Data reduction

- Sufficient statistic
- How to find a sufficient statistic
  - **(Neyman-Fisher) Factorization theorem. Theorem 6.2.6**
  - Exponential family (Theorem 6.2.10)
  - Transformation
- Minimal Sufficient Statistics
  - Theorem 6.2.13
  - Exponential family
- Complete Statistics

Note: Exponential family vs Non-regular family (e.g. Uniform[0,θ])

Methods for finding MLEs

- Look at a graph of the likelihood function (monotonicity).
- Calculus method: Take partial derivative of log-likelihood with respect to each parameter and solve the likelihood equations.
  Note: Don't forget to check the second order condition!
- Invariance property of MLEs: MLE of a function of θ is the function applied to MLE of θ (Theorem 7.2.10)

Important Concepts in Bayes Estimation

- prior distribution of θ: $\theta \sim \pi(\theta)$. Conjugate priors: The posterior is in the same family as the prior regardless of the data observed.
- posterior distribution of θ: $\pi(\theta|\boldsymbol{x}) = f(\boldsymbol{x}|\theta)\pi(\theta)/m(\boldsymbol{x})$
- marginal distribution of $\boldsymbol{x}$: $m(\boldsymbol{x}) = \int f(\boldsymbol{x}|\theta)\pi(\theta)d\theta$
- posterior mean of θ: $E(\theta|\boldsymbol{X}) = \int \theta\pi(\theta|\boldsymbol{x})d\theta$ (Bayes estimator of θ)