1 Comparison Between Two Papers

1.1 Mitsuru Igami and Nathan Yang, 2016

- Data Set: 400 sample path, 35 years (1970-2004)
- Information Term: number of outlets, firms' entry/exit decisions, population, income, (property value)
- Level of Data: market level, 0.8km radius
- Hidden State: market type, hidden to researcher, doesn't change with time

1.2 Jason R. Blevins, Ahmed Khwaja, Nathan Yang, 2018

- Data Set: 31 sample path, 35 years (1970-2004)
- Information Term: number of outlets, firms' entry/exit decisions, population, income, property value, region-specific minimum wage levels, host CFL Grey Cup, anti-smoking regulation
- Level of Data: city level, 60km radius
- Hidden State: profitability, hidden to researcher but known to decision maker, different among each firm

1.3 Our model

- Data Set: 1.Market Level Data set (400×35); 2.City Level Data set (31×35)
- Information Term:
  - number of outlets ($N_i, N_j (or\ N_{-i})$)
  - firms' entry/exit decisions (a_i)
  - population (s_1)
  - income (s_2)
  - property value (s_3)
- New term: (Available from 1970 to 2004, main indicators)
  - GDP Growth Rate (z)
  - Inflation Rate
  - Unemployment Rate
- Hidden State: Economy Condition (s^h), hidden to both researcher and decision maker, different among the time but the same among each sample path.

2 Data Analysis

2.1 New Term Chosen

I choose GDP Growth Rate as the observation for the hidden state, economy condition. The reason is as follow:

- First, only GDP Growth Rate has both negative term and positive term which is shown in the following figure 1, figure 2, and figure 3. This character is consistent with the definition of the 2-dim economy condition (good condition and bad condition).
- Second, GDP Growth Rate is a kind of the coincident indicators, which may be used to identify, after the fact, the dates of peaks and troughs in the business cycle.

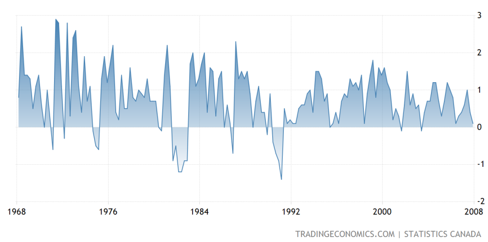

Figure 1: GDP Growth Rate

Figure 2: Inflation Rate

Figure 3: Unemployment Rate

2.2 Linear Regression: Market Level Data

In this part, I assume the linear regression model as:

$$\begin{aligned} a_{mcd,n,t} =& \gamma_0^{mcd} + \gamma_1^{mcd} N_{mcd,n,t} + \gamma_2^{mcd} N_{-mcd,n,t} \\ &+ \gamma_3^{mcd} s_{1,n,t} + \gamma_4^{mcd} s_{2,n,t} + \gamma_5^{mcd} s_{3,n,t} + \gamma_6^{mcd} z_t + v_{mcd,n,t} \end{aligned} \tag{1}$$

$$\begin{aligned} a_{other,n,t} =& \gamma_0^{other} + \gamma_1^{other} N_{other,n,t} + \gamma_2^{other} N_{-other,n,t} \\ &+ \gamma_3^{other} s_{1,n,t} + \gamma_4^{other} s_{2,n,t} + \gamma_5^{other} s_{3,n,t} + \gamma_6^{other} z_t + v_{other,n,t} \end{aligned} \tag{2}$$

Here, $(\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6)$ are the coefficients of base profit, own competition, rival competition, population, property value, income, gdp growth rate. $n \in \{1, 2, ..., 400\}, t \in \{1970, 1971, ...2004\}$. $v_{n,t}$ follows the i.i.d standard normal distribution. The results is as follow:

Agent i	MCD (McDonald's)	Other (A&W, Burger King, Harvey's, Wendy's)
Base Profit (γ_0^i)	1.008916 (0.035318)	1.004104 (0.035333)
Own-store Competition (γ_1^i)	-0.008936 (0.014934)	-0.031355 (0.011642)
Rival-store Competition (γ_2^i)	0.005464 (0.012112)	0.003106 (0.009189)
Population (γ_3^i)	0.000655 (0.007472)	-0.000087 (0.007470)
Property Value(γ_4^i)	0.004360 (0.008445)	0.000359 (0.008372)
Income (γ_5^i)	0.001209 (0.008449)	0.001228 (0.008459)
Gdp Growth Rate(γ_6^i)	0.009179 (0.031140)	0.000043 (0.031139)

Table 1: Regressions Results

From the above regression results, only the parameters $\gamma_1 < 0$ and $\gamma_3^{other} < 0$. Namely, only the own-store competition has the negative influence on decision making. The population term for other stores has the negative influence on the other agents decision making.

3 Recover Results

In my model setting, I choose discount factor to be $\beta = 0.6$. In each case, I need 5 to 6 hours to get one recover results. The initial points are all zeros.

3.1 Complete Cases

I don't parameterize the transition matrix here. I directly use the quantity distribution of data as the estimation of transition probability.

3.1.1 Without GDP Growth Rate Term

In this model, the one-step reward is

$$r_i(N_i, N_{-i}, s_1, s_2, s_3, a_i) = N_i \cdot (\theta_0^i + \theta_1^i N_i + \theta_2^i N_{-i} + \theta_3^i s_1 + \theta_4^i s_2 + \theta_5^i s_3) - \kappa 1_{\{a_i>0\}} \tag{3}$$

Here, $i \in \{mcd, other\}$ and κ is the parameter of sunk cost. The recover results is as follow:

Agent i	MCD	Other
Base Profit (θ_0^i)	5.671596	3.156060
Own-store Competition (θ_1^i)	-0.663133	-0.569137
Rival-store Competition (θ_2^i)	0.161048	0.163654
Population (θ_3^i)	0.022969	-0.002630
Property Value(θ_4^i)	0.139925	0.055765
Income (θ_5^i)	0.023551	0.055568
Net Entry Sunk Cost (κ)	9.883215	7.442349
Log-likelihood	-45731.035078	

Table 2: Recover Results without GDP Growth Rate

3.1.2 With GDP Growth Rate Term

Here, I assume $z \in \{0\text{: GDP Growth Rate is negative, 1: GDP Growth Rate is positive}\}$. In this model, the one-step reward is

$$\begin{aligned} r_i(N_i, N_{-i}, s_1, s_2, s_3, a_i, z) =& N_i \cdot (\theta_0^i + \theta_1^i N_i + \theta_2^i N_{-i} + \theta_3^i s_1 + \theta_4^i s_2 + \theta_5^i s_3 \\ &+ \theta_6^i 1_{\{z=0\}} + \theta_7^i 1_{\{z=1\}}) - \kappa 1_{\{a_i>0\}} \end{aligned} \tag{4}$$

Agent i	MCD	Other
Base Profit (θ_0^i)	3.097545	2.571658
Own-store Competition (θ_1^i)	-0.664163	-0.569556
Rival-store Competition (θ_2^i)	0.160879	0.163932
Population (θ_3^i)	0.022655	-0.002774
Property Value(θ_4^i)	0.142084	0.055763
Income (θ_5^i)	0.025409	0.055539
Negative GDP Growth Rate(θ_6^i)	0.304360	0
Positive GDP Growth Rate(θ_7^i)	2.793059	0.641219
Net Entry Sunk Cost (κ)	9.897543	7.442707
Log-likelihood	-45727.855105	

Table 3: Recover Results with GDP Growth Rate

Here, θ_6^i and θ_7^i are the parameter for $z = 0$ and $z = 1$ respectively. In my recover process, I set $\theta_6^{other} = 0$ because of the correlation issue between θ_6^i, θ_7^i. The recover results is as follow:

By comparing the results in Table 2 and Table 3, the term GDP growth rate can help improve the log-likelihood by 5. Moreover, $\theta_7^i > \theta_6^i \geq 0$ is consistent with the common sense that in good condition ($z = 1$) the agents are more likely open new stores than in bad condition ($z = 0$).

3.2 Partial Cases

In this model, I use z to partially observe hidden state $s^h \in \{$ 0:good economy condition,1:bad economy condition$\}$. The structure of my transition and observation matrix are as follow:

$P_{\eta_1}(z_{t+1}\|s_{t+1}^h)$	$z_{t+1} = 0$	$z_{t+1} = 1$
$s_{t+1}^h = 0$	$\eta_{1,1}$	$1 - \eta_{1,1}$
$s_{t+1}^h = 1$	$\eta_{1,2}$	$1 - \eta_{1,2}$

Table 4: Observation Matrix

$P_{\eta_2}(s_{t+1}^h\|s_t^h)$	$s_{t+1}^h = 0$	$s_{t+1}^h = 1$
$s_t^h = 0$	$\eta_{2,1}$	$1 - \eta_{2,1}$
$s_t^h = 1$	$\eta_{2,2}$	$1 - \eta_{2,2}$

Table 5: Transition Matrix

The one-step reward is as follows:

$$\begin{aligned} r_i(N_i, N_{-i}, s_1, s_2, s_3, a_i, x) = \sum_{s^h=0}^{1} x(s^h)\Big(N_i \cdot (\theta_0^i + \theta_1^i N_i + \theta_2^i N_{-i} \\ + \theta_3^i s_1 + \theta_4^i s_2 + \theta_5^i s_3 + \theta_6^i 1_{\{s^h=0\}} + \theta_7^i 1_{\{s^h=1\}}) - \kappa 1_{\{a_i>0\}}\Big) \end{aligned} \quad (5)$$

Here, x is the belief for s^h. θ_6^i and θ_7^i are the parameter for $s^h = 0$ and $s^h = 1$ respectively. In my recover process, I set $\theta_6^{other} = 0$ because of the correlation issue between θ_6^i, θ_7^i. The recover results is as follow:

$P_{\eta_1}(z_{t+1}\|s_{t+1}^h)$	$z_{t+1} = 0$	$z_{t+1} = 1$
$s_{t+1}^h = 0$	0.984343	0.015657
$s_{t+1}^h = 1$	0.000001	0.999999
$P_{\eta_2}(s_{t+1}^h\|s_t^h)$	$s_{t+1}^h = 0$	$s_{t+1}^h = 1$
$s_t^h = 0$	0.320457	0.679543
$s_t^h = 1$	0.056783	0.943217
Log-likelihood	-3799.594755	

Table 6: Estimation for Dynamic Matrix

Agent i	MCD	Other
Base Profit (θ_0^i)	3.632985	3.063102
Own-store Competition (θ_1^i)	-0.662911	-0.569576
Rival-store Competition (θ_2^i)	0.159071	0.163828
Population (θ_3^i)	0.022867	-0.002772
Property Value(θ_4^i)	0.141855	0.056090
Income (θ_5^i)	0.021585	0.054791
Bad Economy Condition(θ_6^i)	1.605323	0
Good Economy Condition(θ_7^i)	2.027524	0.111978
Net Entry Sunk Cost (κ)	9.758993	7.439714
Log-likelihood	-45728.860381	

Table 7: Recover Results for Partial Cases

Comparing Table 7 and Table 3, the influence of good and bad economy condition is more positive than the gdp growth rate for MCD company.

4 Further Work

- Try to use larger $\beta = 0.7, 0.8, 0.9$ and faster the running time
- Try other main indicators: inflation rate or unemployment rate to replace GDP growth rate
- Try other unrelated indicators: such as health indicators (need collect more data)
- Fix $\theta_{bad} - \theta_{good}$ to see what will happens
- Analysis city level data set (I don't think this part is necessary because the data set is too small, only 31×35)