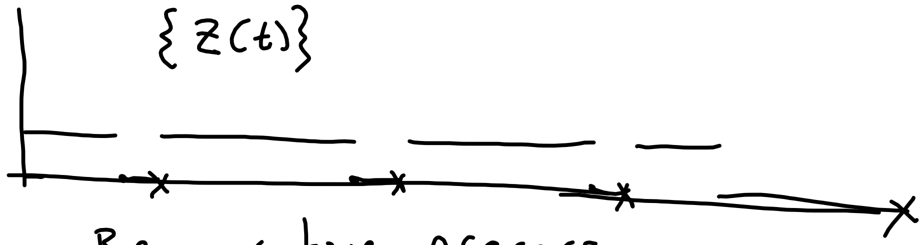

Regenerative process

$$P\{Z(t)=1\} = P\{Z(t)=1, S_1>t\} + P\{Z(t)=1, S_1\le t\}$$
$$= 1-\varphi(t) + \int_{[0,t]} F(ds)\, P\{Z(t-s)=1\}$$
$$= 1-\varphi(t) + \int_{[0,t]} m(ds)\,[1-\varphi(t-s)]$$

$F=\varphi*\psi$ and $m=\sum_{n=1}^{\infty} F_n$

Solution to $h = g + F*h$

is $h = g + m*g$ where $m=\sum_{n=1}^{\infty} F_n$

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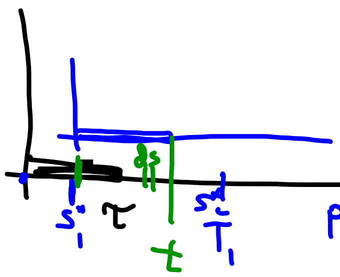
Counter-type II $\Rightarrow$ locked period gets re-locked at an arrival inside locked period



$P\{T_1>t \mid \tau < S_1 < t\} = 0$

$P\{T_1>t, S_1>t\}$

$$P\{T_1>t\} = 1-\varphi(t) + \int_{[0,t]} \varphi(ds)\, I_{[0,\tau]}(s)\, P\{T_1>t-s\}$$

$$\left.\begin{array}{ll} F(ds)=\varphi(ds) & \text{if } s\le\tau \\ \text{and } F(ds)=0 & \text{if } s>\tau \end{array}\right\} \to F(ds)=\varphi(ds)\, I_{[0,\tau]}(s)$$

$$F(t) = \begin{cases} \varphi(t) & \text{for } t\le\tau \\ \varphi(\tau) & \text{for } t>\tau \end{cases}$$

let $m(t) = \sum_{n=1}^{\infty} F_n(t)$

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If $\varphi(\tau) < 1 \Rightarrow F(\infty) < 1 \Rightarrow$ transient renewal process

$m(\infty) < \infty$ and $m(\infty) = \dfrac{F(\infty)}{1 - F(\infty)} = \dfrac{\varphi(\tau)}{1 - \varphi(\tau)}$

$1 - G(t) =$

$$P\{T_1 > t\} = 1 - \varphi(t) + \int_{[0,t]} m(ds)\left[1 - \varphi(t-s)\right]$$

$$\lim_{t \to \infty} P\{T_1 > t\} = m(\infty) \cdot 0 = 0$$

$$\lim_{t\to\infty} m * g(t) = \begin{cases} m(\infty)\, g(\infty) & \text{if } m(\infty) < \infty \\ \dfrac{1}{\mu}\displaystyle\int_0^\infty g(x)\,dx & \text{if } m(\infty) = \infty \end{cases}$$

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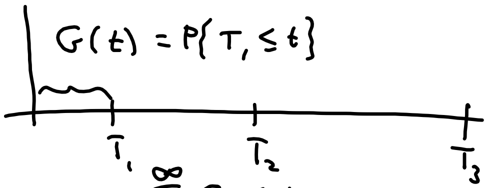
$G(t) = P\{T_1 \le t\}$

$T_1 \geq \tau$



$$r(t) = \sum_{n=1}^{\infty} G_n(t)$$

$\displaystyle\lim_{t\to\infty} \frac{r(t)}{t} \longrightarrow$ avg. # registrations per time

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Elementary renewal thm.

$$\lim_{t\to\infty} \frac{m(t)}{t} = \frac{1}{\mu}$$

Central limit thm

$$\frac{N(t) - \frac{1}{\mu}\cdot t}{\sigma\sqrt{t/\mu^3}} \longrightarrow \text{standard normal}$$

$N(t) \longrightarrow$ Normal with mean $\frac{1}{\mu}\cdot t$

and var. $\dfrac{\sigma^2 t}{\mu^3}$

μ and σ^2 are mean and variance of inter-renewal times

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$$\lim_{t\to\infty} \frac{r(t)}{t} = \frac{1}{E[T_1]} \qquad \left(m(\infty) = \frac{\varphi(\tau)}{1-\varphi(\tau)} \right)$$

$$\int_0^\infty P\{T > t\}\,dt = \int_0^\infty \left\{ 1 - \varphi(t) + \int_{[0,t]} m(ds)\left[1-\varphi(t-s)\right] \right\} dt$$

$$\int_{t=0}^\infty dt \int_{s\in[0,t]} m(ds)\left[1-\varphi(t-s)\right] = \int_{s\in[0,\infty)} m(ds) \int_{t=s}^\infty \left[1-\varphi(t-s)\right] dt$$

$$E[T_1] = \frac{\mu}{1-\varphi(t)} \quad \text{where } \mu = \int_0^\infty [1-\varphi(t)]\,dt$$

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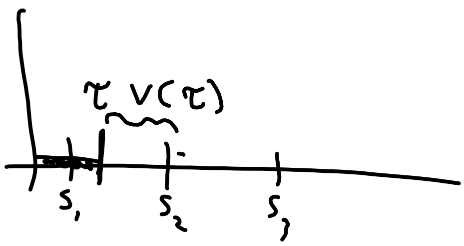

$P\{T_1 > t\} = 1$ if $t \le \tau$

$T_1 > t$ if $t \le \tau$

if $t > \tau$

$T_1 = \tau + V(\tau)$

$$P\{V(t) > y\} = 1 - F(t+y) + \int_{[0,t]} m(ds)\left[1 - F(t+y-s)\right]$$

a general renewal process

For $t > \tau$

$$P\{T_1 > t\} = P\{\tau + V(\tau) > t\} = P\{V(\tau) > t-\tau\}$$

φ is distribution for inter-arrivals

$$m = \sum_{n=1}^\infty \varphi_n$$

$$\hookrightarrow P\{T_1 > t\} = 1 - \varphi(\tau + t - \tau) + \int_{[0,\tau]} m(ds)\left[1 - \varphi(\tau + t - \tau - s)\right)$$

$$P\{T_1 > t\} = 1 - \varphi(t) + \int_{[0,\tau]} m(ds)\left[1 - \varphi(t-s)\right]$$

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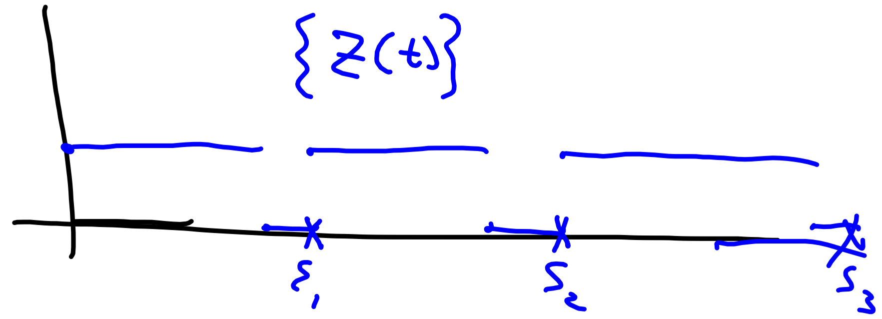

Feb 24-9:02 AM