In[665]:=

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(* ::Package:: *)
(* Title: Einstein Tensor Calculation for 3+1 Conformally Flat Metric *)
(* 1mm Wormhole *)

Print["Defining the potential and metric components..."];



(* Parameters for 1mm wormhole *)
R0 = 1.0*10^-3; (* 1 mm throat *)
A = 1;     (* Example value *)
w = 1.5*R0;   (* Width parameter *)
ϵ = 0.1*R0;   (* Regularization at r=0 *)

(* Potential function Φ(x,y,z) *)
rCoord[x_, y_, z_] := Max[Sqrt[x^2 + y^2 + z^2], ϵ];
Φ[x_, y_, z_] := -A*(1 - R0/rCoord[x, y, z])*Exp[-((rCoord[x, y, z] - R0)^2/w^2)];

(* Metric components in (t,x,y,z) coordinates *)
(* ds² = -e^{2Φ}dt² + e^{-2Φ}(dx² + dy² + dz²) *)
gtt[x_, y_, z_] := -Exp[2*Φ[x, y, z]]
gxx[x_, y_, z_] := Exp[-2*Φ[x, y, z]]
gyy[x_, y_, z_] := Exp[-2*Φ[x, y, z]]
gzz[x_, y_, z_] := Exp[-2*Φ[x, y, z]]

(* Metric tensor: g_{μν} *)
metric = {{gtt[x, y, z], 0, 0, 0},
    {0, gxx[x, y, z], 0, 0},
    {0, 0, gyy[x, y, z], 0},
    {0, 0, 0, gzz[x, y, z]}};

Print["Metric tensor at throat (x=R0,y=0,z=0):"];
MatrixForm[metric /. {x → R0, y → 0, z → 0}]

(* Coordinate system *)
Coordinates = {t, x, y, z};

(* Inverse metric *)
inversemetric = Inverse[metric];

(* Christoffel symbols Γ^μ_{νρ} *)
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Print["\nCalculating Christoffel symbols..."];
Clear[Γ]
Γ[μ_, ν_, ρ_] := Sum[(1/2)*inversemetric[[μ, σ]]*(
  D[metric[[σ, ν]], Coordinates[[ρ]]] +
  D[metric[[σ, ρ]], Coordinates[[ν]]] -
  D[metric[[ν, ρ]], Coordinates[[σ]]]), {σ, 1, 4}];

(* Riemann tensor: R^μ_{νρσ} *)
Print["\nCalculating Riemann tensor..."];
Clear[Riemann]
Riemann[μ_, ν_, ρ_, σ_] :=
 D[Γ[μ, ν, σ], Coordinates[[ρ]]] -
 D[Γ[μ, ν, ρ], Coordinates[[σ]]] +
 Sum[Γ[μ, κ, ρ]*Γ[κ, ν, σ] - Γ[μ, κ, σ]*Γ[κ, ν, ρ], {κ, 1, 4}];

(* Ricci tensor: R_{μν} *)
Print["\nCalculating Ricci tensor and scalar..."];
Clear[Ricci]
Ricci[μ_, ν_] := Sum[Riemann[σ, μ, σ, ν], {σ, 1, 4}];

RicciScalar = Sum[inversemetric[[μ, ν]]*Ricci[μ, ν], {μ, 1, 4}, {ν, 1, 4}];

(* Einstein tensor: G_{μν} *)
Print["\nCalculating Einstein tensor..."];
Clear[Einstein]
Einstein[μ_, ν_] := Ricci[μ, ν] - (1/2)*metric[[μ, ν]]*RicciScalar;

(* Compute at throat for analysis *)
x0 = R0; y0 = 0; z0 = 0;
Print["\nEinstein tensor at throat x=", x0, ", y=", y0, ", z=", z0, ":"];

einsteinAtThroat =
  Table[Einstein[i, j] /. {x → x0, y → y0, z → z0}, {i, 1, 4}, {j, 1, 4}];
MatrixForm[Simplify[einsteinAtThroat]]

(* ADM mass estimate *)
Print["\nADM Mass estimation..."];
PhiRadial[r_] := -A*(1 - R0/r)*Exp[-((r - R0)^2/w^2)]
ADMmass = Limit[r^2*D[PhiRadial[r], r], r → Infinity]
Print["ADM Mass: ", ADMmass]

Print["\nCalculation complete."]
```

Defining the potential and metric components...

Metric tensor at throat (x=R0,y=0,z=0):

Out[678]//MatrixForm=

$$\begin{pmatrix} -1. & 0 & 0 & 0 \\ 0 & 1. & 0 & 0 \\ 0 & 0 & 1. & 0 \\ 0 & 0 & 0 & 1. \end{pmatrix}$$

Calculating Christoffel symbols...

Calculating Riemann tensor...

Calculating Ricci tensor and scalar...

Calculating Einstein tensor...

Einstein tensor at throat x=0.001, y=0, z=0:

Out[697]//MatrixForm=

$$\begin{pmatrix} -1.\times 10^{6} & 0 & 0 & 0 \\ 0 & -1.\times 10^{6} & 0. & 0. \\ 0 & 0. & 1.\times 10^{6} & 0. \\ 0 & 0. & 0. & 1.\times 10^{6} \end{pmatrix}$$

ADM Mass estimation...

Out[700]=

0.

ADM Mass: 0.

Calculation complete.