1 Comparison Between Two Papers

1.1 Mitsuru Igami and Nathan Yang, 2016

- Data Set: 400 sample path, 35 years (1970-2004)
- Information Term: number of outlets, firms' entry/exit decisions, population, income, (property value)
- Level of Data: market level, 0.8km radius
- Hidden State: market type, hidden to researcher, doesn't change with time

1.2 Jason R. Blevins, Ahmed Khwaja, Nathan Yang, 2018

- Data Set: 31 sample path, 35 years (1970-2004)
- Information Term: number of outlets, firms' entry/exit decisions, population, income, property value, region-specific minimum wage levels, host CFL Grey Cup, anti-smoking regulation
- Level of Data: city level, 60km radius
- Hidden State: profitability, hidden to researcher but known to decision maker, different among each firm

1.3 Our model

- Data Set:

  1.Market Level Data set (400×35 where market Type 1: 133×35, market Type 2: 134×35 market Type 3: 133×35);

  2.City Level Data set (31×35, reasons not to use: data set smaller and higher dim, over 60)
- Information Term:
  - number of outlets ($N_i, N_j(or\ N_{-i})$)
  - firms' entry/exit decisions (a_i)
  - population (s_1)
  - income (s_2)
  - property value (s_3)
- New term: (Available from 1970 to 2004, main indicators)
  - GDP Growth Rate (z)
  - Inflation Rate

- Unemployment Rate

- Hidden State: Economy Condition (s^h), hidden to both researcher and decision maker, different among the time but the same among each sample path.

2 Data Analysis

2.1 New Term Chosen

I choose GDP Growth Rate as the observation for the hidden state, economy condition. The reason is as follow:

- First, only GDP Growth Rate has both negative term and positive term which is shown in the following figure 1, figure 2, and figure 3. This character is consistent with the definition of the 2-dim economy condition (good condition and bad condition).

- Second, GDP Growth Rate is a kind of the coincident indicators, which may be used to identify, after the fact, the dates of peaks and troughs in the business cycle.

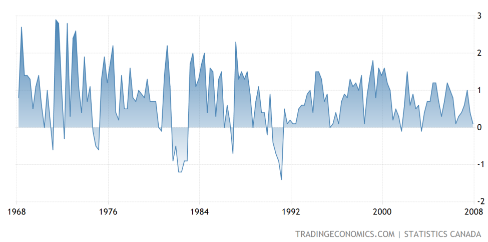

Figure 1: GDP Growth Rate

Figure 2: Inflation Rate

Figure 3: Unemployment Rate

2.2 Linear Regression: Market Level Data

In this part, I assume the linear regression model as:

$$\begin{aligned} a_{mcd,n,t} =& \gamma_0^{mcd} + \gamma_1^{mcd} N_{mcd,n,t} + \gamma_2^{mcd} N_{-mcd,n,t} \\ &+ \gamma_3^{mcd} s_{1,n,t} + \gamma_4^{mcd} s_{2,n,t} + \gamma_5^{mcd} s_{3,n,t} + \gamma_6^{mcd} z_t + v_{mcd,n,t} \end{aligned} \tag{1}$$

$$\begin{aligned} a_{other,n,t} =& \gamma_0^{other} + \gamma_1^{other} N_{other,n,t} + \gamma_2^{other} N_{-other,n,t} \\ &+ \gamma_3^{other} s_{1,n,t} + \gamma_4^{other} s_{2,n,t} + \gamma_5^{other} s_{3,n,t} + \gamma_6^{other} z_t + v_{other,n,t} \end{aligned} \tag{2}$$

Here, $(\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6)$ are the coefficients of base profit, own competition, rival competition, population, property value, income, gdp growth rate. $n \in \{1, 2, ..., 400\}, t \in \{1970, 1971, ...2004\}$. $v_{n,t}$ follows the i.i.d standard normal distribution. The results is as follow:

Agent i	MCD (McDonald's)	Other (A&W, Burger King, Harvey's, Wendy's)
Base Profit (γ_0^i)	1.008916 (0.035318)	1.004104 (0.035333)
Own-store Competition (γ_1^i)	-0.008936 (0.014934)	-0.031355 (0.011642)
Rival-store Competition (γ_2^i)	0.005464 (0.012112)	0.003106 (0.009189)
Population (γ_3^i)	0.000655 (0.007472)	-0.000087 (0.007470)
Property Value(γ_4^i)	0.004360 (0.008445)	0.000359 (0.008372)
Income (γ_5^i)	0.001209 (0.008449)	0.001228 (0.008459)
Gdp Growth Rate(γ_6^i)	0.009179 (0.031140)	0.000043 (0.031139)

Table 1: Regressions Results on Full Data

From the above regression results, only the parameters $\gamma_1 < 0$ and $\gamma_3^{other} < 0$. Namely, only the own-store competition has the negative influence on decision making. The population term for other stores has the negative influence on the other agents decision making. This part is contradict to the common sense. The rival influence should be negative. The reason is that I ignore market type information. Basing on the estimation of market type, the regression results are as follows:

Agent i	Type 1		Type 2		Type 3	
	MCD	Other	MCD	Other	MCD	Other
Base Profit (γ_0^i)	0.979631 (0.060356)	0.997050 (0.029882)	1.004658 (0.059428)	0.997447 (0.028740)	1.028972 (0.061898)	1.01190 (0.030083)
Own-store compete (γ_1^i)	-0.030952 (0.052991)	-0.067641 (0.035970)	-0.027950 (0.034214)	-0.039364 (0.026962)	-0.022387 (0.021203)	-0.028801 (0.015965)
Rival-store compete (γ_2^i)	-0.012310 (0.038070)	-0.004736 (0.016566)	-0.008828 (0.029203)	-0.007065 (0.011955)	-0.000764 (0.016006)	-0.001778 (0.007686)
Population (γ_3^i)	0.004205 (0.014514)	0.001799 (0.007085)	0.007287 (0.013304)	0.001612 (0.006671)	0.001726 (0.014207)	0.002050 (0.007294)
Property Value(γ_4^i)	0.007890 (0.014595)	0.000740 (0.007593)	0.005131 (0.014844)	0.001272 (0.007216)	0.006341 (0.015023)	0.001751 (0.007244)
Income (γ_5^i)	0.004341 (0.014161)	0.000311 (0.007204)	0.005520 (0.013985)	0.003529 (0.007473)	0.003488 (0.014845)	0.003580 (0.007424)
GDP Growth Rate(γ_6^i)	0.012652 (0.050748)	0.002891 (0.025821)	0.004561 (0.055080)	0.005491 (0.027715)	0.016066 (0.052577)	-0.002709 (0.026419)

Table 2: Regressions Results of Market Type 1, 2, 3

3 Recover Results

In my model setting, I choose discount factor to be $\beta = 0.6$. In each case, I need 5 to 6 hours to get one recover results. The initial points are all zeros.

3.1 Complete Cases

I don't parameterize the transition matrix here. I directly use the quantity distribution of data as the estimation of transition probability.

3.1.1 Without GDP Growth Rate Term

In this model, the one-step reward is

$$r_i(N_i, N_{-i}, s_1, s_2, s_3, a_i) = N_i \cdot (\theta_0^i + \theta_1^i N_i + \theta_2^i N_{-i} + \theta_3^i s_1 + \theta_4^i s_2 + \theta_5^i s_3) - \kappa 1_{\{a_i > 0\}} \tag{3}$$

Here, $i \in \{mcd, other\}$ and κ is the parameter of sunk cost. The recover results is as follow:

Agent i	Type 1		Type 2		Type 3	
	MCD	Other	MCD	Other	MCD	Other
Base Profit (θ_0^i)	9.793806	3.005595	10.050077	3.247261	5.826777	3.511965
Own-store compete (θ_1^i)	-6.489731	-1.090149	-1.136730	-0.938922	-0.720928	-0.626513
Rival-store compete (θ_2^i)	-1.617876	-0.560673	-0.387604	-0.603265	-0.014699	-0.032187
Population (θ_3^i)	0.412883	0.213491	0.259861	0.117467	0.031142	0.052789
Property Value(θ_4^i)	0.585137	0.130171	0.250104	0.198102	0.116319	0.065111
Income (θ_5^i)	0.301227	0.028432	0.203034	0.273457	0.046070	0.109699
Net Entry Sunk Cost (κ)	11.067892	7.656504	16.542355	7.519674	9.033867	7.166546
Log-likelihood	-16060.780721		-15459.793982		-13975.366515	

Table 3: Recover Results without GDP Growth Rate

3.1.2 With GDP Growth Rate Term

Here, I assume $z \in \{0\text{: GDP Growth Rate is negative, } 1\text{: GDP Growth Rate is positive}\}$. In this model, the one-step reward is

$$\begin{aligned} r_i(N_i, N_{-i}, s_1, s_2, s_3, a_i, z) =& N_i \cdot (\theta_0^i + \theta_1^i N_i + \theta_2^i N_{-i} + \theta_3^i s_1 + \theta_4^i s_2 + \theta_5^i s_3 \\ &+ \theta_6^i 1_{\{z=0\}} + \theta_7^i 1_{\{z=1\}}) - \kappa 1_{\{a_i>0\}} \end{aligned} \tag{4}$$

Here, θ_6^i and θ_7^i are the parameter for $z = 0$ and $z = 1$ respectively. In my recover process, I set $\theta_6^{other} = 0$ because of the correlation issue between θ_6^i, θ_7^i. The recover results is as follow:

Agent i	Type 1		Type 2		Type 3	
	MCD	Other	MCD	Other	MCD	Other
Base Profit (θ_0^i)	6.421244	1.599276	4.025341	1.606233	3.237952	3.763295
Own-store Compete (θ_1^i)	-8.311125	-1.086504	-1.138331	-0.951178	-0.719809	-0.626466
Rival-store Compete (θ_2^i)	-1.552391	-0.558178	-0.402293	-0.608044	-0.013865	-0.031940
Population (θ_3^i)	0.404332	0.212885	0.261238	0.117457	0.031891	0.052680
Property Value(θ_4^i)	0.585991	0.127276	0.247865	0.198481	0.118556	0.064944
Income (θ_5^i)	0.297753	0.033210	0.213066	0.277768	0.047726	0.109546
GDP Growth Rate ≤ 0 (θ_6^i)	-0.511737	0	0.373825	0	0.456034	0
GDP Growth Rate > 0 (θ_7^i)	6.932958	1.538947	1.606233	1.812223	2.782144	-0.275352
Net Entry Sunk Cost (κ)	12.696170	7.663612	12.569557	7.529290	9.038095	7.166785
Log-likelihood	-16057.570749		-15457.746572		-13974.068321	

Table 4: Recover Results with GDP Growth Rate

By comparing the results in Table 3 and Table 4, the term GDP growth rate can help improve the log-likelihood by 5. Moreover, $\theta_7^i > \theta_6^i \geq 0$ is consistent with the common sense that in good condition ($z = 1$) the agents are more likely open new stores than in bad condition ($z = 0$) in Type 1 and Type 2.

3.2 Partial Cases

In this model, I use z to partially observe hidden state, business cycle $s^h \in \{$ 0:bad economy condition or recession,1:good economy condition or expansion$\}$. The structure of my transition and observation matrix are as follow:

$P_{\eta_1}(z_{t+1}\|s^h_{t+1})$	$z_{t+1}=0$	$z_{t+1}=1$
$s^h_{t+1}=0$	$\eta_{1,1}$	$1-\eta_{1,1}$
$s^h_{t+1}=1$	$\eta_{1,2}$	$1-\eta_{1,2}$

Table 5: Observation Matrix

$P_{\eta_2}(s^h_{t+1}\|s^h_t)$	$s^h_{t+1}=0$	$s^h_{t+1}=1$
$s^h_t=0$	$\eta_{2,1}$	$1-\eta_{2,1}$
$s^h_t=1$	$\eta_{2,2}$	$1-\eta_{2,2}$

Table 6: Transition Matrix

The one-step reward is as follows:

$$r_i(N_i, N_{-i}, s_1, s_2, s_3, a_i, x) = \sum_{s^h=0}^{1} x(s^h)\Big(N_i \cdot (\theta_0^i + \theta_1^i N_i + \theta_2^i N_{-i} + \theta_3^i s_1 + \theta_4^i s_2 + \theta_5^i s_3 + \theta_6^i 1_{\{s^h=0\}} + \theta_7^i 1_{\{s^h=1\}}) - \kappa 1_{\{a_i>0\}}\Big) \tag{5}$$

Here, x is the belief for s^h. θ_6^i and θ_7^i are the parameter for $s^h=0$ and $s^h=1$ respectively. In my recover process, I set $\theta_6^{other}=0$ because of the correlation issue between θ_6^i, θ_7^i. The recover results is as follow:

	Type 1		Type 2		Type 3	
$P_{\eta_1}(z_{t+1}\|s^h_{t+1})$	$z_{t+1}=0$	$z_{t+1}=1$	$z_{t+1}=0$	$z_{t+1}=1$	$z_{t+1}=0$	$z_{t+1}=1$
$s^h_{t+1}=0$	0.9999	0.0001	0.9999	0.0001	0.9999	0.0001
$s^h_{t+1}=1$	0.0001	0.9999	0.0001	0.9999	0.0001	0.9999
$P_{\eta_2}(s^h_{t+1}\|s^h_t)$	$s^h_{t+1}=0$	$s^h_{t+1}=1$	$s^h_{t+1}=0$	$s^h_{t+1}=1$	$s^h_{t+1}=0$	$s^h_{t+1}=1$
$s^h_t=0$	0.2914	0.7085	0.2914	0.7085	0.2914	0.7085
$s^h_t=1$	0.0654	0.9345	0.0654	0.9345	0.0654	0.9345
Log-likelihood	-1259.0825		-1268.5493		-1259.0825	

Table 7: Estimation for Dynamic Matrix

Comparing Table 8 and Table 4, the influence of good and bad economy condition is more positive than the gdp growth rate for MCD company.

Agent i	Type 1		Type 2		Type 3	
	MCD	Other	MCD	Other	MCD	Other
Base Profit (θ_0^i)	3.127312	3.023752	4.671595	2.751361	3.816559	3.518256
Own-store (θ_1^i)	-2.627602	-1.22650374	-1.138465	-0.959743	-0.722695	-0.626421
Rival-store (θ_2^i)	-1.605730	-0.61146276	-0.407380	-0.959743	-0.015570	-0.029809
Population (θ_3^i)	0.412889	0.21369075	0.257286	0.116581	0.031229	0.053599
Property Value(θ_4^i)	0.596188	0.13531777	0.249473	0.198755	0.117912	0.065461
Income (θ_5^i)	0.288357	0.026975	0.208838	0.272448	0.045400	0.109128
Recession (θ_6^i)	0.822616	0	2.119861	0	1.750854	0
Expansion (θ_7^i)	2.304811	0.16650269	2.551779	0.630476	2.065704	-0.019070
Net Entry Sunk Cost (κ)	9.979604	7.6802635	12.186535	7.524737	9.034655	7.158967
Log-likelihood	-16057.9085		-15457.6701		-13974.7679	

Table 8: Recover Results for Partial Cases

4 Indicator

4.1 Estimation for Business Cycle

Cross and Bergevin 2012		Business Cycle Council 2021	
Peak (Quarterly)	Trough (Quarterly)	Peak (Quarterly)	Trough (Quarterly)
DEC 1974 (1974:Q4)	MAR 1975 (1975:Q1)	OCT 1974 (1974:Q3)	MAR 1975 (1975:Q1)
JAN 1980 (1979:Q4)	JUN 1980 (1980:Q2)	*	*
JUN 1981 (1981:Q2)	OCT 1982 (1982:Q4)	JUN 1981 (1981:Q2)	OCT 1982 (1982:Q4)
MAR 1990 (1990:Q1)	APR 1992 (1992:Q2)	MAR 1990 (1990:Q1)	MAY 1992 (1992:Q2)

Table 9: Business Cycle in Canada

ECRI		BBQ	
Peak (Quarterly)	Trough (Quarterly)	Peak (Quarterly)	Trough (Quarterly)
*	*	1980:Q1	1980:Q3
1981:Q1	1982:Q4	1981:Q2	1982:Q4
1990:Q1	1992:Q1	1990:Q1	1991:Q1

Table 10: Business Cycle in Canada

4.2 GDP Growth Rate

Choose GDP growth rate (quarterly data), and average the data (to yearly data).

GDP growth rate < 0 : recession

GDP growth rate > 0 : expansion

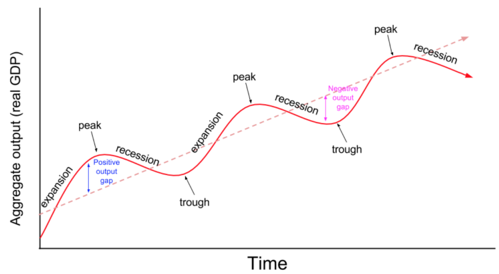

Figure 4: Real GDP and Business Cycle

GDP growth is the percentage change of real GDP between two consecutive periods (relative to some base year).

Limitation of GDP.

5 Future Work

Q1: The improvement of Partial Observable Case in log-likelihood is not too much

Solution:

(1) We can analysis this part with simulation data set.

(2) Try other indicators (Unemployment Rate or Inflation Rate) to compare with GDP Growth Rate – need more literature review

(3) Give a specific dynamic structure for complete cases (instead of directly using frequency as transition matrix). Namely, show that log-likelihood in dynamic part can be improved significantly.