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Distributionally robust hydro-thermal-wind economic dispatch

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HIGHLIGHTS

- A two-stage distributionally robust hydro-thermal-wind model is proposed.
- A semi-definite programing equivalent and its algorithm are developed.
- Cases that demonstrate the effectiveness of the proposed model are included.

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ABSTRACT

With the penetration of wind energy increasing, uncertainty has become a major challenge in power system dispatch. Hydro power can change rapidly and is regarded as one promising complementary energy resource to mitigate wind power fluctuation. Joint scheduling of hydro, thermal, and wind energy is attracting more and more attention nowadays. This paper proposes a distributionally robust hydro-thermal-wind economic dispatch (DR-HTW-ED) method to enhance the flexibility and reliability of power system operation. In contrast to the traditional stochastic optimization (SO) and adjustable robust optimization (ARO) method, distributionally robust optimization (DRO) method describes the uncertain wind power output by all possible probability distribution functions (PDFs) with the same mean and variance recovered from the forecast data, and optimizes the expected operation cost in the worst distribution. Traditional DRO optimized the random parameter in entire space, which is sometimes contradict to the actual situation. In this paper, we restrict the wind power uncertainty in a bounded set, and derive an equivalent semi-definite programming (SDP) for the DR-HTW-ED using S-lemma. A delayed constraint generation algorithm is suggested to solve it in a tractable manner. The proposed DR-HTW-ED is compared with the existing ARO based hydro-thermal-wind economic dispatch (AR-HTW-ED). Their respective features are shown from the perspective of computational efficiency and conservativeness of dispatch strategies.

1. Introduction

Nowadays, renewable energy, with its characteristic of clean, cheap and environmentally friendly, continues to flourish worldwide [1–3]. As [3] shows, renewable energy, such as solar power and wind, has enormous potential to the energy shortage problem solving in China. However, in view of its intermittence and fluctuation nature, increasing penetration of large-scale renewable energy exacerbates the difficulties to maintain power system reliability and security [4,5]. In this context, jointly scheduling of multiple energy resources is regarded as an efficient way to accommodate more renewable energy without compromising power system security and economy. Extensive researches have been dedicated to the coordinated optimization problem in a multi-resource power system. Purvins et al. [5] and Dallinger et al. [6] incorporate electrical vehicles to help balance energy and demand. Wang et al. [7] tries to reduce the risk by advanced thermal-wind unit commitment (UC). Wang et al. [8] further shows the potential of multi-energy with the development of the bidirectional communication in smart grid.

Multi-energy co-optimization is a must in the future power system operation, such as unit commitment (UC) and economic dispatch (ED). Among this topic, the key issue is the modeling of uncertainty. Two prevalent approaches are based on stochastic optimization (SO) and adjustable robust optimization (ARO). SO describes uncertainty by chance-constraint based models [9,10] or generating scenarios [11–13]. Different from SO, ARO uses an uncertainty set to depict the random factor. Bertsimas et al. [14] comes up with the typical form of uncertainty set, which takes into

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Nomenclature

Parameter

- R_i^+ ramp-up limit for thermal unit i
- R_i^- ramp-down limit for thermal unit i
- P_i^{min} minimal output of thermal unit i
- P_i^{max} maximal output of thermal unit i
- w_{jt}^{min} minimal output of wind farm j in period t
- w_{jt}^{max} maximal output of wind farm j in period t
- w_{jt}^{e} forecasted output of wind farm j in period t
- w^e vector $w^e = [w_{jt}^e], \forall j, \forall t$
- σ_{jt} variance of forecast error in wind farm j period t
- Σ covariance matrix of the forecast errors
- E_k available hydro energy in the scheduling horizon
- p_{qt} demand of load q in period t
- π_{il}, π_{kl} line flow distribution factor from thermal unit i and hydro unit k to line l
- π_{jl} line flow distribution factor from wind farm j to line l
- π_{ql} line flow distribution factor from load q to line l
- F_l capacity of transmission line l
- c_i running cost coefficient of thermal unit i
- d_i^+ upward spinning reserve cost coefficient of thermal unit i
- d_i^- downward spinning reserve cost coefficient of thermal unit i
- s_i^+ upward regulation cost coefficient of thermal unit i
- s_i^- downward regulation cost coefficient of thermal unit i
- s_k^+ upward regulation cost coefficient of hydro unit k
- s_k^- downward regulation cost coefficient of hydro unit k
- w_{jt} real-time wind power output, a random parameter
- w vector $w = [w_{jt}], \forall j, \forall t$

Decision variable

- p_{it}^{fg} set point of thermal unit i in period t, vector $p^{fg} = [p_{it}^{fg}], \forall i, \forall t$
- p_{kt}^{fh} set point of hydro unit k in period t, vector $p^{fh} = [p_{kt}^{fh}], \forall k, \forall t$
- r_{it}^{g+} upward spinning reserve capacity offered by thermal unit i, in period t, vector $r^{g+} = [r_{it}^{g+}], \forall i, \forall t$
- r_{it}^{g-} downward spinning reserve capacity offered by thermal unit i, in period t, vector $r^{g-} = [r_{it}^{g-}], \forall i, \forall t$
- r_{kt}^{h+} upward spinning reserve capacity offered by hydro unit k, in period t, vector $r^{h+} = [r_{kt}^{h+}], \forall k, \forall t$
- r_{kt}^{h-} downward spinning reserve capacity offered by hydro unit k, in period t, vector $r^{h-} = [r_{kt}^{h-}], \forall k, \forall t$
- p_{it}^{g+} upward regulation power of thermal unit i, in period t in real-time dispatch, vector $p^{g+} = [p_{it}^{g+}], \forall i, \forall t$
- p_{it}^{g-} downward regulation power of thermal unit i, in period t in real-time dispatch, vector $p^{g-} = [p_{it}^{g-}], \forall i, \forall t$
- p_{kt}^{h+} upward regulation power of hydro unit k, in period t in real-time dispatch, vector $p^{h+} = [p_{it}^{h+}], \forall k, \forall t$
- p_{kt}^{h-} downward regulation power of hydro unit k, in period t in real-time dispatch, vector $p^{h-} = [p_{it}^{h-}], \forall k, \forall t$
- x vector of the first-stage decision, $x = [p^{fg}, p^{fh}, r^{g+}, r^{g-}, r^{h+}, r^{h-}]$
- y vector of the second stage decision, $y = [p^{g+}, p^{g-}, p^{h+}, p^{h-}]$

Abbreviations

- ARO adjustable robust optimization
- DRO distributionally robust optimization
- ED economic dispatch
- HTW-ED hydro-thermal-wind ED
- AR-HTW-ED ARO based HTW-ED
- DR-HTW-ED DRO based HTW-ED
- DCG delayed constraint generation
- PDF probability distribution function
- SDP semi-definite programming
- SO stochastic optimization
- LP linear programming

account the temporal smoothing effect of the wind power output. Wei et al. [15] furthermore considers the spatial clust effect of the wind farms. In order to characterize the temporal and spatial correlations, [16] introduces the concept of dynamic uncertainty sets. Current studies of ARO uncertainty sets are summarized in [17].

SO usually assumes wind generation follow a certain probability distribution, and minimizes the expected total operational cost. ARO considers all possible outcomes of wind generation in a prespecified uncertainty set, and minimizes the cost in the worst scenario. In practical application, the accurate probability distribution function (PDF) may not be available; although an uncertainty set is easier to obtain than a PDF, it ignores the distribution property of the uncertain factor, i.e., the probability of a scenario usually decreases with its distance from the forecast increasing in the sense of statistics. Moreover, the conservativeness of the strategy get by ARO largely depends on the selection of the uncertainty set, which is somehow subjective because the shape and size is manually supplied. To overcome the above difficulties, there is a growing interest to utilize the emerging distributionally robust optimization (DRO) methodology [18–20] in power system operation, and successful application has been made in several aspects. Summers et al. [21] solves the optimal power flow based on conditional value at risk (CVaR) and DRO. Liu et al. [22] proposes a stochastic robust framework to solve the distributional robust UC problem in case that the moments information of uncertainty parameters is unknown. But to the wind power we consider here, those information can be practically obtained from the statistics of historical data. Bian et al. [23] studies a reserve scheduling problem with chance-constraint, and S-lemma is used to get an SDP equivalent, but it does not consider the bound of the uncertainty set.

In DRO, wind power uncertainty is described through an ambiguous PDF, whose statistical characteristics should be consistent with those derived from actual data, such as the mean and variance, and the expected cost in the worst distribution is to be minimized. In this regard, DRO inherits the advantages from both SO and ARO. However, the existing DRO method optimizes the random parameter in the entire space, which is contradict to the reality that the random factors, such as wind power output uncertainty, usually varies in a quite limited range. In this paper, we restrict the uncertainty in a ellipsoid region and deploy S-lemma to circumvent the difficulties in solving the resulted optimization problem.

Aforementioned researches focus on the coordination of thermal and wind power. However, frequent redispatch of thermal energy induces mechanical wear and tear, which is harmful to generators. As hydro power exhibits high flexibility and low cost, it is a desired solution to mitigate wind power uncertainty. Current investigations on hydro-thermal-wind (HTW) co-optimization can be roughly divided into two categories, one based on SO and the other ARO. Garcia-Gonzalez et al. [24] studies the stochastic joint optimization of wind and pumped-hydro. Shukla and Singh [25] models the hydro power characteristic in detail and uses SO method to solve the multi-objective UC problem. Clustering techniques with weight-improved crazy particle swarm optimization is used in [26], and the influence of wind fluctuation is considered by generating scenarios. The above works are all based on SO method. As for ARO, there are relatively scanty works. Jiang et al. [27] solves the robust thermal-wind UC problem with pumped-hydro. In this paper, we propose a distributionally robust hydro-thermal-wind economic dispatch (DR-HTW-ED) model, which fills the gap between SO and ARO for HTW-ED. The main contribution of this paper is threefold:

1. To facilitate the co-optimization of multi-energy sources with different operational constraints and under uncertainties, a two-stage DR-HTW-ED model is proposed. In purpose of making full use of underlying information of wind generation uncertainty, we use DRO method here, which models the uncertain wind generation by a set of multivariate distributions with known expectations and variances that are practically available from the statistics of historical data. To the best of our knowledge, this is the first study of hydro-thermal-wind coordination with DRO method in power system application. This methodology also favors other co-optimization problems among diverse multi-energy sources in the power system that is in a changing landscape.
2. To better formulate wind generation uncertainty, the uncertain wind power output is restricted within an ellipsoid, and an equivalent semi-definite programming (SDP) model is derived based on the outcome in [19] and S-lemma. Then the original two-stage model is converted to be a tractable one, where the master problem becomes a relaxed SDP and is sequentially tightened, while the sub-problem is a biconvex program. A delayed constraint generation (DCG) algorithm is then suggested to solve it.
3. Performance of the distributionally robust based and adjustable robust based hydro-thermal-wind economic dispatch models, including computational time and conservativeness, is compared. The results show the appealing features and consideration potential for DRO under uncertainties. We also empirically reveal the underlying benefit of hydro-thermal-wind co-dispatch versus the traditional thermal-wind one, demonstrating the superiority and necessity of multi-energy coordinated optimization.

The remaining parts of this paper are organized as follows. The mathematical formulation of DR-HTW-ED is described in Section 2. The equivalent SDP as well as the DCG algorithm is presented in Section 3. Case studies and computational results are shown in Section 4. The conclusions are drawn in Section 5. The model of AR-HTW-ED is given in Appendix.

2. Mathematic formulation

2.1. Modeling of wind uncertainty

We will first model the wind uncertainty by a set of multivariate distributions. To capture the probabilistic characteristics, we assume the PDF of wind power is $f(w)$. Meanwhile, we have the wind generation forecast $E[w] = w^e$ and the covariance matrix Σ which can be calculated from limited historical data. In light of this, the PDF $f(w)$ should satisfy the following constraints

$$\Omega = \left\{ f(w) \left| \begin{array}{c} f(w) \geqslant 0, \forall w \in B \\ \int_{w \in B} f(w) \mathrm{d}w = 1 \\ \int_{w \in B} w f(w) \mathrm{d}w = w^e \\ \int_{w \in B} w w^T f(w) \mathrm{d}w = \Sigma + w^e (w^e)^T \end{array} \right. \right\} \tag{1.1}$$

where the support set B restricts the range of nodal wind power injection. In most existing DRO literatures, such as [19,20], $B = \mathbb{R}^{N_W}$ where $\mathbb{R}^{N^W}$ is the space of N_W-dimensional real vector. However, the actual output of a wind farm can neither exceed its capacity nor become negative, it is likely to be near to the forecast. Various support sets can be utilized according to practical requirements, in this paper, we adopt the following ellipsoid

$$B = \left\{ w \middle| (w - w^e)^T Q (w - w^e) \leqslant \Gamma \right\} \tag{1.2}$$

where

$$Q = \begin{bmatrix} \sigma_1^{-2} & & & \\ & \sigma_2^{-2} & & \\ & & \ddots & \\ & & & \sigma_{N^W}^{-2} \end{bmatrix}$$

and Γ is an adjustable parameter.

It can be seen that Ω inherits the advantages from both SO and ARO. On the one hand, the PDF $f(w)$ captures the statistical nature; one the other hand, the support set B plays the role of uncertainty set in ARO. Moreover, it is data-driven and easy to procure. Theoretically, DRO is expected to give less pessimistic dispatch strategies than the corresponding ARO model. The worst case of ARO is an extreme point right on the bound of the uncertainty set that is a polyhedron. The support set of DRO is an inner ellipse of the polyhedron, so all the scenarios of DRO will never be worse than the worst case of ARO. Moreover, DRO takes into account the distributional characteristic of the uncertain factor, and the worst distribution needs to allocate probability among other scenarios to satisfy the mean and moment constraints. Therefore, the scene of DRO is nearer to the expectation, and thus outputs less conservative strategies than ARO.

The inclusion of set B brings difficulty in the model solving, because the quadratic constraint in DRO can no longer be transform directly by the half positive definite condition. S-lemma is used here to help settle this problem, details are in Section 3.2.

2.2. Mathematical formulation of DR-HTW-ED

The whole model of DR-HTW-ED is given as follows. The first stage of DR-HTW-ED is a set-point optimization associating with the wind power forecast, and the second stage deploys corrective actions in response to the wind power realization, including redispatch of thermal units and operation of hydro units. Specifically, according to the rule of reservoir operation, the total water volume used during a certain period is usually fixed in advance. Operators will decide the water usage in the generation plan, such as the day-ahead HTW-UC. Consider that the net head of the reservoir changes a little in the ED problem we study here, we can simply assume the net head is a constant. So the total hydro energy in the scheduling horizon, which is the product of the net head and the total water usage, is fixed. The mathematical formulation of DR-HTW-ED is shown below

$$\min \sum_{t=1}^{N_T} \sum_{i=1}^{N_G} (c_i p_{it}^{fg} + d_i^+ r_{it}^{g+} + d_i^- r_{it}^{g-}) + \max_{f(w) \in \Omega} E[Q(x, w)] \tag{2.1}$$

$$s.t.\ P_i^{min} \leqslant p_{it}^{fg} - r_{it}^{g-},\ p_{it}^{gf} + r_{it}^{g+} \leqslant P_i^{max}, \forall i, \forall t \tag{2.2}$$

$$P_k^{min} \leqslant p_{kt}^{fh} - r_{kt}^{h-},\ p_{kt}^{fh} + r_{kt}^{h+} \leqslant P_k^{max}, \forall k, \forall t \tag{2.3}$$

$$\sum_{t=1}^{N_T} p_{kt}^{fh} = E_k, \forall k \tag{2.4}$$

$$\sum_{i=1}^{N_G} p_{it}^{fg} + \sum_{j=1}^{N_W} w_{jt}^e + \sum_{k=1}^{N_H} p_{kt}^{fh} = \sum_{q=1}^{N_Q} p_{qt}, \forall t \tag{2.5}$$

$$-F_l \leqslant \sum_{i=1}^{N_G} \pi_{il} p_{it}^{fg} + \sum_{j=1}^{N_W} \pi_{jl} w_{jt}^e + \sum_{k=1}^{N_H} \pi_{kl} p_{kt}^{fh} - \sum_{q=1}^{N_Q} \pi_{ql} p_{qt} \leqslant F_l, \forall l, \forall t \quad (2.6)$$

$$0 \leqslant r_{it}^+ \leqslant R_i^+ \Delta t,\ 0 \leqslant r_{it}^- \leqslant R_i^- \Delta t,\ \forall i, \forall t \quad (2.7)$$

$$0 \leqslant r_{kt}^{h+} \leqslant M p_{kt}^{fh},\ 0 \leqslant r_{kt}^{h-} \leqslant M p_{kt}^{fh},\ \forall k, \forall t \quad (2.8)$$

$$Q(x,w) = \min \sum_{t=1}^{N_T} \sum_{i=1}^{NG} (s_i^+ p_{it}^{g+} + s_i^- p_{it}^{g-}) + \sum_{t=1}^{N_T} \sum_{k=1}^{N_H} (s_k^+ p_{kt}^{h+} + s_k^- p_{kt}^{h-}) \quad (2.9)$$

$$s.t.\ 0 \leqslant p_{it}^{g+} \leqslant r_{it}^{g+},\ 0 \leqslant p_{it}^{g-} \leqslant r_{it}^{g-}, \forall i, \forall t \quad (2.10)$$

$$0 \leqslant p_{kt}^{h+} \leqslant r_{kt}^{h+},\ 0 \leqslant p_{kt}^{h-} \leqslant r_{kt}^{h-}, \forall k, \forall t \quad (2.11)$$

$$\sum_{i=1}^{N_G} (p_{it}^{fg} + p_{it}^{g+} - p_{it}^{g-}) + \sum_{j=1}^{N_W} w_{jt} + \sum_{k=1}^{N_H} (p_{kt}^{fh} + p_{kt}^{h+} - p_{kt}^{h-}) = \sum_{q=1}^{N_Q} p_{qt}, \forall t \quad (2.12)$$

$$-F_l \leqslant \sum_{i=1}^{N_G} \pi_{il} (p_{it}^{fg} + p_{it}^{g+} - p_{it}^{g-}) + \sum_{j=1}^{N_W} \pi_{jl} w_{jt} + \sum_{k=1}^{N_H} \pi_{kl} (p_{kt}^{fh} + p_{kt}^{h+} - p_{kt}^{h-}) - \sum_{q=1}^{N_Q} \pi_{ql} p_{qt} \leqslant F_l, \forall l, \forall t \quad (2.13)$$

The objective function (2.1) is to minimize the total expected cost. The first stage decisions are made before uncertainty is known, so is independent of the PDF. The first-stage cost consists of the operating cost and reserve cost of all thermal units, and the cost of hydro units is ignored as it is usually small, but still we consider the reserve cost of hydro units in view of its potential participation in the reserve market. The second stage decision are made with respect to wind power output w and fixed first-stage decision. The expected second-stage cost in the worst wind power PDF is considered in the objective. Constraints (2.2) and (2.3) are the capacity limits of thermal and hydro units. The total hydro energy is restricted in constraint (2.4). Constraint (2.5) is the supply–demand balancing condition. Constraint (2.6) is the security restriction of transmission lines. Constraints (2.7) and (2.8) give the bound of thermal and hydro reserve capacity, respectively; M is a parameter, which restricts the range the real time hydropower can vary in, so called hydro reserve ratio. The committed units and E_k is determined in a previous stage, such as HTW-UC.

In the second stage, the first-stage decision x has been made, the optimal regulation cost for wind power w is given by $Q(x,w)$. Objective (2.9) is the cost for corrective actions in real-time dispatch. Constraints (2.10) and (2.11) restrict the real-time adjustments within the reserve capacity offered in the first stage, constraints (2.12) and (2.13) are the balancing condition and line security restriction in real-time dispatch.

Recall the vector notation in Nomenclature, DR-HTW-ED (2) can be arranged as a compact form

$$\min_x c^T x + \max_{f(w) \in \Omega} E[Q(x,w)] \quad (3)$$

and

$$Q(x,w) = \min_{y \in Y(x,w)} d^T y \quad (4)$$

where the feasible region of real-time dispatch is given by

$$Y(x,w) = \{y \mid By \leqslant b - Ax - Cw\}$$

3. Solution methodology

Several decomposition algorithms are available to solve the traditional ARO problem, such as the Benders decomposition algorithm [14] and the constraint-and-column generation method [28]. As the second stage of DR-HTW-ED requires the maximization of expectation over a functional set, above methods are not applicable. In this section we demonstrate DR-HTW-ED can be reduced to a SDP, which can be solved by commercial solver in a DCG framework.

3.1. Dual formulation of the second-stage problem

Write the second-stage problem explicitly

$$\begin{aligned} &\max\ E[Q(x,w)] = \max_{f(w) \in \Omega} \int_{w \in B} Q(x,w) f(w) dw \\ &\int_{w \in B} f(w) dw = 1 \\ &\int_{w \in B} w f(w) dw = w^e \\ &\int_{w \in B} w w^T f(w) dw = \Sigma + w^e (w^e)^T \\ &f(w) \geqslant 0, \quad \forall w \in B \end{aligned} \quad (5)$$

Its Lagrangian can be written as:

$$L(H,h,h_0) = \int_{w \in B} [Q(x,w) - w^T H w - h^T w - h_0] f(w) dw + tr[H^T(\Sigma + w^e(w^e)^T)] + h^T w^e + h_0$$

where $tr(M)$ stands for the trace of matrix M, h_0, h and H are dual variables of the moment constraints in Ω. The Lagrange dual of problem (5) is

$$\min_{H,h,h_0} \max_{f(w)} L(H,h,h_0) \quad (6)$$

The conclusion on the duality gap between primal problem (5) and dual problem (6) is given in [19,29]. It is known that if the second-order moment matrix $\Sigma + w^e(w^e)^T$ is positive definite, there is no duality gap. According to the probability theory, a covariance matrix is always positive semidefinite. Nevertheless, we assume the covariance matrix Σ derived from actual data is positive definite, as a result, strong duality holds for the primal problem and dual problem.

Next we will first prove Lemma 1 following a similar procedure in [30].

Lemma 1. *Problem (6) has a limited solution if and only if*

$$Q(x,w) - w^T H w - h^T w - h_0 \leqslant 0, \forall w \in B \quad (7)$$

Proof. Suppose

$$Q(x,w) - w^T H w - h^T w - h_0 \nleqslant 0, \quad \forall w \in B$$

As the optimal value function of a linear program is a piece-wise linear function [31], while $w^T H w + h^T w + h_0$ is a quadratic function, it means

$$Q(x,w) - w^T H w - h^T w - h_0 \not\equiv 0, \quad \forall w \in B$$

in other words

$$\exists w^* \in B : Q(x,w^*) - w^{*T} H w^* - h^T w^* - h_0 > 0$$

As $Q(x,w)$ is continuous in w [31], there exists a small neighborhood region $B(w^*, \varepsilon')$ and a strict positive constant ζ such that

$$Q(x,w^*) - w^{*T} H w^* - h^T w^* - h_0 \geqslant \zeta, \quad \forall w \in B(w^*, \varepsilon')$$

Let $f(w) \to \infty, \forall w \in B(w^*, \varepsilon')$, then $L(H,h,h_0)$ is unbounded, so does the dual problem. This is in contradiction with the weak duality theory because $Q(x,w)$ is bounded for any given x, w, and the primal problem (5) is bounded. This observation completes the proof. □

Using Lemma 1, we have Eq. (7) and

$$\max_{f(w)} \int_{w\in B} [Q(x,w) - w^T Hw - h^T w - h_0] f(w) dw = 0$$

Then the dual form of problem (5) can be presented as (8).

$$\begin{aligned} &\max\ E[Q(x,w)] = \min\ tr(H^T(\Sigma + w^e(w^e)^T) + h^T w^e + h_0) \\ &s.t. w^T Hw - h^T w - h_0 \geqslant Q(x,w),\ \forall w \in B \end{aligned} \tag{8}$$

The following two subsections will discuss the equivalent SDP of DR-HTW-ED.

3.2. Equivalent SDP

One difficulty in problem (8) is that the optimal value function $Q(x,w)$ is not given in a closed form. This problem can be circumvented by using duality theory. The dual of LP (4) is

$$\max_{u\in U} u^T(b - Ax - Cw)$$

with the feasible region of dual variable u is

$$U = \left\{ u \middle| B^T u = d, u \leqslant 0 \right\}$$

Duality theory of LP indicates

$$Q(x,w) \geqslant (b - Ax - Cw)^T u, \forall u \in U$$

and

$$\exists u^* \in \text{vert}(U) : Q(x,w) = (b - Ax - Cw)^T u^*$$

We have mentioned that both primal and dual problems are bounded, so the optimal solution can be found at one of the vertices of polyhedron U. In this regard, we are facing the following constraint equivalently

$$w^T Hw + h^T w + h_0 \geqslant (b - Ax - Cw)^T u^i, \quad \forall w \in B, \forall u^i \in \text{vert}(U) \tag{9}$$

where $\text{vert}(U)$ denotes the vertices of polyhedron U.

As w is limited in set B, constraint (9) cannot be transformed directly by the half positive definite condition. To eliminate the enumeration on $\forall w \in B$, we exploit the idea of positivstellensatz. By introducing a positive scalar λ, if the following equation is satisfied

$$\begin{aligned} &w^T Hw + h^T w + h_0 - (b - Ax - Cw)^T u^i \\ &\quad - \lambda[\Gamma - (w - w^e)^T Q(w - w^e)] \geqslant 0,\ \forall w, \forall u^i \end{aligned}$$

Then as $\Gamma - (w - w^e)^T Q(w - w^e) \geqslant 0, \forall w \in B$, we have

$$\begin{aligned} &w^T Hw + h^T w + h_0 - (b - Ax - Cw)^T u^i \\ &\geqslant \lambda[\Gamma - (w - w^e)^T Q(w - w^e)] \geqslant 0, \forall u^i \end{aligned} \tag{10}$$

In fact, this transformation is equivalent to (9) according to the S-lemma [24].

Eq. (10) has a matrix form as

$$\begin{bmatrix} w \\ 1 \end{bmatrix}^T M^i \begin{bmatrix} w \\ 1 \end{bmatrix} \geqslant 0, \quad \forall w, \forall i$$

where

$$M^i = \begin{bmatrix} H + \lambda Q & 0.5(h - (u^i)^T C) - \lambda (w^e)^T Q \\ 0.5(h^T - C^T u^i) - \lambda Q^T w^e & h_0 - (b - Ax) - \lambda(\Gamma + (w^e)^T Q w^e) \end{bmatrix}$$

Then, (10) holds when $M^i \succeq 0, \forall i$.

In this way, DR-HTW-ED is equivalent to the following SDP

$$\begin{aligned} &\min_{x,H,h,h_0,\lambda} c^T x + tr[H^T(\Sigma + w_e(w_e)^T] + h^T w_e + h_0 \\ &s.t.\ x \in X,\ \lambda \geqslant 0, \forall u^i \in \text{vert}(U) : \\ &\begin{bmatrix} H + \lambda Q & 0.5(h - (u^i)^T C) - \lambda (w^e)^T Q \\ 0.5(h^T - C^T u^i) - \lambda Q^T w^e & h_0 - (b - Ax) - \lambda(\Gamma + (w^e)^T Q w^e) \end{bmatrix} \succeq 0 \end{aligned} \tag{11}$$

It is worth noting that the dimension of the positive semidefinite matrices depend on the dimension of uncertainty, and is independent of the size of the power system. However, the polyhedral set U does depend on the power system. Enumerating all the vertices of U may not be practical.

3.3. The DCG algorithm

To solve the problem efficiently, in this subsection, we suggest a DCG algorithm, which dynamically identifies the vertices that will offer binding constraints in SDP (11). The algorithm is outlined below.

DCG Algorithm

Step 1: Analyze historical data and construct the covariance matrix Σ; Predict the wind power output w_e and choose the initial vertex set $VE \subseteq \text{vert}(U)$.

Step 2: Solve the following master problem, which is a relaxed version of SDP (11)

$$\begin{aligned} &\min_{x,H,h,h_0,\lambda} c^T x + tr[H^T(\Sigma + w_e(w_e)^T] + h^T w_e + h_0 \\ &s.t. x \in X, \lambda \geqslant 0, \forall u^i \in VE : \\ &\begin{bmatrix} H + \lambda Q & 0.5(h - (u^i)^T C) - \lambda (w^e)^T Q \\ 0.5(h^T - C^T u^i) - \lambda Q^T w^e & h_0 - (b - Ax) - \lambda(\Gamma + (w^e)^T Q w^e) \end{bmatrix} \succeq 0 \end{aligned}$$

The optimal value is R^*, and the optimal solution is x^*.

Step 3: Solve the following sub-problem with $x = x^*$

$$\min_{w\in B, u\in U} w^T Hw + h^T w + h_0 - (b - Ax^* - Cw)^T u$$

The optimal value is r^*, and the optimal solution is u^* and w^*.

Step 4: If $r^* \geqslant 0$, terminate, report the optimal solution x^* and the optimal value R^* ; otherwise, $VE = VE \cup u^*$, go **Step 2**.

The DCG algorithm terminates in a finite number of iterations which is bounded by $|vert(U)|$. The sub-problem in **Step 3** is a nonlinear program and can be solved by general nonlinear solvers. Nevertheless, by noticing the fact that the constraints on variable u and w are disjoint, we introduce an alternating direction heuristic to solve it. This algorithm sequentially solves the following two problems in variable u or w with the other one being fixed at previous value, until a certain converge criterion is reached.

$$\min_{w\in B} w^T Hw + h^T w + h_0 - (b - Ax^* - Cw)^T u^*$$

$$\min_{u\in U} w^{*T} Hw^* + h^T w^* + h_0 - (b - Ax^* - Cw^*)^T u$$

There is no optimality guarantee for this procedure because the matrix H is not necessarily positive semidefinite and the sub-problem may be non-convex.

As for the computational complexity of ARO and DRO, we analyse it as follows. The master problem of the ARO is a linear program (LP), and the sub-problem is bilinear program (BLP). In our DRO model, the master problem is a semidefinite program (SDP), and the sub-problem is a biconvex program (BCP). Both master problems can be solved in polynomial time, while the latter is more time-consuming. Both sub-problems are biconvex optimizations, which are NP-hard. With an alternating direction oracle, the former can be settled by solving a series of LPs, while the latter by solving LPs and SDPs iteratively. If longer time periods and more complex/real systems are considered, the DRO will lead to higher computational complexity, but it still can be solved in polynomial time. We would like to highlight that the DRO can be solved much faster than the traditional SO, since the latter calculates the average performance over all possible scenarios. Indeed, the DRO is an in-between method that combines the advantages of the ARO and the SO, which can improve the solution quality of the ARO without increasing considerable computational cost.

In practical situations, one can follow the steps below to apply the proposed method. Here, two phases are considered:

(1) The look-ahead phase:

Step 0: Let $\{\tilde{w}_i\}_{i=1}^{M}$ be M independent samples according to the distribution of $\tilde{w}$.

Step 1: Calculate the mean and second-moment matric of the random vector of parameters $\tilde{w}$ by formula (12). Then construct the support set B by formula (13).

$$\mu = \frac{1}{M}\sum_{i=1}^{M}\tilde{w}_i,\ \Sigma = \frac{1}{M}\sum_{i=1}^{M}(\tilde{w}_i - \mu)(\tilde{w}_i - \mu)^T \tag{12}$$

$$\begin{aligned} &(\tilde{w} - \mu)^T\Sigma^{-1}(\tilde{w} - \mu) \leqslant \Gamma \\ &\Gamma^{1/2} = \sup_{\tilde{w}\in B}\left\|\Sigma^{-1/2}(\tilde{w} - \mu)\right\|_2 \end{aligned} \tag{13}$$

Step 2: After obtaining the probabilistic characteristics, build the DR-HTW-ED model up with system statistics.

Step 3: Solve the model to give the first-stage strategy (look-ahead dispatch strategy), determining the set points of generators and the spinning reserve capacity preserved in each generator.

(2) The real-time phase:

After the wind power output is observed, we re-solve the second-stage problem with the observed wind power output to obtain the best corrective actions, including the strategies of generator re-dispatch, renewable generation curtailment and load shedding.

4. Case studies

The proposed DR-HTW-ED is applied to the IEEE 39-bus system and compared with the AR-HTW-ED. The computation platform is a laptop with Intel i7 CPU and 8 GB memory. SDP is solved by MOSEK and NLP is solved by KNITRO.

4.1. Data

The IEEE 39-bus system has 10 thermal units and 46 transmission lines. The data of thermal units are described in Table 1. A wind farm is connected to the grid at bus 29, its forecasted output in the next 4 h are 65 MW, 70 MW, 78 MW, 69 MW, the variance is simply assumed to be $\sigma_j/p_j^e = 0.4$, and the budget $\Gamma = 4$. Other data without particular mention is the same as the IEEE standard data.

4.2. HTW-ED

In this case, we assume unit 1, 2, 4, 5, 8 are committed. The available hydro energy is $E_k = 403$ MW h (obtained from the previous HTW-UC). We allow the real time hydropower to adjust in the 10% of its planned output, and this is implemented by letting the hydro reserve ratio M in constraint (2.8) equal to 0.1. The constraint is as follows

$$0 \leqslant r_{kt}^{h+}, r_{kt}^{h-} \leqslant 0.1p_{kt}^{fh}, \quad \forall k, \forall t$$

Result are shown from Tables 2–5 (in format of DR-HTW-ED/AR-HTW-ED). From Table 2 we can see, the computational time of DR-HTW-ED is longer than that of AR-HTW-ED. We find the majority of computational effort is spent in solving the nonlinear sub-problem. Since the ED decision is made an hour ahead, the efficiency of DR-HTW-ED is still acceptable. The worst expected cost of DR-HTW-ED is $41026 while the worst scenario cost of

Table 2
Comparison of computational time and optimal costs.

Model	Cost			Time (s)
	First stage ($)	Second stage ($)	Total ($)	
DR-HTW-ED	40,733	293	41,026	49
AR-HTW-ED	41,215	354	41,569	4.6

Table 3
Optimal set point.

Period	Set point (MW)					
	G1	G2	G4	G5	G8	Hydro
1	455/439	150/150	20/20	25/25	10/10	125/141
2	432/432	150/150	20/20	25/25	10/10	94/93
3	374/397	150/150	20/20	25/25	10/10	93/45
4	355/325	150/150	20/20	25/25	10/10	91/124

Table 4
Optimal upward spinning reserve.

Period	Upward spinning reserve (MW)					
	G1	G2	G4	G5	G8	Hydro
1	0/15.9	0/0	11.3/26.0	0/0	0/0	10.07/10.07
2	0/20.9	0/0	11.3/26.0	0/0	0/0	9.36/9.10
3	0/29.4	0/0	11.3/26.0	0/0	0/0	9.33/7.03
4	0/19.1	0/0	4.33/26.0	0/0	0/0	9.10/10.1

Table 1
Data of thermal units.

Unit	P_i^{min} (MW)	P_i^{max} (MW)	R_i^+ (MW/h)	R_i^- (MW/h)	c_i ($/MW)	$d_i^{\pm}$ ($/MW)	$s_i^{\pm}$ ($/MW)
G1	150	455	120	120	16.19	1.62	1.62
G2	150	455	120	120	17.26	1.73	1.73
G3	20	130	60	60	16.6	1.66	1.66
G4	20	130	50	50	16.5	1.59	1.59
G5	25	162	70	70	19.7	1.97	1.97
G6	20	80	30	30	22.26	2.22	2.22
G7	25	85	34	34	22.74	2.77	2.77
G8	10	55	22	22	25.92	2.59	2.59
G9	10	55	20	20	27.27	2.73	2.73
G10	10	55	10	10	27.79	1.3	1.3

Table 5
Optimal downward spinning reserve.

Period	Downward spinning reserve (MW)					
	G1	G2	G4	G5	G8	Hydro
1	1.86/41.9	0/0	0/0	0/0	0/0	10.1/10.1
2	12.6/46.9	0/0	0/0	0/0	0/0	9.36/9.10
3	12.3/55.4	0/0	0/0	0/0	0/0	9.33/7.03
4	12.0/45.1	0/0	0/0	0/0	0/0	9.10/10.1

AR-HTW-ED is \$41569, because the former is an expectation represents an average performance. Actually in AR-HTW-ED, wind power uncertainty is modeled by an uncertainty set with every scenario treated with the same importance. The strategy of ARO protects the system against all rare events, so the reserve capacity and operational cost could be unacceptable in practice. DRO method considers the distributional characteristic of the random factor and its support set is modeled as an inner ellipse of the ARO uncertainty set. Meanwhile, because the variance is fixed, a scenario that deviates far away from the forecast would have a low probability. DRO gives a strategy that better reflects the average system performance, and thus, less conservative. As we can see in Tables 4 and 5, the reserve capacity obtained by DRO are all less than ARO, which can improve the economic without compromising the system security.

4.3. Related factor

In this part, we study the influence of serval factors that affect the model's performance.

First, we test the impact of parameter Γ on the optimal value of DR-HTW-ED and AR-HTW-ED, the result is shown in Fig. 1. We can see that, the optimal costs of both methods increase with Γ becoming larger, as wind generation behaves more volatile; however, the cost of DR-HTW-ED grows slower, because it takes the statistical distribution property into account.

Then, We test the optimal value with different variances. We allow σ_j/p_j^e to change from 0 to 0.4, and record the optimal costs and computational time of DR-HTW-ED and AR-HTW-ED. Other factors remain the same as those in Sections 4.1 and 4.2. The results are shown in Figs. 2 and 3.

Fig. 2 demonstrates that the total optimal cost increases with the variance increasing. This is easy to understand, since bigger variance means larger uncertainty. In order to cope with the increasing nodal injection uncertainty, both models need to preserve more reserve capacity and thus the costs increase. We can also see that, the cost of DR-HTW-ED grows faster than that of AR-HTW-ED with the variance increasing, following the same reason we explained for Fig. 1. Fig. 3 shows that the computational time of DR-HTW-ED increases with the variance increasing, because the DCG algorithm takes more iterations to converge. But the computational time is still acceptable. In the ED cases we study here with one wind farm for simplicity, AR-HTW-ED always converges in one or two iterations, so the computational time does not change much. When the number of wind farms increases, both the ARO and the DRO will become more time consuming. However, since both two methods are in polynomial computational

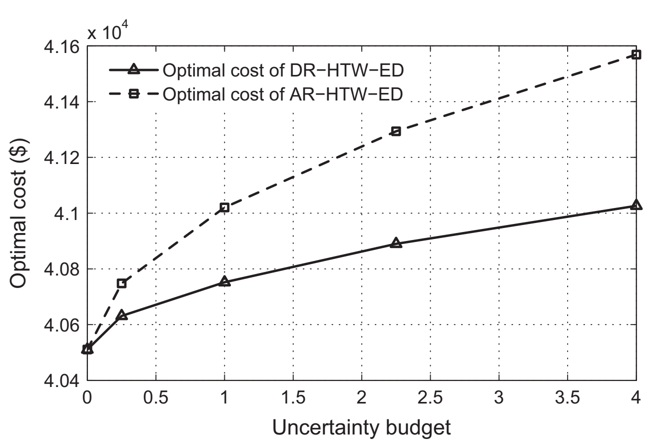

Fig. 1. Optimal value of DR-HTW-ED and AR-HTW-ED under different Γ.

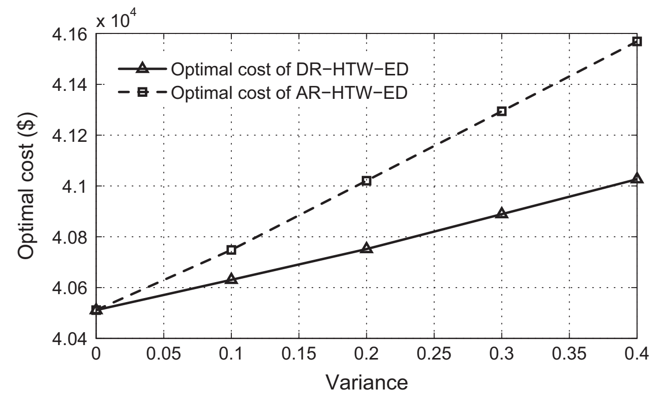

Fig. 2. Optimal value of DR-HTW-ED and AR-HTW-ED under different variance.

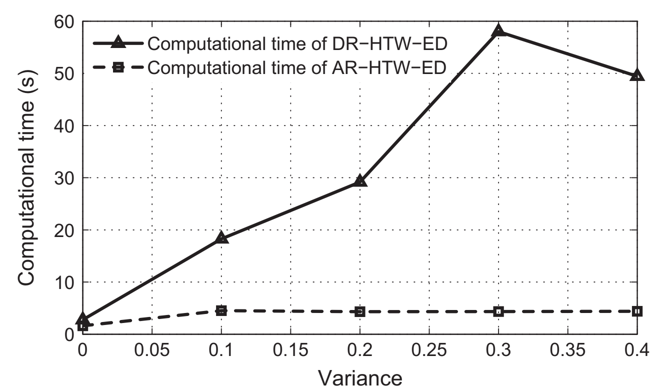

Fig. 3. Computational time of DR-HTW-ED and AR-HTW-ED under different variance.

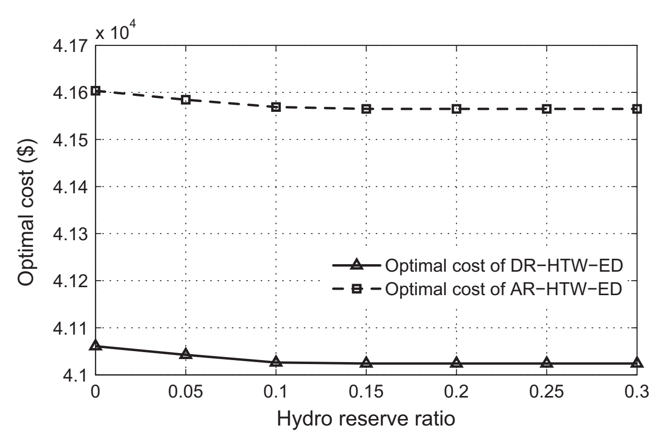

Fig. 4. Optimal value of DR-HTW-ED and AR-HTW-ED under different hydro reserve capacity.

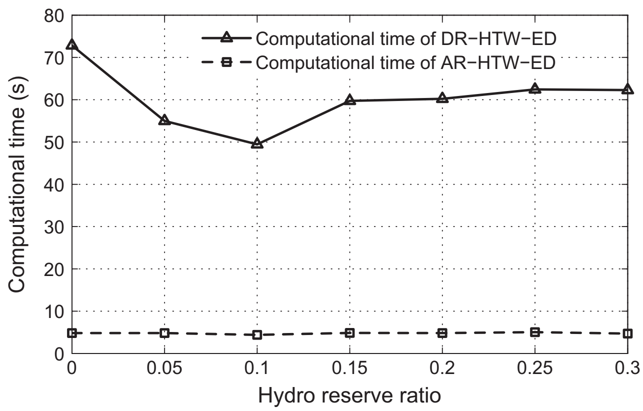

Fig. 5. Computational time of DR-HTW-ED and AR-HTW-ED under different hydro reserve capacity.

complexity, they are still tractable if there are a few more wind farms being considered.

We also test the model performance under different hydropower reserve intervals. We allow the variation range of hydropower to change from 0 to 30 percent of the set point. The optimal costs of DR-HTW-ED and AR-HTW-ED are shown in Figs. 4 and 5.

From Fig. 4 we can see, by expanding the hydropower adjustment range, the total cost can be reduced, because less thermal reserve is needed. It is worth mentioning that the total water used in the scheduling horizon is fixed, that is why we impose a bound constraint on the amount of real-time adjustment. The computational time of both methods are shown in Fig. 5. We can see that the computation time of DR-THW-ED decreases with hydro power becoming more flexible as less iteration is needed.

5. Conclusions

This paper proposes a distributionally robust optimization model for jointly dispatching of hydro, thermal, and wind power. The proposed DR-HTW-ED model considers the statistical property. By optimizing the expected cost in the worst renewable power distribution, DR-HTW-ED combines the advantages of SO and ARO. In contrast to the traditional DRO, we take into account the range of uncertainty and derive an equivalent SDP model by S-lemma. Case studies demonstrate that the strategy of DR-HTW-ED is less conservative than that of the traditional ARO based approach, although it will take longer time to solve. And still the computational efficiency of DRO is acceptable in practical usage.

6. Appendix

Recall the compact formulation in Section 2, the mathematical formulation of AR-HTW-ED is the following tri-level optimization problem

$$\min_{x} c^T x + \max_{w \in W} \min_{y \in Y(x,w)} Q(x,w)$$

where the uncertainty set is defined as all the possible values of w and is given by

$$W = \left\{ w \left| \begin{array}{l} w_{jt}^{min} \leqslant w_{jt} \leqslant w_{jt}^{max}, \forall j \\ \sum_{t=1}^{N_T} \sum_{j=1}^{N_W} \frac{|w_{jt} - w_{jt}^e|}{w_{jt}^h} \leqslant \Gamma \end{array} \right. \right\}$$

where w_{jt}^e is simply assumed to be the mid-point of the forecast interval $[w_{jt}^{min}, w_{jt}^{max}]$ in this study. Non-symmetric forecast is also allowed. Parameter Γ is the "budget of uncertainty", which restricts the total wind power deviation from its nominal value. In the uncertainty set W, every element w is treated with the same importance without considering the statistical characteristic of the uncertain factor. We use the C&CG algorithm proposed in [28] to solve AR-HTW-ED. In general case, ARO method terminates in $O(|vert(W)|)$ iterations [28], where $|vert(W)|$ denotes the number of extreme points of W.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.apenergy.2016.04.060.

References

[1] Connolly D, Lund H, Mathiesen B, Leahy M. A review of computer tools for analysing the integration of renewable energy into various energy systems. Appl Energy 2010;87(4):1059–82.
[2] Mathiesen B, Lund H, Karlsson K. 100% renewable energy systems climate mitigation and economic growth. Appl Energy 2011;88(2):488–501.
[3] Liu W, Lund H, Mathiesen B, Zhang X. Potential of renewable energy systems in china. Appl Energy 2011;88(2):518–25.
[4] González-Aparicio I, Zucker A. Impact of wind power uncertainty forecasting on the market integration of wind energy in Spain. Appl Energy 2015;159:334–49.
[5] Purvins A, Zubaryeva A, Llorente M, Tzimas E, Mercier A. Challenges and options for a large wind power uptake by the european electricity system. Appl Energy 2011;88(5):1461–9.
[6] Dallinger D, Gerda S, Wietschel M. Integration of intermittent renewable power supply using grid-connected vehicles–a 2030 case study for california and Germany. Appl Energy 2013;104:666–82.
[7] Wang J, Botterud A, Bessa R, Keko H, Carvalho L, Issicaba D, et al. Wind power forecasting uncertainty and unit commitment. Appl Energy 2011;88(11):4014–23.
[8] Wang J, Conejo A, Wang C, Yan J. Smart grids, renewable energy integration, and climate change mitigation–future electric energy systems. Appl Energy 2012;96:1–3.
[9] Wu H, Shahidehpour M, Li Z. Chance-constrained day-ahead scheduling in stochastic power system operation. IEEE Trans Power Syst 2014;29(4):1583–91.
[10] Liu X, Xu W. Economic load dispatch constrained by wind power availability: a here-and-now approach. IEEE Trans Sustain Energy 2010;1(1):2–9.
[11] Wang J, Wang J, Liu C. Stochastic unit commitment with sub-hourly dispatch constraints. Appl Energy 2013;105:418–22.
[12] Ahmadi-Khatir A, Conejo A, Cherkaoui R. Multi-area energy and reserve dispatch under wind uncertainty and equipment failures. IEEE Trans Power Syst 2013;28(4):4373–83.
[13] Aghaei J, Shayanfar H, Amjady N. Joint market clearing in a stochastic framework considering power system security. Appl Energy 2009;86(9):1675–82.
[14] Bertsimas D, Litvinov E, Sun X, Zhao J, Zheng T. Adaptive robust optimization for the security constrained unit commitment problem. IEEE Trans Power Syst 2013;28(1):52–63.
[15] Wei W, Liu F, Mei S, Hou Y. Robust energy and reserve dispatch under variable renewable generation. IEEE Trans Smart Grid 2015;6(1):369–80.
[16] Lorca A, Sun X. Adaptive robust optimization with dynamic uncertainty sets for multi-period economic dispatch under significant wind. IEEE Trans Power Syst 2015;30(4):1702–13.
[17] Guan Y, Wang J. Uncertainty sets for robust unit commitment. IEEE Trans Power Syst 2014;3(29):1439–40.
[18] Delage E, Ye Y. Distributionally robust optimization under moment uncertainty with application to data-driven problems. Oper Res 2010;58(3):595–612.
[19] Bertsimas D, Doan X, Natarajan K, Teo C. Models for minimax stochastic linear optimization problems with risk aversion. Math Oper Res 2010;35(3):580–602.
[20] Zymler S, Kuhn D, Rustem B. Distributionally robust joint chance constraints with second-order moment information. Math Program 2013;137(1–2):167–98.

[21] Summers T, Warrington J, Morari M, Lygeros J. Stochastic optimal power flow based on conditional value at risk and distributional robustness. Int J Elec Power 2015;72:116–25.
[22] Liu C, Lee C, Chen H, Mehrotra S. Stochastic robust mathematical programming model for power system optimization. IEEE Trans Power Syst 2016;31(1):821–2.
[23] Bian Q, Xin H, Wang Z, Gan D, Wong K. Distributionally robust solution to the reserve scheduling problem with partial information of wind power. IEEE Trans Power Syst 2015;30(5):2822–3.
[24] Garcia-Gonzalez J, la Muela D, Ruiz R, Santos L, González A. Stochastic joint optimization of wind generation and pumped-storage units in an electricity market. IEEE Trans Power Syst 2008;23(2):460–8.
[25] Shukla A, Singh SN. Multi-objective unit commitment with renewable energy using hybrid approach. IET Renew Power Gener 2016;10(3):327–38.
[26] Shukla A, Singh SN. Clustering based unit commitment with wind power uncertainty. Energy Convers Manage 2016;111:89–102.
[27] Jiang R, Wang J, Guan Y. Robust unit commitment with wind power and pumped storage hydro. IEEE Trans Power Syst 2012;27(2):800–10.
[28] Zeng B, Zhao L. Solving two-stage robust optimization problems using a column-and-constraint generation method. Oper Res Lett 2013;41(5):457–61.
[29] Shapiro A. On duality theory of conic linear problems. In: Semi-infinite programming. p. 135–65.
[30] Zhao C, Guan Y. Data-driven risk-averse two-stage stochastic program with ζ-structure probability metrics. Available in Optimization Online.
[31] Borrelli F, Bemporad A, Morari M. Geometric algorithm for multiparametric linear programming. J Optimiz Theory App 2003;118(3):515–40.