Probability Space

$$(\underbrace{\Omega}_{\text{Sample space}}, \underbrace{\mathcal{F}}_{\text{Event space}}, \underbrace{P}_{\text{Prob. measure}})$$

Outcome: element of sample space

Event: subset of sample space

Event: collection of "legal" events

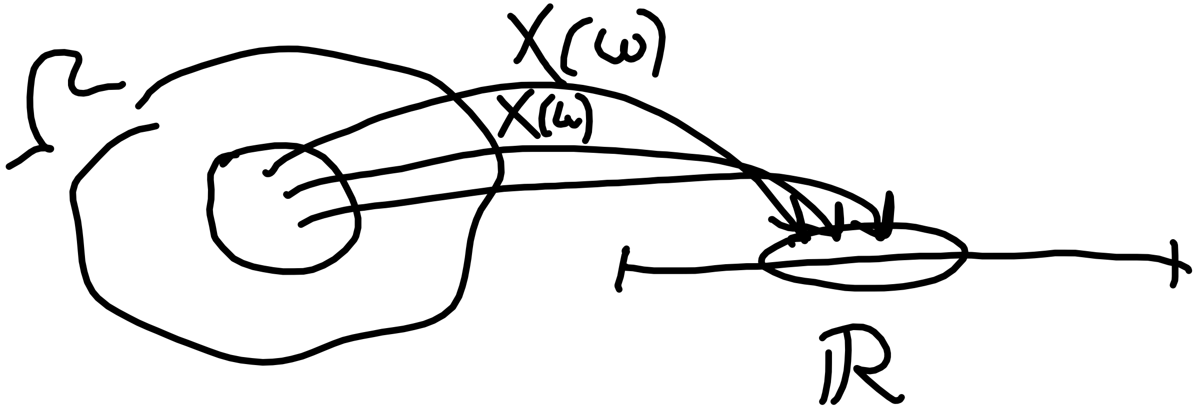
Outcome → element Ω

Event: subset of Ω, element of $\mathcal{F}$

$P : \mathcal{F} \to \mathbb{R}$

$X : \Omega \to \mathbb{R}$

$A \in \mathbb{R}$, $P\{X \in A\} = P(X^{-1}(A))$

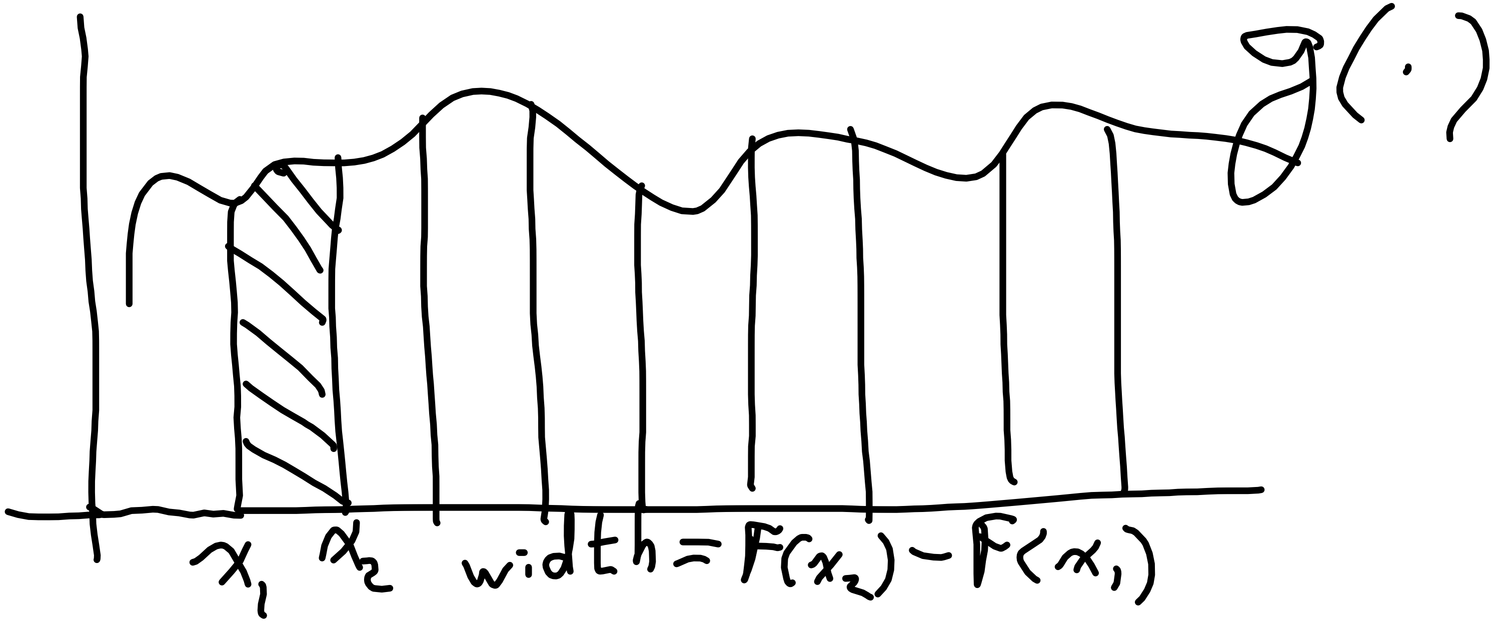


Distribution Function

$$F(x) = P\{X \le x\}$$

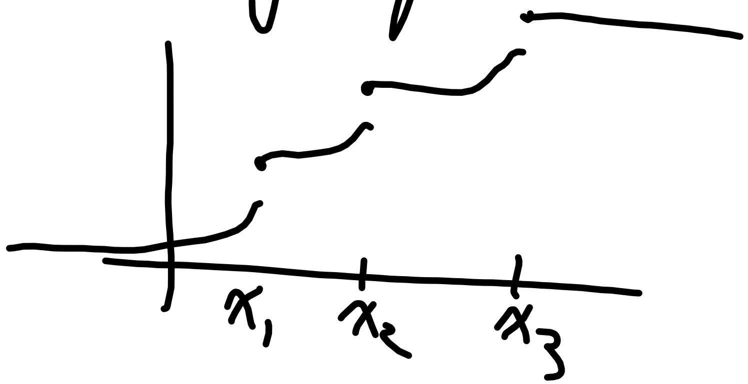
1. nondecreasing +6
2. right-continuous
3. $\lim_{x \to -\infty} F(x) = 0$
4. $\lim_{x \to \infty} F(x) = 1$

$$E[X] = \int_{(-\infty, \infty)} x \, dF(x)$$ Riemann-Stieljes



Let F be a d.f. with jumps at

$x_1, x_2, \dots, x_n$



with jump size

$f_1, f_2, \dots, f_n$

Let $\frac{d}{dx}F(x) = f(x)$

$$\int g(x)\,dF(x) = \sum_{i=1}^{n} g(x_i) f_i + \int g(x) f(x)\,dx$$

$$\int_{(a,b]} dF(x) = P\{a < X \leq b\}$$

Let $X \geq 0$ with d.f. $F(\cdot)$

$$E[X] = \int_{t \in [0,\infty)} t \, dF(t) = \int_t \left[\int_{u=0}^{t} du \right] dF(t)$$

$$= \int_{u=0}^{\infty} du \int_{t \in [u,\infty)} dF(t) = \int_0^{\infty} (1 - F(u)) \, du$$

$$E[X] = \int_0^{\infty} \overline{F(t)} \, dt$$