Homework #3

1. A security consultant has observed that the attempts to breach the security of the company's computer system occurs according to a Poisson process with a mean rate of 2.2 attempts per day. (The system is on 24 hours per day using three 8-hour shifts.)

   Let $\{N(t)\}$ be process of breach attempts and X_1 be the first breach attempt.

   a. What is the probability that there will be exactly 4 breach attempts tomorrow?

   $P\{N_1 = 4\}$ = exp(-2.2)*2.2^4 / 4! = 0.108

   b. What is the probability that there will be no breach attempts during the evening shift?

   $P\{N_{1/3} = 0\}$ = exp(-2.2/3) = 0.480

   c. It is now midnight and the most recent attempt occurred at 10:30PM. What is the probability that the next attempt will occur before 5:30AM?

   $P\{X_1 <= 5.5/24\}$ = 1 – exp(-2.2*5.5/24) = 0.396

2. Consider again the computer security system from the previous problem. Further study of the attempts to breach the security of the computer system has determined that it is a non-stationary Poisson process. To approximate the process, each 24-hour period has been divided into four intervals: 6AM to 8PM, 8PM to midnight, midnight to 4AM, and 4AM to 6AM. Each interval is assumed to have a constant arrival rate for the attempted illegal entries. The mean number of attempts during the time span of each interval is 0.3, 0.6, 1, and 0.3. (Notice that this yields the same daily rate as given in the first problem.)

   a. What is the probability that there will be 4 breach attempts tomorrow?

   Poisson with rate = 2.2 ➔ exp(-2.2)*2.2^4 / 4! = 0.108 (same as problem 1a)

   b. What is the probability that there will be no breach attempts during the evening shift which is from 3PM until 11PM?

   Poisson with rate = 0.3*5/14 + 0.6*3/4 = 0.55714 ➔ exp(-0.55714) = 0.573

   c. It is now midnight and the most recent attempt occurred at 10:30PM. What is the probability that the next attempt will occur before 5:30AM?

   Poisson with rate = 1 + 0.3*1.5/2 = 1.225 ➔ 1 – exp(-1.225) = 0.706

3. There are two common types of failure to a critical electronic element of some machinery: either component A or component B may fail. If either component fails, the machinery goes down. Component A fails according to a Poisson process with mean rate 1.1 failures *per shift*. (The company operates 24/7 using eight-hour shifts.) Component B fails according to a Poisson process with a mean rate of 1.2 failures *per day*. Upon the failure the electronic element is immediately replaced, and for this problem, assume the time to replace the element is negligible.

   a. What is the probability that there will be exactly five failures of the machine within a given day?

   $P\{N_1 = 5\}$ = exp(-4.5)*4.5^5 / 5! = 0.171

   b. What is the probability that there will be no more than one failure of the machine during the next shift?

   $P\{N_{1/3} = 0\} + P\{N_{1/3} = 1\}$ = 0.558

   c. Assume you have the superposition of two Poisson processes $\{N(t)\}$ and $\{M(t)\}$ with mean rates λ_1 and λ_2. Derive the probability that the next occurrence of an arrival is from the

$\{N(t)\}$ process. Use your derivation to give the probability that the next failure of the machine is caused by Component A.

Let X_A be first time of Component A failure and X_B be first time of B failure.

Let $F(t) = 1 - \exp(-\lambda_1 t)$ and $G(t) = 1 - \exp(-\lambda_2 t)$.

$$P\{X_A < X_B\} = E[\ P\{X_A < X_B \mid X_B\}\ ] = \int_{[0,\infty)} P\{X_A < t\}\ dG(t)$$

$$= \int_{[0,\infty)} [1 - \exp(-\lambda_1 t)]\ \lambda_2 \exp(-\lambda_2 t)\ dt = 1 - \lambda_2/(\lambda_1+\lambda_2) = \lambda_1/(\lambda_1+\lambda_2)$$

Applying this formula to the above problem, we have $3.3/(3.3+1.2) = 0.733$