Renewal Reward Process

Let R_n be reward at time of n^{th} renewal with $\{(X_n, R_n)\}$ i.i.d. random vectors

Let $R(t) = \sum_{n=1}^{N(t)} R_n$

$$\lim_{t\to\infty} \frac{R(t)}{t} = \lim_{t\to\infty} \left(\underbrace{\frac{\sum_{n=1}^{N(t)} R_n}{N(t)}}_{E[R_1]} \right)\left(\underbrace{\frac{N(t)}{t}}_{\frac{1}{E(X_1)}} \right) = \frac{E[R_1]}{E[X_1]}$$

S.L.N.

$$\lim_{t\to\infty} \frac{E[R(t)]}{t} = \frac{E[R_1]}{E[X_1]}$$

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Age-replacement Process

$\{U_n\} \to$ i.i.d. failure times

If $U_n \le \tau$, replace at U_n

If $U_n > \tau$, replace at τ

$\{S_n\} \to$ be times of successive replacements

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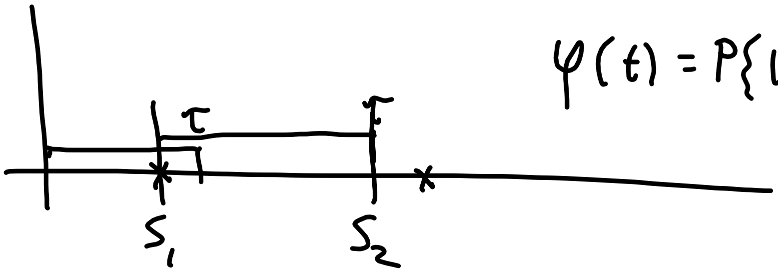

$$\varphi(t) = P\{U_1 \le t\}$$

$$S_1 = U_1 \wedge \tau$$

$$P\{S_1 \le t\} = F(t) = \begin{cases} \varphi(t) & \text{if } t < \tau \\ 1 & \text{if } t \ge \tau \end{cases}$$

$N(t) \to$ number replacements $[0, t]$

$E[N(t)] = m(t)$

$\lim_{t \to \infty} \frac{m(t)}{t} = \frac{1}{\mu}$ where $\mu = \int_0^\tau [1 - \varphi(t)]\,dt$

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$$P\{U \le t\} = \varphi(t)$$

$$\lim_{s \to 0^+} \frac{P\{U \le t+s \mid U > t\}}{s} = \frac{\varphi'(t)}{1-\varphi(t)}$$

$$= \lim_{s \to 0^+} \frac{\varphi(t+s) - \varphi(t)}{[1-\varphi(t)] \cdot s} = \frac{1}{1-\varphi(t)} \underbrace{\boxed{\lim_{s \to 0^+} \frac{\varphi(t+s) - \varphi(t)}{s}}}_{\varphi'(t)}$$

For a continuous d.f. $\varphi(\cdot)$
its hazard rate is defined to be

$$h(t) = \frac{\varphi'(t)}{1-\varphi(t)} \quad \text{for all } t \ge 0.$$

if $h(\cdot)$ is increasing, φ is said to have an IFR.

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The most common d.f. used to describe failures is the Weibull distribution

$$F(t) = 1 - e^{-(t/\beta)^{\alpha}} \quad \text{for } t \geq 0$$

where α is the shape parameter and β is the scale parameter.

$$E[T] = \beta\,\Gamma(1+\tfrac{1}{\alpha})$$

$$E[T^2] = \beta^2\,\Gamma(1+\tfrac{2}{\alpha})$$

$$SCV = \frac{\sigma^2}{\mu^2} = \frac{E[T^2]-E[T]^2}{E[T]^2} = \frac{E[T^2]}{E[T]^2} - 1$$

$$SCV + 1 = \frac{E[T^2]}{E[T]^2} = \frac{\beta^2\,\Gamma(1+\frac{2}{\alpha})}{\beta^2\,\Gamma(1+\frac{1}{\alpha})^2}$$

$$\therefore \boxed{SCV + 1 = \frac{\Gamma(1+\frac{2}{\alpha})}{\Gamma(1+\frac{1}{\alpha})^2}} \Rightarrow \beta = \frac{E[T]}{\Gamma(1+\frac{1}{\alpha})}$$

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Age replacement policy. (Given τ and $\varphi(\cdot)$)

Let $c_r \rightarrow$ cost to replace

Let $c_f \rightarrow$ additional cost if fails

$$\lim_{t\to\infty} \frac{E[R(t)]}{t} = \frac{E[R_1]}{E[X_1]}$$

$$E[R_1] = c_r + c_f\,\varphi(\tau)$$

$$E[X_1] = \int_0^{\tau} [1-\varphi(t)]\,dt$$

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$T \sim F$

$F^{-1}(u) = T$

where u is UNIF(0,1)

$F^{-1}(\text{RAND}())$

$m(t) = \frac{t}{\mu}$

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