Homework #17

Due by 7AM, Monday, April 15

Instructions: Do your work on your own paper and give only the numerical answers in eCampus. Give your answers rounded to **two digits to the right of the decimal**.

A machine has two components a and b whose lifetimes are exponential with mean time to failure of 100 hours and 25 hours, respectively. The machine fails if either component fails, and the repair times are random with a lognormal distribution. The mean and standard deviation of the repair times for component a are 10 hours and 5 hours, respectively. The mean and standard deviation of the repair times for component b are 10 hours and 8 hours, respectively.

Let Y(t) be a, b, or c according as, at time t, the component a is being repaired, the component b is being repaired, or the machine is working. The process {Y(t)} is a semi-Markov process and let $\{X_n,T_n\}$ be the Markov renewal process with state space E={a, b, c} associated with {Y(t)}. Your task is to give the semi-Markov kernel for this process and then find $P_c\{Y(t) = j\}$ for $j \in E$. Answer the questions below based on your derivations. Hint: the semi-Markov kernel has the following form:

$$Q(t) = \begin{matrix} 0 & 0 & \text{Lognorm for a} \\ 0 & 0 & \text{Lognorm for b} \\ 0.2(1-\exp(-0.05t)) & 0.8(1-\exp(-0.05t)) & 0 \end{matrix}$$

a. What is the mean of the normal random variable that generates the lognormal distribution for the repair of component a? Mean and standard deviation are 2.19101 and 0. 47238, respectively.

b. What is the mean of the normal random variable that generates the lognormal distribution for the repair of component b? Mean and standard deviation are 2.05524 and 0.70335, respectively.

c. What is Q(a, c, t) for t=10 hr? Using Excel gives 0.59344; rounded yields 0.59

d. What is Q(b, c, t) for t=10 hr? Using Excel gives 0.63746; rounded yields 0.64

e. What is Q(c, a, t) for t=50 hr? = 0.18358; rounded yields 0.18

f. What is Q(c, a, t) for t=50 hr? = 0.73433; rounded yields 0.73

g. What is the average time between failures of component a? (Why is this answer not equal to 100?) We have the following vectors: ν = (1,4,5) and μ=(10,10,20). The mean time between failures = $\nu\mu/\nu(a)$ = 150 hr. It's greater than 100 because of time spent during repairs.

h. Find the $\lim_{t\to\infty} P_c\{Y(t) = c\} = \nu(c)\mu(c)/\nu\mu = 2/3$; rounded yields 0.67.