_] := λ φ^4;
potentialTerm = V[CurlyPhi[r]];


(*------------------------------------------------------*)
(* Full Lagrangian                                        *)
(*------------------------------------------------------*)


L = Simplify[CurlyPhi[r]^2 RicciScalar - kinetic - potentialTerm];

Print["Ricci scalar R(r) = ", RicciScalar];
Print["Lagrangian ℒ(r) = ", L];

Print["Limit Lagrangian r→R0 = ", Limit[L, r → R0]];
Print["Asymptotic series at infinity: ", Series[L, {r, ∞, 2}]];


(*------------------------------------------------------*)
(* Example numeric plot for a 1 mm wormhole              *)
(*------------------------------------------------------*)

Plot[
 Evaluate[L /. {
   A → 1, R0 → 1*10^-3, w → 0.0005, ε → 10^-6, λ → 1
 }],
 {r, 0.001, 0.01},
 PlotLabel → "Lagrangian vs r (1 mm wormhole)"
]
```

Ricci scalar R(r) =

$$-\frac{1}{r^2 w^4 (r^2 + \varepsilon^2)^3} 2 e^{-2 A e^{-\frac{(R\theta - \sqrt{r^2 + \varepsilon^2})^2}{w^2}}\left(1 - \frac{R\theta}{\sqrt{r^2 + \varepsilon^2}}\right) - \frac{2(R\theta - \sqrt{r^2 + \varepsilon^2})^2}{w^2}} \left(e^{\frac{2(R\theta - \sqrt{r^2 + \varepsilon^2})^2}{w^2}}\left(1 + e^{2 A e^{-\frac{(R\theta - \sqrt{r^2 + \varepsilon^2})^2}{w^2}}\left(1 - \frac{R\theta}{\sqrt{r^2 + \varepsilon^2}}\right)}\right) w^4 (r^2 + \varepsilon^2)^3 -$$

$$2 A^2 r^4 \left(4 r^6 + 4 R\theta^4 \varepsilon^2 + 4 \varepsilon^6 - 4 R\theta^3 \sqrt{r^2 + \varepsilon^2} (w^2 + 4 \varepsilon^2) - 4 R\theta \varepsilon^2 \sqrt{r^2 + \varepsilon^2} (w^2 + 4 \varepsilon^2) + \right.$$

$$R\theta^2 (w^4 + 8 w^2 \varepsilon^2 + 24 \varepsilon^4) - 4 r^4 (-6 R\theta^2 - 3 \varepsilon^2 + 4 R\theta \sqrt{r^2 + \varepsilon^2}) -$$

$$\left. 4 r^2 (-R\theta^4 - 3 \varepsilon^4 + 4 R\theta^3 \sqrt{r^2 + \varepsilon^2} - 2 R\theta^2 (w^2 + 6 \varepsilon^2) + R\theta \sqrt{r^2 + \varepsilon^2} (w^2 + 8 \varepsilon^2))\right) + A e^{\frac{(R\theta - \sqrt{r^2 + \varepsilon^2})^2}{w^2}} r^2$$

$$\left(4 r^8 - 2 r^6 (-6 R\theta^2 + w^2 - 6 \varepsilon^2 + 6 R\theta \sqrt{r^2 + \varepsilon^2}) + w^2 \varepsilon^2 (-2 R\theta^2 \varepsilon^2 - 2 \varepsilon^4 + R\theta \sqrt{r^2 + \varepsilon^2} (w^2 + 4 \varepsilon^2)) - \right.$$

$$2 r^4 (3 \varepsilon^2 (w^2 - 2 \varepsilon^2) + 2 R\theta^3 \sqrt{r^2 + \varepsilon^2} - 2 R\theta^2 (w^2 + 6 \varepsilon^2) + R\theta \sqrt{r^2 + \varepsilon^2} (w^2 + 12 \varepsilon^2)) -$$

$$\left. 2 r^2 (3 w^2 \varepsilon^4 - 2 \varepsilon^6 + 2 R\theta^3 \varepsilon^2 \sqrt{r^2 + \varepsilon^2} - R\theta^2 \varepsilon^2 (w^2 + 6 \varepsilon^2) + R\theta \sqrt{r^2 + \varepsilon^2} (w^4 - w^2 \varepsilon^2 + 6 \varepsilon^4))\right)\right)$$

Lagrangian 𝓛(r) =

$$\frac{1}{2} e^{-4 A e^{-\frac{(R\theta - \sqrt{r^2 + \varepsilon^2})^2}{w^2}}\left(1 - \frac{R\theta}{\sqrt{r^2 + \varepsilon^2}}\right)} \left(-\frac{4\left(1 + e^{2 A e^{-\frac{(R\theta - \sqrt{r^2 + \varepsilon^2})^2}{w^2}}\left(1 - \frac{R\theta}{\sqrt{r^2 + \varepsilon^2}}\right)}\right)}{r^2} - \right.$$

$$\frac{A^2 e^{-\frac{2(R\theta - \sqrt{r^2 + \varepsilon^2})^2}{w^2}} r^2 (2 R\theta^2 \sqrt{r^2 + \varepsilon^2} + 2 \varepsilon^2 \sqrt{r^2 + \varepsilon^2} - R\theta (w^2 + 4 \varepsilon^2) + 2 r^2 (-2 R\theta + \sqrt{r^2 + \varepsilon^2}))^2}{w^4 (r^2 + \varepsilon^2)^3} +$$

$$\frac{1}{w^4 (r^2 + \varepsilon^2)^3} 8 A^2 e^{-\frac{2(R\theta - \sqrt{r^2 + \varepsilon^2})^2}{w^2}} r^2 \left(4 r^6 + 4 R\theta^4 \varepsilon^2 + 4 \varepsilon^6 - 4 R\theta^3 \sqrt{r^2 + \varepsilon^2} (w^2 + 4 \varepsilon^2) - \right.$$

$$4 R\theta \varepsilon^2 \sqrt{r^2 + \varepsilon^2} (w^2 + 4 \varepsilon^2) + R\theta^2 (w^4 + 8 w^2 \varepsilon^2 + 24 \varepsilon^4) - 4 r^4 (-6 R\theta^2 - 3 \varepsilon^2 + 4 R\theta \sqrt{r^2 + \varepsilon^2}) -$$

$$\left. 4 r^2 (-R\theta^4 - 3 \varepsilon^4 + 4 R\theta^3 \sqrt{r^2 + \varepsilon^2} - 2 R\theta^2 (w^2 + 6 \varepsilon^2) + R\theta \sqrt{r^2 + \varepsilon^2} (w^2 + 8 \varepsilon^2))\right) - \frac{1}{w^4 (r^2 + \varepsilon^2)^3} 4 A$$

$$e^{-\frac{(R\theta - \sqrt{r^2 + \varepsilon^2})^2}{w^2}} \left(4 r^8 - 2 r^6 (-6 R\theta^2 + w^2 - 6 \varepsilon^2 + 6 R\theta \sqrt{r^2 + \varepsilon^2}) + w^2 \varepsilon^2 (-2 R\theta^2 \varepsilon^2 - 2 \varepsilon^4 + R\theta \sqrt{r^2 + \varepsilon^2} (w^2 + 4 \varepsilon^2)) - \right.$$

$$2 r^4 (3 \varepsilon^2 (w^2 - 2 \varepsilon^2) + 2 R\theta^3 \sqrt{r^2 + \varepsilon^2} - 2 R\theta^2 (w^2 + 6 \varepsilon^2) + R\theta \sqrt{r^2 + \varepsilon^2} (w^2 + 12 \varepsilon^2)) -$$

$$\left.\left. 2 r^2 (3 w^2 \varepsilon^4 - 2 \varepsilon^6 + 2 R\theta^3 \varepsilon^2 \sqrt{r^2 + \varepsilon^2} - R\theta^2 \varepsilon^2 (w^2 + 6 \varepsilon^2) + R\theta \sqrt{r^2 + \varepsilon^2} (w^4 - w^2 \varepsilon^2 + 6 \varepsilon^4))\right)\right) - 2 \lambda\right)$$

$$\text{Limit Lagrangian } r \to R0 = \frac{1}{2} e^{-4 A e^{-\frac{\left(R0 - \sqrt{R0^2 + \epsilon^2}\right)^2}{w^2}} \left(1 - \frac{R0}{\sqrt{R0^2 + \epsilon^2}}\right)} \left(-\frac{4\left(1 + e^{\frac{2 A e^{-\frac{\left(R0 - \sqrt{R0^2 + \epsilon^2}\right)^2}{w^2}}\left(1 - \frac{R0}{\sqrt{R0^2 + \epsilon^2}}\right)}}\right)}{R0^2} - \right.$$

$$\frac{A^2 e^{-\frac{2\left(R0 - \sqrt{R0^2 + \epsilon^2}\right)^2}{w^2}} R0^2 \left(2 R0^2 \sqrt{R0^2 + \epsilon^2} + 2 \epsilon^2 \sqrt{R0^2 + \epsilon^2} - R0\left(w^2 + 4 \epsilon^2\right) + 2 R0^2\left(-2 R0 + \sqrt{R0^2 + \epsilon^2}\right)\right)^2}{w^4 \left(R0^2 + \epsilon^2\right)^3} +$$

$$\frac{1}{w^4 \left(R0^2 + \epsilon^2\right)^3} 8 A^2 e^{-\frac{2\left(R0 - \sqrt{R0^2 + \epsilon^2}\right)^2}{w^2}} R0^2 \left(4 R0^6 + 4 R0^4 \epsilon^2 + 4 \epsilon^6 - 4 R0^3 \sqrt{R0^2 + \epsilon^2} \left(w^2 + 4 \epsilon^2\right) - \right.$$

$$4 R0 \epsilon^2 \sqrt{R0^2 + \epsilon^2} \left(w^2 + 4 \epsilon^2\right) + R0^2 \left(w^4 + 8 w^2 \epsilon^2 + 24 \epsilon^4\right) - 4 R0^4 \left(-6 R0^2 - 3 \epsilon^2 + 4 R0 \sqrt{R0^2 + \epsilon^2}\right) -$$

$$\left. 4 R0^2 \left(-R0^4 - 3 \epsilon^4 + 4 R0^3 \sqrt{R0^2 + \epsilon^2} - 2 R0^2 \left(w^2 + 6 \epsilon^2\right) + R0 \sqrt{R0^2 + \epsilon^2} \left(w^2 + 8 \epsilon^2\right)\right)\right) -$$

$$\frac{1}{w^4 \left(R0^2 + \epsilon^2\right)^3} 4 A e^{-\frac{\left(R0 - \sqrt{R0^2 + \epsilon^2}\right)^2}{w^2}} \left(4 R0^8 - 2 R0^6 \left(-6 R0^2 + w^2 - 6 \epsilon^2 + 6 R0 \sqrt{R0^2 + \epsilon^2}\right) + \right.$$

$$w^2 \epsilon^2 \left(-2 R0^2 \epsilon^2 - 2 \epsilon^4 + R0 \sqrt{R0^2 + \epsilon^2} \left(w^2 + 4 \epsilon^2\right)\right) -$$

$$2 R0^4 \left(3 \epsilon^2 \left(w^2 - 2 \epsilon^2\right) + 2 R0^3 \sqrt{R0^2 + \epsilon^2} - 2 R0^2 \left(w^2 + 6 \epsilon^2\right) + R0 \sqrt{R0^2 + \epsilon^2} \left(w^2 + 12 \epsilon^2\right)\right) -$$

$$\left. 2 R0^2 \left(3 w^2 \epsilon^4 - 2 \epsilon^6 + 2 R0^3 \epsilon^2 \sqrt{R0^2 + \epsilon^2} - R0^2 \epsilon^2 \left(w^2 + 6 \epsilon^2\right) + R0 \sqrt{R0^2 + \epsilon^2} \left(w^4 - w^2 \epsilon^2 + 6 \epsilon^4\right)\right)\right) - 2 \lambda \right)$$

Asymptotic series at infinity: $e^{-\frac{r^2}{w^2}+\frac{2 R0 r}{w^2}+\frac{-R0^2-\epsilon^2}{w^2}+\frac{R0 \epsilon^2}{w^2 r}+O[\frac{1}{r}]^3}\left(-2 A+\frac{2 A R0}{r}+O[\frac{1}{r}]^3\right)\left(-\frac{2}{r^2}+O[\frac{1}{r}]^3\right)+$

$e^{-\frac{r^2}{w^2}+\frac{2 R0 r}{w^2}+\frac{-R0^2-\epsilon^2}{w^2}+\frac{R0 \epsilon^2}{w^2 r}+O[\frac{1}{r}]^3}\left(-4 A+\frac{4 A R0}{r}+O[\frac{1}{r}]^3\right)\left(-\lambda-\frac{2}{r^2}+O[\frac{1}{r}]^3\right)+e^{-\frac{r^2}{w^2}+\frac{2 R0 r}{w^2}+\frac{-R0^2-\epsilon^2}{w^2}+\frac{R0 \epsilon^2}{w^2 r}+O[\frac{1}{r}]^3}\left(-4 A+\frac{4 A R0}{r}+O[\frac{1}{r}]^3\right)+\left(-\frac{r^2}{w^2}+\frac{2 R0 r}{w^2}+\frac{-R0^2-\epsilon^2}{w^2}+\frac{R0 \epsilon^2}{w^2 r}+O[\frac{1}{r}]^3\right)$

$\left(-\frac{8 A r^2}{w^4}+\frac{24 A R0 r}{w^4}+\frac{4 A\left(-6 R0^2+w^2\right)}{w^4}+\frac{4 A R0\left(2 R0^2+w^2-3 \varepsilon^2\right)}{w^4 r}-\frac{8\left(A R0^2\left(w^2-3 \varepsilon^2\right)\right)}{w^4 r^2}+O[\frac{1}{r}]^3\right)+$

$e^{-\frac{r^2}{w^2}+\frac{2 R0 r}{w^2}+\frac{-R0^2-\epsilon^2}{w^2}+\frac{R0 \epsilon^2}{w^2 r}+O[\frac{1}{r}]^3}\left(-4 A+\frac{4 A R0}{r}+O[\frac{1}{r}]^3\right)+\left(-\frac{2 r^2}{w^2}+\frac{4 R0 r}{w^2}-\frac{2\left(R0^2+\epsilon^2\right)}{w^2}+\frac{2 R0 \epsilon^2}{w^2 r}+O[\frac{1}{r}]^3\right)$

$\left(-\frac{2 A^2 r^2}{w^4}+\frac{8 A^2 R0 r}{w^4}-\frac{12\left(A^2 R0^2\right)}{w^4}+\frac{2 A^2 R0\left(4 R0^2+w^2-2 \varepsilon^2\right)}{w^4 r}-\frac{2\left(A^2 R0^2\left(R0^2+2 w^2-6 \varepsilon^2\right)\right)}{w^4 r^2}+O[\frac{1}{r}]^3\right)+$

$e^{-\frac{r^2}{w^2}+\frac{2 R0 r}{w^2}+\frac{-R0^2-\epsilon^2}{w^2}+\frac{R0 \epsilon^2}{w^2 r}+O[\frac{1}{r}]^3}\left(-4 A+\frac{4 A R0}{r}+O[\frac{1}{r}]^3\right)+\left(-\frac{2 r^2}{w^2}+\frac{4 R0 r}{w^2}-\frac{2\left(R0^2+\epsilon^2\right)}{w^2}+\frac{2 R0 \epsilon^2}{w^2 r}+O[\frac{1}{r}]^3\right)$

$\left(\frac{16 A^2 r^2}{w^4}-\frac{64\left(A^2 R0\right) r}{w^4}+\frac{96 A^2 R0^2}{w^4}-\frac{16\left(A^2 R0\left(4 R0^2+w^2-2 \varepsilon^2\right)\right)}{w^4 r}+\frac{16 A^2 R0^2\left(R0^2+2 w^2-6 \varepsilon^2\right)}{w^4 r^2}+O[\frac{1}{r}]^3\right)$

Out[20]=

Lagrangian vs r (1 mm wormhole)