Distributionally Robust Solution to the Reserve Scheduling Problem With Partial Information of Wind Power

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Distributionally Robust Solution to the Reserve Scheduling Problem With Partial Information of Wind Power

Qiaoyan Bian, *Student Member, IEEE*, Huanhai Xin, *Member, IEEE*, Zhen Wang, *Member, IEEE*, Deqiang Gan, *Senior Member, IEEE*, and Kit Po Wong, *Fellow, IEEE*

***Abstract*—A distributionally robust optimization (DRO) model is presented for the reserve schedule decision-making problem with partial information of wind power, aiming to find a robust solution to the uncertainty of wind power probability distribution. The stochastic problem can be converted into an equivalent deterministic bilinear matrix inequality (BMI) problem. Numerical results verify the effectiveness of the proposed method.**

***Index Terms*—Distributionally robust optimization, reserve schedule, uncertainty, wind power.**

I. Introduction

The uncertainty of wind power poses a big challenge to the system operations including reserve scheduling [1]. Different methodologies, such as scenario approach [2] and probabilistic analysis [3], have been developed to deal with the uncertainty of wind power, based on the assumption that the exact probability distribution of wind power is given.

However, in many situations, the probability distribution (PD) of wind power cannot be exactly obtained; instead, its partial information such as the first- and second-order moments can be collected from the historical data. Thus, the PD of wind power is uncertain, making the aforementioned methodologies less effective.

This letter aims to make robust reserve schedule decisions that minimize the cost of generation and reserve, when the exact PD of wind power is unknown and the mean and variance of wind power are available. To this end, a distributionally robust optimization (DRO) model is presented, which is indeed a distributionally robust joint chance constrained (DRJCC) problem [4]. The DRJCC approach guarantees that the system reliability requirement can be satisfied over all the possible PDs of wind power. The S-lemma and the Schur complements [4] can be used to convert the probabilistic DRO model into an equivalent deterministic BMI-constrained problem. Simulation results verify the effectiveness of the proposed method.

II. Distributionally Robust Optimization Model

System operators have to counter the generation-load imbalances caused by the difference between the actual wind power and its forecast, by reserves procured in a reserve market before real-time operation [5]. To co-optimize the day-ahead generation dispatches, the reserves to be procured and the reserve deploy strategy, with the assumption that the unit commitment problem is solved, the PD of wind power is unknown and the mean vector μ and covariance matrix Γ are available, the DRO reserve scheduling model is proposed as follows:

$$(\textbf{Reserve scheduling})\ \min_{x_t} f = C_G^T P_{G,t} + C_{up}^T R_{up.t} + C_{down}^T R_{down,t} \tag{1a}$$

$$s.t.\quad \mathbf{1}^T P_{G,t} + \mathbf{1}^T P_{w,t}^0 - \mathbf{1}^T P_{d,t} = 0 \tag{1b}$$

$$P_{G,\min} \le P_{G,t} \le P_{G,\max} \tag{1c}$$

$$\inf_{\phi\in\Phi(\mu,\Gamma)} \Pr\begin{pmatrix} -\bar{P}_{line} < TP_t < \bar{P}_{line}, \\ P_{G\min} < P_{G,t} + R_t < P_{G\max}, \\ -R_{down,t} < R_t < R_{up,t} \end{pmatrix} \ge 1-\varepsilon \tag{1d}$$

$$\text{with } R_t = d_t(P_{w,t} - P_{w,t}^0)$$

$$\mathbf{1}^T d_t = -1 \tag{1e}$$

$$R_{up,t} \ge 0, R_{down,t} \ge 0 \tag{1f}$$

where the decision variables $x_t = \{P_{G,t}, d_t, R_{up,t}, R_{down,t}\}$ are in vector form; t is the schedule time period; $P_{G,t}$ is the day-ahead generation dispatch, $R_{up,t}$ and $R_{down,t}$ denote the up and down reserve capacities procured in advance, while R_t is the real-time reserve amount; d_t indicates the reserve strategy, which is a vector consists of the reserve percentage each unit provides; C_{up}, C_{down}, and C_G are reserve and generation cost; $P_{w,t}$, $P_{w,t}^0$, and $P_{d,t}$ denote the actual, forecast wind power, and the load; T is the power network distribution factor matrix, and P_t is the power injection vector; ϕ denotes the PD of wind power, and $\Phi(\mu,\Gamma)$ denotes the set of all the distributions whose mean and covariance are μ and Γ, respectively; $\Pr(\cdot)$ denotes the probability and ε is the risk level; the joint constraint inside $\Pr(\cdot)$ is the intersection of all the individual constraints, which include power flow limit of each line, generation capacity limit of each unit, and reserve purchase estimation.

In this model, the "inf" denotes the infimum of the probability, so constraint (1d) implies that the probability that the system satisfies the constraints should be no smaller than $1-\varepsilon$ over all the possible PDs of wind power. Therefore, when the uncertainty of the PD of wind power is considered, the DRO model (1) is a DRJCC problem.

III. Tractable Problem Reformulations

In this section, the stochastic DRO problem will be converted into a deterministic BMI problem.

Each individual constraint in (1d) can be written in a quadratic form, e.g., the generation capacity limit of the ith unit:

$$\begin{aligned} &P_{G\min}^i < P_{G,t}^i + R_t^i < P_{G\max}^i \\ &\Leftrightarrow \left(\left[d_t^i\, P_{G,t}^i - d_t^i P_{w,t}^0 - 0.5(P_{G\max}^i + P_{G\min}^i) \right] \right. \\ &\quad \left. \cdot [P_{w,t}\ 1]^T \right)^2 < 0.25(P_{G\max}^i - P_{G\min}^i)^2. \end{aligned} \tag{2}$$

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Q. Bian, H. Xin, Z. Wang, and D. Gan are with the Department of Electrical Engineering, Zhejiang University, Hangzhou 310027, China (e-mail: xinhh@zju.edu.cn).

K. P. Wong is with the School of Electrical, Electronic and Computer Engineering, The University of Western Australia, Perth, Australia.

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Thus, (1d) can be written in a general form of (3):

$$P_{wc}(x) = \sup_{\phi \in \Phi(\mu,\Gamma)} \Pr\left(\bigcup_{j=1,2,\ldots,m} \left\{ [L_j(x)z]^2 \geq \gamma_j^2 \right\} \right) \leq \varepsilon \tag{3}$$

where $\delta \in \mathbb{R}^n$ denotes the random vector, $z = [\delta^T\ 1]^T$; $L_j(x)$, $j = 1, \ldots, m$ are $n+1$ dimension vectors whose elements are linear functions of the decision variable x; γ_j are constants.

According to the duality theory, $P_{wc}(x)$ in (3) corresponds to the objective value in (4). By the S-lemma [4, Lemma 2.2], the random vector z in (4) can be eliminated strictly, and by the Schur complements, each constraint in (4) can be further transformed into the form of the constraints in (5). Thus sub-problem (4) is equivalently converted to (5):

$$\begin{aligned} P_{wc}(x) = & \inf_{M=M^T \in \mathbb{R}^{(n+1)\times(n+1)}, M \geq 0} Tr(Q \cdot M) \\ s.t.\ & z^T M z \geq 1, \forall z : [L_j(x)z]^2 \geq \gamma_j^2 \\ & j = 1, 2, \ldots m \end{aligned} \tag{4}$$

$$\begin{aligned} \Leftrightarrow P_{wc}(x) = & \inf_{M=M^T \in \mathbb{R}^{(n+1)\times(n+1)}, M \geq 0} Tr(Q \cdot M) \\ & \exists \tau_j \geq 0, \begin{bmatrix} M - diag(\mathbf{0}_n, 1 - \tau_j \gamma_j^2) & \tau_j L_j^T(x) \\ \tau_j L_j(x) & \tau_j \end{bmatrix} \geq 0, \\ & j = 1, \ldots, m. \end{aligned} \tag{5}$$

where Tr denotes the trace operator, M is defined to be a symmetric matrix consists of the dual variables, and $Q = \begin{bmatrix} \Gamma + \mu\mu^T & \mu \\ \mu^T & 1 \end{bmatrix}$.

From the above derivation, it is obvious that the probabilistic constraint (1d) can be converted into the deterministic BMI form of (6) with the bilinear term $\tau_j L_j(x)$ introduced:

$$\begin{aligned} & \exists M = M^T \in \mathbb{R}^{(n+1)\times(n+1)},\ \tau_j \geq 0,\ j = 1, \ldots, m \\ & Tr(Q \cdot M) \leq \varepsilon,\ M \geq 0 \\ & \begin{bmatrix} M - diag\left(\mathbf{0}_n, 1 - \tau_j \gamma_j^2\right) & \tau_j L_j^T(x) \\ \tau_j L_j(x) & \tau_j \end{bmatrix} \geq 0,\ j = 1, \ldots, m. \end{aligned} \tag{6}$$

Therefore, by replacing (1d) with (6), the DRO problem (1) is converted into a new BMI-constrained problem (the BMI-P). BMI problems are nonconvex and the *sequential convex optimization algorithm* described in [4] is used to solve the BMI-P. The main idea is to optimize $\boldsymbol{\tau} = [\tau_1, \tau_2, \ldots, \tau_m]^T$ and x in alternation, by solving a sequence of linear matrix inequality (LMI) constrained problems. The specific procedures are as follows:

1) Set the initial solution x^0 and objective value f^0, $k = 1$.
2) Solve sub-problem (5) with $x = x^{k-1}$ (which is an LMI problem); the optimal solution obtained is $\boldsymbol{\tau}^k$.
3) Solve the BMI-P with $\boldsymbol{\tau} = \boldsymbol{\tau}^k$ (which is also an LMI problem) and obtain the optimal solution (x^k, f^k).
4) If $|f^k - f^{k-1}|$ is small enough, the final solution is found; otherwise, $k = k+1$, and steps 2) and 3) will be repeated.

In addition, since x is bounded, the sequence of objective values f^k is monotonically decreasing and converges to a finite limit. However, it should be emphasized that the sequential convex algorithm cannot guarantee the solution to be the global optimum due to the nonconvexity of the BMI-P [4].

IV. Numerical Results

The DRO reserve scheduling method is tested on the benchmark IEEE 39-bus system. Without loss of generality, we assume the wind farm is located at bus 16, μ increases by 0.25 from 1.00 to 1.75 (p.u.), Γ increases by 0.003 from 0.002 to 0.011; the units at bus 30, 31, 35, and 38 provide spinning reserves, the corresponding reserve prices are $C_{up}^T = [0.35, 0.45, 0.40, 0.50]$, and $C_{down}^T = [0.50, 0.40, 0.45, 0.35]$.

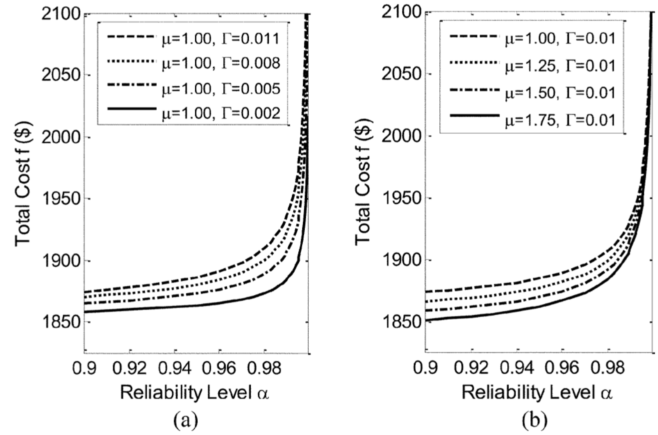

Fig. 1. (a) Total cost with different wind power mean. (b) Total cost with different wind power variance.

The resultant Fig. 1 illustrates the relationship among the uncertainty level (the wind power μ and Γ), the reliability level $\alpha = 1 - \varepsilon$, and the cost level (the total cost f). Both Fig. 1(a) and (b) can show that when μ or Γ are fixed, f has a monotonically increasing relationship with α, especially when α is high $(0.99 \sim 0.9995)$, f increases significantly with the increasing of α. Similarly, when α and μ are fixed, f increases as Γ becomes larger. Two trends can be observed: 1) Higher system reliability requirement will lead to higher cost, and the cost will increase sharply when some reliability level is reached. 2) Higher wind power variation will lead to higher reserve purchase and thus increase the total cost.

It should be noted that f cannot be solved when α is 1, because in that case, the chance constraint becomes a robust constraint, and a scheduling solution cannot be found to cover all the possible wind power output realizations.

V. Conclusion

The DRO model is proposed to solve the reserve scheduling problem. The DRJCC method guarantees that the required system reliability level can be achieved over all the possible probability distributions of wind power. The DRO model can also be applied to other system operation and planning problems, such as reliability assessment and generation expansion with uncertainties.

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