```
In[•]:= (* ::Title::*)(*Two-Sided Wormhole:NEC at Throat+Geodesic Across Universes*)
ClearAll["Global`*"];

(*============================================================*)
(*Parameters scaled for a 2 meter radial throat*)
(*============================================================*)

R0 = 2.0
w = 0.2
A = 0.01
ε = 0.04

(*Proper radial coordinate*)
r[l_] := Sqrt[l^2 + R0^2];

(*Shape function (same formula,different scale)*)
ϕ[l_] := -A (1 - R0 / r[l]) Exp[-(r[l] - R0)^2 / w^2];

(*============================================================*)
(*Metric in (t,l,ϴ,φ) coordinates*)
(*============================================================*)

coords = {t, l, th, ph};

metric = DiagonalMatrix[
   {-Exp[2 ϕ[l]], Exp[-2 ϕ[l]], Exp[-2 ϕ[l]] × r[l]^2, Exp[-2 ϕ[l]] × r[l]^2 × Sin[th]^2}];

Print["Metric ready for R0 = 2 m"];

(*============================================================*)
(*Check asymptotic flatness*)
(*============================================================*)

limitp = Limit[metric[[1, 1]], l → Infinity];
limitm = Limit[metric[[1, 1]], l → -Infinity];

Print["Asymptotic g_tt as l → +∞: ", limitp // N];
Print["Asymptotic g_tt as l → -∞: ", limitm // N];

(*============================================================*)
(*NEC at the throat (l=0)*)
(*============================================================*)

Phi0 = ϕ[0];
Phi1 = D[ϕ[l], l] /. l → 0;
Phi2 = D[ϕ[l], {l, 2}] /. l → 0;
```

```
NECThroat = ((-Phi1^2 R0^2 + Phi2 R0^2 - 1) Exp[4 Phi0]) / (4 Pi R0^2) // N;

Print["Φ(0)   = ", Phi0 // N];
Print["Φ'(0)  = ", Phi1 // N];
Print["Φ''(0) = ", Phi2 // N];
Print["NEC at throat (l = 0): ", NECThroat];

(*============================================================*)
(*Radial timelike geodesic across throat*)
(*============================================================*)

Clear[s, ell, τ];

Lgeo = -Exp[2 ϕ[ell[s]]] × τ'[s]^2 + Exp[-2 ϕ[ell[s]]] × ell'[s]^2;

eqTau = D[D[Lgeo, τ'[s]], s] - D[Lgeo, τ[s]] == 0;
eqEll = D[D[Lgeo, ell'[s]], s] - D[Lgeo, ell[s]] == 0;

(*Start 5 throat radii on left side:l=-10 m*)
geoIC = {τ[0] == 0, ell[0] == -5 R0, (*Start at l=-10 m*)τ'[0] == 1, ell'[0] == 1};

geoSol = NDSolve[{eqTau, eqEll} ~Join~ geoIC, {τ, ell}, {s, 0, 200},
    (*must increase domain because distances are larger*)MaxStepFraction → 1 / 200];

Print["Sample values of l(s):"];
Print[Table[{ss, ell[ss] /. geoSol}, {ss, 0, 200, 40}]];

ParametricPlot[Evaluate[{s, ell[s]} /. geoSol], {s, 0, 200},
 AxesLabel → {"s (affine parameter)", "l"}, PlotRange → All,
 PlotLabel → "Radial Timelike Geodesic Through 2 m Wormhole"]
```

Out[•]=

```
2.
```

Out[•]=

```
0.2
```

Out[•]=

```
0.01
```

Out[•]=

```
0.04
```

```
Metric ready for R0 = 2 m
Asymptotic g_tt as l → +∞: -1.
Asymptotic g_tt as l → -∞: -1.
Φ(0)   = 0.
Φ'(0)  = 0.
Φ''(0) = -0.0025
NEC at throat (l = 0): -0.0200933
Sample values of l(s):
{{0, {-10.}}, {40, {30.}}, {80, {70.}}, {120, {110.}}, {160, {150.}}, {200, {190.}}}
```

Out[•]=