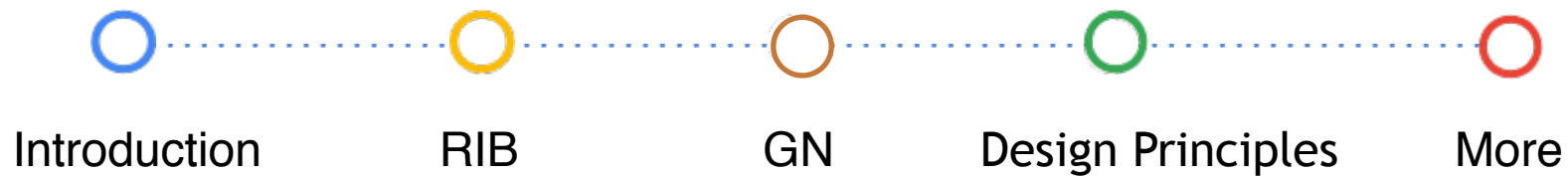


Relational inductive biases, Deep learning, Graph Networks

—
Jarvis

Outline





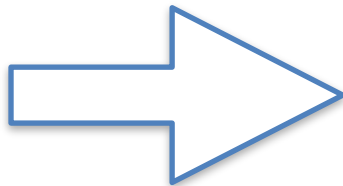
Introduction

Introduction

Hand-engineering:

Sample Efficient

Interpretable



Reasoning Structured Data

End-to-end learning

Minimal priori knowledge

Avoid explicit structure

Relational Biases

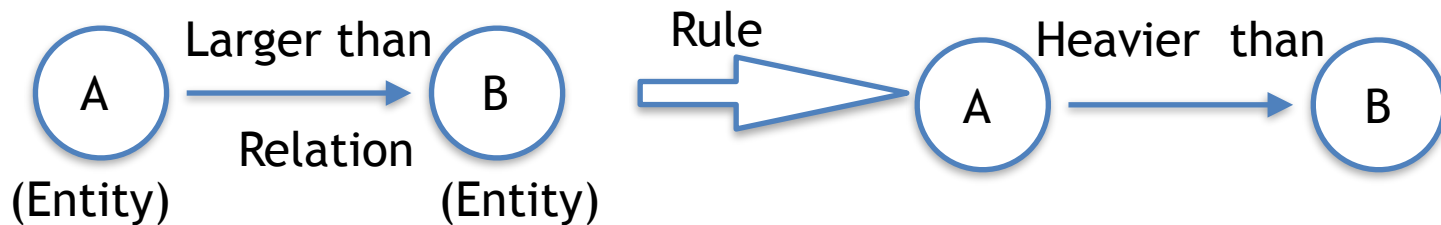
Relational Inductive Biases

Relational reasoning: manipulate entities and relations with rules.

Entity: element with attributes

relation: property between entities

rules: map entities and relations to entities and relations



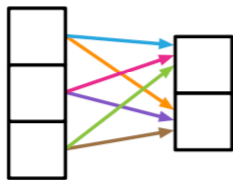
Relational Inductive Biases

Inductive bias: Allows algorithm to prioritize one solution

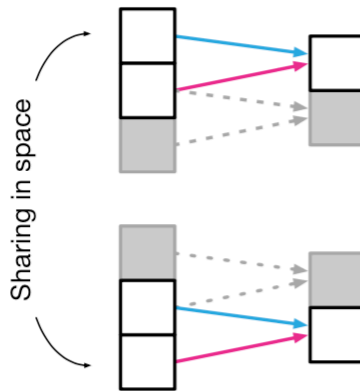
L2 regularizer: prefer small parameters

L1 regularizer: prefer sparse parameters

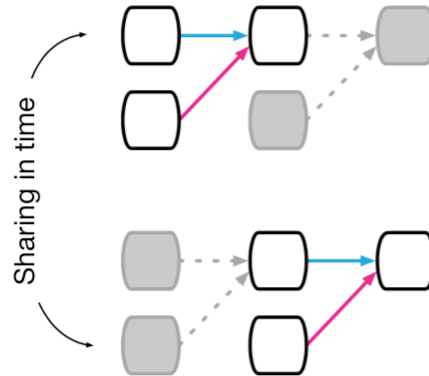
Relational Inductive Biases



(a) Fully connected



(b) Convolutional



(c) Recurrent

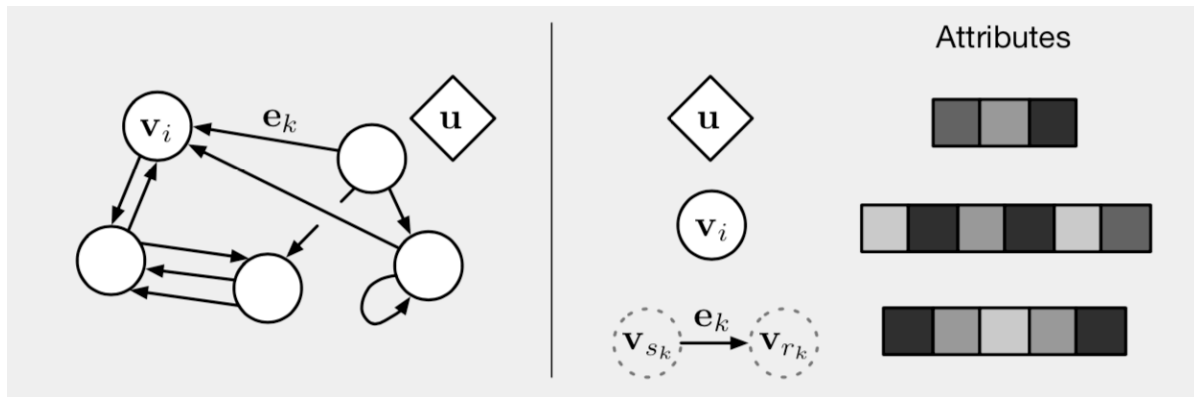
Component	Entities	Relations	Rel. inductive bias	Invariance
Fully connected	Units	All-to-all	Weak	-
Convolutional	Grid elements	Local	Locality	Spatial translation
Recurrent	Timesteps	Sequential	Sequentiality	Time translation
Graph network	Nodes	Edges	Arbitrary	Node, edge permutations



GN

Graph Net

Graph: $G = (\mathbf{u}, V, E)$



Nodes: $V = \{\mathbf{v}_i\}_{i=1:N^v}$

Edges: $E = \{(\mathbf{e}_k, r_k, s_k)\}_{k=1:N^e}$

Global: \mathbf{u}

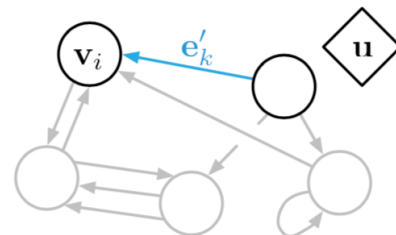
Graph Net

Update function:

$$\mathbf{e}'_k = \phi^e(\mathbf{e}_k, \mathbf{v}_{r_k}, \mathbf{v}_{s_k}, \mathbf{u})$$

$$\mathbf{v}'_i = \phi^v(\bar{\mathbf{e}}'_i, \mathbf{v}_i, \mathbf{u})$$

$$\mathbf{u}' = \phi^u(\bar{\mathbf{e}}', \bar{\mathbf{v}}', \mathbf{u})$$



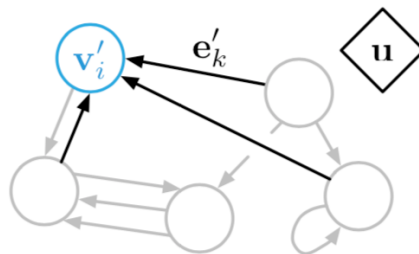
(a) Edge update

Aggregate function:

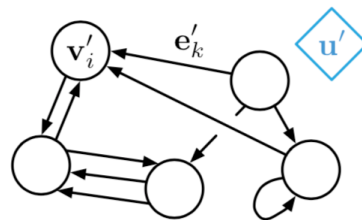
$$\bar{\mathbf{e}}'_i = \rho^{e \rightarrow v}(E'_i)$$

$$\bar{\mathbf{e}}' = \rho^{e \rightarrow u}(E')$$

$$\bar{\mathbf{v}}' = \rho^{v \rightarrow u}(V')$$



(b) Node update



(c) Global update

$$E'_i = \{(\mathbf{e}'_k, r_k, s_k)\}_{r_k=i, k=1:N^e}, V' = \{\mathbf{v}'_i\}_{i=1:N^v}, \text{ and } E' = \bigcup_i E'_i = \{(\mathbf{e}'_k, r_k, s_k)\}_{k=1:N^e}.$$

Graph Net

Algorithm 1 Steps of computation in a full GN block.

function GRAPHNETWORK(E, V, \mathbf{u})

for $k \in \{1 \dots N^e\}$ **do**

$\mathbf{e}'_k \leftarrow \phi^e(\mathbf{e}_k, \mathbf{v}_{r_k}, \mathbf{v}_{s_k}, \mathbf{u})$

▷ 1. Compute updated edge attributes

end for

for $i \in \{1 \dots N^n\}$ **do**

let $E'_i = \{(\mathbf{e}'_k, r_k, s_k)\}_{r_k=i, k=1:N^e}$

$\bar{\mathbf{e}}'_i \leftarrow \rho^{e \rightarrow v}(E'_i)$

▷ 2. Aggregate edge attributes per node

$\mathbf{v}'_i \leftarrow \phi^v(\bar{\mathbf{e}}'_i, \mathbf{v}_i, \mathbf{u})$

▷ 3. Compute updated node attributes

end for

let $V' = \{\mathbf{v}'_i\}_{i=1:N^n}$

let $E' = \{(\mathbf{e}'_k, r_k, s_k)\}_{k=1:N^e}$

$\bar{\mathbf{e}}' \leftarrow \rho^{e \rightarrow u}(E')$

▷ 4. Aggregate edge attributes globally

$\bar{\mathbf{v}}' \leftarrow \rho^{v \rightarrow u}(V')$

▷ 5. Aggregate node attributes globally

$\mathbf{u}' \leftarrow \phi^u(\bar{\mathbf{e}}', \bar{\mathbf{v}}', \mathbf{u})$

▷ 6. Compute updated global attribute

return (E', V', \mathbf{u}')

end function

Graph net

Advantages:

1. GN's input determines how representations interact and are isolated rather than fixed structure
2. Invariant to permutations
3. Combinatorial generalization(reuse update and aggregate func)



Design Principles

Design Principles

Flexible representations:

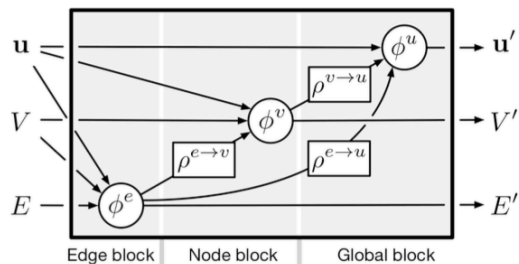
1. Nodes, edges, global features can be arbitrary format.
2. Output block can be edges or nodes or globals
3. The structure of graph can be defined or inferable

Design Principles

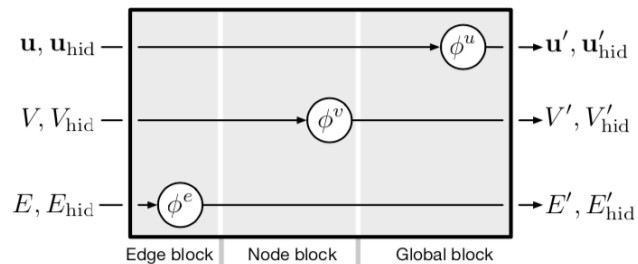
Configurable within-block structure

The within-block can be configured in many different way,
which offers many invariants

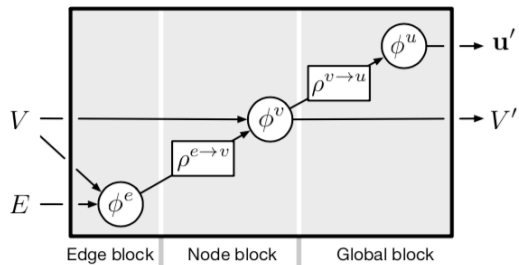
Variants



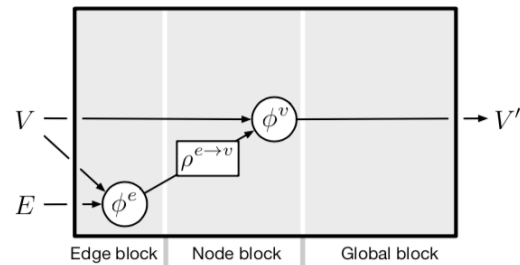
(a) Full GN block



(b) Independent recurrent block



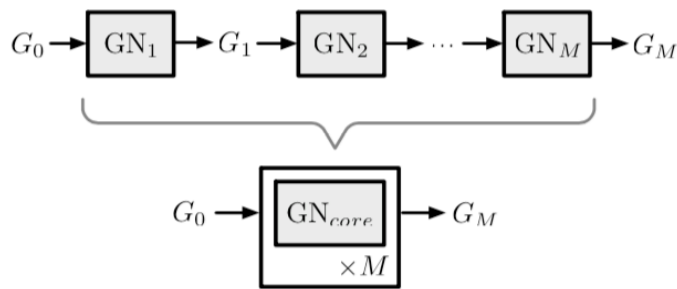
(c) Message-passing neural network



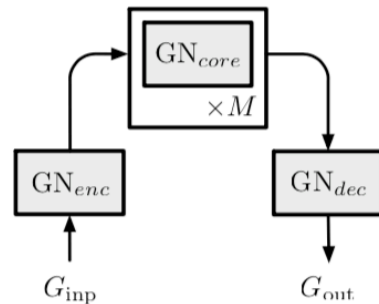
(d) Non-local neural network

Design Principles

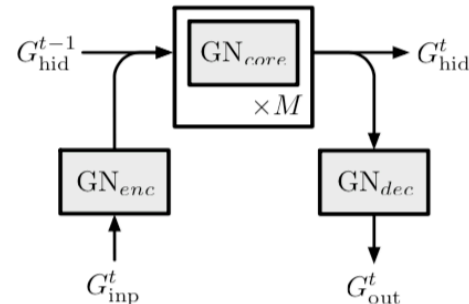
Composable multi-block



(a) Composition of GN blocks



(b) Encode-process-decode

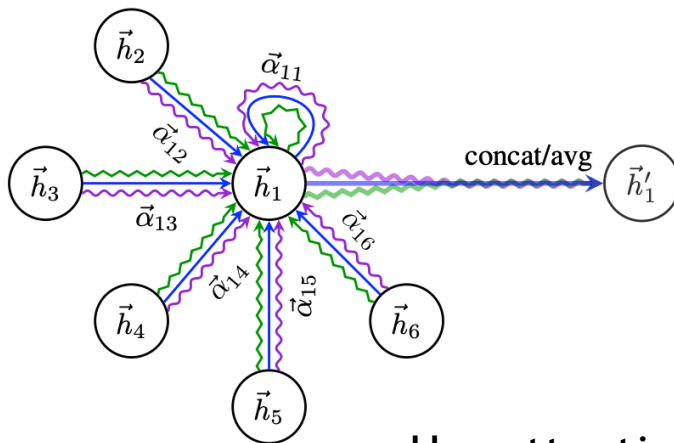
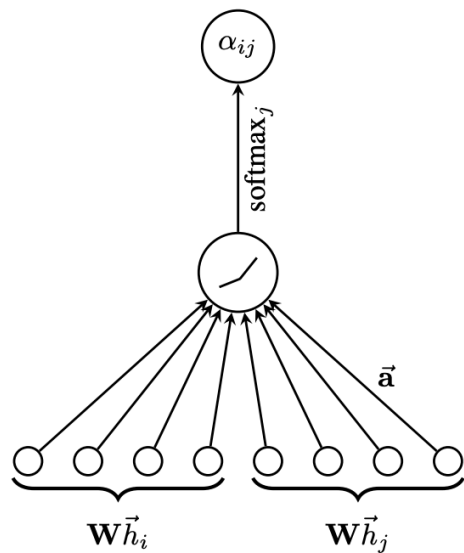


(c) Recurrent GN architecture



More

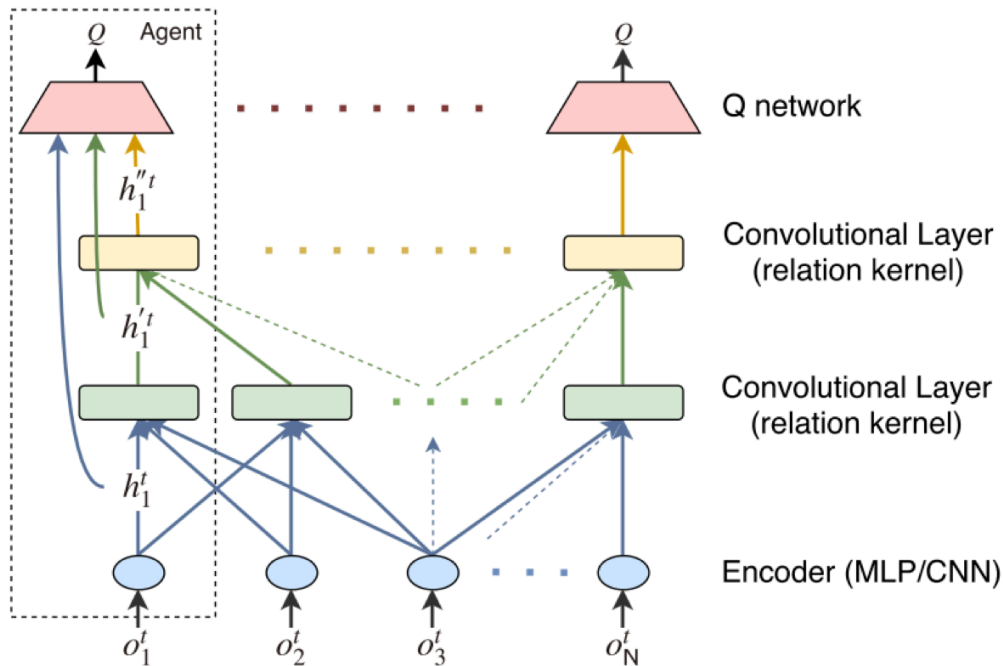
Graph Attention Networks



$$\vec{h}'_i = \sigma \left(\sum_{j \in \mathcal{N}_i} \alpha_{ij} \mathbf{W} \vec{h}_j \right)$$

Use attention network as aggregate func

GCN for MARL



$$\mathcal{L}(\theta) = \frac{1}{S} \sum_S \frac{1}{N} \sum_{i=1}^N (y_i - Q(O_i, a_i; \theta))^2$$

* Graph Convolutional Reinforcement Learning for Multi-Agent Cooperation. Jiechuan Jiang

GCN for MARL

Remarks:

1. Feed every layer into Q-Net to assemble and reuse features from different receptive fields.
2. Encourage self consistent relation representation by add KL to loss

$$\mathcal{L}(\theta) = \frac{1}{S} \sum_S \frac{1}{N} \sum_{i=1}^N ((y_i - Q(O_i, a_i; \theta))^2 + \lambda D_{\text{KL}}(\mathcal{G}^k(O_i; \theta) \parallel z_i))$$

⌋ $z_i = \mathcal{G}^k(O'_i; \theta)$ is the attention weight distribution at conv layer k