Homework #1

Class assignment given Jan 13, 2020

Round all answers to two digits to the right of the decimal.

1. Let X be a random variable with CDF given by

$$F(x) = \begin{cases} 0 & \text{for } x \le 2 \\ (x-2)^2 & \text{for } 2 < x < 3 \\ 1 & \text{for } x \ge 3 \end{cases}$$

a. Find $P\{X \le 2.5\} = (2.5 - 2)^2 = 0.25$

b. Find $P\{X = 2.75\} = 0$

c. Find $P\{2.5 \le X < 3.5\} = 1 - P\{X \le 2.5\} = 0.75$

d. Find E[X] = integrate from 2 to 3 of $2x(x-2)dx = 8/3 = 2.67$

2. The dispatcher at a central fire station has observed that the time between calls is an exponential random variable with a mean of 30 minutes.

a. A call has just arrived. What is the probability that the next call will arrive within the next half hour?

$P\{T <= 30min\} = 1 - \exp\{-1\} = 0.63$

b. What is the probability that there will be exactly three calls during the next hour?

$P\{N=3\} = 2^3 \exp\{-2\} / 3! = 0.18$

3. Let N and K be two random variables whose joint pmf is given by

	1	2
10	0.1	0.1
20	0.3	0.2
30	0.2	0.1

where N can take the values 10, 20, and 30; and K can take the values 1 and 2.

a. E[K | N=20]. Note: $P\{N=20\} = 0.5$, $P\{K=1 \mid N=20\} = 0.3/0.5 = 0.6$ and $P\{K=2 \mid N=20\} = 0.2/0.5 = 0.4$; therefore, $E[K \mid N=20] = 1*0.6 + 2*0.4 = 1.4$.

b. Find $E[NK^2] = 10*1^2*0.1 + 20*1^2*0.3 + 30*1^2*0.2 + 10*2^2*0.1 + 20*2^2*0.2 + 30*2^2*0.1 = 45$

4. . Let X be a Markov chain with state space {a,b,c,d} and transition probabilities given by

P =				
a	0.0	0.7	0.2	0.1
b	0.3	0.5	0.0	0.2
c	0.2	0.0	0.2	0.6
d	0.1	0.1	0.8	0.0

Define a profit function as f = (10, -20, -30, 40). In other words, each visit to state a yields a profit of

$10, each visit to state b yields a loss of $20, etc. Find the following:

a. $P\{X_2 = d \mid X_0 = c\} = P2(c, d) = 0.14$

b. $P\{X_3 = d \mid X_2 = c\} = P(c, d) = 0.6$

c. $P\{X_2 = d, X_1 = a \mid X_0 = c\} = P(c, a)\, P(a, d) = 0.2*0.1 = 0.02$

d. $P\{X_2 = c \mid X_1 = d, X_0 = c\} = P(d, c) = 0.8$

e. $E[f(X_1) \mid X_0 = c] = Pf(\ c\ ) = 20$