Difference between

$\lim_{n\to\infty} P_i\{X_n = a\} = 0.56$ ✓ jump times — tally at jump time

$\lim_{t\to\infty} P_i\{Y(t) = a\} = 0.39$ ← arbitrary time — time weighted avg — state statistic

M/G/1 • at arrival pts same

G/M/1 ← at arrivals different

Lifetimes of components: $S_1, S_2, \dots, S_n$
$K = j$ if j component caused failure
$T = \min\{S_1, \dots, S_n\}$
Let Z_j be R.V. such that $P\{Z_j \le t\} = \varphi(j, t)$

$$P\{T > t\} = P\{S_1 > t, S_2 > t, \dots, S_n > t\} = e^{-(\lambda_1 + \lambda_2 + \dots + \lambda_n)t}$$

let $\lambda = \lambda_1 + \dots + \lambda_n \Rightarrow P\{T > t\} = 1 - e^{-\lambda t}$ for $t \geq 0$

$P\{K = j\}$ — Assume S_1 and S_2
$$P\{K = 1\} = P\{S_1 \le S_2\} = E[P\{S_1 \le S_2 \mid S_2\}]$$

$$P\{S_1 \le S_2 \mid S_2 = s\} = \underbrace{1 - e^{-\lambda_1 s}} \quad \text{for } s \geq 0$$

$$E[P\{S_1 \le S_2 \mid S_2\}] = \int_0^\infty (1 - e^{-\lambda_1 s})\lambda_2 e^{-\lambda_2 s}\, ds = \dots = \frac{\lambda_1}{\lambda}$$

$$P\{K = 1, T \le t\} = \frac{\lambda_1}{\lambda}(1 - e^{-\lambda t})$$

$$P\{K=j, T\le t\} = \frac{\lambda_j}{\lambda} P\{T \le t\} \qquad 1-e^{-\lambda t} \xrightarrow{pdf} \lambda e^{-\lambda t}$$

$$P_i\{X_1 = j, T_1 \le t\} = \left(\frac{\lambda_j}{\lambda}\right) \underline{P\{Z_i + T \le t\}}$$

$$Q(i,j,t) = \int_0^t \lambda_j e^{-\lambda s} \varphi(i, t-s)\, ds$$

$$P(i,j) = \frac{\lambda_j}{\lambda} \longrightarrow \text{all rows are the same}$$

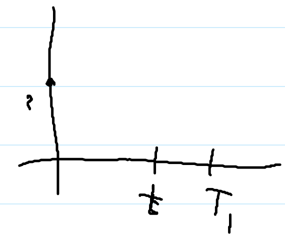
$$\nu = (\lambda_1, \lambda_2, \cdots, \lambda_n) \text{ is invariant}$$

$$\mu(i) = E_i[T_1] = E_i[Z_i + T] = E[Z_i] + \frac{1}{\lambda}$$

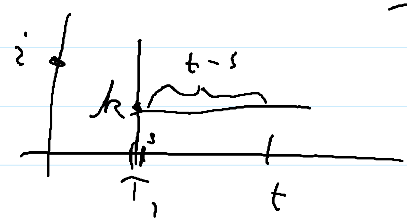
$$\nu \cdot \mu = \sum_j \lambda_j E[Z_j] + \sum_j \frac{\lambda_j}{\lambda} = 1 + \sum_j \lambda_j E[Z_j]$$

$$h(i,t) = P_i\{Y(t)=j,\ W(t)=0\} \quad \text{for a fixed } j$$

$$P_i\{Y(t)=j,\ W(t)=0,\ T_1 > t\} = I(i,j)\left[1-\varphi(i,t)\right]$$



$$P_i\{Y(t)=j,\ W(t)=0,\ T_1 \le t\}$$



$$P_i\{Y(t)=j,\ W(t)=0\} = I(i,j)\left[1-\varphi(i,t)\right] + \sum_k \int_{[0,t]} Q(i,k,ds)\, P_k\{Y(t-s)=j,\ W(t-s)=0\}$$

$$P_i\{Y(t)=j,\ W(t)=0\} = \sum_k \int_{[0,t)} R(i,k,ds)\, I(k,j)\left[1-\varphi(k,t-s)\right]$$

$$= \int_{[0,t)} R(i,j,ds)\left[1-\varphi(j,t-s)\right]$$

$$\lim_{t\to\infty} P_i\{Y(t)=j,\ W(t)=0\} = \frac{\nu(j)}{\nu\cdot\mu}\int_0^\infty \left[1-\varphi(j,t)\right]dt$$

$$\lim_{t\to\infty} P_i\{Y(t)=j,\ W(t)=0\} = \frac{\lambda_j E[Z_j]}{1+\sum_k \lambda_k E[Z_k]}$$

Lognormal

$X \to$ Normal mean $= \mu_N$ var $= \sigma_N^2$

$Y = e^X$ with $E[Y] = \mu_L$ and $Var(Y) = \sigma_L^2$

Y is called lognormal, $\ln(Y)$ is normal

Assume know μ_L and σ_L^2

$$\sigma_N^2 = \ln\left(\frac{\sigma_L^2}{\mu_L^2} + 1\right) \quad \mu_N = \ln(\mu_L) - \frac{1}{2}\sigma_N^2$$

Assume know μ_N and σ_N^2

$$\mu_L = \exp\left(\mu_N + \frac{1}{2}\sigma_N^2\right) \text{ and } \sigma_L^2 = \mu_L^2 \times (\exp(\sigma_N^2) - 1)$$