One little Practice about Newton Method

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Problem

求

$$argmin f(x,y) = x^2 + y^2$$

从$(x,y)=(1,1)$开始迭代，函数的求导用差分近似, 间隔0.001,

最终输出误差在`1e-6`范围内即可

具体要求: 写一个函数 gradient(f),然后main() 调用该函数

https://turingjudge.com/contest/76/problem/1001

Newton Method

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2 + \ldots + \frac{1}{n!} f^{(n)}(x_0)(x - x_0)^n \ldots$$

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2$$

$$f'(x) = f'(x_0) + f''(x_0)(x - x_0) = 0$$

$$x = x_0 - \frac{f'(x_0)}{f''(x_0)}$$

Iteration

$$x_{t+1} = x_t - \frac{f'(x_t)}{f''(x_t)}$$

https://zhuanlan.zhihu.com/p/37588590

Hesse Matrix

$$\mathbf{H} = \begin{bmatrix} \dfrac{\partial^2 f}{\partial x_1^2} & \dfrac{\partial^2 f}{\partial x_1\,\partial x_2} & \cdots & \dfrac{\partial^2 f}{\partial x_1\,\partial x_n} \\[2.2ex] \dfrac{\partial^2 f}{\partial x_2\,\partial x_1} & \dfrac{\partial^2 f}{\partial x_2^2} & \cdots & \dfrac{\partial^2 f}{\partial x_2\,\partial x_n} \\[2.2ex] \vdots & \vdots & \ddots & \vdots \\[2.2ex] \dfrac{\partial^2 f}{\partial x_n\,\partial x_1} & \dfrac{\partial^2 f}{\partial x_n\,\partial x_2} & \cdots & \dfrac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

$$\mathbf{H}_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j}$$

https://en.wikipedia.org/wiki/Hessian_matrix

Hesse Matrix

$$f(x, y) = x^2 + y^2$$

$$H = \begin{bmatrix} \dfrac{\partial^2 f}{\partial^2 x} & \dfrac{\partial^2 f}{\partial x \partial y} \\ \dfrac{\partial^2 f}{\partial y \partial x} & \dfrac{\partial^2 f}{\partial^2 y} \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} = 2y$$

Hesse Matrix

$$f(x, y) = x^2 + y^2$$

$$H = \begin{bmatrix} \dfrac{\partial^2 f}{\partial^2 x} & \dfrac{\partial^2 f}{\partial x \partial y} \\ \dfrac{\partial^2 f}{\partial y \partial x} & \dfrac{\partial^2 f}{\partial^2 y} \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} = 2y$$

$$= \begin{bmatrix} \dfrac{\partial(2x)}{\partial x} & \dfrac{\partial(2y)}{\partial x} \\ \dfrac{\partial(2x)}{\partial y} & \dfrac{\partial(2y)}{\partial y} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Define in code

```python
def f(x):
    return x[0] ** 2 + x[1] ** 2
def gradient(x):
    return np.array([2*x[0], 2*x[1]])
hessian_inv = np.array([[0.5, 0.0],
                        [0.0, 0.5]])
```

inv

Main loop

```
def newton(x):
    '''
    x -= f'(x) / f''(x)
      -= hessian_inv * f'(x)
    '''
    while True:
        new_x = x - np.dot(hessian_inv, gradient(x))
        if abs(f(x) - f(new_x)) < 0.000001:
            break
        x = new_x
    return x
```

if __name__ == '__main__':

```
• def main():
•     x = np.array([1, 1])
•     ans = newton(x)
•     print("{:.8f} {:.8f}".format(ans[0], ans[1]))
```


https://turingjudge.com/article/83

Thank you

- End