```
In[∘]:= (*********************************************************)
(*2. Scalar/Vector/Tensor sectors via NDEigensystem*)
(*********************************************************)
(*---Geometry& potential parameters (1 mm wormhole)---*)ClearAll["Global`*"];

(*Physical scale:take r in meters,with R0=1 mm*)
A = 0.01;         (*potential strength*)
R0 = 2.0      (*2m throat radius*)
w = 0.2 R0;       (*Gaussian width-adjust as needed*)
ε = 0.1 R0;    (*small regulator scale,r≥ϵ*)

(*Radial domain for the eigenproblem*)
rmin = ε;
rmax = 10 R0;
nModes = 6;

(*---Scalar potential Φ(r)---*)

(*3D definition (for reference,not used directly in 1D problem):r[x_,y_,z_]:=
 Sqrt[x^2+y^2+z^2];
Φ3D[x_,y_,z_]:=Module[{rr=Max[r[x,y,z],ϵ]},-A (1-R0/rr) Exp[-(rr-R0)^2/w^2]];*)

(*1D radial restriction:r≥rmin≥ϵ,so no explicit Max needed*)
Φ[r_] := -A (1 - R0 / r) Exp[-(r - R0) ^2 / w^2];

(*This Φ(r) is the same one that appears in the metric:ds^2=
 -Exp[2 Φ[r]] dt^2+Exp[-2 Φ[r]] dr^2+r^2 dΩ^2*)

(*---2a.Scalar sector---*)

potentialS[r_] := Φ[r];

Ls = -D[ψs[r], {r, 2}] + potentialS[r] × ψs[r];
bcS = DirichletCondition[ψs[r] == 0, r == rmin || r == rmax];

{valsS, funsS} = NDEigensystem[{Ls, bcS}, ψs, {r, rmin, rmax}, nModes];

Print["\nScalar ω^2 = ", valsS // N];
If[AllTrue[valsS, # > 0 &],
  Print["☑  Scalar sector stable"], Print["✖  Scalar instability"]];

(*---3. Vector/Tensor sectors---*)

(*Numeric first& second derivatives of Φ*)
dΦ = Function[{x}, Evaluate[D[Φ[r], r] /. r → x]];
```

```
d2Φ = Function[{x}, Evaluate[D[Φ[r], {r, 2}] /. r → x]];

(*Illustrative effective potentials for perturbations*)
Vv[r_] := -d2Φ[r] + dΦ[r] / r; (*vector potential*)
Vt[r_] := d2Φ[r] + dΦ[r] / r; (*tensor potential*)

Lv = -D[ψv[r], {r, 2}] + Vv[r] × ψv[r];
Lt = -D[ψt[r], {r, 2}] + Vt[r] × ψt[r];

bcV = DirichletCondition[ψv[r] == 0, r == rmin || r == rmax];
bcT = DirichletCondition[ψt[r] == 0, r == rmin || r == rmax];

{valsV, funsV} = NDEigensystem[{Lv, bcV}, ψv, {r, rmin, rmax}, nModes];
{valsT, funsT} = NDEigensystem[{Lt, bcT}, ψt, {r, rmin, rmax}, nModes];

Print["\nVector ω^2 = ", valsV // N];
Print["Tensor ω^2 = ", valsT // N];

If[AllTrue[valsV, # > 0 &],
  Print["☑  Vector sector stable"], Print["✖  Vector instability"]];
If[AllTrue[valsT, # > 0 &],
  Print["☑  Tensor sector stable"], Print["✖  Tensor instability"]];

(*******************************************************)
(*4. Plots:scalar,vector,tensor eigenfunctions*)
(*******************************************************)

(*---Scalar eigenfunctions---*)
Plot[Evaluate@Table[funsS[[k]][r], {k, nModes}],
 {r, rmin, rmax}, PlotLabel → "Scalar eigenfunctions psi_s[n](r)",
 PlotLegends → Table["n = " <> ToString[k], {k, nModes}], AxesLabel → {"r", "psi_s[n]"}]

(*---Vector eigenfunctions---*)
Plot[Evaluate@Table[funsV[[k]][r], {k, nModes}],
 {r, rmin, rmax}, PlotLabel → "Vector eigenfunctions psi_v[n](r)",
 PlotLegends → Table["n = " <> ToString[k], {k, nModes}], AxesLabel → {"r", "psi_v[n]"}]

(*---Tensor eigenfunctions---*)
Plot[Evaluate@Table[funsT[[k]][r], {k, nModes}],
 {r, rmin, rmax}, PlotLabel → "Tensor eigenfunctions psi_t[n](r)",
 PlotLegends → Table["n = " <> ToString[k], {k, nModes}], AxesLabel → {"r", "psi_t[n]"}]
```

Out[∘]=

```
2.
```

Scalar ω^2 = {0.0251737, 0.100697, 0.226585, 0.402884, 0.629705, 0.90727}

☑ Scalar sector stable

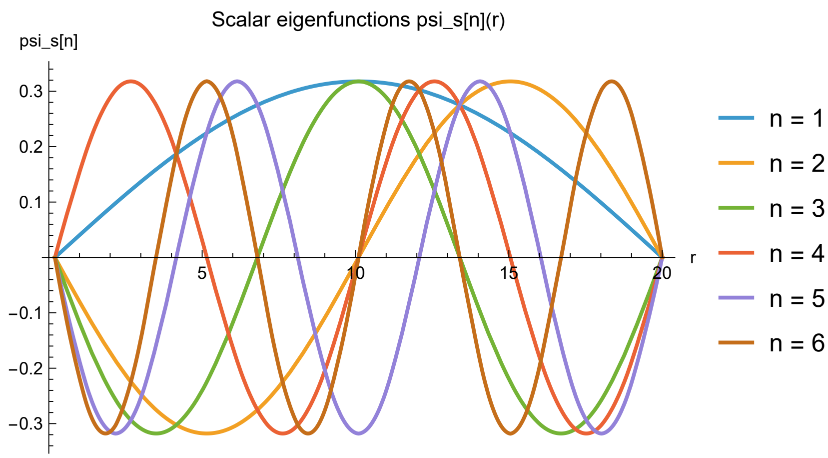
Vector ω^2 = {0.025171, 0.100685, 0.226557, 0.402844, 0.629679, 0.907308}

Tensor ω^2 = {0.0251787, 0.100716, 0.226617, 0.402915, 0.6297, 0.90718}

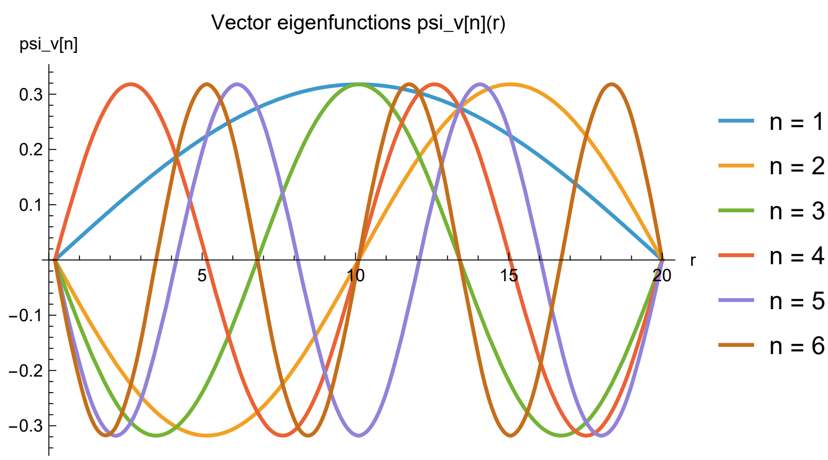
☑ Vector sector stable

☑ Tensor sector stable

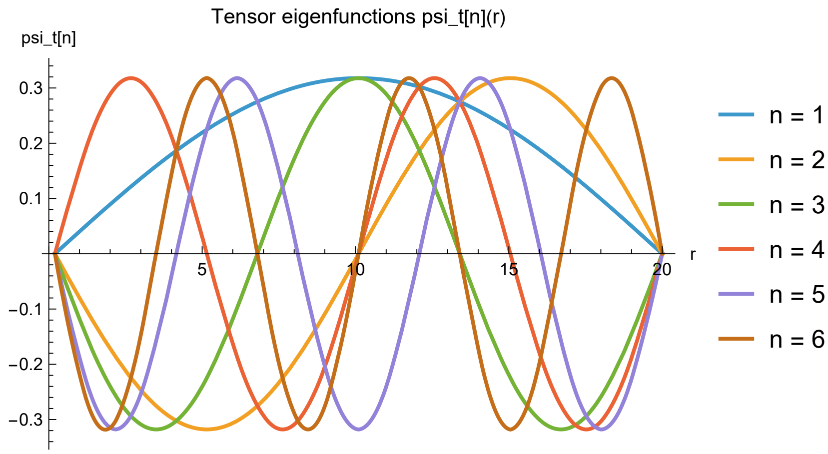
Out[#]=



Out[#]=



Out[#]=


Tensor eigenfunctions psi_t[n](r)
psi_t[n]
0.3
0.2
0.1
−0.1
−0.2
−0.3
5
10
15
20
r
n = 1
n = 2
n = 3
n = 4
n = 5
n = 6