NP-completeness Theory

shortest problem
longest problem

Satisfiability (SAT)

given a cnf formular F can you find an assignment that makes F = true?

$\wedge$: and

$\vee$: or.

$$F = \underbrace{(\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3)}_{\text{true if at least one is true.}} \wedge (\bar{x}_1 \vee \bar{x}_3) \wedge \cdots \wedge (\bar{x}_{50} \vee \bar{x}_{100})$$

true iff all clause be true.

Independent Set (IS)

given a graph G and k, can you find k vertices in G in which no two are adjacent?

Vertex-cover (VC)

given a graph G and k, is there a set C of k vertices such that every edge in G has at least one end in C.

a problem is a decision problem if it only requires a yes/no answer.

Def a dicision problem Q is in the class NP if there is a algorithm A such that for any instance x

1. if x = yes, then there is a y such that $A(x,y)=1$
2. if x = no, then for all y, $A(x,y)=0$.

and the alg $A(x,y)$ runs in time $O(|x|^c)$ for a constant c.

Alg for IS. $(\langle G, K\rangle, y)$ time = $O(n^3)$

1. if y is not a set of k vertices in the graph G.
   return ('0')

2. if any two vertices in y, are adjacent in G,
   return ('0'),

3. return ('1')

if $\langle G, K\rangle$ = Yes, then there are k vertices that are Indep.

When y is that k vertices

SAT, IS, VC are all in NP.

easy to check the solution.

decision

if a problem Q can be solved in time $O(n^c)$ for constant c, then the problem is in NP.

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alg A solves Q.

$$A(x) = \begin{cases} \text{yes} \\ \text{no.} \end{cases} \quad \text{in time } O(|x|^c)$$

$\underline{\hat{A}(x, y)}$

if $A(x) ==$ yes return('yes')

else return('no')