Distributionally Robust Congestion Management With Dynamic Line Ratings

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***Abstract*—Dynamic line rating based on real time meteorological data has been shown to be useful in transmission line capacity management. Based on a binary rating forecast, we propose a distributionally robust congestion management model that selectively uses dynamic ratings on critical lines and keeps the risk of thermal overloading below a prescribed level. A case study illustrates that the proposed model can effectively alleviate transmission congestion with a low error rate.**

***Index Terms*—Chance constraint, congestion management, distributionally robust, dynamic line rating, robust optimization.**

I. Introduction

DYNAMIC line rating (DLR) constantly monitors the ambient environment to estimate the thermal rating of a transmission line [1]. However, as DLR is an estimation technique in nature, it is subjected to uncertainties in underlying ambient factors such as temperature and wind speed. Forecasting tools have been developed to predict the line ratings based on ambient conditions and line tension, etc. [1]. Although a line rating forecast can be made available, the inherent forecasting errors may render the system insecure and under thermal overloading risk. Inaccurate information on line ratings may lead to misoperations and potential power outages. Hence, in this paper we focus on the thermal overloading risk of transmission lines in short-term power system operations caused by rating forecast errors. Instead of examining the overloading risk on a single line as in existing research, we expand the study to evaluate the probability of overloading on multiple lines as we are addressing a system-wide congestion problem. We propose a new distributionally robust optimization based congestion management model that selectively uses DLR to alleviate system congestion while keeping the overloading risk at the system level within a safe range. As the formulated model is computationally challenging to solve, we exploit the structure of the problem and utilize the latest development in stochastic programming to convert the original nonlinear problem to a readily solvable MILP problem.

II. Probabilistic Congestion Management With DLR

In this paper, we use the binary line rating method developed in [2], which has been applied in a practical setting in the industry, for predicting line capacities. The binary line rating forecast predicts the probability (confidence level) that the rating exceeds a given threshold value. The forecast may be given in the following form: line ℓ will have 15% or more extra capacity above the static rating with a probability of 0.95 in time period t, in which 15% is the given threshold value and 0.95 is the confidence level. Therefore, given a threshold value, α, we can model the binary rating for line ℓ in time period t as a Bernoulli random number $\tilde{a}_{\ell,t}$. The forecasted probability for $\tilde{a}_{\ell,t} = 1$ is denoted as $r_{\ell,t}$. Clearly, line ℓ also has $1 - r_{\ell,t}$ of chance of having less than α extra capacity over its static rating.

TABLE I
Outcome Table

		$x_{\ell,t}$ = 1	$x_{\ell,t}$ = 0
$\tilde{a}_{\ell,t}$	1	correct	missed
	0	error	correct

Let the binary variable $x_{\ell,t}$ be the decision of whether to use the dynamic rating for line ℓ in time period t. Considering the combination of the actual outcomes of the binary rating and the decision whether to use the rating forecast as a random event (parameterized by $x_{\ell,t}$), we have the following four outcomes listed in Table I: Note that, when $\tilde{a}_{\ell,t} = 0$ and $x_{\ell,t} = 1$, line ℓ turns out to have no extra capacity and we decide to use the forecasted extra capacity. In this situation, there is a risk of thermal overloading on line ℓ, which is undesirable for system operations.

We define an overloading risk measurement at the system level as the probability that more than k lines are at an overloading risk and require this probability be no greater than a prescribed level, say, ϵ, i.e.,

$$\mathbb{P}(\text{more than } k \text{ out of } |L| \ (\tilde{a}_{\ell,t} = 0, x_{\ell,t} = 1) \text{ occur}) \leq \epsilon \tag{1}$$

where L is the set of lines monitored for DLR and $\mathbb{P}(E)$ evaluates the probability of event E. ϵ is a small fractional. The value of k reflects the system's tolerance for overloading and can be specified by a system operator. This risk measure could be used together with other security assessments, such as the one in [3], to obtain a comprehensive understanding of the risk of certain operation plans. Constraint (1) is often called the *chance constraint* or *probabilistic constraint*.

The probability in (1) is parameterized by decision variable $x_{\ell,t}$ and is equivalent to the following:

$$\mathbb{P}\left(\sum_{\ell \in L}(1 - \tilde{a}_{\ell,t})x_{\ell,t} \geq k + 1\right) \leq \epsilon. \tag{2}$$

Note that, unless $k = 1$ and $\tilde{a}_{\ell,t}$ are independent, the calculation of the probability in (2) requires all joint probabilities, which is of exponential size and often unavailable in practice. In case of incomplete distribution, e.g., only marginal and pairwise joint distributions are available, we can only identify a family of probability distributions, $\mathcal{P}$, instead of a specific distribution ξ. The value of $\mathbb{P}(E)$ could be dramatically different with regard to different ξ [4]. To ensure that each solution x be feasible under **every** possible distribution ξ in $\mathcal{P}$, we employ a distributionally robust model, i.e., $\sup_{\xi \in \mathcal{P}}\{\mathbb{P}_\xi(\sum_{\ell \in L}(1-\tilde{a}_{\ell,t})x_{\ell,t} \geq k$

Manuscript received May 02, 2014; revised August 15, 2014; accepted September 15, 2014. Date of publication October 09, 2014; date of current version June 16, 2015. This work was supported by the DOE Office of Electricity Delivery and Energy Reliability Advanced Grid Modeling program. Paper no. PESL-00064-2014.

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Digital Object Identifier 10.1109/TPWRS.2014.2361012

$+ 1)\} \le \epsilon$, where $\mathbb{P}_\xi(E)$ evaluates the probability of event E with regard to ξ.

Now we incorporate the dynamic rating and overloading risk requirement into an economic dispatch model by the following formulation:

$$\min \sum_{g\in G}\sum_{t\in T} c_g(p_{g,t}) + \sum_{n\in N}\sum_{t\in T} h_{n,t}q_{n,t} \tag{3}$$

$$\text{s.t. } G(p_{g,t}, p_{\ell,t}, q_{n,t}) \ge 0 \tag{4}$$

$$\sup_{\xi\in\mathcal{P}} \left(\mathbb{P}_\xi\Big(\sum_{\ell\in L}(1-\tilde{a}_{\ell,t})x_{\ell,t} \ge k+1\Big) \right) \le \epsilon \ \forall t \in T \tag{5}$$

$$\begin{aligned} & - SLR_\ell \cdot (1+\alpha_{\ell,t}x_{\ell,t}) \le p_{\ell,t} \\ & \le SLR_\ell \cdot (1+\alpha_{\ell,t}x_{\ell,t}) \quad \forall \ell \in L \quad \forall t \in T \end{aligned} \tag{6}$$

where $c_g(p_{g,t})$ is the cost of generator g in time period t, $h_{n,t}q_{n,t}$ is the cost of load shedding $q_{n,t}$ at bus n in time period t. $p_{\ell,t}$ is the power flow through line ℓ. $G(p_{g,t}, p_{\ell,t}, q_{n,t}) \ge 0$ represents the common linear constraints of a dispatch problem such as power balance, voltage angle, and ramping constraints. SLR_ℓ is the static rating for line ℓ. Constraints (6) provide options of using the extra capacity predicted by DLR. Note that this model produces solutions that are unlikely to have more than k overloading risk. Whether the solution satisfies other security requirement can be checked by other security assessment programs.

An exact characterization of the set defined by (5), denoted by X, requires an exponential-size formulation, hence not tractable [4]. Therefore, we develop an inner approximation of X that has a polynomial-size formulation and still yields distributionally robust solutions.

Proposition 1: Let $U(x)$ be a function of x such that $U(x) \ge F(x) := \sup_{\xi\in\mathcal{P}}\{\mathbb{P}_\xi(\sum_{\ell\in L}(1-\tilde{a}_{\ell,t})x_{\ell,t} \ge k+1)\}$ for all x of interest. Let $X := \{x \in \{0,1\}^{|L|} : F(x) \le \epsilon\}$ and $\bar{X} := \{x \in \{0,1\}^{|L|} : U(x) \le \epsilon\}$. Then $\bar{X} \subseteq X$.

Proposition 1 shows that an upper bound function can be used to construct an inner approximation. We next present a tractable upper bound function $U(x)$. Assume that the only available distribution information is the marginal distributions and joint distributions of up to m events, i.e., $p_C = \mathbb{P}(\bigcap_{i\in C}\tilde{a}_{i,t} = 0)$ for all $C \subseteq L$ such that $|C| \le m$. An upper bound $U(x)$ can be calculated using the following linear program [5]:

$$U(x) = \max\left\{ \sum_{j=k+1}^{|L|} v_j : \sum_{j=i}^{|L|} \binom{j}{i} v_j = s_i(x) \ i = 0 \ldots m \right\} \tag{7}$$

where $s_0(x) = 1$, $s_i(x) = \sum_{C\subseteq L:|C|=i} p_C \prod_{j\in C} x_j$, and $\binom{j}{0} = 1$; $v_i \ge 0$ are auxiliary variables and can be interpreted as the probability that exactly i events occur. We denote this linear program as $\max\{e_k^\top v : T^\top v = S(x), v \ge 0\}$, where $e_k \in \mathbb{R}_+^{|L|+1}$ is a vector with the first $k+1$ components equal to zero and the rest equal to one; $S(X) := (s_0(x), \ldots, s_m(x))^\top$; T is the coefficient matrix. Note that using (7) as $U(x)$ will introduce an optimization problem in the definition of $\bar{X}$. We use the dual of (7) to remove the optimization problem and express $\bar{X}$ in an explicit form as follows:

Proposition 2: Let $\pi \in \mathbb{R}^{m+1}$ be the dual multipliers for the maximization problem, then

$$\bar{X} = \{x \in \{0,1\}^{|L|} : \exists \pi \in \mathbb{R}^{m+1} : \pi^\top S(x) \le \epsilon, \pi^\top T \ge e_k^\top\}.$$

TABLE II
COMPARISON OF LOAD SHEDDING REDUCTIONS

Threshold (α)	ϵ	L.S. Reduction	# of lines	k-Overloading Risk
0.15	0.01	67%	5.25	0.007
	0.05	69%	6.25	0.016
0.30	0.01	68%	3.25	0.009
	0.05	80%	5.00	0.030

Note that the formulation in Proposition 2 consists of products of variables, i.e., $\pi_i \prod_{j\in C} x_j$. We introduce auxiliary variables $y_C = \pi_i \prod_{j\in C} x_j$ and linearize them using the McCormick linearization technique as follows:

$$\begin{aligned} & y_C \le M^+ x_j \quad \forall j \in C, \\ & y_C \ge -M^- x_j \quad \forall j \in C, \\ & y_C \le \pi_t + M^+(|C| - \sum_{j\in C} x_j) \\ & y_C \ge \pi_t - M^-(|C| - \sum_{j\in C} x_j) \end{aligned}$$

where M^+ and M^- are sufficiently large positive numbers and can be properly bounded [4]. Note that the number of variables and constraints introduced in the linearization is polynomial in the size of the number of lines monitored by DLR. After approximation and linearization, we obtain a mixed integer linear program (MILP), which is readily solvable by commercial MILP solvers.

III. Case Study

We study a four-hour dispatch problem on a revised IEEE 73 (RTS 96)-bus system. We assume that every line is monitored for dynamic rating forecast. Since the binary rating forecast itself is out of the scope of this paper, we assume that the binary rating forecast is given and $\tilde{a}_{\ell,t}$ are independent only for the sake of convenience to generate joint distributions. We generate two sets of ratings: lower ratings but with higher confidence levels ($\alpha = 0.15$ and $r_{\ell,t} \in [0.80, 0.99]$); and higher ratings but with lower confidence levels ($\alpha = 0.30$ and $r_{\ell,t} \in [0.75, 0.95]$). We use only marginal and pairwise joint distributions, i.e., $m = 2$, and fix k at 3.

Table II presents the results under different settings of α and ϵ. Column 3 presents load shedding reduction in percentage; Column 4 presents the average number of lines in each hour that use the extra capacity predicted by DLR; Column 5 presents upper bounds on the probability that more than k lines are at overloading risks, calculated by the linear program (7). All instances are solved within 10 to 20 min. Computational time can be significantly reduced when only critical lines are modeled with the DLR option. Table II shows that for both rating data sets, the proposed model with DLR can significantly reduce load shedding and keep the overloading risk under the prescribed level. For the same risk level requirement, the lower rating data set has a larger set of lines to utilize the extra capacity predicted by DLR than the higher rating data set.

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