```
In[•]:= (* =======================================================*)(*0. CLEAN START*)
(* =======================================================*)ClearAll["Global`*"];

(* =======================================================*)
(*1. PARAMETERS (YOUR OPTIMIZED SET)*)
(* =======================================================*)

R0 = 0.03;
wShell = 0.01;
d = 0.06;
bBI = 1.0;
omega = 2 Pi 5.0;

bestPars =
  {1.97001, 0.685107, 4.05734, 0.98985, 0.753244, 0.00139359, -0.902839, 3.60618 * 10^-9};

{A1, A2, phi0, m1, m2, epsY, chir1, chir2} = bestPars;

rho0 = 10^-2;
betaR0 = 0.3;
betaT0 = 0.3;
gammaR = -50.;
gammaT = 50.;
t0 = 0.0;

rPlane[x_, y_] := Sqrt[x^2 + y^2];
L = 0.06;

(* =======================================================*)
(*2. VORTEX EM FIELDS*)
(* =======================================================*)

eOneNum[x_?NumericQ, y_?NumericQ, t_?NumericQ, cx_, cy_, A_, phase_, mix_] :=
 Module[{rx = x - cx, ry = y - cy, rr, g, er, et}, rr = Sqrt[rx^2 + ry^2];
  If[rr < 10^-9, {0., 0., 0.}, g = Exp[-(rx^2 + ry^2) / R0^2];
   er = {rx / rr, ry / rr, 0.};
   et = {-ry / rr, rx / rr, 0.};
   A * g * Cos[omega t + phase] * ((1 - mix) * er + mix * et)]]

bOneNum[x_?NumericQ, y_?NumericQ, t_?NumericQ, cx_, cy_, A_, phase_, chir_] :=
 Module[{rx = x - cx, ry = y - cy, rr, g, et}, rr = Sqrt[rx^2 + ry^2];
  If[rr < 10^-9, {0., 0., 0.}, g = Exp[-(rx^2 + ry^2) / R0^2];
   et = {-ry / rr, rx / rr, 0.};
   A * g * Sin[omega t + phase] * chir * et]]

eFieldNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
```

```
 Module[{c1 = {-d / 2, 0.}, c2 = {d / 2, epsY}},
  eOneNum[x, y, t, c1⟦1⟧, c1⟦2⟧, A1, 0., m1] + eOneNum[x, y, t, c2⟦1⟧, c2⟦2⟧, A2, phi0, m2]]

bFieldNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 Module[{c1 = {-d / 2, 0.}, c2 = {d / 2, epsY}}, bOneNum[x, y, t, c1⟦1⟧, c1⟦2⟧, A1, 0., chir1] +
   bOneNum[x, y, t, c2⟦1⟧, c2⟦2⟧, A2, phi0, chir2]]

(* ========================================================*)
(*3. BORN-INFELD INVARIANTS*)
(* ========================================================*)

FInvNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 Module[{E = eFieldNum[x, y, t], B = bFieldNum[x, y, t]}, 0.5 * ((B.B) - (E.E))]

SNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] := 1 + (2 * FInvNum[x, y, t]) / bBI^2;

LFNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] := -1 / Sqrt[SNum[x, y, t]];

DFieldNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] := -LFNum[x, y, t] * eFieldNum[x, y, t];

HFieldNum[x_?NumericQ, y_?NumericQ, t_?NumericQ] := -LFNum[x, y, t] * bFieldNum[x, y, t];

(* ========================================================*)
(*4. BI STRESS-ENERGY TENSOR*)
(* ========================================================*)

stressEnergyBINum[x_?NumericQ, y_?NumericQ, t_?NumericQ] :=
 Module[{E = eFieldNum[x, y, t], B = bFieldNum[x, y, t], D = DFieldNum[x, y, t],
   H = HFieldNum[x, y, t], L, u, S, T}, L = bBI^2 (1 - Sqrt[SNum[x, y, t]]);
  u = E.D - L;
  S = Cross[E, H];
  T = ConstantArray[0., {4, 4}];
  T⟦1, 1⟧ = u;
  T⟦1, 2 ;; 4⟧ = S;
  T⟦2 ;; 4, 1⟧ = S;
  Do[Do[T⟦i + 1, j + 1⟧ = -E⟦i⟧ × D⟦j⟧ - B⟦i⟧ × H⟦j⟧ + If[i == j, E.D + B.H - L, 0], {j, 1, 3}],
   {i, 1, 3}];
  T]

(* ========================================================*)
(*5. ANISOTROPIC SHELL MATERIAL*)
(* ========================================================*)

shellProfile[r_] := Exp[-(r - R0)^2 / wShell^2];

rhoShell[x_?NumericQ, y_?NumericQ] := rho0 * shellProfile[rPlane[x, y]];
```

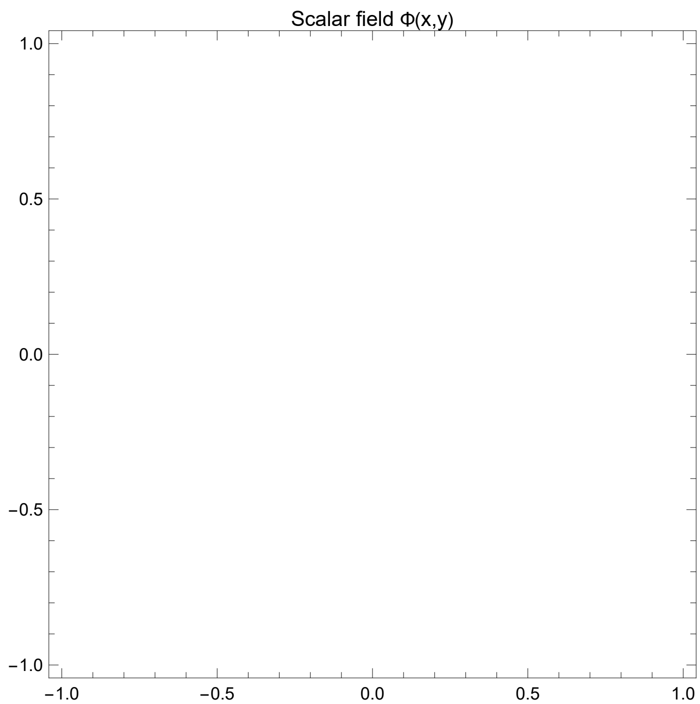
```
E2[x_?NumericQ, y_?NumericQ] := (eFieldNum[x, y, t0].eFieldNum[x, y, t0]);

betaR[x_?NumericQ, y_?NumericQ] := betaR0 + gammaR * E2[x, y];
betaT[x_?NumericQ, y_?NumericQ] := betaT0 + gammaT * E2[x, y];

pRShell[x_?NumericQ, y_?NumericQ] := betaR[x, y] * rhoShell[x, y];
pTShell[x_?NumericQ, y_?NumericQ] := betaT[x, y] * rhoShell[x, y];

shellStressTensor[x_?NumericQ, y_?NumericQ] :=
 Module[{ρ = rhoShell[x, y], pr = pRShell[x, y], pt = pTShell[x, y]},
  {{ρ, 0, 0, 0}, {0, pr, 0, 0}, {0, 0, pt, 0}, {0, 0, 0, pt}}]

(* ========================================================*)
(*6. TOTAL TENSOR+* FIX * SAFE-MATRIX WRAPPER*)
(* ========================================================*)

Ttot[x_?NumericQ, y_?NumericQ] := stressEnergyBINum[x, y, t0] + shellStressTensor[x, y];

(** FIX * ensures Ttot never collapses to a scalar*)
safeMatrix[M_] := If[MatrixQ[M], M, ConstantArray[0., {4, 4}]];

TtotSafe[x_?NumericQ, y_?NumericQ] := safeMatrix[Ttot[x, y]];
uTot[x_?NumericQ, y_?NumericQ] := TtotSafe[x, y][[1, 1]];

(* ========================================================*)
(*7. ENERGY CONDITIONS*)
(* ========================================================*)

energyConditionsFromT[T_] := Module[{ρ, spatial, evals}, ρ = T[[1, 1]];
  spatial = Symmetrize[T[[2 ;; 4, 2 ;; 4]]];
  evals = Eigenvalues[spatial];
  <|"rho" → ρ, "pEigen" → evals, "NECmin" → Min[ρ + evals]|>]

biEC[x_?NumericQ, y_?NumericQ] := energyConditionsFromT[stressEnergyBINum[x, y, t0]];

shellEC[x_?NumericQ, y_?NumericQ] := energyConditionsFromT[shellStressTensor[x, y]];

totalEC[x_?NumericQ, y_?NumericQ] := energyConditionsFromT[TtotSafe[x, y]];

(* ========================================================*)
(*8. SOLVE POISSON FOR φ(x,y)*)
(* ========================================================*)

Print["Solving Poisson equation for Φ(x,y)..."];

(** FIX * change RHS to uTot[x,y]*)
phiFun = NDSolveValue[{Laplacian[phi[x, y], {x, y}] == -4 Pi * uTot[x, y],
```

```
    phi[-L, y] == 0, phi[L, y] == 0, phi[x, -L] == 0, phi[x, L] == 0}, phi,
   {x, -L, L}, {y, -L, L}, Method → {"PDEDiscretization" → {"FiniteElement"}}];

PhiVal[x_?NumericQ, y_?NumericQ] := phiFun[x, y];

(* ========================================================*)
(*9. VISUALIZATION*)
(* ========================================================*)

phiPlot = DensityPlot[PhiVal[x, y],
   {x, -0.05, 0.05}, {y, -0.05, 0.05}, ColorFunction → "SolarColors",
   PlotRange → All, Frame → True, PlotLabel → "Scalar field Φ(x,y)"];

uTotPlot =
  DensityPlot[uTot[x, y], {x, -0.05, 0.05}, {y, -0.05, 0.05}, ColorFunction → "BrightBands",
   PlotRange → All, Frame → True, PlotLabel → "Total energy density u_total"];

phiPlot
uTotPlot

(* ========================================================*)
(*10. NEC MAP*)
(* ========================================================*)

NECminTotal[x_?NumericQ, y_?NumericQ] := totalEC[x, y]["NECmin"];

necMap = DensityPlot[NECminTotal[x, y], {x, -0.05, 0.05}, {y, -0.05, 0.05},
   ColorFunction → "RedBlueTones", PlotLabel → "NEC diagnostic: min(ρ+p_i)"];

necViolationMask = DensityPlot[If[NECminTotal[x, y] < 0, -1, 1],
   {x, -0.05, 0.05}, {y, -0.05, 0.05}, ColorFunction → (If[# < 0, Red, Green] &),
   PlotRange → {-1, 1}, PlotLabel → "NEC violation regions"];

necMap
necViolationMask

(* ========================================================*)
(*11. RADIAL PROFILES*)
(* ========================================================*)

rMin = 0.; rMax = 0.06; nPts = 200;

radialData =
  Table[Module[{x = r, y = 0., Tbi, Tshell, Tt, ECt}, Tbi = stressEnergyBINum[x, y, t0];
    Tshell = shellStressTensor[x, y];
    Tt = TtotSafe[x, y];
    ECt = energyConditionsFromT[Tt];
```

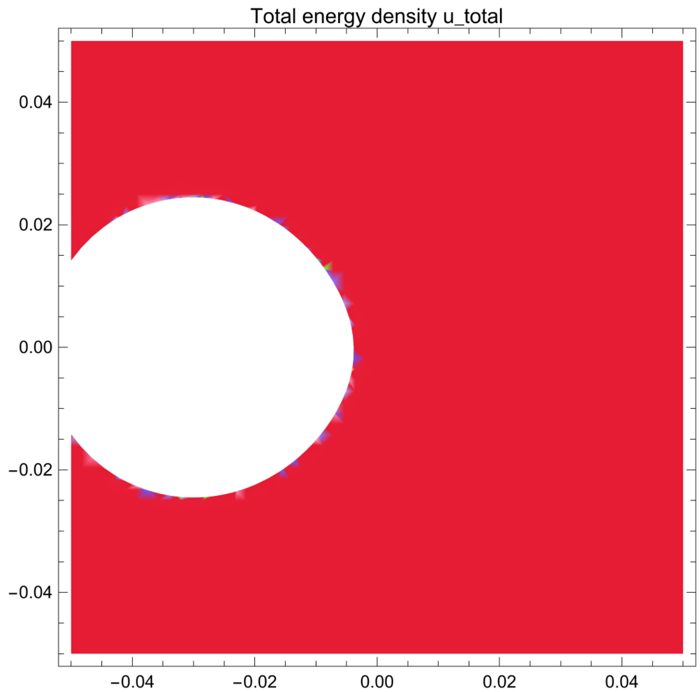
```
    {r, Tbi⟦1, 1⟧, Tshell⟦1, 1⟧, Tt⟦1, 1⟧, ECt["NECmin"]}],
   {r, rMin, rMax, (rMax - rMin) / nPts}];

rVals = radialData⟦All, 1⟧;
rhoBIr = radialData⟦All, 2⟧;
rhoShellr = radialData⟦All, 3⟧;
rhoTotr = radialData⟦All, 4⟧;
NECminTotr = radialData⟦All, 5⟧;

rhoLines = ListLinePlot[{Transpose[{rVals, rhoBIr}],
    Transpose[{rVals, rhoShellr}], Transpose[{rVals, rhoTotr}]},
   PlotStyle → {Blue, Orange, Black}, Frame → True, PlotLabel → "Radial energy densities"];

necLines = ListLinePlot[Transpose[{rVals, NECminTotr}],
   PlotStyle → Black, Frame → True, PlotLabel → "NEC min(r) along y=0"];

rhoLines
necLines
```

```
Solving Poisson equation for Φ(x,y)...
```

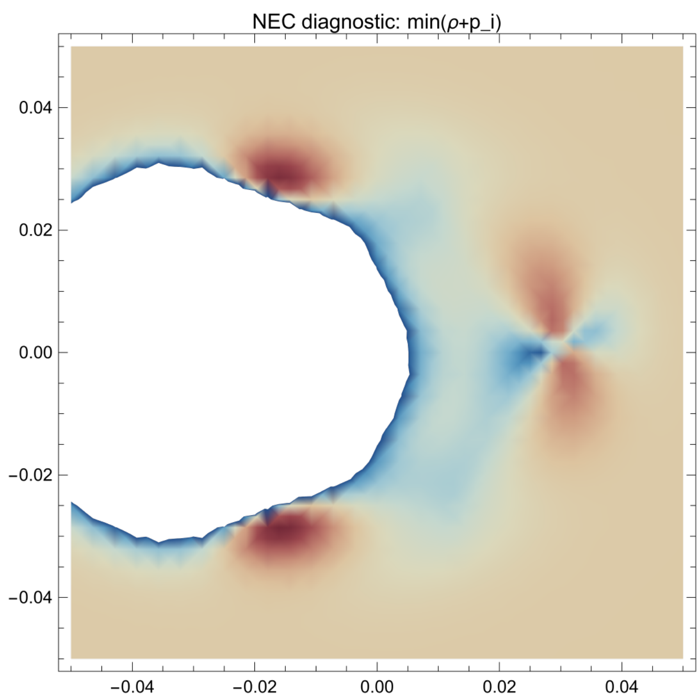
Out[ ]=



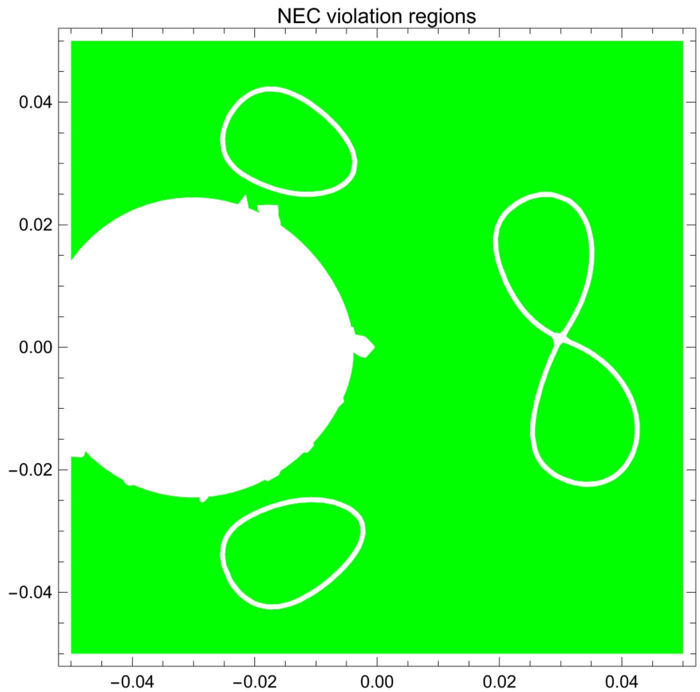
Out[ ]=


Total energy density u_total
0.04
0.02
0.00
−0.02
−0.04
−0.04
−0.02
0.00
0.02
0.04

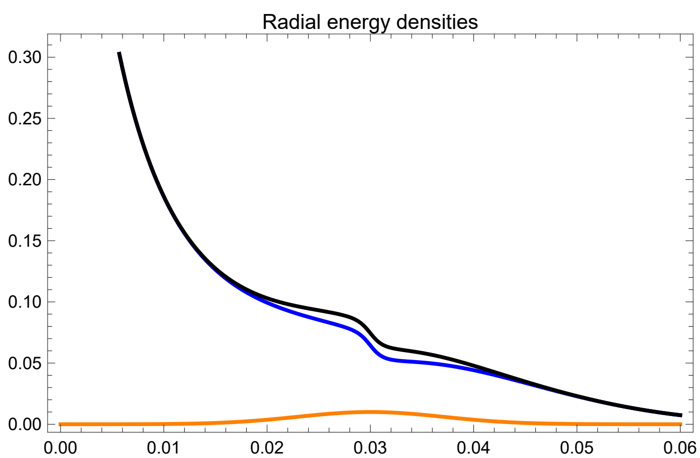
Out[ ]=


NEC diagnostic: min(ρ+p_i)
0.04
0.02
0.00
−0.02
−0.04
−0.04
−0.02
0.00
0.02
0.04

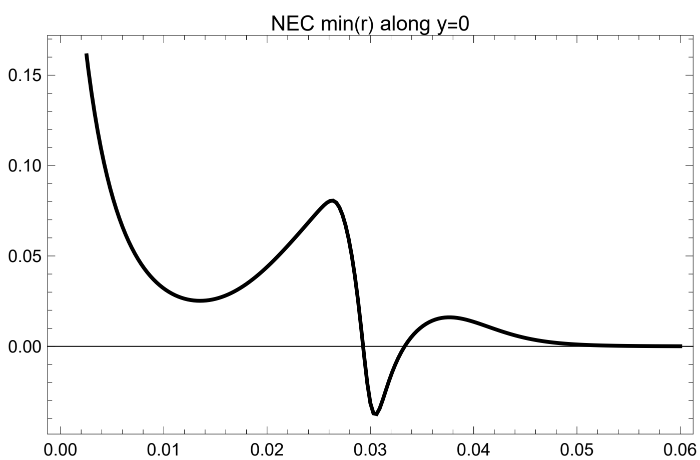
Out[#]=


NEC violation regions
0.04
0.02
0.00
−0.02
−0.04
−0.04
−0.02
0.00
0.02
0.04

Out[#]=


Radial energy densities
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.00
0.01
0.02
0.03
0.04
0.05
0.06

Out[#]=


NEC min(r) along y=0
0.15
0.10
0.05
0.00
0.00
0.01
0.02
0.03
0.04
0.05
0.06