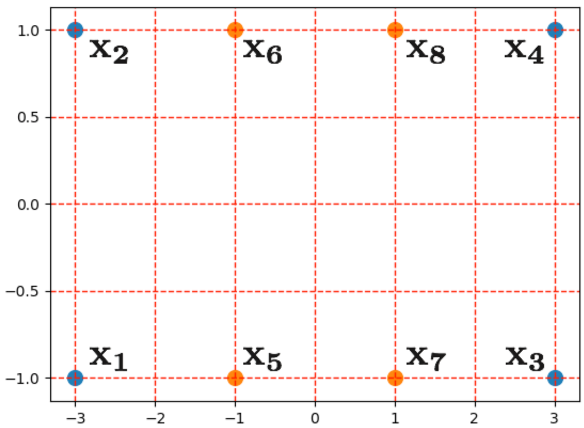
Consider the following 2-class binary problem. We will run Adaboost for two iterations.

Class 0 ($y = -1$): $\mathbf{x_1} = [-3, -1]$, $\mathbf{x_2} = [-3, 1]$, $\mathbf{x_3} = [3, -1]$, $\mathbf{x_4} = [3, 1]$
Class 1 ($y = 1$): $\mathbf{x_5} = [-1, -1]$, $\mathbf{x_6} = [-1, 1]$, $\mathbf{x_7} = [1, -1]$, $\mathbf{x_8} = [1, 1]$



(a) Assuming that the first classifier is $h_1(\mathbf{x}) = \begin{cases} -1 & \text{if } \mathbf{x} < -2 \\ +1 & \text{if } \mathbf{x} > -2 \end{cases}$, compute the classification error ϵ_1 and new weights $w_2(n)$ for every sample $n = 1, \ldots, 8$ that result from the first iteration ($t = 1$) of Adaboost.

$w_1(n) = \frac{1}{8}$, $n = 1, \ldots, 8$

$\epsilon_1 = w_1(3) + w_1(4) = \frac{2}{8} = \frac{1}{4}$ (samples $\mathbf{x_3}$ and $\mathbf{x_4}$ are incorrectly classified based on h_1)

$\beta_1 = log\frac{1-\epsilon_1}{\epsilon_1} = log\frac{1-\frac{1}{4}}{\frac{1}{4}} = log3$

Using the weight update equation we have: $w_2(n) = \frac{1}{8}$ for $n = 1, 2, 5, 6, 7, 8$ (since those samples were classified correctly) $w_2(n) = \frac{1}{8}exp(\beta_1) = \frac{3}{8}$ for $n = 3, 4$ (since those samples were classified incorrectly)

After normalizing the weights (i.e., dividing by their total sum $\sum n = 1^8 w_2(n) = \frac{3}{2}$), we get:

$$w_2(n) = \begin{cases} \frac{\frac{1}{8}}{\frac{3}{2}} = \frac{1}{12} & \text{if } n = 1, 2, 5, 6, 7, 8 \\ \frac{\frac{3}{8}}{\frac{3}{2}} = \frac{1}{4} & \text{if } n = 3, 4 \end{cases}$$

We observe that the weights of the correctly classified samples decreased, while the weights of the incorrectly classified samples have increased.

(b) Based on the error that you computed in **(a)**, what would be a reasonable classification boundary $h_2(\mathbf{x})$ for the second iteration of Adaboost?

A reasonable classifier would make sure that the samples with larger weight (i.e., $\mathbf{x_3}$ and $\mathbf{x_4}$) are correctly classified, therefore a reasonable choice would be: $h_2(\mathbf{x}) = \begin{cases} -1 & \text{if } \mathbf{x} > 2 \\ +1 & \text{if } \mathbf{x} < 2 \end{cases}$

(c) Compute the classification error ϵ_2 and new weights $w_3(n)$ for every sample $n = 1, \ldots, 8$ that result from the second iteration ($t = 2$) of Adaboost.
$\epsilon_2 = w_2(2) + w_2(2) = \frac{2}{12} = \frac{1}{6}$ (samples $\mathbf{x_1}$ and $\mathbf{x_2}$ are incorrectly classified based on h_2)

$\beta_2 = log\frac{1-\epsilon_2}{\epsilon_2} = log\frac{1-\frac{1}{6}}{\frac{1}{6}} = log6$

Using the weight update equation we have: $w_2(n) = \frac{1}{12}$ for $n = 5, 6, 7, 8$ (since those samples were classified correctly) $w_2(n) = \frac{1}{4}$ for $n = 3, 4$ (since those samples were classified correctly)

$w_2(n) = \frac{1}{12}exp(\beta_2) = \frac{5}{12}$ for $n = 1, 2$ (since those samples were classified incorrectly)
After normalizing the weights (i.e., dividing by their total sum $\sum n = 1^8 w_2(n) = \frac{3}{2}$), we get:

$$w_2(n) = \begin{cases} \frac{\frac{1}{12}}{\frac{5}{3}} = \frac{1}{20} & \text{if } n = 5, 6, 7, 8 \\ \frac{\frac{1}{4}}{\frac{5}{3}} = \frac{3}{20} & \text{if } n = 3, 4 \\ \frac{\frac{5}{12}}{\frac{5}{3}} = \frac{1}{4} & \text{if } n = 1, 2 \end{cases}$$

(d) What would be the final decision rule based on the first two iterations of Adaboost and the computed variables h_1, h_2, β_1, and β_2? Would this result in all samples being correctly classified? What would you do if not?

$h(\mathbf{x}) = sign\left(\beta_1 h_1(\mathbf{x}) + \beta_2 h_2(\mathbf{x})\right) = sign\left(log3 \cdot h_1(\mathbf{x}) + log5 \cdot h_2(\mathbf{x})\right)$
$\mathbf{x_1} = [-1, 3]^T$: $h(\mathbf{x_1}) = -log3 + log5 > 0 \rightarrow$ Class 1 (incorrect)
$\mathbf{x_2} = [-3, 1]^T$: $h(\mathbf{x_2}) = -log3 + log5 > 0 \rightarrow$ Class 1 (incorrect)
$\mathbf{x_3} = [3, -1]^T$: $h(\mathbf{x_2}) = log3 - log5 < 0 \rightarrow$ Class -1 (correct)
$\mathbf{x_4} = [3, 1]^T$: $h(\mathbf{x_4}) = log3 - log5 < 0 \rightarrow$ Class -1 (correct)
$\mathbf{x_5} = [-1, -1]^T$: $h(\mathbf{x_5}) = -log3 + log5 > 0 \rightarrow$ Class 1 (correct)
$\mathbf{x_6} = [-1, 1]^T$: $h(\mathbf{x_6}) = -log3 + log5 > 0 \rightarrow$ Class 1 (correct)
$\mathbf{x_7} = [1, -1]^T$: $h(\mathbf{x_7}) = log3 - log5 < 0 \rightarrow$ Class -1 (incorrect)
$\mathbf{x_8} = [1, -1]^T$: $h(\mathbf{x_8}) = log3 - log5 < 0 \rightarrow$ Class -1 (incorrect)
We will need to perform more iterations in order to be able to classify correctly every sample.