Homework #9

1. Consider a Poisson process with mean rate λ, and let $U(t) = t - S_{N(t)}$ for each $t>0$; in other words, U(t) is the time since the last renewal. Also, let $V(t) = S_{N(t)+1} - t$, i.e., the forward renewal time.

    a. Find an expression for E[U(t)]. Hint: use $E[U(t)] = \int_0^\infty P\{U(t) > x\}dx$.

First get expression for P{U(t)>x} for Poisson process, then obtain expected value. For Poisson process, $m(s) = \lambda s$; therefore,

$P\{U(t)>x\} = e^{-\lambda t} + \int_0^{t-x} \lambda e^{-\lambda(t-s)}\, ds = e^{-\lambda x}$ for $x <= t$

$E[U(t)] = \int_0^t e^{-\lambda x}\, dx = (1 - e^{-\lambda t})/\lambda$

b. Let λ=2/hr with t = 1.5 hr. $E[V(t)] = 1/\lambda$ = 0.5 hr.

c. Let λ=2/hr with t = 1.5 hr. $E[U(t)] = (1 - e^{-3})/2$ = 0.4751

2. *Geiger counters of type II*. Particles arrive at the counter according to a renewal process. At time t=0, a particle arrives locks the counter for a fixed time τ. The first arrival after $[0, \tau]$ gets registered. If a particle arrives during a locked period, it does not get registered and it has no effect on the length of the locked period.

    Let $\{X_n\}$ be times of arriving particles with distribution function φ and let m(t) be the renewal function associated with $\{X_n\}$. Let $\{T_n\}$ be the times of successive registrations with G being the distribution function for the time between registrations, and let r(t) be the renewal function associated with $\{T_n\}$.

    a. Can you justify the following expression:

        $1 - G(t) = 1 - \varphi(t) + \int_{s\in[0,\tau]} m(ds)[1 - \varphi(t - s)]$?

        Hint: think about the {V(t)} process.

    b. Let p(t) be the probability that the counter is locked at time t. Can you derive an expression for p(t)?

    Problem number 2 is not to be handed in. We will cover this in class on Monday, but if you can figure it out on your own, you will be well on your way to becoming an renewal processes expert.

    Covered in class.