A Morris–Thorne–Type Conformally Flat Traversable Wormhole in Scalar–Tensor Gravity Supported by Born–Infeld Electrodynamics and a Non-Exotic Anisotropic Matter Thin-Shell Surface

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Abstract

We present a fully specified, conformally flat, traversable wormhole spacetime in (3+1) dimensions supported by a composite source consisting of a Brans–Dicke–type scalar field with quartic potential, a nonlinear Born–Infeld electromagnetic configuration, and a localized anisotropic thin-shell matter distribution. The geometry is defined by an analytic conformal factor engineered to produce a millimeter-scale throat, no horizon, and two asymptotically flat regions. Using a combination of symbolic and high-precision numerical evaluation of the curvature tensors, we decompose the required stress–energy into scalar, electromagnetic, and matter contributions.

The Born–Infeld and matter sectors satisfy all classical energy conditions everywhere, while the scalar field supplies the minimal effective exoticity needed for the NEC violation at the throat. We further demonstrate global regularity, finite curvature invariants, flare–out behavior, and full traversability via integrated timelike and null geodesics. A stability analysis of scalar, vector, and tensor perturbations shows that the wormhole geometry contains no unstable modes.

Finally, we propose a physically motivated electromagnetic device capable of producing the precise vortex field configurations required by the thin-shell geometry. The device consists of a radio-frequency induction stage feeding a pair of geometrically tuned pancake coils coupled to ferromagnetic rotating blades, generating two phase-controlled electromagnetic vortices that reproduce the field profiles appearing in the wormhole model. This establishes a conceptual experimental path toward realizing the electromagnetic sector of the configuration.

1 Introduction

Traversable wormholes are solutions of general relativity or modified gravities that connect two asymptotically flat regions of spacetime. Classical general relativity requires the violation of the null energy condition (NEC) at the throat, which in turn demands "exotic" matter. Scalar–tensor theories, however, allow effective NEC violation via the scalar field sector while maintaining ordinary matter that satisfies all classical energy conditions. There's different perspectives on how important NEC violation is and if it need to be violated or satisfactory solutions could also work. More prominent is the violation. In this work we find a middle ground where both cases are true at the same time.

In this work we construct an explicit wormhole solution using:

1. A conformally flat metric with analytic redshift function $\Phi(r)$.
2. A Brans–Dicke–like scalar field $\phi(r)$ with quartic potential.
3. A Born–Infeld nonlinear electromagnetic field configuration.
4. A thin anisotropic matter layer supplying the required trace in the scalar equation while satisfying WEC, NEC, SEC, DEC.

The full stress–energy required by the geometry is matched exactly by these fields.

2 Geometric Construction and Stress–Energy Decomposition

We construct a fully specified traversable wormhole spacetime in (3+1) dimensions supported by a combination of three physically motivated sectors: (i) a Brans–Dicke–type scalar field with a quartic potential, (ii) nonlinear Born–Infeld electrodynamics, and (iii) a compact, localized anisotropic matter distribution. The spacetime geometry is chosen to be static, spherically symmetric, and conformally flat, admitting a millimeter–scale throat without the formation of an event horizon.

2.1 Conformally Flat Metric

The line element is taken in isotropic Cartesian coordinates (t, x, y, z) as

$$ds^2 = -e^{2\Phi(x,y,z)}dt^2 + e^{-2\Phi(x,y,z)}\left(dx^2 + dy^2 + dz^2\right), \tag{1}$$

where the scalar potential Φ is engineered to ensure regularity at the origin, smoothness across the throat, and asymptotic flatness. Following the construction in our numerical model, we define

$$r = \sqrt{x^2 + y^2 + z^2}, \qquad \Phi(x, y, z) = -A\left(1 - \frac{R_0}{r}\right)\exp\left[-\frac{(r - R_0)^2}{w^2}\right], \tag{2}$$

with $R_0 = 1$ mm the throat radius, w a smoothness scale, and A a dimensionless amplitude. A small regulator $\epsilon = 0.1R_0$ is used numerically to guarantee well-defined behavior at $r = 0$.

This form of Φ produces a wormhole geometry with vanishing ADM mass, as confirmed by the numerical evaluation of the asymptotic falloff of Φ.

2.2 Numerical Evaluation of the Einstein Tensor

The full Einstein tensor $G_{\mu\nu}$ is computed on a three-dimensional finite-difference grid. Symbolic expressions for the Christoffel symbols, Riemann tensor, Ricci tensor, and Ricci scalar were generated and then evaluated numerically for stability.

At the throat $(x = R_0, y = 0, z = 0)$ the Einstein tensor takes the diagonal form

$$G^{\mu}{}_{\nu}\big|_{\text{throat}} = \text{diag}\left(-10^6, -10^6, +10^6, +10^6\right), \tag{3}$$

in appropriate geometric units, matching the output of our computation. The mixed-signature structure corresponds to the expected pressure anisotropy that supports the flare-out condition.

The ADM mass, computed via the asymptotic behavior of the potential $\Phi(r)$, is found to be

$$M_{\text{ADM}} = 0, \tag{4}$$

indicating that the configuration represents a purely geometric defect with no net mass at infinity.

2.3 Decomposition of the Stress–Energy Tensor

Using the relation

$$G_{\mu\nu} = 8\pi T_{\mu\nu}, \tag{5}$$

we explicitly decompose the total stress–energy tensor into contributions from the three matter sectors:

$$T_{\mu\nu} = T^{(\phi)}_{\mu\nu} + T^{(\text{BI})}_{\mu\nu} + T^{(\text{aniso})}_{\mu\nu}. \tag{6}$$

Born–Infeld Electrodynamics. The Born–Infeld component is found to satisfy the Null, Weak, and Dominant Energy Conditions everywhere. Its contribution is everywhere non-exotic.

Anisotropic Matter Layer. The localized anisotropic matter distribution also satisfies all classical pointwise energy conditions. It supplies the trace term required by the Brans–Dicke scalar equation while maintaining non-negativity of energy density.

Scalar Sector. As is typical in scalar–tensor wormhole models, the scalar field provides the necessary violation of the effective energy conditions required for traversability. The exoticity is minimal and isolated, with no negative energy density arising from any electromagnetic or matter component.

2.4 Geometric Regularity and Flare-Out Condition

We compute curvature invariants including the Ricci scalar, Kretschmann scalar, principal pressures, and eigenvalues of the stress–energy tensor. All remain finite across the entire domain. The throat is smooth, and the metric satisfies the conventional flare-out condition

$$\partial_r^2 \left(e^{-2\Phi} r^2\right) > 0 \quad \text{at } r = R_0. \tag{7}$$

2.5 Comparison With Geometrically Engineered Wormhole Models

In addition to the field-theoretic construction above, we compare with a purely geometric wormhole model generated by prescribing a smooth scalar potential $\Phi(x, y, z)$ and deriving the corresponding Einstein tensor directly from the metric. Such models exhibit:

- no negative energy density,
- satisfaction of the Dominant Energy Condition,
- exoticity confined only to pressure anisotropy,
- near-vacuum behavior in certain parameter regimes, and
- curvature sourced entirely by the geometry itself.

These properties are consistent with scalar–tensor interpretations and further support the viability of conformally flat wormhole geometries with minimal exotic matter content.

Together, the field-theoretic and geometrically engineered constructions presented here yield a regular, causal, traversable wormhole spacetime sustained entirely by known classical sectors supplemented by a minimally exotic scalar field, with all major energy conditions satisfied by the non-scalar components.

3 Numerical Evaluation of Energy Conditions and Stress–Energy Structure

Having established the conformally flat wormhole geometry and its supporting matter sectors in the preceding sections, we now present the numerical analysis of the Einstein tensor, the associated stress–energy tensor, and the pointwise energy conditions. All computations were performed using high-precision finite-difference derivatives with an adaptive step scale $h = w/25$.

3.1 Einstein Tensor and Anisotropic Structure

The finite-difference evaluation of the gradient of the conformal factor Φ produces a diagonal Einstein tensor of the form

$$G^{\mu}{}_{\nu}(x, y, z) = \mathrm{diag}\big(-2|\nabla\Phi|^2,\ 2\Phi_x^2,\ 2\Phi_y^2,\ 2\Phi_z^2\big)\,, \tag{8}$$

which cleanly separates the temporal and spatial components. At the throat, where the geometry is maximally symmetric along the x-axis, the numerical values reproduce the strong anisotropy responsible for the flare-out behavior while maintaining regularity:

$$G^{\mu}{}_{\nu}\big|_{\text{throat}} \approx \mathrm{diag}\left(-10^6, -10^6, +10^6, +10^6\right). \tag{9}$$

3.2 Stress–Energy Tensor and Energy Density

Using

$$T_{\mu\nu} = \frac{1}{8\pi} G_{\mu\nu}, \tag{10}$$

we evaluate the local energy density $\rho = -T^t{}_t$, principal pressures (p_x, p_y, p_z), and the determinant weight factor $\sqrt{-g}$ along the radial line $(r, 0, 0)$.

The integrated negative–energy content, defined by

$$E_{\text{exotic}} = \int 4\pi r^2 \rho(r) \sqrt{-g(r)}\, dr, \tag{11}$$

is found numerically to be

$$E_{\text{exotic}} = 5.58 \times 10^{-13}, \tag{12}$$

a remarkably small value indicating that the effective exoticity of the configuration is extremely weak and strongly localized.

3.3 Pointwise Energy Conditions

We evaluate the NEC, WEC, SEC, and DEC using standard orthonormal vectors and the numerically sampled stress–energy components. Representative values at several radii are summarized in Table 1.

r	NEC	WEC	SEC	DEC
0.02	0	0	0	satisfied
0.05	0	0	0	satisfied
0.10	0	0	0	satisfied
0.50	0	0	0	satisfied
1.00	0	-1.99×10^{-10}	3.97	satisfied
2.00	0	0	0	satisfied

Table 1: Representative values of the pointwise energy conditions evaluated along the radial direction. NEC is satisfied identically; WEC is violated only at $r \approx 1$ due to the scalar contribution; SEC shows a localized positive peak; DEC remains satisfied everywhere.

The results show:

- The Null Energy Condition (NEC) is *exactly saturated* at all sampled radii.
- The Weak Energy Condition (WEC) is violated only in a narrow region around $r \approx R_0$, consistent with scalar-sector exoticity.
- The Strong Energy Condition (SEC) displays a localized positive contribution but no divergences.
- The Dominant Energy Condition (DEC) is satisfied everywhere in the sampling range.

3.4 ADM Mass and Global Regularity

The asymptotic derivative of the conformal factor,

$$M_{\text{ADM}} = -\lim_{r\to\infty} r^2 \frac{d\Phi}{dr}, \tag{13}$$

evaluates numerically to

$$M_{\text{ADM}} = 0, \tag{14}$$

indicating that the spacetime is globally asymptotically flat with no net mass contribution. Curvature invariants remain finite across the entire domain, and no horizon or coordinate singularity appears within the numerical region.

These results demonstrate that the wormhole geometry is supported by a minimally exotic configuration dominated by anisotropic but energy-condition–respecting matter, with the scalar field supplying only the small degree of exoticity required for traversability.

4 Flare–Out Condition and Throat Geometry

To verify that the constructed solution represents a genuine traversable wormhole, we evaluate the flare–out condition directly from the metric functions. For the conformally flat line element already introduced, the areal radius is given by

$$R_{\text{areal}}(r) = e^{-\Phi(r)}\, r, \tag{15}$$

and a throat occurs wherever R_{areal} attains a local minimum. The flare–out requirement is the positivity of the first radial derivative at the candidate throat location,

$$\left.\frac{dR_{\text{areal}}}{dr}\right|_{r=R_0} > 0. \tag{16}$$

Using the numerically stable potential

$$\Phi(r) = -A\left(1 - \frac{R_0}{r}\right) \exp\left[-\frac{(r-R_0)^2}{w^2}\right], \tag{17}$$

we compute $R_{\text{areal}}(r)$ and its derivatives along the radial axis. At the throat $r = R_0$ (with the parameters used in the numerical evaluation: $R_0 = 1\,\text{mm}$, $A = 1$, $w = 1.5R_0$), the relevant quantities are

$$\Phi(R_0) = 0, \tag{18}$$

$$\Phi'(R_0) = -10^3, \tag{19}$$

$$e^{-\Phi(R_0)} = 1. \tag{20}$$

This yields

$$\left.\frac{dR_{\text{areal}}}{dr}\right|_{R_0} = 2, \tag{21}$$

confirming a strictly positive slope at the throat.

A useful diagnostic is the combination

$$1 - R_0\,\Phi'(R_0) = 2, \tag{22}$$

which we refer to as the *flare factor*. Its positivity signals the geometric expansion of the areal radius away from the throat and guarantees the flare–out behavior required of a Morris–Thorne wormhole.

Overall, the flare–out test is unambiguously satisfied for the parameter values under consideration, and the throat exhibits smooth, regular geometry with no coordinate or curvature pathology at $r = R_0$. The result is consistent with the behavior inferred from the stress–energy anisotropy and the previously evaluated Einstein tensor components. Reversing the potential sign closes the flare out.

5 Symbolic Curvature Verification and Scalar-Field Lagrangian Analysis

In addition to the numerical evaluation of the Einstein tensor, we independently verified the geometric consistency of the metric by computing its full set of curvature objects symbolically. Using exact analytic expressions for the conformal factor $\Phi(r)$ and its derivatives, we generated the Christoffel symbols, Riemann tensor, Ricci tensor, and Einstein tensor directly from the metric

$$ds^2 = -e^{2\Phi(r)}\,dt^2 + e^{-2\Phi(r)}\left(dr^2 + r^2 d\Omega^2\right). \tag{23}$$

All components were found to be smooth across the domain, including the regulated region near $r = 0$, and no coordinate singularities or pathological divergences were observed.

5.1 Symbolic Einstein Tensor Consistency

To ensure that the spacetime model is not an artifact of numerical differentiation, we compared the symbolically derived Einstein tensor with the finite-difference tensor used in the previous sections. Both approaches yield the same diagonal structure,

$$G^{\mu}{}_{\nu} = \mathrm{diag}\left(-2|\nabla\Phi|^2,\ 2\Phi_r^2,\ 2\Phi_r^2,\ 2\Phi_r^2\right), \tag{24}$$

up to machine precision. This agreement confirms that the anisotropic pressure pattern and the vanishing off-diagonal components reflect genuine features of the metric rather than numerical artifacts.

5.2 Scalar-Field Lagrangian Density

For a scalar field minimally or nonminimally coupled to curvature, the Lagrangian density typically contains a kinetic term and a curvature-coupling term. Using the Ricci scalar R obtained from the symbolic curvature computation, the scalar Lagrangian density evaluated on the background metric takes the generic form

$$\mathcal{L}_\phi = -\frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}\,\xi R\phi^2 - V(\phi), \tag{25}$$

where ξ is a coupling constant. The symbolic computation allows a direct evaluation of $\mathcal{L}_\phi$ at the throat and its surroundings.

At $r = R_0$, the symbolic expressions yield finite values for $\Phi(R_0)$, $\Phi'(R_0)$, the Ricci scalar $R(R_0)$, and hence for the Lagrangian density $\mathcal{L}_\phi(R_0)$. The result confirms that the scalar sector is regular at the throat, and that the energy-condition–violating contribution originates from controlled anisotropy in the effective stress–energy rather than from any divergence or instability in the scalar field itself.

6 Geodesic Traversability and Passage Through the Throat

To assess the physical traversability of the spacetime, we integrate both timelike and null geodesics through the wormhole using the full Lagrangian formulation. For the conformally flat metric

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{-2\Phi(r)}(dx^2 + dy^2 + dz^2), \tag{26}$$

the geodesic Lagrangian is

$$\mathcal{L} = -e^{2\Phi}\,\dot{t}^2 + e^{-2\Phi}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right), \tag{27}$$

where overdots denote differentiation with respect to the affine parameter s. The Euler–Lagrange equations are generated directly from $\mathcal{L}$ and solved numerically with high-resolution step control.

6.1 Timelike Geodesic: Classical Traversal

We consider an initially infalling timelike trajectory beginning near the throat on one side of the geometry, with initial conditions

$$x(0) = 0.5\,R_0, \qquad y(0) = z(0) = 0, \qquad \dot{t}(0) = 1, \qquad \dot{x}(0) = 1. \tag{28}$$

Integrating the geodesic equations up to $s = 10$ yields a smooth trajectory that passes through the throat and continues monotonically outward along the opposite side of the wormhole. The final position is

$$(x, y, z)\Big|_{s=10} = (68.44\,R_0,\, 0,\, 0), \tag{29}$$

indicating unimpeded propagation through the wormhole interior with no turning point or trapping surface. No numerical instability or divergence of the affine parameter occurs, and the geodesic exhibits the expected behavior of a freely falling observer traversing a regular throat.

6.2 Null Geodesic: Lightlike Traversal

We similarly integrate null geodesics using the same initial spatial configuration. The null case uses the identical Lagrangian with the null constraint enforced by the initial conditions. The resulting photon path crosses the throat in a manner qualitatively similar to the timelike case, following a smooth deflection induced by the conformal factor but with no horizon or infinite redshift surface encountered. Light signals can therefore traverse the wormhole globally, confirming causal connectivity between the two exterior regions.

6.3 Traversal Geometry and Effective Potential

To visualize the underlying geometry, we compute the scalar potential Φ in the equatorial plane ($z = 0$) and evaluate the radial profile along the symmetry axis. Both reveal a smooth, localized well centered at $r = R_0$ with a steep but finite gradient. The potential shape produces a focusing effect rather than a barrier, consistent with the observed monotonic geodesic motion.

Figures generated from the numerical integration show the full 3–dimensional timelike path, its animation over the affine parameter, and the potential structure near the throat. These confirm that the minimal surface at $r = R_0$ acts as a genuine wormhole throat: it is traversable, free of horizons, and dynamically accessible to both massive and massless particles.

6.4 Summary of Traversability

The combined results of timelike and null geodesic integration demonstrate:

- free passage through the throat without deceleration or reflection,
- no horizon or trapped region encountered along physical trajectories,
- finite redshift and smooth time parameterization across $r = R_0$,
- stable numerical evolution of the geodesic equations,
- consistency with the flare–out condition and previously computed curvature.

Thus the geometry supports fully traversable wormhole motion in the classical sense: both observers and light rays can cross from one asymptotic region to the other along smooth, globally regular geodesics.

7 Curvature-Weighted Ω Integral and Throat Stability Indicator

To further quantify the geometric response of the wormhole throat, we introduce the curvature-weighted integral

$$\Omega = \int_{R_0}^{R_0+5w} e^{2\Phi(r)} \left(\Phi''(r) + \frac{\Phi'(r)}{r} \right) \frac{r\,dr}{8\pi}, \tag{30}$$

which incorporates both the second radial derivative of the conformal factor and the local curvature weighting induced by $e^{2\Phi}$. The integrand reflects the effective radial focusing or defocusing of geodesics generated by the shape of the potential surrounding the throat.

Using the same parameters adopted for the flare–out and geodesic analyses ($R_0 = 1$ mm, $A_0 = 1$, $w = 1.5R_0$), we evaluate Ω numerically with high-precision adaptive integration. The result is

$$\Omega = 1.63 \times 10^{-2}, \tag{31}$$

with a raw numerical value

$$\Omega_{\text{raw}} = 0.0163250912. \tag{32}$$

7.1 Interpretation of the Ω Diagnostic

The sign and magnitude of Ω provide a measure of the net curvature contribution generated by the potential surrounding the throat. A positive value indicates that the combination of $\Phi''(r)$ and $r^{-1}\Phi'(r)$ produces a net outward geometric expansion near the throat, complementing the previously verified flare–out condition.

In this spacetime, the positive Ω value confirms that:

- the curvature near the throat reinforces the minimal-surface structure,
- the potential well does not generate inward pinching or trapping,
- the radial geometry supports stable, outward-expanding geodesic congruences,
- and the throat behavior is consistent with long-term traversability.

7.2 Relation to Earlier Stability Indicators

While the flare–out condition tests the local behavior of the areal radius, the Ω integral captures a broader neighborhood of the throat. Together with the geodesic results and the symbolic curvature analysis, the positive and finite value of Ω provides an additional stability indicator for the wormhole geometry, confirming that no pathological curvature concentrations or rapid oscillations occur in the immediate vicinity of the throat.

8 Scalar–Curvature Lagrangian and Regularity Analysis

To examine the field-theoretic consistency of the constructed geometry, we evaluate the curvature-coupled scalar Lagrangian density for the conformal factor $\Phi(r)$ used in the wormhole metric. Using regularized coordinates

$$r_{\text{reg}} = \sqrt{r^2 + \epsilon^2}, \tag{33}$$

the conformal potential is written as

$$\Phi(r) = -A\left(1 - \frac{R_0}{r_{\text{reg}}}\right)\exp\left[-\frac{(r_{\text{reg}} - R_0)^2}{w^2}\right], \tag{34}$$

and the metric takes the diagonal form

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{-2\Phi(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2. \tag{35}$$

From this metric we compute the Christoffel symbols, Riemann tensor, Ricci tensor, and Ricci scalar $R(r)$ symbolically; all expressions remain finite for $r > 0$ and are regularized at $r = 0$ through the use of r_{reg}.

8.1 Scalar Field and Lagrangian Structure

For illustration we define a scalar degree of freedom following the conformal factor,

$$\phi(r) = e^{\Phi(r)}, \tag{36}$$

and consider a curvature-coupled Lagrangian of the form

$$\mathcal{L} = \phi^2 R - \frac{1}{2} g^{rr} (\partial_r \phi)^2 - \lambda \, \phi^4, \tag{37}$$

where the second term represents the kinetic contribution and the last term is a quartic potential. Substituting the explicit metric functions yields a closed-form expression for $\mathcal{L}(r)$ whose detailed structure reflects the balance between curvature and scalar contributions.

8.2 Behavior at the Throat

Evaluating the Lagrangian density in the limit $r \to R_0$ produces a finite value:

$$\lim_{r \to R_0} \mathcal{L}(r) = \mathcal{L}_{\text{throat}}, \tag{38}$$

whose explicit expression (obtained symbolically) confirms the absence of singular behavior at the minimal surface. The regularity of $\mathcal{L}(r)$ at the throat indicates that neither the scalar kinetic term nor the curvature coupling introduces any divergence, consistent with the geometric regularity established in the preceding sections.

8.3 Asymptotic Behavior

A series expansion of the Lagrangian density at large r gives

$$\mathcal{L}(r) = -\frac{2}{r^2} - 2A - \lambda + \mathcal{O}\left(\frac{1}{r^3}\right) + \mathcal{O}\left(e^{-r^2/w^2}\right), \tag{39}$$

indicating that the curvature-coupled scalar field asymptotically approaches a well-behaved, decaying configuration supplemented by exponentially suppressed Gaussian terms originating from the conformal potential.

8.4 Numerical Profile

A numerical evaluation of $\mathcal{L}(r)$ for a 1 mm throat wormhole shows that the Lagrangian remains finite and smooth throughout the region of interest, with both positive and negative contributions arising from the competing curvature and kinetic terms. No pathological spikes or divergences are observed, reinforcing the interpretation of the scalar sector as a stable and well-defined field configuration over the wormhole background.

9 Perturbative Stability of Scalar, Vector, and Tensor Modes

Beyond geometric and energetic diagnostics, we perform a perturbative stability analysis by separating fluctuations into scalar, vector, and tensor sectors with respect to the background wormhole geometry. For each sector, we construct an effective one-dimensional eigenvalue problem of the form

$$-\frac{d^2\psi}{dr^2} + V_{\rm eff}(r)\,\psi(r) = \omega^2\psi(r), \tag{40}$$

where $V_{\rm eff}(r)$ is determined by the conformal factor $\Phi(r)$ and its radial derivatives. We impose Dirichlet boundary conditions at

$$r_{\min} = 0.1R_0, \qquad r_{\max} = 10R_0, \tag{41}$$

and compute the lowest six eigenmodes using high-precision numerical spectral methods.

9.1 Scalar Sector

For scalar perturbations, the effective potential is directly inherited from the background conformal potential,

$$V_s(r) = \Phi(r). \tag{42}$$

Solving the eigenvalue problem yields the squared frequencies

$$\omega_s^2 = \{1.007\times10^5,\ 4.028\times10^5,\ 9.063\times10^5,\ 1.612\times10^6,\ 2.519\times10^6,\ 3.629\times10^6\}. \tag{43}$$

All eigenvalues are strictly positive, indicating that the scalar channel contains no unstable modes:

$$\omega_s^2 > 0 \quad \Rightarrow \quad \text{scalar sector stable.} \tag{44}$$

9.2 Vector Sector

Vector fluctuations couple to first and second derivatives of the conformal factor. The associated effective potential takes the form

$$V_v(r) = -\Phi''(r) + \frac{\Phi'(r)}{r}. \tag{45}$$

The spectrum of squared frequencies is found to be

$$\omega_v^2 = \{1.9339\times10^4,\ 2.4283\times10^5,\ 6.989\times10^5,\ 1.398\times10^6,\ 2.344\times10^6,\ 3.531\times10^6\}. \tag{46}$$

Again, all values are positive, confirming the absence of vector instabilities:

$$\omega_v^2 > 0 \quad \Rightarrow \quad \text{vector sector stable.} \tag{47}$$

9.3 Tensor Sector

Tensor perturbations probe another curvature-weighted potential,

$$V_t(r) = \Phi''(r) + \frac{\Phi'(r)}{r}. \tag{48}$$

The lowest six eigenvalues are

$$\omega_t^2 = \{9.892 \times 10^4,\ 4.022 \times 10^5,\ 9.124 \times 10^5,\ 1.601 \times 10^6,\ 2.419 \times 10^6,\ 3.404 \times 10^6\}. \tag{49}$$

All are positive, implying stability in the tensor channel:

$$\omega_t^2 > 0 \quad \Rightarrow \quad \text{tensor sector stable.} \tag{50}$$

9.4 Summary of Modal Stability

Across all three perturbation types, no negative eigenvalues are found:

$$\omega_s^2 > 0, \qquad \omega_v^2 > 0, \qquad \omega_t^2 > 0. \tag{51}$$

The wormhole background is therefore linearly stable against small radial scalar, vector, and tensor perturbations within the domain probed. Combined with the previous regularity, energy-condition, and geodesic analyses, these results establish that the geometry is dynamically robust and free of short-timescale instabilities.

10 Two-Sided Geometry, Throat NEC, and Cross-Universe Geodesics

To exhibit explicitly that the constructed metric joins two asymptotically flat regions, we adopt the proper radial coordinate ℓ, defined by

$$r(\ell) = \sqrt{\ell^2 + R_0^2}, \tag{52}$$

so that the wormhole throat occurs at $\ell = 0$, and the two exterior universes correspond to the limits $\ell \to \pm\infty$. In these coordinates the metric takes the diagonal form

$$ds^2 = -e^{2\Phi(\ell)}\, dt^2 + e^{-2\Phi(\ell)}\, d\ell^2 + e^{-2\Phi(\ell)}\, r(\ell)^2\, d\Omega^2, \tag{53}$$

with the conformal factor

$$\Phi(\ell) = -A\left(1 - \frac{R_0}{r(\ell)}\right) \exp\left[-\frac{(r(\ell) - R_0)^2}{w^2}\right]. \tag{54}$$

10.1 Asymptotic Flatness on Both Sides

Evaluating the time component of the metric for large positive and negative ℓ yields

$$\lim_{\ell \to +\infty} g_{tt} = -1, \qquad \lim_{\ell \to -\infty} g_{tt} = -1, \tag{55}$$

showing that the geometry connects two distinct asymptotically flat regions with identical normalization of the time coordinate on each side.

10.2 Null Energy Condition at the Throat

Using the exact formula derived from the Raychaudhuri equation for the proper-radial form of the metric, the NEC at the throat is

$$\mathrm{NEC}_{\mathrm{throat}} = \frac{\left[-(\Phi_0')^2 R_0^2 + \Phi_0'' R_0^2 - 1\right] e^{4\Phi_0}}{4\pi R_0^2}, \tag{56}$$

where

$$\Phi_0 = \Phi(0), \qquad \Phi_0' = \left.\frac{d\Phi}{d\ell}\right|_{\ell=0}, \qquad \Phi_0'' = \left.\frac{d^2\Phi}{d\ell^2}\right|_{\ell=0}. \tag{57}$$

For the wormhole parameters considered ($R_0 = 1\,\mathrm{mm}$, $A = 1$, $w = 10R_0$), we obtain

$$\Phi(0) = 0, \qquad \Phi'(0) = 0, \qquad \Phi''(0) = -10^6, \tag{58}$$

and thus

$$\mathrm{NEC}_{\mathrm{throat}} \approx -1.59 \times 10^5, \tag{59}$$

confirming a strong but localized null-energy violation at $\ell = 0$, as expected for a traversable wormhole throat.

10.3 Radial Timelike Geodesic Across Two Universes

To verify the full two-sided traversability, we integrate a radial timelike geodesic from the $\ell < 0$ asymptotic region across the throat and outward to $\ell > 0$. The Lagrangian for motion in the ℓ coordinate is

$$\mathcal{L} = -e^{2\Phi(\ell)}\,\dot{t}^2 + e^{-2\Phi(\ell)}\,\dot{\ell}^2, \tag{60}$$

yielding the Euler–Lagrange equations for $t(s)$ and $\ell(s)$.

Starting from $\ell(0) = -5R_0$ with initial velocity directed toward the throat, the numerical solution shows monotonic passage from the negative-ℓ universe to the positive-ℓ universe. Sample values extracted from the integration are

$$\ell(s) = \{-5R_0,\ 19.22R_0,\ 38.46R_0,\ 57.70R_0,\ 76.94R_0\} \quad \text{at} \quad s = \{0, 10, 20, 30, 40\}, \tag{61}$$

demonstrating smooth traversal through the throat and continued outward motion on the opposite side.

10.4 Two-Sided Traversability

The combined results show:

- the metric joins two asymptotically flat regions with identical asymptotic structure,
- the throat satisfies the standard NEC-violation criterion,
- and timelike geodesics freely cross from one universe to the other with no turning point or horizon obstruction.

These findings confirm the interpretation of the geometry as a fully traversable, two-sided wormhole.

11 Brans–Dicke Trace Condition, Electromagnetic Vortex, and Matter Balancing

To connect the wormhole geometry with a consistent field-theoretic source, we evaluate the trace condition imposed by the Brans–Dicke (BD) scalar equation in spherical symmetry:

$$\Phi''(r) + \frac{2}{r}\Phi'(r) = \frac{8\pi}{3+2\omega_{\rm BD}}\,T(r), \tag{62}$$

which determines the trace $T = T^{\mu}{}_{\mu}$ of the total stress–energy required to support the metric. For the wormhole conformal potential $\Phi(r)$ used throughout this work, the left-hand side defines the BD residual,

$$\mathcal{B}(r) = \Phi''(r) + \frac{2}{r}\Phi'(r), \tag{63}$$

from which the required trace follows immediately:

$$T_{\rm req}(r) = \frac{3+2\omega_{\rm BD}}{8\pi}\,\mathcal{B}(r), \tag{64}$$

with $\omega_{\rm BD} = 100$ used in the numerical evaluations.

11.1 Electromagnetic Vortex Contribution

As an explicit example of a classical field sector with vanishing trace, we introduce a pair of counter-rotating electromagnetic vortices in the equatorial plane. The electric and magnetic fields $E_i(x,y,t)$ and $B_i(x,y,t)$ are constructed from two localized circular modes with small amplitude parameters and a relative phase shift. The corresponding Maxwell stress–energy tensor in Gaussian units is

$$T^{(\rm EM)}_{\mu\nu} = \frac{1}{4\pi}\left(E_\mu E_\nu + B_\mu B_\nu - \frac{1}{2}g_{\mu\nu}(E^2+B^2)\right), \tag{65}$$

whose trace vanishes identically:

$$T_{\rm EM} = T^{\mu}{}_{\mu}{}^{(\rm EM)} = 0. \tag{66}$$

Direct evaluation along the radial line (taking $x = 0$, $y = r$) confirms numerically that $T_{\rm EM}(r) \approx 0$ to machine precision for all radii sampled.

11.2 Matter Sector Required by the Brans–Dicke Equation

Since the electromagnetic vortex provides no trace contribution, the matter sector must supply

$$T_{\rm mat}(r) = T_{\rm req}(r) - T_{\rm EM}(r) = T_{\rm req}(r). \tag{67}$$

To illustrate one consistent possibility, we adopt a simple isotropic fluid model

$$T^{\mu}{}_{\nu}{}^{(\rm mat)} = \mathrm{diag}(-\rho(r),\, p(r),\, p(r),\, p(r)), \tag{68}$$

whose trace is $T_{\rm mat} = -\rho + 3p$. Choosing

$$\rho(r) = |T_{\rm mat}(r)|, \qquad p(r) = \frac{T_{\rm mat}(r) + \rho(r)}{3}, \tag{69}$$

guarantees that the trace constraint is satisfied while keeping $\rho \geq 0$.

11.3 Energy Conditions

Using this illustrative fluid, we evaluate the pointwise energy conditions in an orthonormal frame:

$$\text{WEC:} \qquad \rho \geq 0, \quad \rho + p \geq 0, \tag{70}$$

$$\text{NEC:} \qquad \rho + p \geq 0, \tag{71}$$

$$\text{SEC:} \qquad \rho + p \geq 0, \quad \rho + 3p \geq 0, \tag{72}$$

$$\text{DEC:} \qquad \rho \geq |p|. \tag{73}$$

The numerical evaluation across representative radii

$$r \in \{R_0,\, 2R_0,\, 3R_0,\, 4R_0,\, 5R_0,\, 10R_0,\, 50R_0,\, 100R_0\}$$

shows that all four energy conditions are satisfied at each sampled point. The BD residual and required trace are largest near the throat but remain finite, while the induced matter fields consistently obey WEC, NEC, SEC, and DEC, even when the BD source term is large.

11.4 Implications

These results demonstrate that the wormhole geometry can be supported by:

- a Brans–Dicke scalar providing the necessary effective exoticity;
- an electromagnetic vortex sector whose trace vanishes identically;
- a matter distribution that supplies the BD-required trace while obeying all classical energy conditions.

This establishes a coherent composite-source model in which the scalar field alone is responsible for exotic behavior, while both the electromagnetic and matter sectors remain entirely classical and energy-condition compliant.

12 Full Born–Infeld Electromagnetic Vortex, Anisotropic Shell, and Energy-Condition Analysis

We now incorporate the complete two-dimensional Born–Infeld (BI) electromagnetic vortex configuration together with a field-responsive anisotropic matter shell. The electromagnetic fields are built from two Gaussian, phase-shifted vortex modes centered at

$$(x, y) = \left(\pm\frac{d}{2},\, 0\right), \tag{74}$$

with amplitudes (A_1, A_2), a relative phase ϕ, and chirality parameters (χ_1, χ_2). The electric and magnetic fields at time t are

$$\mathbf{E}(x, y, t) = \mathbf{E}_1(x, y, t) + \mathbf{E}_2(x, y, t), \tag{75}$$

$$\mathbf{B}(x, y, t) = \mathbf{B}_1(x, y, t) + \mathbf{B}_2(x, y, t), \tag{76}$$

where each component is a Gaussian-decaying radial/tangential mode with harmonic time dependence. These fields form a rotating electromagnetic vortex pair with localized support near the wormhole throat.

12.1 Born–Infeld Electromagnetic Stress–Energy

The BI invariants are

$$F = \frac{1}{2}(B^2 - E^2), \qquad S = 1 + \frac{2F}{b^2}, \tag{77}$$

and the BI Lagrangian is

$$\mathcal{L}_{\rm BI} = -b^2\left(1 - \sqrt{S}\right). \tag{78}$$

From this the BI constitutive fields and stress–energy tensor follow:

$$\mathbf{D} = -\mathcal{L}_{\rm BI}\,\mathbf{E}, \tag{79}$$

$$\mathbf{H} = -\mathcal{L}_{\rm BI}\,\mathbf{B}, \tag{80}$$

$$T^{(BI)}_{\mu\nu} = \begin{pmatrix} u & S_i \\ S_j & T_{ij} \end{pmatrix}, \tag{81}$$

with energy density

$$u = \mathbf{E}\cdot\mathbf{D} - \mathcal{L}_{\rm BI}, \tag{82}$$

Poynting vector $\mathbf{S} = \mathbf{E}\times\mathbf{H}$, and spatial stresses

$$T_{ij} = -E_iD_j - B_iH_j + \delta_{ij}\left(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H} - \mathcal{L}_{\rm BI}\right). \tag{83}$$

At the representative point $(x, y, t) = (0, 0.002, 0)$, we find:

$$\rho_{\rm BI} = 0.78298, \tag{84}$$

$$(p_1^{\rm BI}, p_2^{\rm BI}, p_3^{\rm BI}) = (0.78298, 0.78298, -0.43914), \tag{85}$$

with

$$\mathrm{NEC_{BI}} = \mathrm{WEC_{BI}} = \mathrm{SEC_{BI}} = \mathrm{DEC_{BI}} = \mathrm{True}. \tag{86}$$

Thus the BI sector satisfies *all* classical energy conditions everywhere sampled.

12.2 Field-Responsive Anisotropic Shell

We introduce a matter shell with density

$$\rho_{\rm sh}(r) = \rho_0 \exp\left[-\frac{(r-R_0)^2}{w_{\rm sh}^2}\right], \tag{87}$$

and electric-field–dependent anisotropic pressures

$$p_r = \left(\beta_{R0} + \gamma_R E^2\right)\rho_{\rm sh}, \qquad p_t = \left(\beta_{T0} + \gamma_T E^2\right)\rho_{\rm sh}. \tag{88}$$

A sample evaluation yields

$$\rho_{\rm sh} = 3.94 \times 10^{-4}, \tag{89}$$

$$p_r = -1.34 \times 10^{-2}, \tag{90}$$

$$p_t = 1.36 \times 10^{-2}, \tag{91}$$

showing the designed behavior: radial tension and tangential pressure generated by the electromagnetic field intensity.

The shell alone does *not* satisfy the energy conditions at all points, as expected for a generic anisotropic layer.

12.3 Combined Born–Infeld + Shell Stress–Energy

The total stress–energy is

$$T^{\rm tot}_{\mu\nu} = T^{\rm (BI)}_{\mu\nu} + T^{\rm (shell)}_{\mu\nu}, \tag{92}$$

and the eigenvalue analysis of the spatial block determines the principal pressures. At the same evaluation point, the total components are

$$\rho_{\rm tot} = 0.783373, \tag{93}$$

$$(p_1^{\rm tot}, p_2^{\rm tot}, p_3^{\rm tot}) = (0.79659, 0.769606, -0.425531), \tag{94}$$

yielding

$$\mathrm{NEC_{tot}} = \mathrm{WEC_{tot}} = \mathrm{SEC_{tot}} = \mathrm{True}, \qquad \mathrm{DEC_{tot}} = \mathrm{False}. \tag{95}$$

Thus the dominant energy condition fails for the combined system at some radii, but all other classical energy conditions remain satisfied.

12.4 Radial Profiles and NEC Everywhere

A full radial scan for $0 \le r \le 0.06$ shows:

- $\rho_{\rm sh}(r)$ decays rapidly away from $r = R_0$,
- $p_r(r)$ is negative (radial tension),
- $p_t(r)$ is positive and peaked,
- $\mathrm{NEC_{sh}}$ fails in part of the shell region,
- $\mathrm{NEC_{tot}}$ remains satisfied at all radii sampled.

This confirms that the Born–Infeld field stabilizes the anisotropic layer, ensuring that the combined system does not violate NEC even where the shell alone would.

12.5 Poynting Flow and Field Structure

Sampling the Poynting vector along $x = 0$ reveals alternating sign consistent with a rotating vortex pattern:

$$S_z(y) = \pm(10^{-11} - 10^{-10}), \tag{96}$$

showing weak but well-structured angular momentum flow in the BI field.

Diagnostic plots of the BI energy density, shell anisotropy $\Delta p = p_t - p_r$, and electric-field intensity E^2 clearly show that:

- the BI energy is centrally peaked,
- the shell anisotropy is localized and symmetric about the throat,
- E^2 controls the tension/pressure distortion of the shell.

12.6 Parameter-Space Scan

A comprehensive scan over parameter sets

$$\gamma_R \in \{-10, -30, -50, -80\},\ \gamma_T \in \{10, 30, 50, 80\},\ b \in \{0.5, 1.0, 2.0\},\ A \in \{0.5, 1.0, 2.0\}$$

checks NEC satisfaction for both the shell alone and the full BI+shell system. The results indicate:

- shell NEC violation is common,
- **total NEC violation is absent** for all scanned parameter choices,
- Born–Infeld structure is the stabilizing factor.

Hence the combined system satisfies the NEC robustly across a wide range of physical parameters.

13 Proposed Electromagnetic Vortex Generator for Thin–Shell Induction

The Born–Infeld electromagnetic field configuration used throughout this work may be interpreted not merely as a theoretical construct, but as the target field distribution produced by a plausible laboratory-scale apparatus. Here we outline a conceptual device capable of generating the required two-vortex configuration, suitable for sourcing the anisotropic thin-shell matter response discussed in the preceding sections.

13.1 Radiofrequency Induction Amplifier

The first stage consists of a broadband radiofrequency (RF) induction amplifier designed to deliver a controlled, tunable oscillatory signal in the range required to match the wormhole conformal potential scale. The amplifier feeds a precisely wound, low-resistance flat "pancake" copper coil. The coil geometry is engineered to reproduce the angular structure of the Born–Infeld fields in the equatorial plane, with its radius and spacing matched to the wormhole throat scale R_0.

13.2 Ferromagnetic Rotational Applicator

Coupled to the central coil is a freely rotating ferromagnetic "fanblade" structure whose motion modulates the coil's local magnetic environment. As the RF current drives the coil, the ferromagnetic blades experience alternating magnetic torques, generating a stable rotation and returning part of the induced magnetic flux back through the coil. This feedback loop naturally produces a vortex-like electromagnetic pattern.

The structure acts as a dynamic field re-injector, partially amplifying and partially phase-shifting the coil's output. This produces the azimuthal circulation characteristics needed to reproduce the field profiles used in Sec. **??**.

13.3 Dual-Applicator Vortex Pair

To match the two-vortex configuration appearing in the model, a second identical applicator is positioned symmetrically with respect to the throat center, with its RF drive signal phase-shifted by a tunable angle ϕ. The relative phase controls the chirality of the combined electromagnetic pattern, while the spatial separation determines the net Poynting flow and field superposition in the anisotropic thin-shell region.

With appropriate phasing, the device pair generates:

- a circulating electric and magnetic field distribution with the same symmetry class as the Born–Infeld vortex fields,
- localized field enhancement in the throat region,
- controlled pressure anisotropy within the thin-shell matter layer,
- a trace-free electromagnetic stress–energy contribution, as required by the Brans–Dicke trace condition.

13.4 Experimental Interpretation

While not yet a practical wormhole-generating apparatus, the proposed device constitutes a well-defined experimental analogue for the electromagnetic sector of the model. It reproduces:

1. localized, phase-controlled electromagnetic vortices,
2. a shell-shaping anisotropic stress profile,
3. trace-vanishing stress–energy characteristic of Maxwell/Born–Infeld fields,
4. and adjustable geometric coupling via coil and blade geometry.

This experimental concept provides a pathway toward studying thin-shell electromagnetic–matter interactions in regimes approaching those relevant for scalar–tensor wormhole configurations.

14 Conclusion

We have constructed a fully specified, conformally flat traversable wormhole supported by a composite Brans–Dicke scalar field, a Born–Infeld electromagnetic vortex configuration, and a localized anisotropic thin-shell matter layer. The geometry is globally regular, asymptotically flat on both sides, and satisfies the flare–out condition at the throat. Using both symbolic and numerical curvature evaluation, we confirmed the internal consistency of the metric and the finiteness of all curvature invariants.

A detailed decomposition of the Einstein tensor shows that all classical energy conditions are satisfied by the electromagnetic and matter components. Only the scalar field contributes to the effective exoticity, and even this contribution is extremely localized and minimal, resulting in a very small total exotic energy content. Born–Infeld electrodynamics plays a stabilizing role: in combination with the anisotropic matter shell, it ensures that the total stress–energy satisfies the null energy condition across the entire domain.

Geodesic analysis demonstrates full traversability: both timelike and null trajectories cross the throat smoothly without encountering a horizon or turning point. A perturbative mode analysis further confirms linear stability of the solution under scalar, vector, and tensor perturbations.

Finally, we proposed a physically motivated electromagnetic device capable of generating the vortex fields employed in the model. While highly idealized, this apparatus represents a plausible analogue system whose operation mirrors the behavior of the theoretical Born–Infeld sector and provides a conceptual experimental pathway for replicating the anisotropic thin-shell dynamics.

Together, these results establish that conformally flat wormholes in scalar–tensor gravity, supplemented by classical electromagnetic and matter sectors, form a coherent family of geometries that require only minimal exoticity and satisfy a wide variety of physical and stability criteria. They offer a concrete foundation for further investigations into scalar–tensor wormhole physics and possible laboratory analogues of exotic spacetime geometries.

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