In[421]:=

```
(* ======================================== *)
(*  1 mm Static Wormhole: Potential & Metric *)
(* ======================================== *)

(* --- Clear all previous definitions --- *)
ClearAll["Global`*"];



(* --- Basic parameters (SI units) --- *)
R0 = 1.*10^-3;   (* throat radius: 1 mm = 10^-3 m *)
A  = 1;      (* potential amplitude (dimensionless for now) *)
w  = 1.5*R0;    (* width of transition region, choose ~ R0 scale *)
eps = 10^-6*R0;   (* small regulator for r → 0 in 3D version *)

(* ---------------------------------- *)
(* 1D RADIAL SCALAR POTENTIAL Φ(r)       *)
(* ---------------------------------- *)

(* Φ(r) = -A (1 - R0/r) Exp[-(r - R0)^2 / w^2] *)
Φ[r_] := -A (1 - R0 / r) Exp[-((r - R0)^2) / w^2];

(* Flare-out condition: d/dr[Exp[-2 Φ(r)]] at r = R0 *)
flareOutExpr   = Exp[-2 Φ[r]];
flareOutDerivative = D[flareOutExpr, r] /. r → R0 // FullSimplify;

Print["\n--- Flare-out Condition at r = R0 = ", R0, " m ---"];
Print["d/dr[Exp[-2 Φ(r)]]|_(r=R0) = ", N[flareOutDerivative, 10]];
If[flareOutDerivative > 0,
 Print["☑ Flare-out condition satisfied at the throat."],
 Print["☒ Flare-out condition violated at the throat."]
];

(* Scan over different w values (in units of R0) *)
Print["\n--- Scanning flare-out over w (multiples of R0) ---"];
Table[
 Module[{wval, φloc, ddr},
  wval = fac*R0;
  φloc[r_] := -A (1 - R0 / r) Exp[-((r - R0)^2) / wval^2];
  ddr = D[Exp[-2 φloc[r]], r] /. r → R0 // N;
  {wval, ddr, If[ddr > 0, "☑ OK", "☒ Violated"]}
```

```
 ],
 {fac, 0.5, 3, 0.1}
] // TableForm

(* ---------------------------------- *)
(* 3D SCALAR POTENTIAL Φ(x,y,z)       *)
(* ---------------------------------- *)

(* Regularized radius:
   r(x,y,z) = max(Sqrt[x^2 + y^2 + z^2], eps) *)
rReg[x_, y_, z_] := Max[Sqrt[x^2 + y^2 + z^2], eps];

(* Φ(x,y,z) built from rReg *)
Φ3D[x_, y_, z_] := Module[{r = rReg[x, y, z]},
 -A (1 - R0/r) Exp[-((r - R0)^2)/w^2]
];

(* Quick checks on the 3D potential *)
Print["\n--- 3D potential checks ---"];
Print["Φ3D[0,0,0] = ", N[Φ3D[0, 0, 0], 10]];
Print["Φ3D[R0,0,0] = ", N[Φ3D[R0, 0, 0], 10]];
Print["Φ3D[2 R0,0,0] = ", N[Φ3D[2 R0, 0, 0], 10]];

(* ---------------------------------- *)
(* METRIC FROM Φ(r)                   *)
(* ---------------------------------- *)

(* Spherical coordinates (t, r, θ, φ) *)
coords = {t, r, θ, φ};

(* Metric components:
   ds^2 = -e^{2Φ(r)} dt^2 + e^{-2Φ(r)} dr^2 + r^2 dθ^2 + r^2 sin^2θ dφ^2
*)
g = {
 {-Exp[2 Φ[r]],     0,          0,              0},
 {0,          Exp[-2 Φ[r]],     0,              0},
 {0,          0,          r^2,             0},
 {0,          0,          0,          r^2 Sin[θ]^2}
};

Print["\n--- Metric components g_{μν}(r,θ) ---"];
```

```
MatrixForm[g] // Print;

(* Check determinant and signature at the throat r = R0 *)
gAtThroat[θ0_] := g /. {r → R0, θ → θ0};

detgAtThroat = Det[gAtThroat[Pi / 2]] // FullSimplify;
eigsAtThroat = Eigenvalues[gAtThroat[Pi / 2]] // N;

Print["\nDet[g]|_(r=R0, θ=π/2) = ", detgAtThroat];
Print["Eigenvalues[g]|_(r=R0, θ=π/2) = ", eigsAtThroat];
Print["(Expect one negative, three positive for Lorentzian signature.)"];
```

```
--- Flare-out Condition at r = R0 = 0.001 m ---
d/dr[Exp[-2 Φ(r)]]|_(r=R0) = 2000.
☑ Flare-out condition satisfied at the throat.

--- Scanning flare-out over w (multiples of R0) ---
```

Out[433]//TableForm=

0.0005	2000.	☑ OK
0.0006	2000.	☑ OK
0.0007	2000.	☑ OK
0.0008	2000.	☑ OK
0.0009	2000.	☑ OK
0.001	2000.	☑ OK
0.0011	2000.	☑ OK
0.0012	2000.	☑ OK
0.0013	2000.	☑ OK
0.0014	2000.	☑ OK
0.0015	2000.	☑ OK
0.0016	2000.	☑ OK
0.0017	2000.	☑ OK
0.0018	2000.	☑ OK
0.0019	2000.	☑ OK
0.002	2000.	☑ OK
0.0021	2000.	☑ OK
0.0022	2000.	☑ OK
0.0023	2000.	☑ OK
0.0024	2000.	☑ OK
0.0025	2000.	☑ OK
0.0026	2000.	☑ OK
0.0027	2000.	☑ OK
0.0028	2000.	☑ OK
0.0029	2000.	☑ OK
0.003	2000.	☑ OK

--- 3D potential checks ---

$\Phi 3D[0,0,0] = 641\,180.$

$\Phi 3D[R0,0,0] = 0.$

$\Phi 3D[2\ R0,0,0] = -0.32059$

--- Metric components $g_{\mu\nu}(r,\theta)$ ---

$$\begin{pmatrix} -e^{-2\,e^{-444\,444.(-0.001+r)^2}\left(1-\frac{0.001}{r}\right)} & 0 & 0 & 0 \\ 0 & e^{2\,e^{-444\,444.(-0.001+r)^2}\left(1-\frac{0.001}{r}\right)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\,\text{Sin}[\theta]^2 \end{pmatrix}$$

$\text{Det}[g]_{(r=R0,\ \theta=\pi/2)} = -1.\times 10^{-12}$

$\text{Eigenvalues}[g]_{(r=R0,\ \theta=\pi/2)} = \{-1.,\ 1.,\ 1.\times 10^{-6},\ 1.\times 10^{-6}\}$

(Expect one negative, three positive for Lorentzian signature.)