Homework #11

***Instructions**:* This is a derivation so hand in a *neat* copy of your derivation at the start of class on Monday, March 2.

Write on one side only. Put your name in the top right on the first page and the top left on the back side of the last page. You receive 5 points for simply trying. There will also be a deduction of 2 points if you don't follow instructions (write on one side only, name on front of first page and back of last page.)

1. Packages arrive at a mailing depot in accordance with a Poisson process having rate λ. Trucks, picking up all waiting packages, arrive in accordance to a renewal process with aperiodic inter-arrival distribution F. Let X(t) denote the number of packages waiting to be picked up at time t.

   a. What type of process is $\{X(t);\ t \geq 0\}$? Regenerative process.

   b. Obtain an expression for $P\{X(t) = k\}$ for $t \geq 0$ and $k \geq 0$.

Let {Sn} be the process of truck arrival times.

$$P\{X(t) = k\} = P\{X(t) = k, S1 > t\} + P\{X(t) = k, S1 <= t\}$$

$$= (\exp(-\lambda t)(\lambda t)^k/k!)\,[1 - F(t)] + \int_{[0,t]} F(ds) P\{X(t-s) = k\}$$

$$= (\exp(-\lambda t)(\lambda t)^k/k!)\,[1 - F(t)] + \int_{[0,t]} m(ds) \left(\frac{\exp(-\lambda t)(\lambda(t-s))^{\wedge}k}{k!}\right) [1 - F(t-s)]$$

Where $m = \Sigma Fn$.

   c. Obtain an expression for $\lim_{t\to\infty} P\{X(t) = k\}$ for $k \geq 0$.

$$\lim_{t\to\infty} P\{X(t) = k\} = (1/\mu) \int_0^\infty \left(\frac{\exp(-\lambda t)(\lambda t)^{\wedge}k}{k!}\right) [1 - F(t)]dt$$

where $\mu = \int_0^\infty [1 - F(t)]dt$

   d. Use your expression to obtain a numerical value for $\lim_{t\to\infty} P\{X(t) = 0\}$ for λ=5/hr and the time between truck arrivals is distributed according to a uniform distribution between 0 and 1 hour.

$$2 \times (1/\mu) \int_0^1 \left(\frac{\exp(-\lambda t)}{1}\right) [1 - t]dt = 2\times\{(1-e^{-5})/5 + (1/25 - 6e^{-5}/25)\} = 0.3205$$