Practice Prelim 1, STAT 611, Spring 2021

Name:

1. Let $X_1, \ldots, X_n$ be iid random variables from a half normal distribution $HN(\mu, \sigma^2)$. The PDF is $f_X(x) = \sqrt{2/(\pi\sigma^2)}\exp\{-(x-\mu)^2/(2\sigma^2)\}$ for $x > \mu$ and $f_X(x) = 0$ for $x \leq \mu$.

   (a) Find a sufficient statistic for $(\mu, \sigma^2)^T$. Justify your answer.

   (b) Find a minimal sufficient statistic $(\mu, \sigma^2)^T$. Justify your answer.

   (c) Supposed μ is known, find a complete statistic for σ^2. Justify your answer.

2. Let $X_1, \ldots, X_n$ be iid random variables from the Pareto(a, b) distribution. The PDF is $f_X(x) = ba^b x^{-b-1}$ for $x > a$ and $f_X(x) = 0$ for $x \leq a$. The parameters a, b satisfy $a > 0, b > 2$.

   (a) Find the moment estimator for a, b.

   (b) Find the MLE for a, b.

3. Let $X_1, \cdots, X_n$ be independent random variables, and $X_i \sim$ Exponential($scale = \lambda$), i.e., $f(x) = \dfrac{1}{\lambda}\exp(-x/\lambda)$, for $x \geq 0$

   (a) Find a MLE for λ.

   (b) Show that the MLE is an unbiased estimator for λ .

   (c) Calculate the mean squared error of the MLE.

   (d) Find the Fisher information $I(\lambda)$ for n observations.

   (e) Find a sufficient statistics for λ.

   (f) Find a MLE for $P(X_1 < 1)$.

4. Practice with distributions that do not belong to the exponential family.