

```

In[=]:= (* =====*)
(*CORRECT FLARE-OUT TEST FOR YOUR EXACT METRIC*)
(* =====*)ClearAll["Global`*"];

(*---Your exact parameters---*)
R0 = 2.0
w = 0.2
A = 0.01
ε = 0.04

(*---Your exact potential---*)
rCoord[x_, y_, z_] := Max[Sqrt[x^2 + y^2 + z^2], ε];
Φ[r_] := -A (1 - R0 / r) Exp[-((r - R0)^2) / w^2];

(*---Your exact metric---*)
(*ds²=-e^{2Φ(r)}dt²+e^{-2Φ(r)}(dr²+r²dΩ²)*)

(*---Areal radius---*)
Rareal[r_] := Exp[-Φ[r]] * r;

(*---Flare-out condition---*)
dRareal = D[Rareal[r], r] /. r → R0 // N;

Print["\n--- Flare-Out Test for YOUR Exact Metric ---"];
Print["Metric: ds² = -e^{2Φ}dt² + e^{-2Φ}(dr² + r²dΩ²)"];
Print["Potential: Φ(r) = -A(1-R0/r)Exp[-(r-R0)²/w²]"];
Print["Parameters: R0 = ", R0, ", A = ", A, ", w = ", w];
Print["Areal radius R_areal(r) = e^{-Φ(r)} r"];
Print["dR_areal/dr at r = R0 = ", dRareal];

If[dRareal > 0, Print["☒ FLARE-OUT SATISFIED at throat."],
Print["☒ FLARE-OUT VIOLATED at throat."]];

(*---Detailed analysis at throat---*)
Print["\n--- Detailed Throat Analysis ---"];
Print["Φ(R0) = ", Φ[R0] // N];
Print["Φ'(R0) = ", D[Φ[r], r] /. r → R0 // N];
Print["e^{-Φ(R0)} = ", Exp[-Φ[R0]] // N];
Print["Flare factor = 1 - R0*Φ'(R0) = ", 1 - R0 * (D[Φ[r], r] /. r → R0) // N];

Out[=]=
2.

Out[=]=
0.2

Out[=]=
0.01

```

```

Out[=]=
0.04

--- Flare-Out Test for YOUR Exact Metric ---
Metric:  $ds^2 = -e^{2\Phi} dt^2 + e^{-2\Phi} (dr^2 + r^2 d\Omega^2)$ 
Potential:  $\Phi(r) = -A(1-R0/r) \text{Exp}[-(r-R0)^2/w^2]$ 
Parameters:  $R0 = 2.$ ,  $A = 0.01$ ,  $w = 0.2$ 
Areal radius  $R_{\text{areal}}(r) = e^{-\Phi(r)} r$ 
 $dR_{\text{areal}}/dr$  at  $r = R0 = 1.01$ 
 FLARE-OUT SATISFIED at throat.

--- Detailed Throat Analysis ---
 $\Phi(R0) = 0.$ 
 $\Phi'(R0) = -0.005$ 
 $e^{-\Phi(R0)} = 1.$ 
Flare factor =  $1 - R0 * \Phi'(R0) = 1.01$ 

```