Modern Portfolio Theory and the Markowitz Model

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Abstract

In an attempt to better understand the world of finance and portfolio management, I decided to research the Markowitz model's approach to Modern Portfolio Theory and the benefit to a diversified portfolio. In this report, I will summarize the functions of the mathematical framework and attempt an example application of the model on some excel data from this video and use Power BI for some data visualizations.

1 Introduction to the Theory

Modern Portfolio Theory first introduced by Harry Markowitz, for whom he won the Nobel Prize in Economic Sciences. According to Wikipedia, the theory can also be referred to as mean-variance analysis, which is a mathematical framework that will maximize returns variable to a level of risk. The theory ultimately favors a diversified portfolio over one homogenized [1]. Markowitz equated an asset's volatility (σ) with its level of risk: the more volatile the price of an asset, the more risky the asstes. However, riskier assets would sometimes lead to higher payouts. On the flip-side, they could lead to bigger losses. He also noted that the correlations between assets proved important to the model.

2 Constructing the Model

2.1 Representing correlations between assets

We can think of the negative, positive, or uncorrelated relationships between assets as a covariance matrix:

$$\Sigma = \begin{bmatrix} cov(A,A) & cov(A,B) & cov(A,C) \\ cov(B,A) & cov(B,B) & cov(B,C) \\ cov(C,A) & cov(C,B) & cov(C,C) \end{bmatrix}$$

As a reminder, the covariance between two random variables X and Y is given by:

$$Cov(X,Y) = E[(X-\mu_x)(Y-\mu_y)]$$

where E is the mean, μ_x and μ_y are the means of X and Y respectively. A positive covariance means the variables increase together, negative means one increases while the other decreases, and a covariance of 0 means no linear relationship.

The assets can be represented in time series by their returns (n_n being the price at end of day, average day price, etc...):

$$A = \begin{bmatrix} n_1 \\ n_2 \\ \dots \\ n_n \end{bmatrix} B = \begin{bmatrix} n_1 \\ n_2 \\ \dots \\ n_n \end{bmatrix} C = \begin{bmatrix} n_1 \\ n_2 \\ \dots \\ n_n \end{bmatrix}$$

2.2 Efficient Frontier Graph

Markowitz was also aware that there was an infinite number of possible allocations for assets. He theorized that there were specific subsets within this infinite number of combinations that would have the most optimized returns. These combinations of assets can be represented with the following graph:

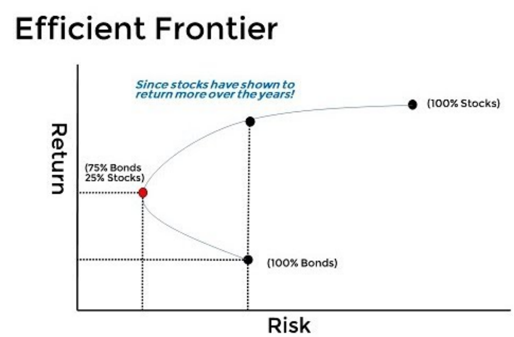

Figure 1

The area above the line are portfolios considered impossible to create. The allocations of assets lying on the curve is considered efficient, and those underneath the curve are considered inefficient.

2.3 Constructing Graph with covariance matrix

We know that we have set portfolio weights for our assets. For example, we have 50% stocks, 30% bonds, 10% currency and 10% gold. We can construct a vector with the weights fror each asset:

$$w = [w_1, w_2, w_3, \dots, w_n]$$

If we multiply the weights vector by the covariance matrix by the transpose of our weight vector, we get volatility squared term σ^2:

$$\sigma^2 = [w_1, w_2, w_3, \dots, w_n] \times \begin{bmatrix} cov(A,A) & cov(A,B) & cov(A,C) \\ cov(B,A) & cov(B,B) & cov(B,C) \\ cov(C,A) & cov(C,B) & cov(C,C) \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \dots \\ w_n \end{bmatrix}$$

Our goal is to minimize the value of σ. So what we want to find out is what given level of returns will the level of risk (again, σ) minimized. The leads to a question, how is the level of returns supposed to be calculated? The answer is – it can be customized. For example, let's call this level or returns R. We will have a vector, μ, of the return values we expect to see from our portfolio's assets:

$$\mu = [r_1, r_2, r_3, \dots, r_n]$$

If we multiply Mu by the transpose of our weights we will get a portfolio return of R:

$$R = [r_1, r_2, r_3, \dots, r_n] \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \dots \\ w_n \end{bmatrix}$$

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in other words (a dot product):

$$R = r_1 w_1 + r_2 w_2 + r_3 w_3 + \cdots + r_n w_n$$

At this point we have our two equations: σ^2 and R. We want to minimize our level of risk σ^2 and maximize our returns (R). Here's is a cleaner representation of our equations:

$$\sigma^2 = w \Sigma w_T \text{ and } R = \mu w_T$$

You can solve for any point on the curve using this function where λ is the risk tolerance $[0, \infty]$, :

$$w = \lambda \Sigma^{-1} \mu$$

3 The art of calculating expected returns

One approach to this problem is by using the CAPM (Capital Asset Pricing Model). And has three components when calculating expected return. This model predicts that the expected return will be equal to the risk-free rate plus the assets volatility rate compared to the market's, times the expected market return , minus the risk free rate. The risk free rate can be found online and represents the rate which you can invest and receive back government t-bills.

$$r = R_f + \beta \times (R_m - R_f)$$

You can then apply this formula for each point in your return vector μ.

4 Dictionary

- **Allocation** - the process of dividing an investment portfolio across different asset classes to achieve a desired balance of risk and return
- Second item
- Third item

References

https://www.youtube.com/watch?v=VsMpw-qnPZY
https://en.wikipedia.org/wiki/Modern_portfolio_theory