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THE COOPER UNION
ALBERT NERKEN SCHOOL OF ENGINEERING

A Deep Partitioned Autoencoder
for De-Noising Live Audio

by
Ethan Lusterman

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Engineering

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Professor Sam Keene, Advisor

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Acknowledgements

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Abstract

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Contents

1	Introduction	1
2	Background	2
2.1	Machine Learning	2
2.1.1	Regression	3
2.1.2	Overfitting and Curse of Dimensionality	3
2.1.3	Loss functions and Regularization	3
2.1.4	Gradient Stuff?	3
2.2	Neural Networks	3
2.2.1	Dense Layer	3
2.2.2	Convolutional Layer	3
2.2.3	Nonlinearity Choice	3
2.3	Signals and Systems	3
2.3.1	Signals	3
2.3.2	Convolution	4
2.3.3	Frequency Transforms	5
2.3.4	Windowing and Perfect Reconstruction	6
2.3.5	Window Size and Frequency v. Time Resolution Tradeoff	7
2.3.6	Noise and Signal-to-Noise Ratio	7
2.3.7	Magnitude and Phase Spectrum	7
3	Signal Model and Data	8
3.0.1	Network Input and Output	8
3.0.2	Signal and Noise Choices	9
3.0.3	Other Hyperparameter Choices	9
4	De-noising Architectures	10
5	Results	11
6	Conclusions and Future Work	12

270		
271	6.1	Conclusions 12
272	6.2	Future Work 12
273		
274	6.2.1	Models 12
275	6.2.2	Data 12
276		
277		
278		
279		
280		
281		
282		
283		
284		
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286		
287		
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List of Figures

378	Table of Nomenclature
379	
380	
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1 Introduction

Advances in smartphone technology have led to smaller devices with more powerful audio hardware, allowing for common consumers to make higher quality recordings. However, recorded speech and music are subject to noisy conditions, often hampering intelligibility and listenability. The goal of denoising audio recordings is to improve intelligibility and perceived quality. A variety of applications of audio denoising exist, including listening to a recording of a band or an artist’s live performance in a noisy crowd, or listening to a recorded conversation or speech under noisy conditions.

A common technique for denoising involves the use of deep neural networks (DNN). [PARIS] Advances in parallel graphics processing units (GPU) and in machine learning algorithms have allowed for training deeper networks faster, utilizing more hidden layers with more neurons.

Prior work in denoising audio has involved access to noise-free training data. Since common consumers do not often have access to clean audio, we seek to denoise without the use of clean audio.

In this thesis, we compare several neural network architectures and problem scenarios, ranging from data input types, level of noise, depth of network, training objectives, and more. In Chapter 2, we present background information on machine learning, neural networks, and signal processing as well as prior work in audio denoising. In Chapter 3, we detail all considered network architectures. In Chapter 4, we compare results from different data inputs, levels of noise, network architectures, and training objectives and discuss methods of evaluation. Finally, we make conclusions and recommendations for future work in Chapter 5.

2 Background

2.1 Machine Learning

Machine learning involves the use of computer algorithms to make decisions based on training data. Generally, this falls into categorizing input data (classification) or determining a mathematical function to determine a continuous output given an input (regression). Popular classification examples include recognizing handwritten digits (MNIST) as well as determining whether an image contains a cat or a dog. (REF) An example of a regression problem is determining the temperature given a set of input features (humidity, latitude, longitude, date, etc.).

Problems where training data contain input data vectors as well as the correct output vectors (targets) are known as supervised learning problems. Training a model to denoise audio where noise was introduced to the clean audio would be a supervised learning problem. On the other hand, training a model to denoise audio where the underlying clean signal is not known is an unsupervised learning problem. Different loss functions and neural network architectures can be exploited to accomplish denoising without the clean data.

For the purposes of this thesis, we use machine learning to determine an underlying nonlinear function that removes noise from time slices of audio (i.e. regression). These slices can then be pieced back together through overlap-add resynthesis. To clarify, this is a general linear model that maps an input noisy audio vector $y[n] = x[n] + N[n]$ to $\tilde{x}[n]$, a target denoised audio vector, where $x[n]$ is the underlying clean signal and $N[n]$ is the additive background noise.

2.1.1 Regression

A classical regression technique is linear regression, where one or more independent variables x_i are used to determine a scalar dependent variable y . The case of a single independent variable x is known as simple linear regression. On the other hand, the case of estimating A canonical example would be estimating a sine wave $x[n]$ given noisy samples

2.1.2 Overfitting and Curse of Dimensionality

2.1.3 Loss functions and Regularization

2.1.4 Gradient Stuff?

2.2 Neural Networks

2.2.1 Dense Layer

2.2.2 Convolutional Layer

2.2.3 Nonlinearity Choice

2.3 Signals and Systems

Domain knowledge of discrete audio signals and systems better informs our decisions for an audio denoising system, so some background information on signals and systems as it pertains to this thesis is detailed below.

2.3.1 Signals

We deal exclusively with discrete-time audio signals in this thesis. A discrete-time audio signal $x[n]$ is represented as a sequence of numbers (samples), where each integer-valued slot n in the sequence corresponds to a unit of time based on the sampling frequency f_s . This comes from sampling the continuous-time audio signal $x_c(t)$:

$$x[n] = x_c(nT) \quad (1)$$

where $T = 1/f_s$. For example, a 1-second speech signal sampled at 8kHz has 8000 samples. Furthermore, digital signals also have discrete valued sample amplitudes. For the purposes of this thesis, the bit depths of computers we use for analysis are high enough to allow for perfect reconstruction between continuous-time signals and digital signals.

We also assume signals collected have been properly sampled according to the Nyquist-Shannon sampling theorem, which states that a discrete-time signal must be sampled at at least twice the highest frequency present in the signal to prevent aliasing of different frequencies. For example, speech signals generally have information up to 8kHz, so many speech signals are sampled at 16kHz. Music is more complex in that signals often span up to about 20kHz, so CD quality recordings are often sampled at 44.1kHz or higher. For this thesis, we use recordings sampled at 44.1kHz or lower.

2.3.2 Convolution

The discrete-time convolution operation takes two sequences $x[n]$ and $h[n]$ and outputs a third sequence $y[n] = x[n] * h[n]$:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \quad (2)$$

Convolution is commutative, so $x[n] * h[n] = h[n] * x[n]$ holds true.

A linear, time-invariant (LTI) system is characterized by its impulse response $h[n]$, which allows us to determine samples $y[n]$ when $x[n]$ is subject to $h[n]$. For the purposes of this thesis, our underlying clean signal $x[n]$ might be

subject to the conditions of an acoustic environment $h[n]$ and crowd noise $N[n]$:

$$y[n] = h[n] * x[n] + N[n] \quad (3)$$

In this scenario, our system would attempt to recover $h[n] * x[n]$ and possibly even $x[n]$ if the acoustic environment were deemed “noisy enough” due to echo and reverberation.

One of our proposed systems also incorporates convolutional neural networks (CNN) which use convolutions between frames of samples instead of simple linear combinations (discussed later).

2.3.3 Frequency Transforms

In some of our proposed systems, we use a frequency transformed version of the input signal as a preprocessing step to the system input. While no new information is gained from transforming the input, networks often respond better to determining the value of the magnitude of varying frequencies at a time slice instead of the individual time samples.

The frequency transform we use in this thesis is the discrete-time Fourier transform (DTFT). A sequence of N discrete-time samples is transformed into another sequence of N samples where each index then corresponds to a frequency bin. The DTFT $X[k]$ of a signal $x[n]$ is given by the following:

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} \quad (4)$$

where the twiddle factor W_N is given by $W_N = e^{-j(2\pi/N)}$. Then the reconstruction of $x[n]$ from $X[k]$ is given by:

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-kn} \quad (5)$$

In this thesis, we also exploit the main duality between the time and frequency domain using the convolution theorem, which states that convolution in time is equivalent to multiplication in frequency and vice versa:

$$\mathcal{F}\{h[n] * x[n]\} = H[k]X[k] \quad (6)$$

$$\mathcal{F}^{-1}\{H[k] * X[k]\} = h[n]x[n] \quad (7)$$

This allows us to effectively treat our network as a non-linear filter that can denoise small time/frequency slices of our noisy signal, which can then be pieced back together using overlap-add resynthesis. We detail this in the next section.

2.3.4 Windowing and Perfect Reconstruction

To window a signal is to multiply a window function $w[n]$ by the frame, i.e. $w[n]x[n]$ over the frame length N . Because we are training a network to denoise small segments of a larger audio signal, we window the signal segments. This accommodates the finite-length requirement of the DTFT and helps to prevent spectral leakage. [DSPBOOK]

Also, to be able to properly reconstruct our signal, we use a window function and corresponding overlapping frame percentage to accomplish perfect reconstruction. The corresponding overlapping frame percentage is set such that the window sums to a constant for all time. For example, a rectangular window $w[n] = 1$ over an interval of length N has an overlap of 0% to sum to a constant 1 for all time. Another popular window is the Hanning window, defined over an interval N by the following:

$$w[n] = \frac{1}{2} \left(1 - \cos \left(\frac{2\pi n}{N-1} \right) \right) \quad (8)$$

For the Hann window, the perfect reconstruction overlap is a frame length of $N = 50\%$.

2.3.5 Window Size and Frequency v. Time Resolution Tradeoff

We must consider window size as a hyperparameter to our system. In general, shorter windows give rise to better time resolution at the cost of frequency resolution. On the other hand, longer windows give rise to better frequency resolution at the cost of time resolution. To illustrate, consider FIGURE.

2.3.6 Noise and Signal-to-Noise Ratio

Since we are trying to denoise audio signals, we must discuss how we measure noise. One of the most common measures of degradation of signal quality from additive noise is signal-to-noise ratio (SNR), defined as the ratio of signal variance to noise variance. [DSP] For the signal $y[n] = x[n] + N[n]$, where $x[n]$ is the signal of interest and $N[n]$ is the additive noise, the SNR is defined as

$$SNR = \frac{\sigma_x^2}{\sigma_n^2} \quad (9)$$

where σ^2 refers to the variance of the signal in question over some time interval. For the purposes of this thesis, we achieve desired a desired SNR for a simulation by scaling the noise to match the variance to the signal, then scaling the noise or the signal to achieve the desired SNR.

2.3.7 Magnitude and Phase Spectrum

3 Signal Model and Data

3.0.1 Network Input and Output

To simulate an audio denoising scheme, we define the following inputs and outputs. We take a known clean signal $x[n]$ which we subject to additive noise $N[n]$ using a specified SNR, resulting in the following noisy signal $y[n]$:

$$y[n] = x[n] + N[n] \quad (10)$$

To achieve a particular average SNR per simulation, we take the average signal energy for each minibatch of size B to determine a multiplicative scale factor k on the noise signal $N[n]$:

$$\frac{1}{BN} \sum_{b=0}^{B-1} \sum_{n=0}^{N-1} x_b^2[n] \quad (11)$$

In supervised scenarios, we allow the network to train with access to the ground truth $x[n]$. On the other hand, in semi-supervised scenarios, we only allow the network to train with access to a “soft label” indicating if the signal is (1) noise-only or (2) noise and possibly signal. [DanStowell] However, in both supervised and semi-supervised scenarios, our neural network input is one of the following:

1. Frames of $y[n]$
2. Frames of $\|Y[k]\|$
3. Magnitude spectrogram frames of $Y[k]$
4. Complex spectrogram frames of $Y[k]$

We choose the frame length L , time-domain window $w[n]$, and frame overlap percentage p as hyperparameters. Since we want to evaluate the level of denoising in the time domain, we recombine the network outputs with the noisy phase components of the spectrum if necessary to obtain an estimate $\hat{x}[n]$. We then compare $\hat{x}[n]$ to $x[n]$, in general using the mean squared error (MSE). For example, when our network outputs frames of $\|\hat{X}[k]\|$, we take the inverse Fast Fourier transform (IFFT) using the noisy phase $\angle Y[k]$ and use overlap-add to recombine the frames. (Anything to add about phase denoising and failures here?)

3.0.2 Signal and Noise Choices

Our choice of signals include the following:

1. Sine waves with multiple frequencies and random amplitudes and phases
2. Clean speech signals
3. Studio music recordings
4. Live concert recordings

Similarly, our choice of noise signals include the following:

1. Additive white Gaussian noise (AWGN)
2. Restaurant noise

3.0.3 Other Hyperparameter Choices

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4 De-noising Architectures

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6 Conclusions and Future Work

6.1 Conclusions

While more work is needed, deep partitioned neural network architectures using time and frequency data seem promising in long-term solutions for denoising speech and music signals.

6.2 Future Work

6.2.1 Models

Make network deeper. Consider gradual partitioning instead of hard.

6.2.2 Data

Get more data. Consider different noise levels and types of signals.

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