Determinacy and Indeterminacy of Equilibria

Abstract

This essay discusses work on the determinacy and indeterminacy of equilibrium in models of competitive markets. Determinacy typically refers to situations in which equilibria are finite in number, and local comparative statics can be precisely described. This essay describes basic results on generic determinacy for exchange economies and the general underlying principles, together with various applications and extensions including incomplete financial markets and markets with infinitely many commodities.

1 Introduction

The Arrow-Debreu model of competitive markets is one of the cornerstones of economics. Part of the explanatory power of this model stems from its flexibility in capturing price-taking behavior in many different markets, and from the predictive power arising from the great generality under which equilibrium can be shown to exist. This predictive power is significantly enhanced when equilibria are determinate, meaning that equilibria are locally unique and local comparative statics can be precisely described. Instead, when equilibria are indeterminate, even arbitrarily precise local bounds on variables might not suffice to give a unique equilibrium prediction, the model might exhibit infinitely many equilibria, and each might be infinitely sensitive to arbitrarily small changes in parameters.

Simple exchange economies cast in an Edgeworth box with two agents and two goods illustrate the possibility of indeterminacy in equilibrium. One easy example arises when agents view the goods as perfect substitutes. In this case, every profile of initial endowments leads to a continuum of equilibria. Another example comes from the opposite extreme, in which each agent views the goods as perfect complements. Every profile of initial endowments dividing equal social endowments of the two goods leads to a continuum of equilibria. These examples may seem degenerate, since they involve individual demand behavior either extremely responsive to prices, or extremely unresponsive to prices. Similar examples can be constructed using preferences that are less extreme, however, and that can be chosen to satisfy a number of regularity conditions including strict concavity, strict monotonicity, and smoothness. Problems from standard graduate texts illustrate this possibility. In fact, indeterminacy is unavoidable, at least for some endowment profiles, in almost any model that may exhibit multiple equilibria for some choices of endowments. The conditions leading to unique equilibria or unambiguous global comparative statics are well-known to be very restrictive, suggesting that equilibrium indeterminacy may be a widespread phenomenon.

In a deeper sense, however, these examples of indeterminacy remain knife-edge. Under fairly mild conditions on primitives, if an initial endowment profile leads to indeterminacy in equilibrium, arbitrarily small perturbations in endowment profiles must restore the determinacy of equilibrium. More powerfully, the set of endowment profiles for which equilibria are determinate is generic, that is, an open set of full Lebesgue measure. Explaining this remarkable result, originally postulated and established by Debreu (1970), and its many extensions and generalizations is the focus of this essay. The following section lays out the basic question of determinacy of equilibrium in finite exchange economies, and sketches the results. Section 3 describes the general underlying principles, together with various applications and extensions. Section 4 concludes by examining recent work on determinacy in markets with infinitely many commodities.

2 Determinacy in Finite Exchange Economies

Imagine a family of exchange economies, each with a fixed set of L commodities and a fixed set of m agents, $i = 1, \ldots, m$, with given preferences $\{\succeq_i\}_{i=1,\ldots,m}$, indexed by varying individual endowments $(e_1, \ldots, e_m) \in \mathbf{R}^{mL}_{++}$. Denote the social endowment $\bar{e} := \sum_i e_i$ and a particular profile of individual endowments by $e := (e_1, \ldots, e_m) \in \mathbf{R}^{mL}_{++}$. An *economy* $\mathcal{E}(e)$ then refers to the exchange economy with preferences $\{\succeq_i\}_{i=1,\ldots,m}$ and endowment profile e.[1]

The crucial departure in Debreu (1970) is to view each economy as a member of this parameterized family, and to ask whether perhaps almost no economies exhibit indeterminacy or pathological comparative statics when indexed this way. To formalize this, Debreu (1970) summarizes an agent's choice behavior by a C^1 demand function $x_i : \mathbf{R}^L_{++} \times \mathbf{R}^L_{++} \to \mathbf{R}^L_{+}$ satisfying basic properties such as homogeneity of degree 0 in prices, Walras' Law, and boundary conditions as prices converge to zero. This leads to the familiar characterization of equilibria as zeros of excess demand:

$$0 = z(p, e) := \sum_i x_i(p, e_i) - \bar{e}$$

Two simplifying normalizations are then commonly adopted. Demand functions derived from optimal choices of price-taking agents are homogeneous of degree zero in prices, so normalize by setting $p_1 \equiv 1$. Normalized prices thus can be taken to range over $\mathbf{R}^{L-1}_{++}$. Next, Walras' Law ensures that excess demand functions are not independent across markets, as $p \cdot z(p, e) = 0$ for each $p \in \mathbf{R}^{L-1}_{++}$. This renders one market clearing equation redundant, and leads to the characterization of equilibria by normalized price vectors $p \in \mathbf{R}^{L-1}_{++}$ such that

$$z_{-L}(p, e) = 0$$

where, adopting common conventions, the subscript $-L$ refers to all goods except L, so $z_{-L}(p, e) = (z_1(p, e), \ldots, z_{L-1}(p, e))$. Using these normalizations, the equilibrium correspondence can be defined by

$$E(e) := \{(x, p) \in \mathbf{R}^{mL}_{+} \times \mathbf{R}^{L-1}_{++} : z_{-L}(p, e) = 0,\ x_i = x_i(p, e) \text{ for } i = 1, \ldots, m\}$$

[1] For simplicity this essay focuses on exchange economies. Mas-Colell (1985) is a comprehensive reference that includes discussion of extensions allowing for production.

Fix a particular equilibrium price vector p^* in the economy $\mathcal{E}(e)$. One way to answer local comparative statics questions at this equilibrium is to apply the classical implicit function theorem. If $D_p z_{-L}(p^*, e)$ is invertible, then the implicit function theorem provides several immediate predictions: the equilibrium price p^* is locally unique; locally, on neighborhoods W of e and V of p^*, the equilibrium price set is described by the graph of a C^1 function $p : W \to \mathbf{R}^{L-1}_{++}$; and local comparative statics are given by the formula

$$Dp(e) = -\left[D_p z_{-L}(p^*, e)\right]^{-1} D_e z_{-L}(p^*, e)$$

If this analysis can be performed for each equilibrium, then there are only finitely many equilibria, because the equilibrium set is compact. Moreover, for each equilibrium $(x, p) \in E(e)$ there is a neighborhood U of (x, p) for which $E(\cdot) \cap U$ has a unique, C^1 selection on a neighborhood W of e, with the comparative statics derived from the preceding formula. Call such a correspondence *locally* C^1 *at* e. The following definition offers a convenient way to summarize these properties.

Definition 1 *The economy $\mathcal{E}(e)$ is* regular *if it has finitely many equilibria, and E is locally C^1 at e.*

An alternative way to describe the problem uses the language of differential topology. For a C^1 function $f : \mathbf{R}^m \to \mathbf{R}^n$, $y \in \mathbf{R}^n$ is a *regular value* of f if $Df(x)$ has full rank for every $x \in f^{-1}(y)$. Notice that this is precisely the condition identified above, for the case of equilibrium prices, under which local uniqueness and local comparative statics could be derived from the implicit function theorem. Whenever 0 is a regular value of $z_{-L}(\cdot, e)$, the corresponding economy $\mathcal{E}(e)$ is regular. For a fixed function f, a given value y may fail to be a regular value, but almost every other value is regular: this is the conclusion of Sard's Theorem. Dually, the fixed value y may fail to be a regular value for a particular function f, but is a regular value for almost every other function. When the set of functions is limited to those drawn from a particular parameterized family, the conclusion remains valid for almost all members of this family provided the parameterization is sufficiently rich. This idea of a rich parameterization can be expressed by requiring y to be a regular value of the parameterized family, and this parametric version of Sard's Theorem is typically called the transversality theorem. Figure 1 depicts this idea for smooth excess demand functions.

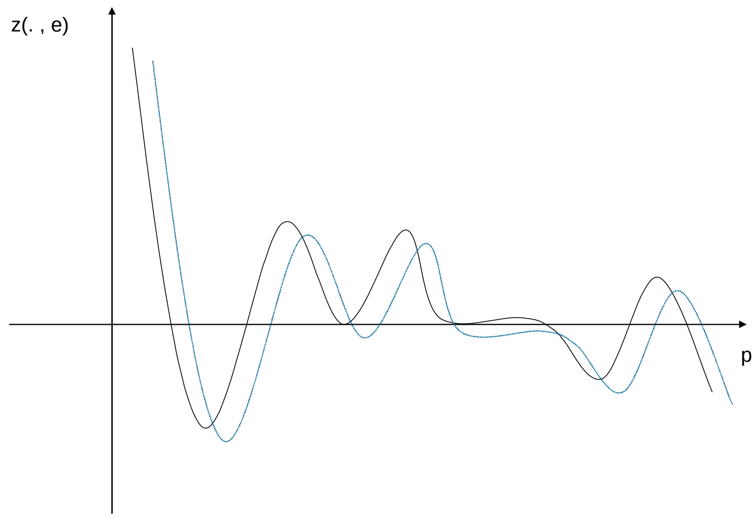

Figure 1: generic determinacy for smooth excess demand

These observations suggest that while extremely restrictive assumptions might be required to ensure that *every* economy is regular, generic regularity might follow simply from the differentiability of demand functions once the problem is framed this way. Straightforward calculations verify that 0 is a regular value of the excess demand function (viewed as a function of both prices and initial endowment parameters). From the transversality theorem we conclude that there is a subset $R^* \subset \mathbf{R}^{mL}_{++}$ of full Lebesgue measure such that for all $e \in R^*$, $\mathcal{E}(e)$ is regular. Using additional properties of excess demand and equilibria, it is similarly straightforward to show that the set of regular economies is also open, giving a strong genericity result for regular economies.

This discussion follows Debreu's original development very closely.[2] This approach takes demand functions as primitives, and gives conditions on individual demand functions under which regularity is a generic feature of exchange economies. To take a step back and start with preferences as primitives, we seek conditions on preferences sufficient to guarantee the individual demand is suitably differentiable. Debreu (1972) addresses this point by introducing a class of "smooth preferences", depicted in Figure 2.

Definition 2 *The preference order $\succeq$ on $\mathbf{R}^L_+$ is* smooth *if it is represented by a utility function U such that*

- $U : \mathbf{R}^L_+ \to \mathbf{R}$ *is C^2 on $\mathbf{R}^L_{++}$*

[2] Debreu (1970) uses a different characterization of equilibrium, to which Sard's Theorem can be directly applied to establish that the set of regular economies has full Lebesgue measure. The argument sketched here instead follows Dierker (1972) in applying the transversality theorem to the standard aggregate excess demand characterization of equilibria.

- *for each* $x \in \mathbf{R}^L_{++}$, $\{y \in \mathbf{R}^L_+ : y \sim x\} \subset \mathbf{R}^L_{++}$
- *for each* $x \in \mathbf{R}^L_{++}$, $DU(x) \gg 0$
- *for each* $x \in \mathbf{R}^L_{++}$, $D^2U(x)$ *is negative definite on* $\ker DU(x) := \{z \in \mathbf{R}^L : DU(x) \cdot z = 0\}$

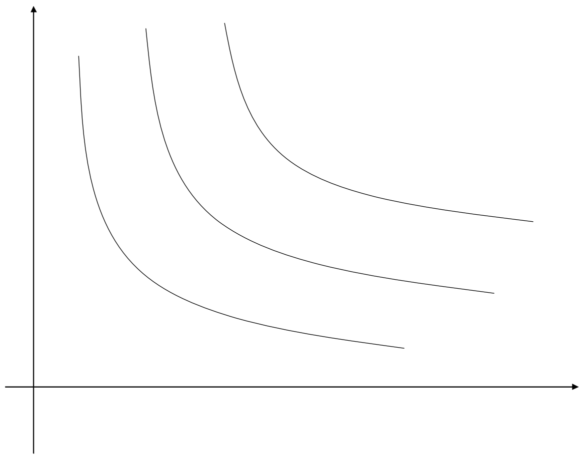

Figure 2: smooth preferences

Fairly straightforward arguments, again using the implicit function theorem, establish that individual demand functions derived from smooth preferences are C^1. Putting all of these results together yields:

Theorem 1 *Let* $\succeq_i$ *be a smooth preference order on* $\mathbf{R}^L_+$ *for each* $i = 1, \ldots, m$. *There exists an open set* $R^* \subset \mathbf{R}^{mL}_{++}$ *of full Lebesgue measure such that for all* $e \in R^*$, $\mathcal{E}(e)$ *is regular.*

3 Determinacy and Indeterminacy: A New Approach to Many Problems

Behind this result for equilibria in finite exchange economies is a broad, powerful, and simple principle that has found many important and ingenious applications in the thirty-five years since Debreu's original 1970 paper. To cast the problem more generally, take a parameterized family of equations, captured by a function $f : \mathbf{R}^m \times \mathbf{R}^k \to \mathbf{R}^n$. This describes a problem with m variables and k parameters simultaneously entering n different equations. Imagine that for each parameter value $r \in \mathbf{R}^k$,

$$E(r) := \{x \in \mathbf{R}^m : f(x, r) = 0\}$$

gives the set of objects of interest. Moreover, imagine that the equations are sufficiently independent in determining the solutions, in the sense that 0 is a regular value of f. Counting the number of equations and unknowns produces three distinct cases, corresponding in turn to three different sorts of applications.

In the canonical case exemplified by the simple exchange economy described above, the number of relevant endogenous variables, m, is equal to the number of equations, n. In this case, 0 being a regular value of f characterizes exactly the case in which the equations are sufficiently independent that the loose "counting equations and unknowns" heuristic corresponds with the precise technical result of generic determinacy. One prominent illustration of this case is given by two-period incomplete markets models with real assets, that is, assets that pay off in bundles of commodities. In these models, there are as many distinct budget equations as there are states. Letting S denote the number of states, this means there are $S+1$ distinct Walras' Law statements, leading to $S+1$ redundant market clearing equations. Because asset payoffs are in real terms, all budget constraints are homogeneous of degree 0 in state prices. This generates $S+1$ distinct normalizations of state prices, compensating exactly for the drop in independent market clearing equations determining equilibrium. Generic determinacy in this case is established by Geanakoplos and Polemarchakis (1987).

When $m < n$, there are fewer equations than unknowns, and the regularity of the system of equations means that it is generically overdetermined. In this case, generically it is impossible to satisfy the equations simultaneously, that is, generically $E(r)$ is empty. As a simple example of this argument, consider the prevalence of trade at equilibrium in an Edgeworth box economy. One market clearing condition in one (normalized) price characterizes equilibria, and standard arguments show that this excess demand function has 0 as a regular value. In fact, varying the endowment of the first agent alone is enough. How often does equilibrium involve trade in some goods? With only two agents, trade occurs in equilibrium if and only if $x_2 \neq e_2$, so the additional two equations $x_2(p,e) - e_2 = 0$ characterize endowment and price combinations for which there is no trade in equilibrium. A simple calculation shows that 0 is a regular value of $f(p,e) := (z_{-2}(p,e), x_2(p,e) - e_2)$. Fixing the endowment profile e, however, this is a problem with three equations in a single variable, so there must be a set $R^{**} \subset \mathbf{R}^{mL}_{++}$ of full Lebesgue measure such that for every $e \in R^{**}$, there are no solutions to the equation $f(p,e) = 0$. For every endowment profile $e \in R^{**}$, every equilibrium then must involve trade, as every equilibrium price solves the first equation $z_{-2}(p^*,e) = 0$, so cannot also involve no trade, $x_2(p^*,e) \neq e_2$. Similar logic but more involved calculations show that equilibrium allocations are generically inefficient in incomplete markets models, and generically constrained inefficient in multi-good incomplete markets models. Geanakoplos and Polemarchakis (1987) pioneered this approach to efficiency with incomplete markets.

Finally, when $m > n$, generically indeterminacy arises, as generically the solution set $E(r)$ is an $(m-n)$-dimensional manifold.[3] In this case, generically there are a continuum of solutions, and the set of solutions is locally, up to diffeomorphism, a set of dimension $m-n$. An important example of this case is provided by two-period incomplete financial markets models with nominal assets. Here, asset payoffs are in nominal terms, in some specified unit of account. As in the

[3] A subset $M \subset \mathbf{R}^m$ is a d-dimensional C^ℓ manifold if for each $x \in M$ there exist open sets $V \subset \mathbf{R}^m$ and $W \subset \mathbf{R}^d$, where V is a neighborhood of x, and a C^ℓ diffeomorphism $\phi : V \to W$ such that $\phi(V \cap M) = W$.

case of real assets described above, there are $S + 1$ independent budget constraints when there are S possible states of nature, so there are $S + 1$ redundant market clearing equations. Because asset payoffs are nominal, however, budget constraints are not all homogeneous of degree zero, and price levels matter. With only two homogeneity conditions, one for period one prices and one relating all commodity and asset prices, this leaves $S - 1$ dimensions of indeterminacy in equilibria generically. The detailed result is established by Geanakoplos and Mas-Colell (1989).

These three cases, and the generic properties of solution sets that follow, are collected below.

Theorem 2 *Let $f : \mathbf{R}^m \times \mathbf{R}^k \to \mathbf{R}^n$ be a C^ℓ function, where $\ell > \max\{m - n, 0\}$, and suppose 0 is a regular value of f.*

(a) Suppose $m = n$. There exists a set $R^* \subset \mathbf{R}^k$ of full Lebesgue measure such that for every $r \in R^*$, $E(r)$ contains only isolated points, $E(r)$ is finite when compact, and E is locally C^1 at r.

(b) Suppose $m < n$. There exists a set $R^* \subset \mathbf{R}^k$ of full Lebesgue measure such that for every $r \in R^*$, $E(r)$ is empty.

(c) Suppose $m > n$. There exists a set $R^* \subset \mathbf{R}^k$ of full Lebesgue measure such that for every $r \in R^*$, $E(r)$ is an $(m - n)$ -dimensional C^ℓ manifold.

The techniques pioneered by Debreu have found widespread applications, and have proven to be remarkably powerful. Nonetheless, the smoothness needed to study determinacy using the tools of differential topology does stem from assumptions that often carry real economic content. These assumptions restrict both the nature of admissible preferences, and the nature of admissible constraints.

For example, to avoid problems arising when nonnegativity constraints on consumption may become binding, these results rest on "boundary" restrictions, both on endowments, because individual endowments are strictly positive, or on equilibrium consumption via boundary conditions on preferences that imply individual demands are strictly positive at all prices. Unless goods are aggregated extremely coarsely, neither pattern is supported by observations on consumer behavior or characteristics. Relaxing the constraint on endowments turns out, perhaps surprisingly, to generate indeterminacy much more readily than relaxing the assumptions on positive consumptions, or incorporating other more general constraints on choices. Minehart (1997) shows by means of an example that for one natural case of restricted endowments, in which each agent is constrained to hold a single, individual-specific, good, an open subset of such parameters leads to indeterminacy in equilibrium.[4] If the assumption that individual endowments of every good are positive is maintained, the restriction to positive individual demand for every good can

[4] Mas-Colell (1985) shows that this conclusion is not robust to perturbations in preferences; generic determinacy, in a topological sense, is restored by considering variations in preferences as well as constrained endowments.

be relaxed. For example, Mas-Colell (1985) provides generic determinacy results for exchange economies allowing for boundary consumptions; Figure 3 depicts such preferences.

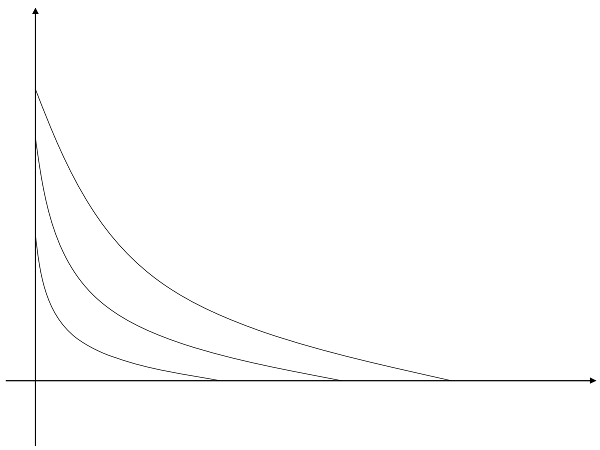

Figure 3: preferences allowing boundary consumption

Smooth preferences, as defined by Definition 2 above, obviously rule out preferences with nondifferentiabilities in level sets, a restriction that also has important behavioral content. Kinks have arisen as central manifestations of various behavioral phenomena, including loss aversion, ambiguity aversion, and reference dependence.[5] Such kinks typically lead to excess demand functions that fail to be differentiable for some prices. Rader (1973), Pascoa and Werlang (1999), Shannon (1994), and Blume and Zame (1993) all develop methods to address such cases. With the exception of Blume and Zame (1993), these techniques can be roughly understood as expanding differential notions by adding to "regularity" the condition that the function (e.g. excess demand) is differentiable at every solution, and establishing that analogues of implicit function theorems, Sard's theorem or the transversality theorem remain valid for sufficiently nice non-smooth functions, such as Lipschitz continuous functions.[6] Blume and Zame (1993) instead use results that exploit the structure of algebraic sets to establish generic determinacy for utilities that are, roughly, finitely piecewise analytic, and need not be strictly concave. Examples in which determinacy has been studied using techniques along these various lines include asset market models with restricted participation (for example, see Cass, Siconolfi, and Villanacci (2001)) and models of ambiguity aversion (for example, see Rigotti and Shannon (2006)).

[5] Examples inlcude Kahneman and Tversky (1979), Tversky and Kahneman (1991), Koszegi and Rabin (2006), Sagi (2006), and Gilboa and Schmeidler (1989).

[6] In particular, see Shannon (1994, 2005).

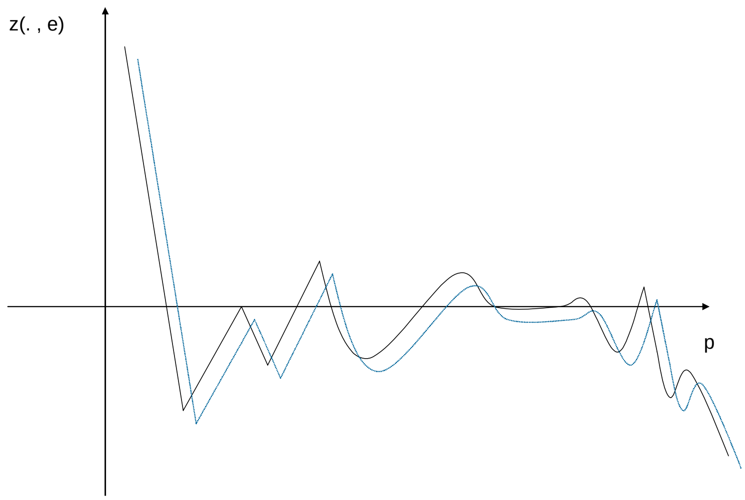

Figure 4: generic determinacy for nonsmooth excess demand

4 Determinacy in Infinite-Dimensional Economies

Many economic models require an infinite number of marketed commodities. Important examples include dynamic infinite horizon economies, continuous-time trading in financial markets, and markets with differentiated commodities. Such infinite-dimensional models present big obstacles to studying determinacy, starting with the fact that individual demand is not defined for most prices, precluding any straightforward parallel of Debreu's arguments for finite economies. In addition, the positive cone in most infinite-dimensional spaces has empty interior in the relevant topologies, meaning individual consumption sets are "all boundaries", and existence of equilibrium typically requires conditions, such as uniform properness or variants, that effectively bound marginal rates of substitution. Boundary conditions akin to those in Debreu's smooth preferences are likely either to be impossible to satisfy or to contradict equilibrium existence in many important applications.

Provided there are finitely many agents and no market distortions, using the welfare theorems and Negishi's argument provides an alternative characterization of equilibria, replacing excess demand with "excess savings". Some version of this characterization of equilibria provides the framework for much of the existing equilibrium analysis in economies with infinitely many commodities, including the seminal work on existence of Mas-Colell (1986) and Aliprantis, Brown, and Burkinshaw (1987), and the approach to determinacy for discrete-time infinite horizon models with time separability pioneered by Kehoe and Levine (1985). To explain this, let X denote the commodity space. The efficient allocations are the solutions to a social planner's problem of the following form: given $\lambda \in \Lambda := \{\lambda \in \mathbf{R}^m_+ : \sum_{i=1}^m \lambda_i = 1\}$, choose a feasible allocation $x(\lambda)$ to solve:

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} \lambda_i U_i(x_i) \\ \text{s.t.} \quad & \sum_{i=1}^{m} x_i \le \bar{e} \end{aligned}$$

Under standard assumptions, the solution $x(\lambda)$ to this problem is well-defined and unique for each $\lambda \in \Lambda$, and a unique price $p(\lambda)$ supporting $x(\lambda)$ can be characterized. Equilibria then correspond to the solutions λ to the budget equations

$$\begin{aligned} p(\lambda) \cdot (x_2(\lambda) - e_2) &= 0 \\ &\vdots \\ p(\lambda) \cdot (x_m(\lambda) - e_m) &= 0 \end{aligned}$$

where Walras' Law accounts for the missing equation. In parallel with excess demand, define the *excess savings map* $s : \Lambda \times X_+^m \to \mathbf{R}^{m-1}$

$$s(\lambda, e) := (p(\lambda) \cdot (x_2(\lambda) - e_2), \ldots, p(\lambda) \cdot (x_m(\lambda) - e_m))$$

Through this construction, the question of determinacy for infinite-dimensional economies can be cast in close parallel to finite economies, with the only change that the set of parameters is now infinite-dimensional. This raises several technical issues, most importantly the choice between topological and measure-theoretic notions of genericity due to the impossibility of defining a suitable analogue of Lebesgue measure in infinite-dimensional spaces.[7] This construction also makes imperative the need to link conditions on excess savings used to imply determinacy with conditions on preferences since, in contrast with excess demand, excess savings depends on artificial and unobservable constructs. Somewhat surprisingly, Shannon (1999) and Shannon and Zame (2002) show that generic determinacy follows from conditions on preferences that closely resemble Debreu's (1972) smooth preferences, after suitable renormalization. As in the finite case, these conditions can roughly be understood as strengthened notions of concavity, requiring that near feasible bundles utility differs from a linear approximation by an amount quadratic in the distance to the given bundle. These notions of concavity thus rule out preferences displaying local or global substitutes. Shannon and Zame (2002) provide a simple geometric argument showing that the excess spending mapping is Lipschitz continuous. Generic determinacy then follows by arguments similar to those sketched above for other problems with nondifferentiabilities.[8] The direct, geometric nature of these arguments render them applicable in a wide range of examples, including models of continuous-time trading, trading in differentiated commodities, and trading over an infinite horizon.

[7] See Hunt, Sauer, and Yorke (1992) and Anderson and Zame (2001) for a discussion of these issues.

[8] Shannon (2005) develops comparative statics and a version of the transversality theorem for this setting.

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