Name: ____

Print Full name

Test Three

Open Book, Open Notes

***Instructions**:* This test represents your independent work. You may not talk to anyone else about this test until after you have turned it in. Answer the questions neatly so that I can read your work. Please return a scan or a photo of the test to me at richf@tamu.edu after insuring that the scan or photo will be readable. The test must be sent to me before 10:00 AM; thus, you have 2 hours for the test. (Every minute past 10:00 will cost five points.) If you have any questions regarding the meaning of a problem, make what you feel is an appropriate assumption and write down your assumption. You will then be graded on the correctness of the problem based on your assumption and the appropriateness of the assumption. Since I cannot see what you enter into your calculator, it is important to show all your work on the test. Guesses do not count, so if your work is not present, I'll assume a lucky guess which deserves no credit. Answers to the parts of Problem 1 should be rounded and accurate to four digits to the right of the decimal. Put a box around your answers to insure I see your results.

Sign the statement below and when you return the test, please return this page with your test. (If you do not have a printer, copy the statement below on your own paper and then sign it.)

Aggie Honor Code: "An Aggie does not lie, cheat, or steal or tolerate those who do." On my honor as an Aggie, I have neither given nor received unauthorized aid on this test.

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signature

1. A certain piece of electronic equipment has two components. The time until failure for component A is described by an exponential distribution function with a mean time of 100 hours. Component B has a mean life until failure of 160 hours and is also described by an exponential distribution. When one component fails, the equipment is turned off and maintenance is performed. (With the equipment off, the other component cannot fail.) The time to fix the component is exponentially distributed with a mean time of 5 hours if it were A that failed and 10 hours if it were B that failed. Let Y be a Markov process with state space E = {w, a, b}, where State w denotes that the equipment is working, State a denotes that component A is under repair, and State b denotes that component B is under repair. For a fixed time t, the random variable Y(t) gives the state of the equipment at time t. In writing your matrices, let the first row represent the state w, the second row represent the state a, and the third row represent the state b.

   a. Give the generator matrix for this process.

   b. Using the generator matrix, determine the long-run probability that the equipment is working?

   c. Every Markov process is also a semi-Markov process so for now, forget that you know this is a Markov process and structure it as a semi-Markov process with an imbedded Markov renewal process. Give the semi-Markov kernel, Q(t), for this process in preparation for analyzing the process using Markov renewal theory.

   d. Give the matrix Q(t) evaluated at t=10 hr. Round to four digits to right of decimal. (Notice, the Matrix Q(t) will have only numerical components – no expressions.)

   e. Using the methodology developed for semi-Markov processes, determine the long-run probability that the equipment is working? Be sure to show your work so that I can tell you know how to apply the results from Markov renewal processes to this problem. The answer to this part is a numerical answer, not an expression. Round to four digits to the right of the decimal. You may also need some of these calculations for Problem 2c.

2. Let $\{X_n, T_n\}$ be a Markov renewal process with state space E, semi-Markov kernel Q and Markov renewal functions $t \rightarrow R(i, j, t)$ for $i,j \in E$. Let $\{Y(t)\}$ be the associated semi-Markov process. Let $V(t) = T_{n+1} - t$ for $T_n <= t < T_{n+1}$. (In other words, V(t) is the time interval from t until the next change of state of the Markov renewal process, or equivalently, the amount of time from t until the next change of state of the semi-Markov process.)

   a. For a fixed $y > 0$ and fixed $j \in E$, derive an expression for $P\{Y(t)=j, V(t) > y \mid X_0=i\}$, or equivalently, $P_i\{Y(t)=j, V(t) > y\}$.

   b. For a fixed y and j, derive an expression for $\lim_{t\rightarrow\infty} P\{Y(t)=j, V(t) > y \mid X_0=i\}$.

   c. Assume we have returned to the situation described in Problem 1 and the current time is 1000 hours after start-up (i.e., t=1000). Use your expression in (b) to obtain the (approximate) probability that the equipment is now working and it will not fail within the next 10 hours. (The answer here is numerical, not an expression.)