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In[*]:= (* ::Title::*) (*Geodesic Traversal and Traversability Check in a 3+
1D Wormhole Metric (Optimized Parameters)*)
(* ::Section::*) (*Define Parameters and Potential*) R0 = 2.0;
A = 0.01;
w = 0.2 R0;
ε = 0.04 R0;

Safe[x_, y_, z_] := Module[{r}, r = Max[Sqrt[x^2 + y^2 + z^2], ε];
-A (1 - R0 / r) Exp[-(r - R0)^2 / w^2]];

(* ::Section::*) (*Define the Metric Components*)

f[x_, y_, z_] := Exp[2 Safe[x, y, z]];
invf[x_, y_, z_] := Exp[-2 Safe[x, y, z]];

(* ::Section::*) (*Define the Lagrangian for Geodesic Motion*)

Clear[t, x, y, z, s];

L = -f[x[s], y[s], z[s]] × t'[s]^2 + invf[x[s], y[s], z[s]] (x'[s]^2 + y'[s]^2 + z'[s]^2);

(* ::Section::*) (*Compute Euler-Lagrange Equations*)

eqt = D[D[L, t'[s]], s] - D[L, t[s]] == 0;
eqx = D[D[L, x'[s]], s] - D[L, x[s]] == 0;
eqy = D[D[L, y'[s]], s] - D[L, y[s]] == 0;
eqz = D[D[L, z'[s]], s] - D[L, z[s]] == 0;

(* ::Section::*) (*Initial Conditions and Numerical Integration for Timelike Geodesic*)

timelikeIC =
{t[0] == 0, x[0] == 0.5 R0, y[0] == 0, z[0] == 0, t'[0] == 1, x'[0] == 1, y'[0] == 0, z'[0] == 0};

timelikeSol = NDSolve[{eqt, eqx, eqy, eqz} ~Join~ timelikeIC,
{t, x, y, z}, {s, 0, 10}, MaxStepFraction → 1 / 100];

(* ::Section::*) (*Plot the Geodesic Path (3D)*)

ParametricPlot3D[Evaluate[{x[s], y[s], z[s]} /. timelikeSol],
{s, 0, 10}, PlotRange → All, AxesLabel → {"x (m)", "y (m)", "z (m)"},
PlotLabel → "Timelike Geodesic Path Through Wormhole (Optimized Parameters)"]

(* ::Section::*) (*Output Final Position*)

Print["Final position after s = 10: ", {x[10], y[10], z[10]} /. timelikeSol];

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(* ::Section::*) (*Null Geodesic Case*)

Lnull = L;

eqtN = eqt;
eqxN = eqx;
eqyN = eqy;
eqzN = eqz;

nullIC =
  {t[0] == 0, x[0] == 0.5 R0, y[0] == 0, z[0] == 0, t'[0] == 1, x'[0] == 1, y'[0] == 0, z'[0] == 0};

nullSol = NDSolve[{eqtN, eqxN, eqyN, eqzN} ~Join~ nullIC,
  {t, x, y, z}, {s, 0, 10}, MaxStepFraction -> 1 / 100];

ParametricPlot3D[Evaluate[{x[s], y[s], z[s]} /. nullSol],
  {s, 0, 10}, PlotRange -> All, AxesLabel -> {"x (m)", "y (m)", "z (m)"},
  PlotLabel -> "Null Geodesic Path Through Wormhole"]

(* ::Section::*) (*Animated Traversal (Timelike Geodesic)*)

frames3D =
  Table[Graphics3D[{Red, PointSize[Large], Point[{x[s], y[s], z[s]} /. timelikeSol]},
    PlotRange -> {{-3 R0, 12 R0}, {-3 R0, 3 R0}, {-3 R0, 3 R0}}, Axes -> True,
    AxesLabel -> {"x (m)", "y (m)", "z (m)"}, BoxRatios -> {4, 1, 1}, {s, 0, 10, 0.5}];

ListAnimate[frames3D]

(* ::Section::*) (*Potential Visualization (Equatorial Plane z=0)*)

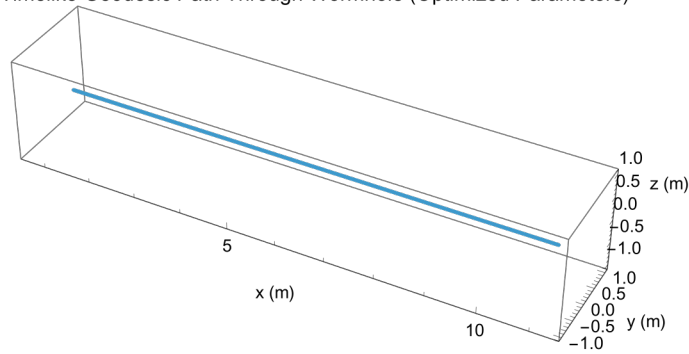
Plot3D[ϕSafe[x, y, 0], {x, -3 R0, 3 R0}, {y, -3 R0, 3 R0},
  PlotLabel -> "Scalar Potential ϕ(x,y,z=0)", AxesLabel -> {"x (m)", "y (m)", "ϕ"},
  PlotRange -> All, PlotPoints -> 200, MaxRecursion -> 8, Mesh -> None]

(*Optional radial potential plot*)

Plot[ϕSafe[r, 0, 0], {r, 0.1 R0, 5 R0},
  PlotLabel -> "Radial Profile of ϕ(r) Along x-Axis", AxesLabel -> {"r (m)", "ϕ(r)"}]
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Out[8]=

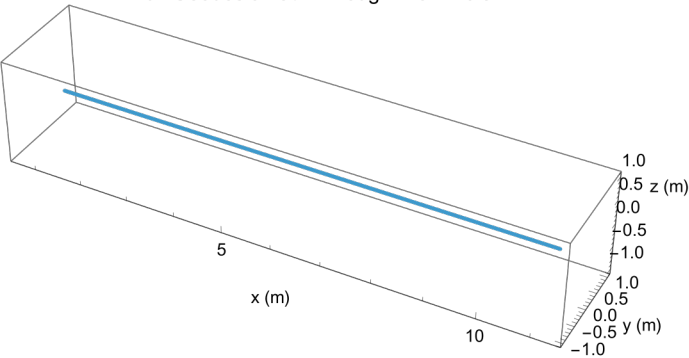
Timelike Geodesic Path Through Wormhole (Optimized Parameters)



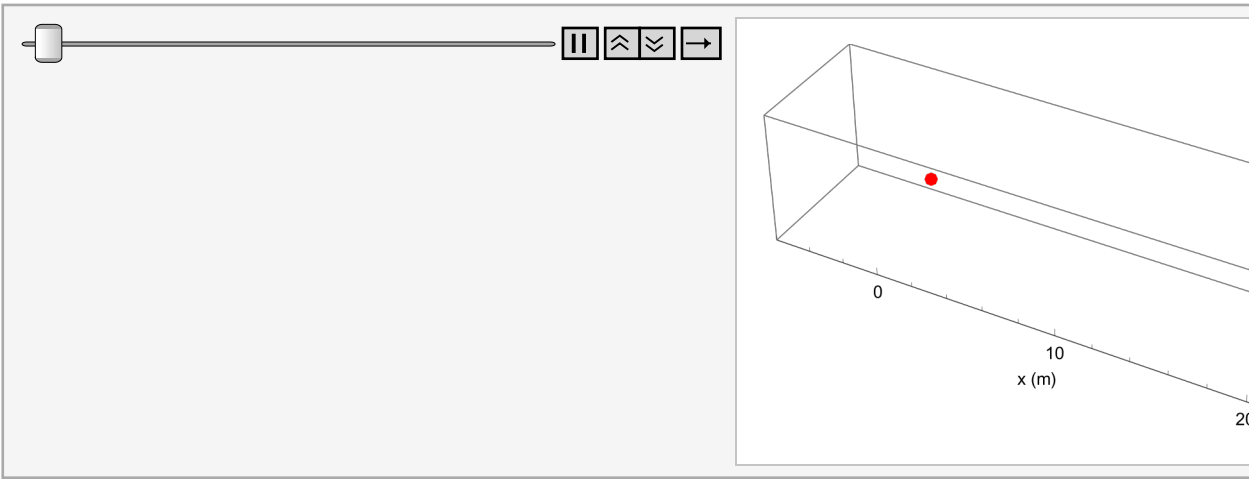
Final position after $s = 10$: $\{\{11., 0., 0.\}\}$

Out[9]=

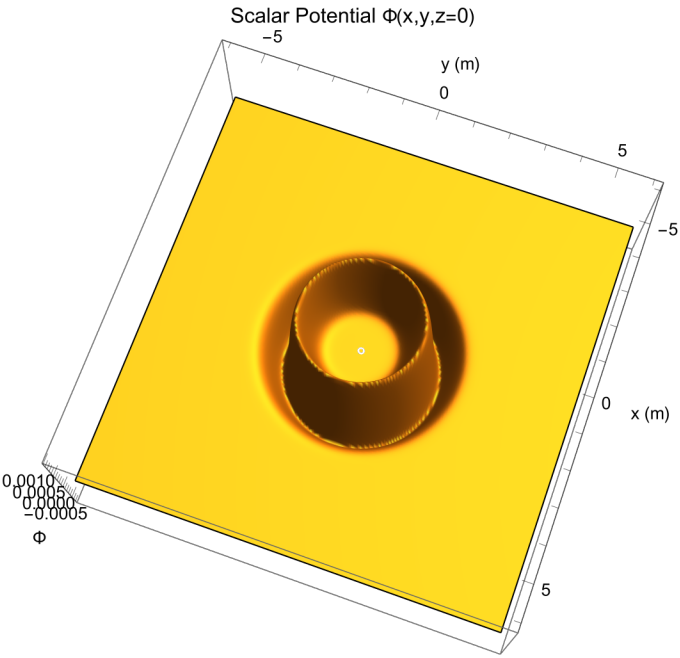
Null Geodesic Path Through Wormhole



Out[10]=



Out[*#*]=



Out[*#*]=

