In[391]:=

```
(* ================================================== *)
(* PARAMETERS *)
(* ================================================== *)

ClearAll["Global`*"];



d   = 0.06;
R0  = 0.03;
omega = 2 Pi 5.0;

bBI  = 1.0;

bestPars = {
 1.97001, 0.685107, 4.05734, 0.98985,
 0.753244, 0.00139359, -0.902839, 3.60618*10^-9
};

{A1, A2, phi, m1, m2, epsY, chir1, chir2} = bestPars;

rho0  = 1.0;
betaR0 = 0.3;
betaT0 = 0.3;
gammaR = -50.0; (* much stronger radial tension from E^2 *)
gammaT =  50.0; (* much stronger tangential pressure from E^2 *)
wShell = 0.01;


t0 = 0.0;

(* ================================================== *)
(* NUMERIC-SAFE HELPERS *)
(* ================================================== *)

rPlane[x_?NumericQ, y_?NumericQ] := Sqrt[x^2 + y^2];

(* ================================================== *)
(* VORTEX FIELDS *)
(* ================================================== *)

eOneNum[
  x_?NumericQ, y_?NumericQ, t_?NumericQ,
  cx_?NumericQ, cy_?NumericQ, A_?NumericQ,
```

```
 phase_?NumericQ, mix_?NumericQ
] :=
Module[{rx = x - cx, ry = y - cy, rr, g, er, et},
 rr = Sqrt[rx^2 + ry^2];
 If[rr < 10^-12,
  {0., 0., 0.},
  g  = Exp[-(rx^2 + ry^2)/R0^2];
  er = {rx/rr, ry/rr, 0.};
  et = {-ry/rr, rx/rr, 0.};
  A*g*Cos[omega*t + phase]*((1 - mix)*er + mix*et)
 ]
];

bOneNum[
  x_?NumericQ, y_?NumericQ, t_?NumericQ,
  cx_?NumericQ, cy_?NumericQ, A_?NumericQ,
  phase_?NumericQ, chir_?NumericQ
] :=
Module[{rx = x - cx, ry = y - cy, rr, g, et},
 rr = Sqrt[rx^2 + ry^2];
 If[rr < 10^-12,
  {0., 0., 0.},
  g  = Exp[-(rx^2 + ry^2)/R0^2];
  et = {-ry/rr, rx/rr, 0.};
  A*g*Sin[omega*t + phase]*chir*et
 ]
];

eFieldNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
Module[{c1 = {-d/2, 0}, c2 = {d/2, epsY}, e1, e2},
 e1 = eOneNum[x, y, t, c1[[1]], c1[[2]], A1, 0.,  m1];
 e2 = eOneNum[x, y, t, c2[[1]], c2[[2]], A2, phi, m2];
 e1 + e2
];

bFieldNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
Module[{c1 = {-d/2, 0}, c2 = {d/2, epsY}, b1, b2},
 b1 = bOneNum[x, y, t, c1[[1]], c1[[2]], A1, 0.,  chir1];
 b2 = bOneNum[x, y, t, c2[[1]], c2[[2]], A2, phi, chir2];
```

```
 b1 + b2
];

(* ==================================================== *)
(* BORN-INFELD INVARIANTS *)
(* ==================================================== *)

FInvNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 Module[{E = eFieldNum[x, y, t], B = bFieldNum[x, y, t]},
  0.5*((B.B) - (E.E))
 ];

SNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 1 + (2*FInvNum[x, y, t])/bBI^2;

LFNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 -1/Sqrt[SNum[x, y, t]];

DFieldNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 Module[{E = eFieldNum[x, y, t], lf = LFNum[x, y, t]},
  -lf*E
 ];

HFieldNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 Module[{B = bFieldNum[x, y, t], lf = LFNum[x, y, t]},
  -lf*B
 ];

(* ==================================================== *)
(* STRESS-ENERGY TENSOR *)
(* ==================================================== *)

stressEnergyBINum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 Module[{E = eFieldNum[x, y, t], B = bFieldNum[x, y, t],
    D, H, L, u, Sflow, Tij},

  D = DFieldNum[x, y, t];
  H = HFieldNum[x, y, t];

  L = bBI^2 (1 - Sqrt[SNum[x, y, t]]);
  u = E.D - L;
```

```
 Sflow = Cross[E, H];

 Tij = ConstantArray[0., {4, 4}];

 Tij[[1, 1]]  = u;
 Tij[[1, 2 ;; 4]] = Sflow;
 Tij[[2 ;; 4, 1]] = Sflow;

 Do[
  Do[
   Tij[[i + 1, j + 1]] =
    -E[[i]]*D[[j]] - B[[i]]*H[[j]] +
    If[i == j, E.D + B.H - L, 0],
   {j, 1, 3}],
  {i, 1, 3}
 ];

 Tij
];

(* ==================================================== *)
(* ANISOTROPIC SHELL *)
(* ==================================================== *)

shellProfile[r_] := Exp[-(r - R0)^2/wShell^2];

rhoShell[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 rho0*shellProfile[rPlane[x, y]];

E2Num[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 Module[{E = eFieldNum[x, y, t]},
  E.E
 ];

betaR[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 betaR0 + gammaR*E2Num[x, y, t];

betaT[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 betaT0 + gammaT*E2Num[x, y, t];

pRShell[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
```

```
 betaR[x, y, t]*rhoShell[x, y, t];

pTShell[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 betaT[x, y, t]*rhoShell[x, y, t];

shellStressTensor[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 Module[{rho = rhoShell[x, y, t],
    pr  = pRShell[x, y, t],
    pt  = pTShell[x, y, t]},
  {
   {rho, 0,  0,  0},
   {0,  pr, 0,  0},
   {0,  0,  pt, 0},
   {0,  0,  0,  pt}
  }
 ];

(* ==================================================== *)
(* ENERGY CONDITIONS *)
(* ==================================================== *)

energyConditionsTensor[T_] :=
 Module[{rho, spatial, evals},
  rho   = T[[1, 1]];
  spatial = (T[[2 ;; 4, 2 ;; 4]] + Transpose[T[[2 ;; 4, 2 ;; 4]]])/2;
  evals  = Eigenvalues[spatial];
  <|
   "rho" → rho,
   "p1"  → evals[[1]],
   "p2"  → evals[[2]],
   "p3"  → evals[[3]],
   "NEC" → And @@ (rho + # ≥ 0 & /@ evals),
   "WEC" → (rho ≥ 0 && And @@ (rho + # ≥ 0 & /@ evals)),
   "SEC" → (And @@ (rho + # ≥ 0 & /@ evals) && rho + Total[evals] ≥ 0),
   "DEC" → (rho ≥ 0 && And @@ (rho ≥ Abs[#] & /@ evals))
  |>
 ];

shellEC[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 energyConditionsTensor[shellStressTensor[x, y, t]];
```

```
biEC[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 energyConditionsTensor[stressEnergyBINum[x, y, t]];

totalEC[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 energyConditionsTensor[
  stressEnergyBINum[x, y, t] + shellStressTensor[x, y, t]
 ];

(* ==================================================== *)
(* SAMPLE OUTPUT *)
(* ==================================================== *)

pt = {0., 0.002, t0};

Print["--- At point ", pt[[{1, 2}]], " ---"];
Print["E: ", eFieldNum @@ pt];
Print["B: ", bFieldNum @@ pt];
Print["Shell rho, pr, pt: ",
 {rhoShell @@ pt, pRShell @@ pt, pTShell @@ pt}];
Print["BI EC: ",  biEC @@ pt];
Print["Shell EC: ", shellEC @@ pt];
Print["Total EC: ", totalEC @@ pt];

(* ============================================== *)
(*  DIAGNOSTIC FIELDS AT t = t0                        *)
(* ============================================== *)

uBI[x_?NumericQ, y_?NumericQ] := stressEnergyBINum[x, y, t0][[1, 1]]
uShell[x_?NumericQ, y_?NumericQ] := rhoShell[x, y, t0]
uTot[x_?NumericQ, y_?NumericQ] := uBI[x, y] + uShell[x, y]

(* Anisotropy: Δp = p_t - p_r *)
deltaPShell[x_?NumericQ, y_?NumericQ] :=
 pTShell[x, y, t0] - pRShell[x, y, t0]

(* Field intensity *)
E2[x_?NumericQ, y_?NumericQ] := E2Num[x, y, t0]

(* ============================================== *)
(*  PLOTS                                              *)
(* ============================================== *)
```

```
range = 0.04; (* adjust if you want bigger view *)

Print["Plot: BI energy density u_BI(x,y)"];
biEnergyPlot = DensityPlot[
 uBI[x, y],
 {x, -range, range}, {y, -range, range},
 PlotPoints → 40, ColorFunction → "ThermometerColors",
 FrameLabel → {"x", "y"},
 PlotLabel → "Born–Infeld energy density u_BI"
];

Print["Plot: shell anisotropy Δp = p_t - p_r"];
shellAnisoPlot = DensityPlot[
 deltaPShell[x, y],
 {x, -range, range}, {y, -range, range},
 PlotPoints → 40, ColorFunction → "SolarColors",
 FrameLabel → {"x", "y"},
 PlotLabel → "Shell anisotropy Δp = p_t - p_r"
];

Print["Plot: E^2 field intensity"];
e2Plot = DensityPlot[
 E2[x, y],
 {x, -range, range}, {y, -range, range},
 PlotPoints → 40, ColorFunction → "AvocadoColors",
 FrameLabel → {"x", "y"},
 PlotLabel → "E^2(x,y) of vortex"
];

(* ============================================ *)
(*  Poynting vector sampling along a line            *)
(* ============================================ *)

samplePoynting = Table[
 Module[{x = 0., yy = y, E = eFieldNum[0., y, t0],
    H = HFieldNum[0., y, t0], S},
  S = Cross[E, H];
  {yy, S}
 ],
 {y, -range, range, range/8.}
```

```
];

Print["Sample Poynting vectors S(y) along x=0:"];
samplePoynting

(* ================================================== *)
(* RADIAL SCAN OF SHELL + TOTAL PROPERTIES            *)
(* ================================================== *)

rMin = 0.0;
rMax = 0.06;
pts  = 200;

radialData = Table[
 Module[{x = r, y = 0,
    Tshell, Ttot,
    rhoS, prS, ptS, necS,
    rhoT, prT, ptT, necT},

  Tshell = shellStressTensor[x, y, t0];
  Ttot  = stressEnergyBINum[x, y, t0] + Tshell;

  rhoS = Tshell[[1, 1]];
  prS  = Tshell[[2, 2]];
  ptS  = Tshell[[3, 3]]; (* same as [[4,4]] *)

  rhoT = Ttot[[1, 1]];
  (* diagonalize spatial part to get principal pressures for total *)
  With[{spatT = (Ttot[[2 ;; 4, 2 ;; 4]] + Transpose[Ttot[[2 ;; 4, 2 ;; 4]]])/2},
   {prT, ptT, (* trash *)} = Eigenvalues[spatT];
  ];

  necS = If[(rhoS + prS ≥ 0) && (rhoS + ptS ≥ 0), 1, 0];
  necT = If[(rhoT + prT ≥ 0) && (rhoT + ptT ≥ 0), 1, 0];

  {r, rhoS, prS, ptS, necS, rhoT, prT, ptT, necT}
 ],
 {r, rMin, rMax, (rMax - rMin)/pts}
];

radialR  = radialData[[All, 1]];
```

```
rhoShellR = radialData⟦All, 2⟧;
prShellR = radialData⟦All, 3⟧;
ptShellR = radialData⟦All, 4⟧;
necShellR = radialData⟦All, 5⟧;
rhoTotR  = radialData⟦All, 6⟧;
prTotR  = radialData⟦All, 7⟧;
ptTotR  = radialData⟦All, 8⟧;
necTotR  = radialData⟦All, 9⟧;

(* ==================================================== *)
(* PLOTS: SHELL ONLY                                    *)
(* ==================================================== *)

Print["Radial profiles (shell): rho_s(r), p_r^s(r), p_t^s(r)"];

rhoShellPlot = ListLinePlot[
 Transpose[{radialR, rhoShellR}],
 PlotStyle → Thick,
 Frame → True,
 FrameLabel → {"r", "rho_s"},
 PlotLabel → "Shell energy density rho_s(r)"
];

prShellPlot = ListLinePlot[
 Transpose[{radialR, prShellR}],
 PlotStyle → {Red, Thick},
 Frame → True,
 FrameLabel → {"r", "p_r^s"},
 PlotLabel → "Shell radial pressure p_r^s(r)"
];

ptShellPlot = ListLinePlot[
 Transpose[{radialR, ptShellR}],
 PlotStyle → {Blue, Thick},
 Frame → True,
 FrameLabel → {"r", "p_t^s"},
 PlotLabel → "Shell tangential pressure p_t^s(r)"
];

Print["Plot: shell rho_s(r)"];  rhoShellPlot
Print["Plot: shell p_r^s(r)"];  prShellPlot
```

```
Print["Plot: shell p_t^s(r)"];  ptShellPlot

Print["Plot: Shell NEC_s(r) (1 = satisfied, 0 = violated)"];

necShellPlot = ListStepPlot[
 Transpose[{radialR, necShellR}],
 Frame → True,
 FrameLabel → {"r", "NEC_s"},
 PlotRange → {0, 1.2},
 PlotLabel → "Shell NEC_s(r): 1 = NEC OK, 0 = NEC violated",
 Filling → Axis
];

necShellPlot

(* ==================================================== *)
(* PLOTS: TOTAL (BI + shell)                             *)
(* ==================================================== *)

Print["Radial profiles (total): rho_tot(r), p_r^tot(r), p_t^tot(r)"];

rhoTotPlot = ListLinePlot[
 Transpose[{radialR, rhoTotR}],
 PlotStyle → Thick,
 Frame → True,
 FrameLabel → {"r", "rho_tot"},
 PlotLabel → "Total energy density rho_tot(r)"
];

prTotPlot = ListLinePlot[
 Transpose[{radialR, prTotR}],
 PlotStyle → {Red, Thick},
 Frame → True,
 FrameLabel → {"r", "p_r^tot"},
 PlotLabel → "Total radial pressure p_r^tot(r)"
];

ptTotPlot = ListLinePlot[
 Transpose[{radialR, ptTotR}],
 PlotStyle → {Blue, Thick},
 Frame → True,
```

```
 FrameLabel → {"r", "p_t^tot"},
 PlotLabel → "Total tangential pressure p_t^tot(r)"
];

Print["Plot: total rho_tot(r)"]; rhoTotPlot
Print["Plot: total p_r^tot(r)"]; prTotPlot
Print["Plot: total p_t^tot(r)"]; ptTotPlot

Print["Plot: NEC_tot(r) (1 = satisfied, 0 = violated)"];

necTotPlot = ListStepPlot[
 Transpose[{radialR, necTotR}],
 Frame → True,
 FrameLabel → {"r", "NEC_tot"},
 PlotRange → {0, 1.2},
 PlotLabel → "Total NEC_tot(r): 1 = NEC OK, 0 = NEC violated",
 Filling → Axis
];

necTotPlot
```

```
--- At point {0., 0.002} ---
E: {1.16388×10^-7, 0.827911, 0.}
B: {1.45604×10^-11, 7.20325×10^-10, 0.}
Shell rho, pr, pt: {0.000393669, -0.0133737, 0.0136099}
BI EC: <|rho → 0.78298, p1 → 0.78298, p2 → 0.78298,
  p3 → -0.439141, NEC → True, WEC → True, SEC → True, DEC → True|>
Shell EC: <|rho → 0.000393669, p1 → 0.0136099, p2 → 0.0136099,
  p3 → -0.0133737, NEC → False, WEC → False, SEC → False, DEC → False|>
Total EC: <|rho → 0.783373, p1 → 0.79659, p2 → 0.769606,
  p3 → -0.425531, NEC → True, WEC → True, SEC → True, DEC → False|>
Plot: BI energy density u_BI(x,y)
Plot: shell anisotropy Δp = p_t - p_r
Plot: E^2 field intensity
Sample Poynting vectors S(y) along x=0:
```

Out[450]=

$\{\{-0.04, \{0., 0., 1.31047\times10^{-11}\}\}$,
$\{-0.035, \{0., 0., 3.19406\times10^{-11}\}\}$, $\{-0.03, \{0., 0., 6.93916\times10^{-11}\}\}$,
$\{-0.025, \{0., 0., 1.3406\times10^{-10}\}\}$, $\{-0.02, \{0., 0., 2.28765\times10^{-10}\}\}$,
$\{-0.015, \{0., 0., 3.38465\times10^{-10}\}\}$, $\{-0.01, \{0., 0., 4.12378\times10^{-10}\}\}$,
$\{-0.005, \{0., 0., 3.53788\times10^{-10}\}\}$, $\{0., \{0., 0., 1.02628\times10^{-10}\}\}$,
$\{0.005, \{0., 0., -1.79819\times10^{-10}\}\}$, $\{0.01, \{0., 0., -3.07091\times10^{-10}\}\}$,
$\{0.015, \{0., 0., -2.9359\times10^{-10}\}\}$, $\{0.02, \{0., 0., -2.18166\times10^{-10}\}\}$,
$\{0.025, \{0., 0., -1.36823\times10^{-10}\}\}$, $\{0.03, \{0., 0., -7.46112\times10^{-11}\}\}$,
$\{0.035, \{0., 0., -3.58245\times10^{-11}\}\}$, $\{0.04, \{0., 0., -1.52351\times10^{-11}\}\}\}$

Set: Symbol Null is Protected.

Set: Symbol Null is Protected.

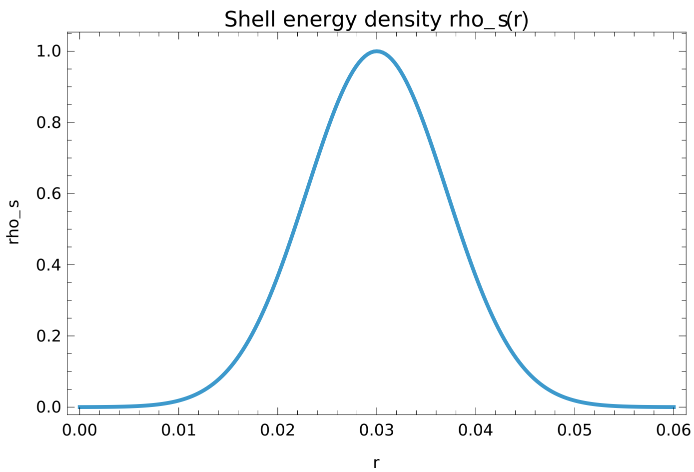
Set: Symbol Null is Protected.

General: Further output of Set::wrsym will be suppressed during this calculation.

Radial profiles (shell): rho_s(r), p_r^s(r), p_t^s(r)

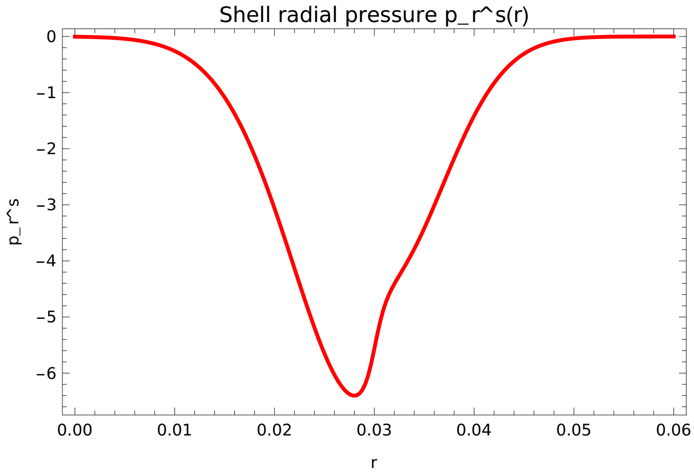
Plot: shell rho_s(r)

Out[468]=



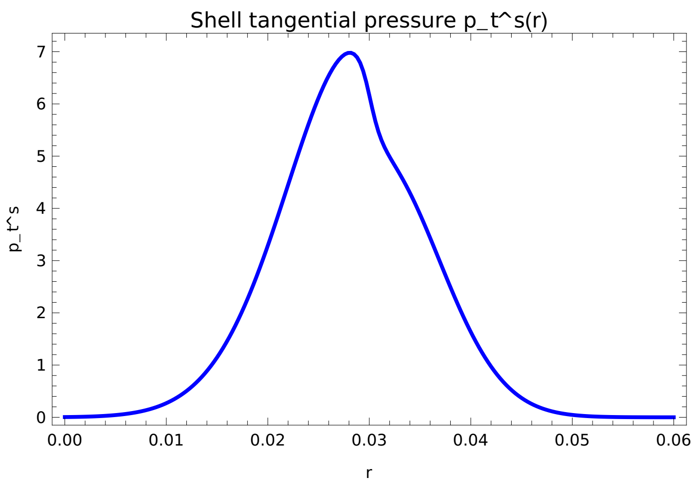
Plot: shell p_r^s(r)

Out[469]=



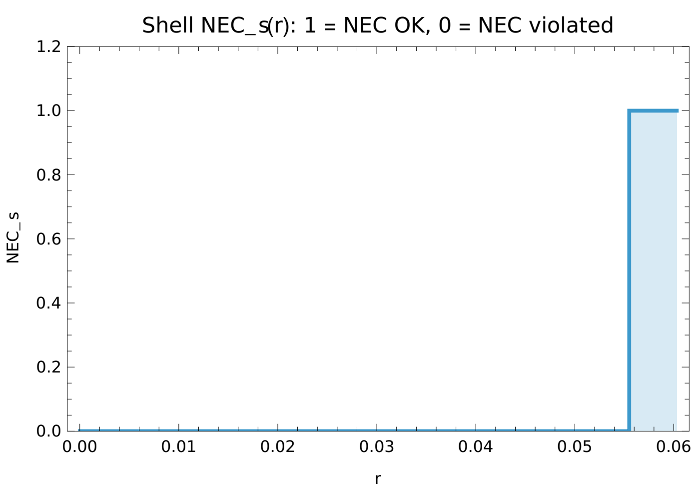
Plot: shell p_t^s(r)

Out[470]=



Plot: Shell NEC_s(r) (1 = satisfied, 0 = violated)

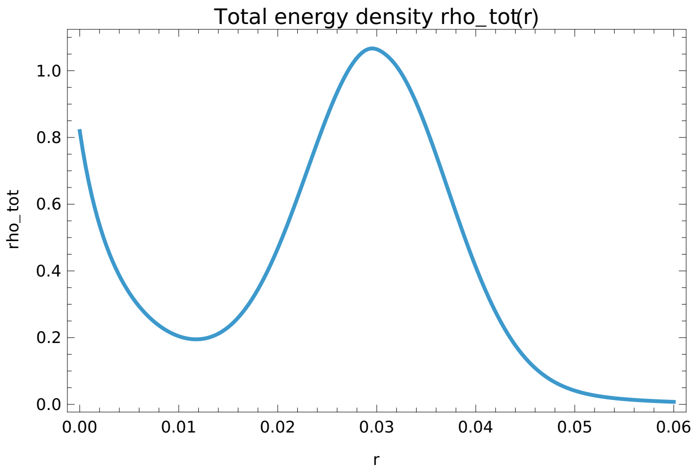
Out[473]=



Radial profiles (total): rho_tot(r), p_r^tot(r), p_t^tot(r)

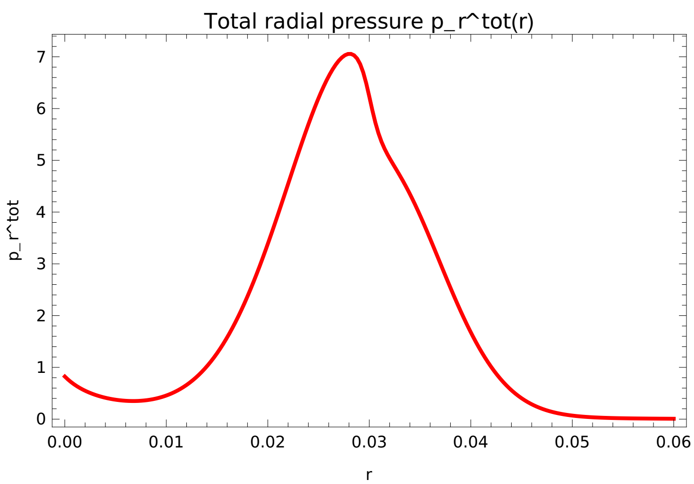
Plot: total rho_tot(r)

Out[478]=



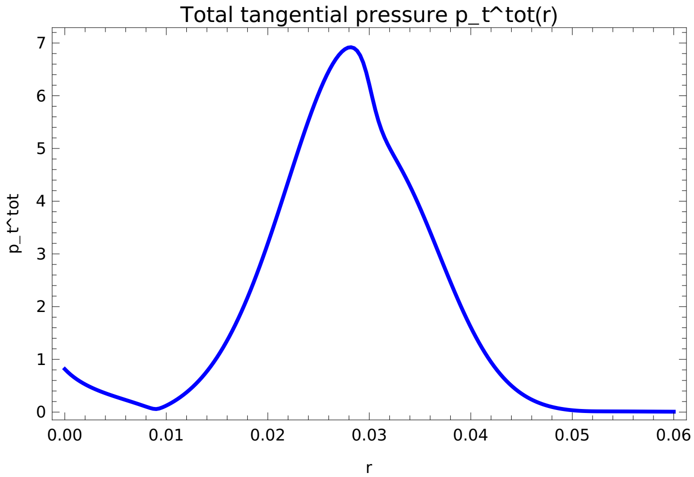
Plot: total p_r^tot(r)

Out[479]=



Plot: total p_t^tot(r)

Out[480]=



Plot: NEC_tot(r) (1 = satisfied, 0 = violated)

Out[483]=