A Web Appendix

This is a Web Appendix to

> **"Firm Expansion, Size Spillovers and Market Dominance in Retail Chain Dynamics."**

A.1 First Stage Estimation

In the first stage, we maximize the log likelihood function with respect to the first-stage parameters, denoted ϕ. For each value of ϕ, the algorithm for evaluating the log likelihood function is as follows:

1. For each market $m = 1, \dots, M$:

   (a) Draw R particles, denoted $\tilde{Z}^r_{m,0}$, from the initial distribution of firm-specific spillover levels. Each draw $\tilde{Z}^r_{m,0}$ is a vector of length I, with one component for each firm.

   (b) For each $t = 1, \dots, T$:

      i. Transition each of the R particles $\tilde{Z}^r_{m,t-1}$ from period $t-1$ through the joint transition equation as determined by (3), given the current value of ϕ, to obtain a new collection of particles denoted[37] Z^r_{mt}.

      ii. Calculate and store the joint likelihood (the product of the firm-specific choice probabilities) associated with each particle, given the values of n_{mt} and X_{mt} from the data. Denote these values by p^r_{mt} for $r = 1, \dots, R$.

      iii. Calculate and store the log likelihood value, given by log of the averaged joint particle-specific likelihood values: $l_{mt} = \ln\left(\frac{1}{R}\sum_{r=1}^{R} p^r_{mt}\right)$.

      iv. Assign the likelihood p^r_{mt} as an importance weight for each particle Z^r_{mt}.

      v. Draw R new particles, denoted $\tilde{Z}^r_{mt}$ for $r = 1, \dots, R$ by sampling with replacement from the particles $\{Z^r_{mt}\}_{r=1}^{R}$ in proportion to the assigned importance weights.[38]

2. Return the value of the log-likelihood function for the trial value of ϕ, $L_M(\phi) = \sum_{m=1}^{M}\sum_{t=1}^{T} l_{mt}$.

A.2 Second Stage Estimation

In the second stage, we choose structural parameters α in order to maximize the minimum distance objective function $Q(\alpha)$. Initially, we construct a collection of B "inequalities" as follows:

[37]The distinction between $\tilde{Z}^r_{mt}$ and Z^r_{mt} is intentional. The former are draws from the period $t-1$ filtering distribution (for the latent states at time $t-1$ given period $t-1$ information) while the latter are draws from the period t prediction distribution (for the latent states at time t using period $t-1$ information).

[38]Again, the tilde denotes that these are draws for period t updated with period t information.

1. Randomly draw B initial market structures, denoted (N_1^b, X_1^b, Z_1^b) for $b = 1, \ldots, B$, consisting of exogenous state variables, firm-specific observable variables, and firm-specific unobservable spillover levels.

2. For each initial market structure indexed by $b = 1, \ldots, B$, choose a single firm i and draw a random alternative policy function for that firm by adding a random vector $\varrho^b \sim \mathrm{N}(0, \sigma_\varrho^2 I)$ to the subvector of parameters in $\hat{\phi}$ related to the first-stage policy functions. Let $\tilde{\sigma}^b$ denote the policy profile where firm i is using the alternative policy while all other firms use the estimated policies from $\hat{\sigma}$.

Once the inequalities are determined, calculate the objective function $Q(\alpha)$ for any α as follows:

1. Given the estimated first-stage parameters $\hat{\phi}$ and a vector of structural parameters α, repeat the following steps for each of the initial market structures indexed by $b = 1, \ldots, B$:

   (a) Draw S sample paths of length T, each starting at (N_1^b, X_1^b, Z_1^b), using the laws of motion determined by the given parameters $\hat{\phi}$ and α. Store the discounted profits for the chosen firm i for the simulated path.

   Specifically, for each simulated path $s = 1, \ldots, S$, for each time $t = 1, \ldots, T$:

      i. Using the law of motion estimated in the first stage, draw shocks to simulate a new vector of firm-specific spillover levels, $Z_t^{b,s}$.

      ii. Using the fitted SUR model, draw shocks and simulate new values for each of the exogenous variables, $X_t^{b,s}$.

      iii. Using the new spillover levels and exogenous variables, draw structural shocks and evaluate the estimated policies $\hat{\sigma}$ in order to simulate each firm's expansion or contraction decision, $n_{it}^{b,s}$.

      iv. Calculate the stock of outlets for each firm, $N_{it}^{b,s} = N_{i,t-1}^{b,s} + n_{it}^{b,s}$.

      v. Calculate the structural period profits for firm i in period t under $\hat{\sigma}$, denoted $\hat{\pi}_{it}^{b,s}$:

$$\hat{\pi}_{it}^{b,s} = \Pi(N_{it}^{b,s}, N_{-it}^{b,s}, X_t^{b,s}, Z_t^{b,s}, \zeta_t^{b,s}, \alpha).$$

   (b) Calculate the discounted profits for firm i from the perspective of the initial state for each simulated path, $V^{b,s}(\hat{\sigma}, \alpha) = \sum_{t=1}^{T} \rho^{t-1} \hat{\pi}_{it}^{b,s}$.

(c) Estimate the ex-ante value of being in state (N_1^b, X_1^b, Z_1^b) for firm i (when firms use the estimated policies in the profile $\hat{\sigma}$) by averaging the discounted profits over all S paths: $\bar{V}^b(\hat{\sigma}, \alpha) = \frac{1}{S}\sum_{s=1}^{S} \hat{V}^{b,s}(\hat{\sigma}, \alpha)$.

(d) Repeat steps in part 1a to simulate S alternative paths of length T, also starting at (N_1^b, X_1^b, Z_1^b) and using the parameters $\hat{\phi}$ and α, but using the alternative policy from $\tilde{\sigma}^b$ for firm i. Let $\tilde{\pi}_{it}^{b,s}$ denote the profits earned by firm i in period t along path s.

(e) Calculate the discounted profits for firm i from the perspective of the initial state for the alternative paths, $V^{b,s}(\tilde{\sigma}^b, \alpha) = \sum_{t=1}^{T} \rho^{t-1} \tilde{\pi}_{it}^{b,s}$.

(f) Estimate the ex-ante value of being in state (N_1^b, X_1^b, Z_1^b) for firm i (when using the alternative policy against the estimimated policies of the rival firms) by averaging the discounted profits over all S paths: $\bar{V}^b(\tilde{\sigma}^b, \alpha) = \frac{1}{S}\sum_{s=1}^{S} \tilde{V}^{b,s}(\tilde{\sigma}^b, \alpha)$.

(g) Calculate the difference in the ex-ante valuations, $g_b(\hat{\sigma}, \alpha) = \bar{V}^b(\hat{\sigma}, \alpha) - \bar{V}^b(\tilde{\sigma}^b, \alpha)$.

2. Use the values $g_b(\hat{\sigma}, \alpha)$ for each of the B initial states and alternative policies to calculate the value of the minimum distance function, $Q(\alpha) = \frac{1}{B}\sum_{b=1}^{B}(\min\{g_b(\hat{\sigma}, \alpha), 0\})^2$.

A.3 Seemingly Unrelated Regressions (SUR) Model

To model the joint evolution of the exogenous state variables, we employ an SUR model. One justification for this approach is that all of these variables may be correlated at some level. For example, income and property value often move along similar trends. The SUR specification is:

$$\begin{bmatrix} X_{1t} \\ X_{2t} \\ \dots \\ X_{kt} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \dots \\ c_k \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1k} \\ A_{21} & A_{22} & \dots & A_{2k} \\ \dots & & & \\ A_{k1} & A_{k2} & \dots & A_{kk} \end{bmatrix} \cdot \begin{bmatrix} X_{1t-1} \\ X_{2t-1} \\ \dots \\ X_{kt-1} \end{bmatrix} + \begin{bmatrix} e_{1t-1} \\ e_{2t-1} \\ \dots \\ e_{kt-1} \end{bmatrix}$$

where $E[e_t e_t'] = \Omega$ and where $c = (c_1, \dots, c_k)$, $A = (a_{ij})$, and Ω are parameters to be estimated. Estimates of the intercepts c and the coefficients A are reported in Table 11 and estimates for the covariances are reported in Table 12.

A.4 Reduced Form Policy and Payoff Estimates

Table 13 contains the reduced form policy estimates from the first stage for each specification as described in Section 4.1. As indicated in the table we include either city fixed effects or city-specific drift terms, denoted η_m, and firm fixed effects or firm-specific drift terms in all specifications. For

Table 11: SUR Model Estimates

	Population$_t$		Income$_t$		Property Value$_t$		Minimum Wage$_t$	
Population$_{t-1}$	1.0043	(0.0031)	0.0007	(0.0003)	0.0035	(0.0020)	-0.0194	(0.0142)
Income$_{t-1}$	0.0148	(0.1586)	0.8499	(0.0174)	0.4498	(0.1041)	1.4761	(0.7366)
Property Value$_{t-1}$	0.0118	(0.0147)	0.0018	(0.0016)	0.9359	(0.0096)	0.1401	(0.0682)
Minimum Wage$_{t-1}$	0.0010	(0.0008)	0.0002	(0.0001)	-0.0010	(0.0005)	0.9869	(0.0037)
Intercept	-0.0061	(0.0096)	0.0072	(0.0011)	-0.0076	(0.0063)	0.1136	(0.0448)

Table 12: Estimated SUR Covariance Matrix

	Population$_t$	Income$_t$	Property Value$_t$	Minimum Wage$_t$
Population$_{t-1}$	2.2420	-0.0060	-0.0017	-0.3005
Income$_{t-1}$	-0.0060	0.0271	0.0011	-0.0102
Property Value$_{t-1}$	-0.0017	0.0011	0.9651	0.2625
Minimum Wage$_{t-1}$	-0.3005	-0.0102	0.2625	48.3756

Note: All entries have been multiplied by 10^3 for easier comparison.

specifications which include the Z process the inclusion of drift terms is denoted as "Z" while in the specification with only i.i.d. unobservables, the inclusion of fixed effects is denoted as "Yes". For both specifications including the Z process, we used 1000 particles to approximate the distribution of Z_{imt} for each firm i, market m, and time period t. Finally, Table 14 reports the remaining second-stage estimates, which are the reduced form portion of payoffs involving market characteristics.

A.5 Additional Simulation Analysis Details

To implement the model simulations and counterfactuals, we employ a similar forward simulation approach as in Benkard, Bodoh-Creed, and Lazarev (2010), which does not require one to solve a computationally intractable dynamic model. First, we estimate the different specifications (full model with Z process, model without Z process, model with Z process but no spillovers) and store the first and second stage estimates.

We then initialize the market characteristics (i.e., population, income, property value, and minimum wage) and market structure (i.e., the initial number of outlets for each chain) using data for the first year for each market. The unobserved and serially correlated Z process also needs to be initialized in our simulations. We draw the first period chain-market-specific $Z_{i,m,1}$'s from the corresponding steady-state distributions at time period $t = 1$ under the assumption of no size spillovers from $t = 0$ as by definition $N_{i,m,0} = 0$ for all firms. Using the estimated Z process, under these assumptions for each chain i in market m in the initial period $t = 0$, the stationary mean is $(\mu_i + \eta_m)/(1 - \delta_i)$, and the stationary variance is $\omega_i^2/(1 - \delta_i^2)$ (see e.g., Hamilton, 1994, p. 53).

Table 13: Reduced Form Policy Estimates

Parameter	Z (Spillovers)		Z (No Spillovers)		No Z	
Population	0.0674	(0.0006)	-0.1457	(0.0030)	-0.1416	(0.0948)
Income	0.0759	(0.0010)	0.2019	(0.0017)	0.2081	(0.0197)
Property Value	-0.0328	(0.0004)	-0.1472	(0.0048)	-0.1398	(0.0486)
Grey Cup Host	0.3742	(0.0072)	0.1436	(0.0044)	0.1479	(0.0755)
Smoking Regulation	-0.1738	(0.0014)	-0.0810	(0.0015)	-0.1395	(0.1196)
Minimum Wage	-0.1746	(0.0017)	-0.1334	(0.0018)	-0.1252	(0.0412)
Population^2	0.0505	(0.0009)	0.0679	(0.0013)	0.0553	(0.0271)
Population $\times$ Income	0.0420	(0.0007)	0.0481	(0.0013)	0.0487	(0.0257)
Population $\times$ Prop. Value	-0.0611	(0.0005)	-0.0677	(0.0011)	-0.0646	(0.0325)
Population $\times$ Min. Wage	-0.0398	(0.0004)	-0.0539	(0.0011)	-0.0476	(0.0188)
Income^2	-0.0214	(0.0002)	-0.0219	(0.0003)	-0.0233	(0.0035)
Income $\times$ Prop. Value	-0.1038	(0.0012)	-0.1287	(0.0004)	-0.1323	(0.0088)
Income $\times$ Min. Wage	0.0359	(0.0006)	0.0334	(0.0006)	0.0354	(0.0062)
Property Value^2	0.1278	(0.0015)	0.1700	(0.0014)	0.1709	(0.0205)
Property Value $\times$ Min. Wage	0.0123	(0.0001)	0.0140	(0.0005)	0.0139	(0.0150)
Minimum Wage^2	-0.0150	(0.0002)	-0.0145	(0.0003)	-0.0146	(0.0088)
Own Z	1.0000	–	1.0000	–	–	–
Own Z $\times$ Population	0.2967	(0.0076)	0.0353	(0.0017)	–	–
Own Z $\times$ Income	-0.0055	(0.0001)	-0.0112	(0.0001)	–	–
Own Z $\times$ Prop. Value	-0.0006	(0.0000)	-0.0001	(0.0000)	–	–
Own Z $\times$ Min. Wage	0.0064	(0.0001)	0.0050	(0.0001)	–	–
Rival Z	-0.1856	(0.0015)	-0.1433	(0.0052)	–	–
Rival Z $\times$ Population	0.0045	(0.0000)	0.0083	(0.0002)	–	–
Rival Z $\times$ Income	0.0079	(0.0001)	-0.0114	(0.0003)	–	–
Rival Z $\times$ Prop. Value	-0.0023	(0.0000)	-0.0115	(0.0002)	–	–
Rival Z $\times$ Min. Wage	0.0379	(0.0004)	0.0079	(0.0002)	–	–
Cutoff 1 (ϑ_1)	-7.1358	(0.1725)	-4.0207	(0.1255)	-3.4663	(0.8414)
Cutoff 2 (ϑ_2)	-4.5462	(0.1044)	-3.1793	(0.1034)	-2.8467	(0.1905)
Cutoff 3 (ϑ_3)	-3.3214	(0.0969)	-2.3914	(0.1055)	-2.1811	(0.1437)
Cutoff 4 (ϑ_4)	0.9318	(0.0146)	0.8660	(0.0220)	0.8186	(0.0776)
Cutoff 5 (ϑ_5)	1.9484	(0.0212)	1.6257	(0.0246)	1.5274	(0.0734)
Cutoff 6 (ϑ_6)	2.5782	(0.0415)	2.0906	(0.0418)	1.9487	(0.0805)
Cutoff 7 (ϑ_7)	3.7805	(0.0928)	2.8829	(0.1073)	2.6372	(0.1204)
City Fixed Effects (η_m)	Z		Z		Yes	
Firm Fixed Effects (μ_i)	Z		Z		Yes	
Observations	5580		5580		5580	
Particles	1000		1000		-	
Log Likelihood	-3918.23		-4044.55		-4059.40	
AIC	8018.45		8233.10		8232.80	
BIC	8159.03		8344.33		8320.85	

log

Table 14: Reduced Form Payoff Estimates

Parameter	Z (Spillovers)		Z (No spillovers)		No Z	
Population ($\theta_{1,1}$)	0.0372	(0.0190)	0.0096	(0.0145)	-0.1169	(0.1197)
Income ($\theta_{1,2}$)	-0.0041	(0.0056)	-0.0040	(0.0083)	0.0319	(0.0348)
Property Value ($\theta_{1,3}$)	0.0139	(0.0110)	-0.0060	(0.0131)	-0.2162	(0.0616)
Grey Cup Host ($\theta_{1,4}$)	0.0034	(0.2181)	0.0910	(0.2968)	0.8161	(1.4234)
Smoking Regulation ($\theta_{1,5}$)	0.0476	(0.0344)	0.0488	(0.0392)	0.0036	(0.3305)
Minimum Wage ($\theta_{1,6}$)	0.0071	(0.0106)	0.0319	(0.0136)	0.1940	(0.0552)

Using the estimates, inferred policy functions from the first stage estimation, and SUR process for the exogenous market characteristics, along with the initializations, we then forward simulate the evolution of the number of stores and per-period profits across all markets m, for each of the specifications. In each market, we simulate 250 sample paths (with length of 36 years) given the initial market conditions, distribution of Z's, and inferred policy functions.