

Homework #15

Due by 7AM, Monday, April 6

Instructions: Do your work on your own paper and give only the numerical answers in eCampus. Give your answers rounded to **four digits to the right of the decimal** unless the answer is exact with less than four digits.

Let $\{X_n, T_n\}$ be a Markov renewal process with state space $\{a, b\}$ and semi-Markov kernel Q given as

$$Q(t) = \begin{matrix} & \begin{matrix} 0.6(1 - e^{-5t}) & 0.4 - 0.4e^{-2t} \end{matrix} \\ \begin{matrix} 0.5 - 0.2e^{-3t} - 0.3e^{-5t} & 0.5 - 0.5e^{-2t} - te^{-2t} \end{matrix} \end{matrix}$$

where t represents *days*.

- If the process is currently in state a, what is the probability that the next jump will be back to itself (i.e., state a)?
- If the process is currently in state b, what is the probability that the next jump will be to state a?
- Given that the process has just made a jump to state a, what is the probability that the next jump will occur within six hours given that the next jump will be a return to state a?
- Given that the process has just made a jump to state a, what is the probability that the next jump will occur within six hours given that the next jump will be to state b?
- Given that the process starts in state a, then next moves to state b, and then back to state a, what is the probability that both initial sojourn times in state a and state b were less than six hours?