```
In[•]:= (* ======================================================*) (*PARAMETERS*)
(* ======================================================*)ClearAll["Global`*"];

d = 0.06;
R0 = 0.03;
omega = 2 Pi 5.0;

bBI = 1.0;

bestPars =
  {1.97001, 0.685107, 4.05734, 0.98985, 0.753244, 0.00139359, -0.902839, 3.60618 * 10^-9};

{A1, A2, phi, m1, m2, epsY, chir1, chir2} = bestPars;

rho0 = 10 * 10^-3;
betaR0 = 0.3;
betaT0 = 0.3;
gammaR = -50.0;  (*much stronger radial tension from E^2*)
gammaT = 50.0;  (*much stronger tangential pressure from E^2*)
wShell = 0.01;

t0 = 0.0;

(* ======================================================*)
(*NUMERIC-SAFE HELPERS*)
(* ======================================================*)

rPlane[x_?NumericQ, y_?NumericQ] := Sqrt[x^2 + y^2];

(* ======================================================*)
(*VORTEX FIELDS*)
(* ======================================================*)

eOneNum[x_?NumericQ, y_?NumericQ, t_?NumericQ, cx_?NumericQ,
   cy_?NumericQ, A_?NumericQ, phase_?NumericQ, mix_?NumericQ] :=
  Module[{rx = x - cx, ry = y - cy, rr, g, er, et}, rr = Sqrt[rx^2 + ry^2];
   If[rr < 10^-12, {0., 0., 0.}, g = Exp[-(rx^2 + ry^2) / R0^2];
    er = {rx / rr, ry / rr, 0.};
    et = {-ry / rr, rx / rr, 0.};
    A * g * Cos[omega * t + phase] * ((1 - mix) * er + mix * et)]];

bOneNum[x_?NumericQ, y_?NumericQ, t_?NumericQ, cx_?NumericQ,
   cy_?NumericQ, A_?NumericQ, phase_?NumericQ, chir_?NumericQ] :=
  Module[{rx = x - cx, ry = y - cy, rr, g, et}, rr = Sqrt[rx^2 + ry^2];
   If[rr < 10^-12, {0., 0., 0.}, g = Exp[-(rx^2 + ry^2) / R0^2];
    et = {-ry / rr, rx / rr, 0.};
```

```
    A * g * Sin[omega * t + phase] * chir * et]];

eFieldNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
  Module[{c1 = {-d / 2, 0}, c2 = {d / 2, epsY}, e1, e2},
   e1 = eOneNum[x, y, t, c1[[1]], c1[[2]], A1, 0., m1];
   e2 = eOneNum[x, y, t, c2[[1]], c2[[2]], A2, phi, m2];
   e1 + e2];

bFieldNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
  Module[{c1 = {-d / 2, 0}, c2 = {d / 2, epsY}, b1, b2},
   b1 = bOneNum[x, y, t, c1[[1]], c1[[2]], A1, 0., chir1];
   b2 = bOneNum[x, y, t, c2[[1]], c2[[2]], A2, phi, chir2];
   b1 + b2];

(* ========================================================*)
(*BORN-INFELD INVARIANTS*)
(* ========================================================*)

FInvNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
  Module[{E = eFieldNum[x, y, t], B = bFieldNum[x, y, t]}, 0.5 * ((B.B) - (E.E))];

SNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] := 1 + (2 * FInvNum[x, y, t]) / bBI^2;

LFNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] := -1 / Sqrt[SNum[x, y, t]];

DFieldNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
  Module[{E = eFieldNum[x, y, t], lf = LFNum[x, y, t]}, -lf * E];

HFieldNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
  Module[{B = bFieldNum[x, y, t], lf = LFNum[x, y, t]}, -lf * B];

(* ========================================================*)
(*STRESS-ENERGY TENSOR (Born-Infeld)*)
(* ========================================================*)

stressEnergyBINum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
  Module[{E = eFieldNum[x, y, t], B = bFieldNum[x, y, t], D, H, L, u, Sflow, Tij},
   D = DFieldNum[x, y, t];
   H = HFieldNum[x, y, t];
   L = bBI^2 (1 - Sqrt[SNum[x, y, t]]);
   u = E.D - L;
   Sflow = Cross[E, H];
   Tij = ConstantArray[0., {4, 4}];
   Tij[[1, 1]] = u;
   Tij[[1, 2 ;; 4]] = Sflow;
   Tij[[2 ;; 4, 1]] = Sflow;
   Do[Do[Tij[[i + 1, j + 1]] = -E[[i]] * D[[j]] - B[[i]] * H[[j]] + If[i == j, E.D + B.H - L, 0], {j, 1, 3}],
```

```
    {i, 1, 3}];
   Tij];

(* ====================================================*)
(*ANISOTROPIC SHELL*)
(* ====================================================*)

shellProfile[r_] := Exp[-(r - R0)^2 / wShell^2];

rhoShell[x_?NumericQ, y_?NumericQ, t_?NumericQ] := rho0 * shellProfile[rPlane[x, y]];

E2Num[x_?NumericQ, y_?NumericQ, t_?NumericQ] := Module[{E = eFieldNum[x, y, t]}, E.E];

betaR[x_?NumericQ, y_?NumericQ, t_?NumericQ] := betaR0 + gammaR * E2Num[x, y, t];

betaT[x_?NumericQ, y_?NumericQ, t_?NumericQ] := betaT0 + gammaT * E2Num[x, y, t];

pRShell[x_?NumericQ, y_?NumericQ, t_?NumericQ] := betaR[x, y, t] * rhoShell[x, y, t];

pTShell[x_?NumericQ, y_?NumericQ, t_?NumericQ] := betaT[x, y, t] * rhoShell[x, y, t];

shellStressTensor[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
  Module[{rho = rhoShell[x, y, t], pr = pRShell[x, y, t], pt = pTShell[x, y, t]},
   {{rho, 0, 0, 0}, {0, pr, 0, 0}, {0, 0, pt, 0}, {0, 0, 0, pt}}];

(* ====================================================*)
(*ENERGY CONDITIONS HELPER*)
(* ====================================================*)

energyConditionsTensor[T_] := Module[{rho, spatial, evals}, rho = T[[1, 1]];
   spatial = (T[[2 ;; 4, 2 ;; 4]] + Transpose[T[[2 ;; 4, 2 ;; 4]]]) / 2;
   evals = Eigenvalues[spatial];
   <|"rho" → rho, "p1" → evals[[1]], "p2" → evals[[2]], "p3" → evals[[3]],
    "NEC" → And @@ (rho + # ≥ 0 & /@ evals), "WEC" → (rho ≥ 0 && And @@ (rho + # ≥ 0 & /@ evals)),
    "SEC" → (And @@ (rho + # ≥ 0 & /@ evals) && rho + Total[evals] ≥ 0),
    "DEC" → (rho ≥ 0 && And @@ (rho ≥ Abs[#] & /@ evals))|>];

shellEC[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
  energyConditionsTensor[shellStressTensor[x, y, t]];

biEC[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
  energyConditionsTensor[stressEnergyBINum[x, y, t]];

totalEC[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
  energyConditionsTensor[stressEnergyBINum[x, y, t] + shellStressTensor[x, y, t]];

(* ====================================================*)
```

```
(*SAMPLE OUTPUT*)
(* ===================================================*)

pt = {0., 0.002, t0};

Print["--- At point ", pt[[{1, 2}]], " ---"];
Print["E: ", eFieldNum @@ pt];
Print["B: ", bFieldNum @@ pt];
Print["Shell rho, pr, pt: ", {rhoShell @@ pt, pRShell @@ pt, pTShell @@ pt}];
Print["BI EC: ", biEC @@ pt];
Print["Shell EC: ", shellEC @@ pt];
Print["Total EC: ", totalEC @@ pt];

(* ================================================*)
(*DIAGNOSTIC FIELDS AT t=t0*)
(* ================================================*)

uBI[x_?NumericQ, y_?NumericQ] := stressEnergyBINum[x, y, t0][[1, 1]];
uShell[x_?NumericQ, y_?NumericQ] := rhoShell[x, y, t0];
uTot[x_?NumericQ, y_?NumericQ] := uBI[x, y] + uShell[x, y];

deltaPShell[x_?NumericQ, y_?NumericQ] := pTShell[x, y, t0] - pRShell[x, y, t0];

E2[x_?NumericQ, y_?NumericQ] := E2Num[x, y, t0];

(* ================================================*)
(*PLOTS*)
(* ================================================*)

range = 0.04;  (*adjust if you want bigger view*)

Print["Plot: BI energy density u_BI(x,y)"];
biEnergyPlot = DensityPlot[uBI[x, y], {x, -range, range},
   {y, -range, range}, PlotPoints → 40, ColorFunction → "ThermometerColors",
   FrameLabel → {"x", "y"}, PlotLabel → "Born-Infeld energy density u_BI"];

Print["Plot: shell anisotropy Δp = p_t - p_r"];
shellAnisoPlot = DensityPlot[deltaPShell[x, y], {x, -range, range},
   {y, -range, range}, PlotPoints → 40, ColorFunction → "SolarColors",
   FrameLabel → {"x", "y"}, PlotLabel → "Shell anisotropy Δp = p_t - p_r"];

Print["Plot: E^2 field intensity"];
e2Plot = DensityPlot[E2[x, y], {x, -range, range},
   {y, -range, range}, PlotPoints → 40, ColorFunction → "AvocadoColors",
   FrameLabel → {"x", "y"}, PlotLabel → "E^2(x,y) of vortex"];

(* ================================================*)
```

```
(*POYNTING VECTOR SAMPLING ALONG A LINE*)
(* ===================================================*)

samplePoynting = Table[Module[
    {x = 0., yy = y, E = eFieldNum[0., y, t0], H = HFieldNum[0., y, t0], S}, S = Cross[E, H];
    {yy, S}], {y, -range, range, range / 8.}];

Print["Sample Poynting vectors S(y) along x = 0:"];
samplePoynting

(* ========================================================*)
(*RADIAL SCAN OF SHELL+TOTAL PROPERTIES*)
(* ========================================================*)

rMin = 0.0;
rMax = 0.06;
pts = 200;

radialData = Table[Module[{x = r, y = 0, Tshell, Ttot, rhoS, prS, ptS, necS,
     rhoT, prT, ptT, necT, spatT, vals}, Tshell = shellStressTensor[x, y, t0];
    Ttot = stressEnergyBINum[x, y, t0] + Tshell;
    rhoS = Tshell[[1, 1]];
    prS = Tshell[[2, 2]];
    ptS = Tshell[[3, 3]]; (*same as[[4,4]]*) rhoT = Ttot[[1, 1]];
    (*diagonalize spatial part to get principal pressures for total*)
    spatT = (Ttot[[2 ;; 4, 2 ;; 4]] + Transpose[Ttot[[2 ;; 4, 2 ;; 4]]]) / 2;
    vals = Eigenvalues[spatT];
    prT = vals[[1]];
    ptT = vals[[2]];
    necS = If[(rhoS + prS ≥ 0) && (rhoS + ptS ≥ 0), 1, 0];
    necT = If[(rhoT + prT ≥ 0) && (rhoT + ptT ≥ 0), 1, 0];
    {r, rhoS, prS, ptS, necS, rhoT, prT, ptT, necT}], {r, rMin, rMax, (rMax - rMin) / pts}];

radialR = radialData[[All, 1]];
rhoShellR = radialData[[All, 2]];
prShellR = radialData[[All, 3]];
ptShellR = radialData[[All, 4]];
necShellR = radialData[[All, 5]];
rhoTotR = radialData[[All, 6]];
prTotR = radialData[[All, 7]];
ptTotR = radialData[[All, 8]];
necTotR = radialData[[All, 9]];

(* ========================================================*)
(*PLOTS:SHELL ONLY*)
(* ========================================================*)
```

```
Print["Radial profiles (shell): rho_s(r), p_r^s(r), p_t^s(r)"];

rhoShellPlot = ListLinePlot[Transpose[{radialR, rhoShellR}], PlotStyle → Thick, Frame → True,
   FrameLabel → {"r", "rho_s"}, PlotLabel → "Shell energy density rho_s(r)"];

prShellPlot =
  ListLinePlot[Transpose[{radialR, prShellR}], PlotStyle → {Red, Thick}, Frame → True,
   FrameLabel → {"r", "p_r^s"}, PlotLabel → "Shell radial pressure p_r^s(r)"];

ptShellPlot =
  ListLinePlot[Transpose[{radialR, ptShellR}], PlotStyle → {Blue, Thick}, Frame → True,
   FrameLabel → {"r", "p_t^s"}, PlotLabel → "Shell tangential pressure p_t^s(r)"];

Print["Plot: shell rho_s(r)"]; rhoShellPlot
Print["Plot: shell p_r^s(r)"]; prShellPlot
Print["Plot: shell p_t^s(r)"]; ptShellPlot

Print["Plot: Shell NEC_s(r) (1 = satisfied, 0 = violated)"];

necShellPlot = ListStepPlot[Transpose[{radialR, necShellR}],
   Frame → True, FrameLabel → {"r", "NEC_s"}, PlotRange → {0, 1.2},
   PlotLabel → "Shell NEC_s(r): 1 = NEC OK, 0 = NEC violated", Filling → Axis];

necShellPlot

(* ======================================================*)
(*PLOTS:TOTAL (BI+SHELL)*)
(* ======================================================*)

Print["Radial profiles (total): rho_tot(r), p_r^tot(r), p_t^tot(r)"];

rhoTotPlot = ListLinePlot[Transpose[{radialR, rhoTotR}], PlotStyle → Thick, Frame → True,
   FrameLabel → {"r", "rho_tot"}, PlotLabel → "Total energy density rho_tot(r)"];

prTotPlot =
  ListLinePlot[Transpose[{radialR, prTotR}], PlotStyle → {Red, Thick}, Frame → True,
   FrameLabel → {"r", "p_r^tot"}, PlotLabel → "Total radial pressure p_r^tot(r)"];

ptTotPlot =
  ListLinePlot[Transpose[{radialR, ptTotR}], PlotStyle → {Blue, Thick}, Frame → True,
   FrameLabel → {"r", "p_t^tot"}, PlotLabel → "Total tangential pressure p_t^tot(r)"];

Print["Plot: total rho_tot(r)"]; rhoTotPlot
Print["Plot: total p_r^tot(r)"]; prTotPlot
Print["Plot: total p_t^tot(r)"]; ptTotPlot

Print["Plot: NEC_tot(r) (1 = satisfied, 0 = violated)"];
```

```
necTotPlot = ListStepPlot[Transpose[{radialR, necTotR}],
   Frame → True, FrameLabel → {"r", "NEC_tot"}, PlotRange → {0, 1.2},
   PlotLabel → "Total NEC_tot(r): 1 = NEC OK, 0 = NEC violated", Filling → Axis];

necTotPlot
```

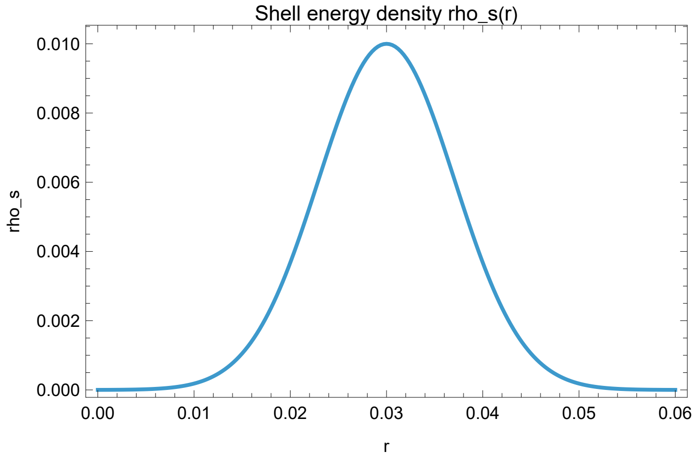
```
--- At point {0., 0.002} ---

E: {1.16388×10^-7, 0.827911, 0.}

B: {1.45604×10^-11, 7.20325×10^-10, 0.}

Shell rho, pr, pt: {3.93669×10^-6, -0.000133737, 0.000136099}

BI EC: <|rho → 0.78298, p1 → 0.78298, p2 → 0.78298,
  p3 → -0.439141, NEC → True, WEC → True, SEC → True, DEC → True|>

Shell EC: <|rho → 3.93669×10^-6, p1 → 0.000136099, p2 → 0.000136099,
  p3 → -0.000133737, NEC → False, WEC → False, SEC → False, DEC → False|>

Total EC: <|rho → 0.782984, p1 → 0.783116, p2 → 0.782846,
  p3 → -0.439005, NEC → True, WEC → True, SEC → True, DEC → False|>

Plot: BI energy density u_BI(x,y)

Plot: shell anisotropy Δp = p_t - p_r

Plot: E^2 field intensity

Sample Poynting vectors S(y) along x = 0:
```

Out[•]=

```
{{-0.04, {0., 0., 1.31047×10^-11}},
 {-0.035, {0., 0., 3.19406×10^-11}}, {-0.03, {0., 0., 6.93916×10^-11}},
 {-0.025, {0., 0., 1.3406×10^-10}}, {-0.02, {0., 0., 2.28765×10^-10}},
 {-0.015, {0., 0., 3.38465×10^-10}}, {-0.01, {0., 0., 4.12378×10^-10}},
 {-0.005, {0., 0., 3.53788×10^-10}}, {0., {0., 0., 1.02628×10^-10}},
 {0.005, {0., 0., -1.79819×10^-10}}, {0.01, {0., 0., -3.07091×10^-10}},
 {0.015, {0., 0., -2.9359×10^-10}}, {0.02, {0., 0., -2.18166×10^-10}},
 {0.025, {0., 0., -1.36823×10^-10}}, {0.03, {0., 0., -7.46112×10^-11}},
 {0.035, {0., 0., -3.58245×10^-11}}, {0.04, {0., 0., -1.52351×10^-11}}}
```

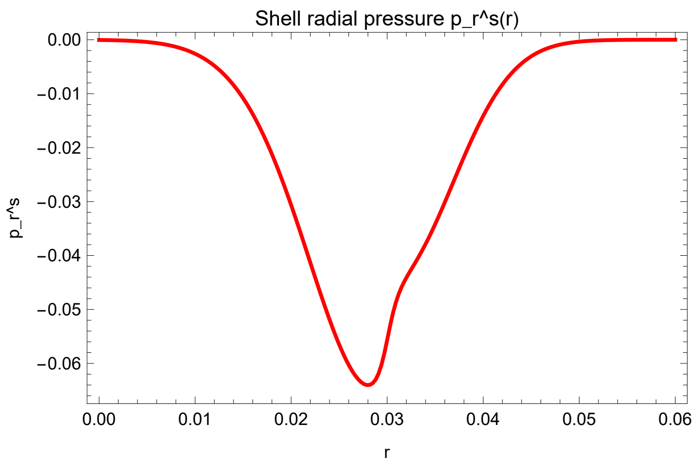
```
Radial profiles (shell): rho_s(r), p_r^s(r), p_t^s(r)

Plot: shell rho_s(r)
```

Out[*]=



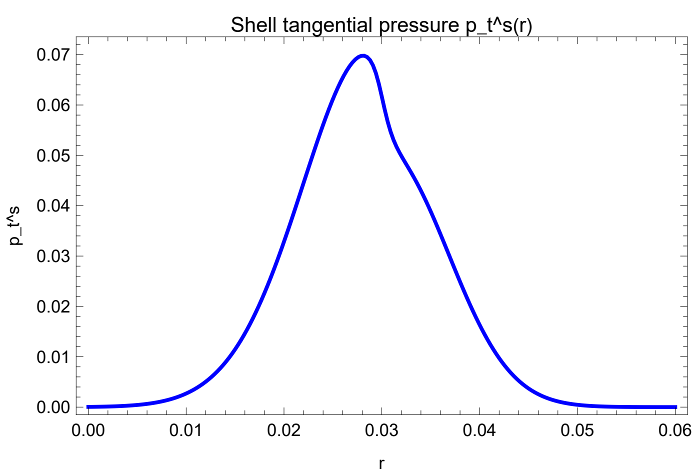
Plot: shell p_r^s(r)

Out[*]=



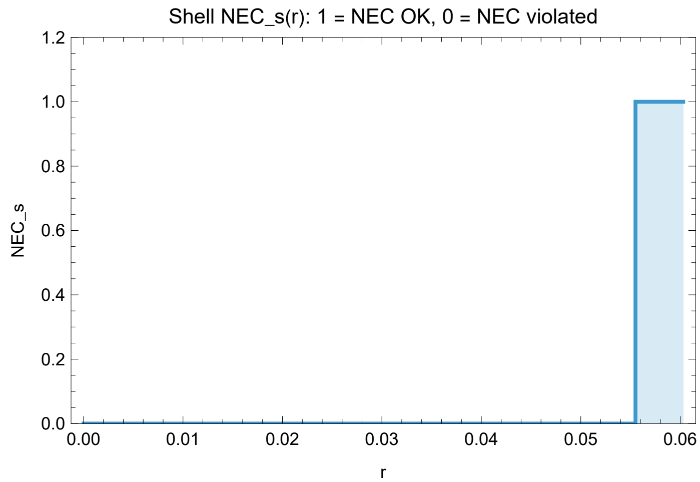
Plot: shell p_t^s(r)

Out[*]=



Plot: Shell NEC_s(r) (1 = satisfied, 0 = violated)

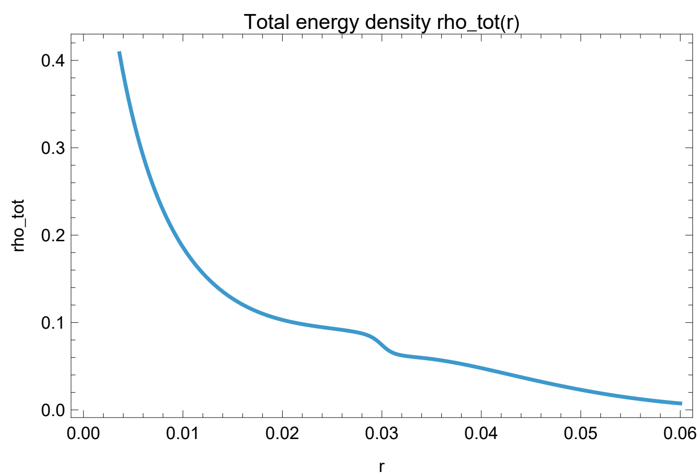
Out[ ]=



Radial profiles (total): rho_tot(r), p_r^tot(r), p_t^tot(r)

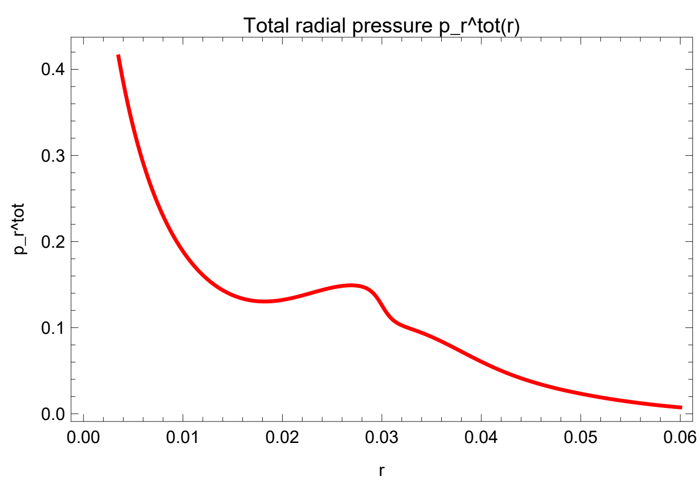
Plot: total rho_tot(r)

Out[ ]=



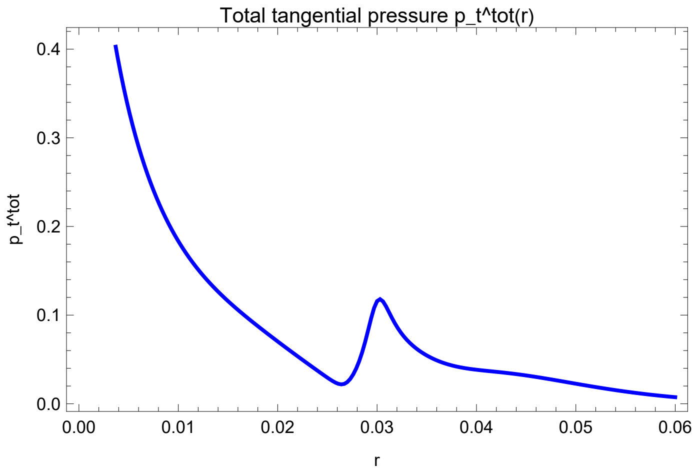
Plot: total p_r^tot(r)

Out[ ]=



```
Plot: total p_t^tot(r)
```

Out[#]=




```
Plot: NEC_tot(r) (1 = satisfied, 0 = violated)
```

Out[#]=