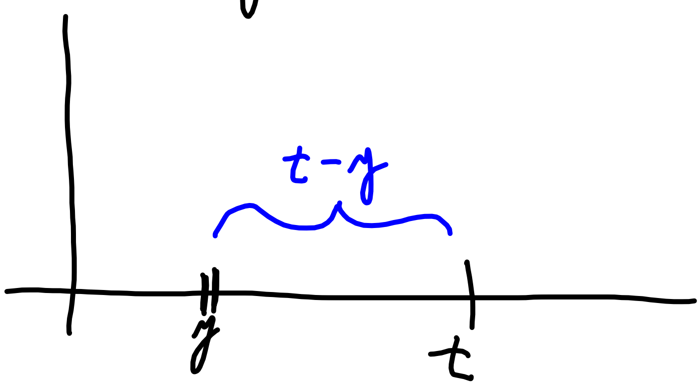
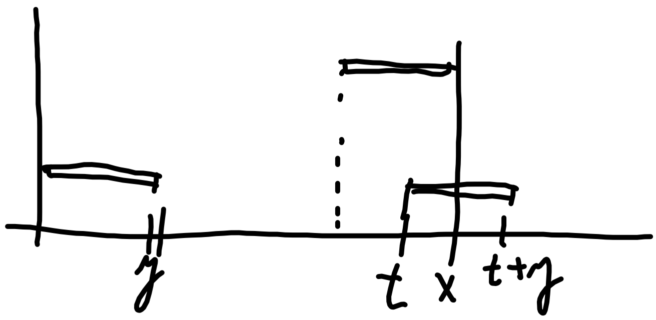
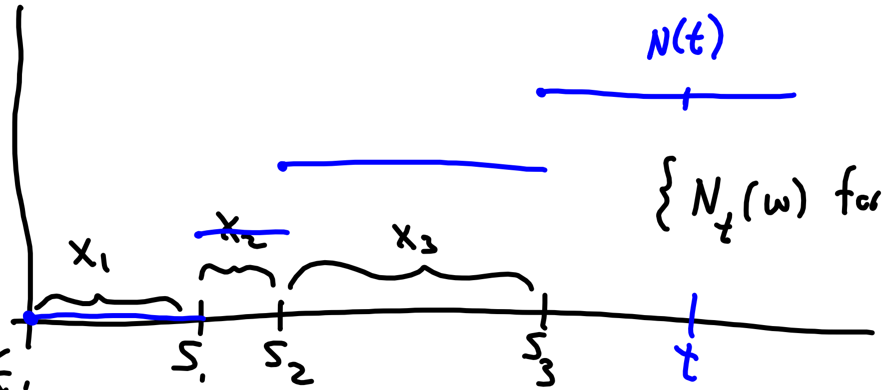
$$P\{X+Y \le t\} = \int_{y \in [0,t]} dG(y)\, \overbrace{F(t-y)}^{P\{X \le t-y\}}$$



$$P\{X - Y \le t\} = \int_{y \in [0,\infty)} dG(y)\, \overbrace{F(t+y)}^{P\{X \le t+y\}}$$





$\{N_t(\omega)$ for $t \geq 0$ and $\omega \in \Omega\}$

Def.

$\{N(t)\}$ is a counting process

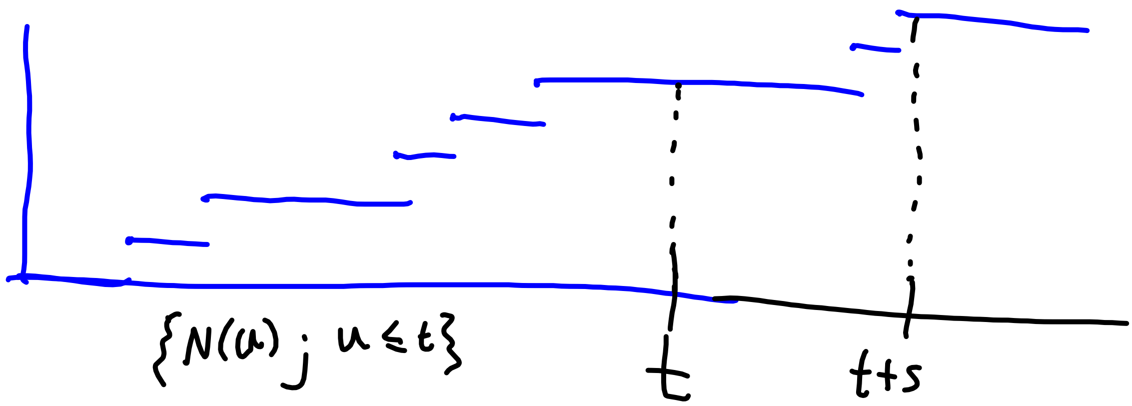
1) $N(t) \geq 0$
2) $N(t)$ is integer valued
3) $t \to N(t)$ is non-decreasing (and rt. continuous)
4) For $t, s \geq 0$, $N(t+s) - N(t)$ is number of events occur in $(t, t+s]$

Def. The counting process $\{N(t)\}$ is a Poisson process with rate $\lambda > 0$ iff

1) $N(0) = 0$
2) The process has independent increments
3) For $t, s \geq 0$, $P\{N(t+s) - N(t) = n\} = e^{-\lambda s} (\lambda s)^n / n!$ for $n = 0, 1, \ldots$

For any $t, s \geq 0$, $N(t+s) - N(t)$ is independent of $\{N(u);\ u \leq t\}$

A stationary property



Def. Counting process is Poisson

1) $N(0) = 0$
2) Process has stationary and independent increments
3) $P\{N(h) = 1\} = \lambda h + o(h)$
4) $P\{N(h) \geq 2\} = o(h)$

where a function $f(\cdot)$ is said to be $o(h)$ if
$\lim_{h \to 0} f(h)/h = 0$

Note $\{X_1 > t\} = \{N(t) = 0\}$

$P\{X_1 > t\} = P\{N(t) = 0\} = e^{-\lambda t}$

$\{N(t)\}$ be Poisson process

We have an item with exponential life with $\lambda = 0.0002$/hr.

1. What is $E[S_3]$ and st. dev. (S_3)

squared coefficient of variation of S_3

X & Y independent

$$E[aX + bY] = aE[X] + bE[Y]$$

$$Var(aX + bY) = a^2 Var(X) + b^2 Var(Y)$$

Superposition

$\{N(t)\}$ and $\{M(t)\}$ be two independent Poisson processes with rates λ_1 and λ_2

For each t, let $Y(t) = N_t + M_t$, then $\{Y(t)\}$ is a Poisson process with rate $\lambda_1 + \lambda_2$

Decomposition

Let $\{N(t)\}$ be Poisson with rate λ

Let Y_1, Y_2 be i.i.d. with Y_1 being a Bernoulli: R.V. with $P\{Y_1 = 1\} = p$.

Form the process $\{M(t)\}$ where the n^{th} arrival to $\{N(t)\}$ is also an arrival to $\{M(t)\}$ if $Y_n = 1$.

Then $\{M(t)\}$ is Poisson with rate λp.

The counting process $\{N(t)\}$ with

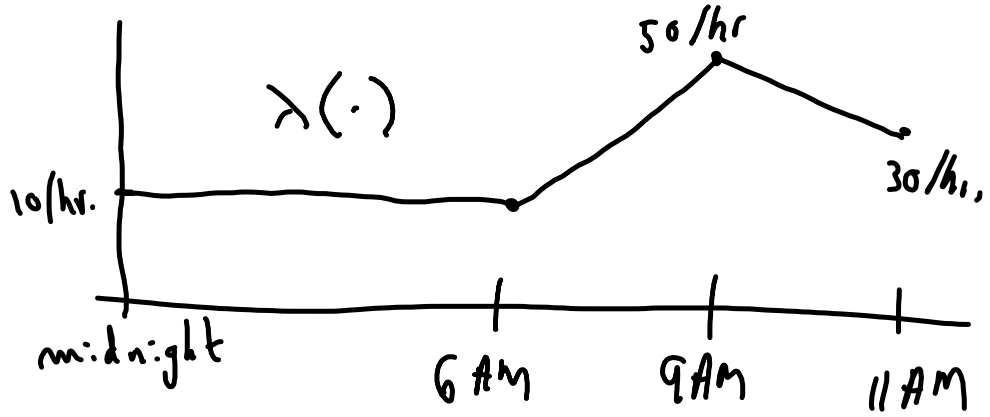
1) $N(0) = 0$
2) $\{N(t)\}$ has independent increments
3) $P\{N(t+s) - N(t) \geq 2\} = o(s)$
4) $P\{N(t+s) - N(t) = 1\} = \lambda(t) \cdot s + o(h)$

is called a non-stationary Poisson process.

Let $m(t) = \int_0^t \lambda(u)\, du$ for $t \geq 0$

$$P\{N(t) = k\} = e^{-m(t)} (m(t))^k / k! \quad \text{for } k = 0, 1, \cdots$$

$$E[N(t+s) - N(t)] = \int_t^{t+s} \lambda(u)\, du = m(t+s) - m(t)$$



1. What is the expected number of arrivals between midnight and 9AM?

$E[N_9] = 150$

2. What is (approximate) probability that there are more than 100 arrivals between 6AM and 9AM?

$P\{N_9 - N_6 > 100\}$?

$E[N_9 - N_6] = 90$

Let Y be a normal R.V. $E[Y] = 90$, $st.dev(Y) = \sqrt{90}$

$P\{Y \geq 100.5\}$