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Stochastic Robust Mathematical Programming Model for Power System Optimization

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***Abstract*—This letter presents a stochastic robust framework for two-stage power system optimization problems with uncertainty. The model optimizes the probabilistic expectation of different worst-case scenarios with different uncertainty sets. A case study of unit commitment shows the effectiveness of the proposed model and algorithms.**

***Index Terms*—Distributionally robust, power system optimization, robust optimization, stochastic, unit commitment.**

I. Introduction

TRADITIONALLY, robust optimization models uncertainty use a deterministic set (e.g., a set of possible scenarios or range of possible values for the uncertain parameters) without any probabilistic description. It provides a robust solution that is immune to any possible scenario of the uncertainty set, which is an important aspect in the security-constrained scheduling or planning of electric power systems. Reference [1] models unit commitment as the two-stage robust optimization problem. Reference [2] adopts the column and constraints algorithm to solve the two-stage robust optimization problem. Reference [3] adds critical transmission line constraints and their dual variables on the fly to accelerate the solution speed.

Distributionally robust optimization is the associated robust version of the stochastic programming. As a substitute that models the exact probability distribution for the uncertain parameters in stochastic programming, it considers the ambiguity set for the probability distributions and obtains the solution and the expected cost under the worst-case probability distribution. Here, the term “worst-case” should not be confused with the worst-case scenario, because it refers to the worst-case probability distribution within the ambiguity set. Reference [4] developed an algorithm for distributionally robust optimization problems in which the uncertainty set consists of probability distributions with given bounds on their moments.

In this letter, instead of modeling the expected cost of each candidate probability distribution as the inner problem and the worst-case probability distribution as the outer problem of the second-stage problem in the traditional distributionally robust optimization model, we propose to switch the inner problem and the outer problem of the second-stage problem. The new distributionally robust optimization model optimizes the probabilistic expectation cost of the different worst-case scenarios under different uncertainty sets (can be box, polyhedra, or ellipsoid) used for describing uncertainties. The parameters of uncertainty sets correspond to a discrete probability distribution. In other words, the proposed model is a stochastic minmax optimization model. References [5] and [6] describe similar approaches by considering weighted summation of performances over multiple uncertainty sets. However, their models are deduced from different ways and trajectory compared to this letter. The formulation in this letter is motivated by the variants of distributionally robust model. Reference [5] is motivated by decision maker's risk consideration and temporal and spatial correlations. The model in [6] is deduced from solving multi-objective robust optimization. In addition, we minimize the expected cost while weight-based summation are used in the objective in [5] and [6].

II. Mathematical Models

Let us consider the traditional two-stage distributionally robust program (1)–(4). The first-stage variables can be binary variables. Equations (2) and (3) bound the first-stage feasible set. Both the first-stage cost function, $\boldsymbol{c}^T(\cdot)$, and the second-stage cost function, $\boldsymbol{q}(\boldsymbol{\xi})^T(\cdot)$, are linear. The second-stage variables are bounded by (4). We do not know the distribution P of the underlying uncertain parameter ξ exactly. We collect the possible probability distributions and denote them as Ω. Next, we consider the following associated robust version of stochastic programming. The second-stage problem finds the expectation cost of the worst-case probability distribution within the given collected probability distribution set Ω. The framework integrates the notion of expectation and the worst-case cost of the robust optimization method:

$$\min \boldsymbol{c}^T\boldsymbol{x} + \max_{\mathrm{P}\in\Omega} E_{\mathrm{P}(\xi)}[\min_{y\in\chi(\boldsymbol{\xi})} \boldsymbol{q}(\boldsymbol{\xi})^T\boldsymbol{y}] \tag{1}$$

$$\textbf{s.t.}\quad \boldsymbol{Fx} \le \boldsymbol{g} \tag{2}$$

$$\boldsymbol{x} \in \mathbf{X} = span\{\boldsymbol{R}^{m_1} \times \{0,1\}^{n_1}\} \tag{3}$$

$$\chi(\boldsymbol{\xi}) = \{\boldsymbol{Gy} \le \boldsymbol{d}(\boldsymbol{\xi}) - \boldsymbol{H}(\boldsymbol{\xi})\boldsymbol{x},\ \boldsymbol{y} \in \mathbf{Y} = span\{\boldsymbol{R}^{m_2}\}\}. \tag{4}$$

The most advantageous feature of the traditional distributionally robust optimization is that the modelers can utilize partial probabilistic information of uncertain parameters such as moments.

In some cases, we may not have moments information of all uncertain parameters. We only know the probability distribution of some uncertain parameters and these parameters will determine the bounds and shape of the uncertainty sets in the second stage. For example, the power generation of a solar PV farm is directly related to radiation level. On the basis of probabilistic weather forecast, we may obtain different solar power generation intervals and the corresponding probability. With this motivation, we generate an idea to switch the inner problem and outer problem of the second-stage problem. The inner problem

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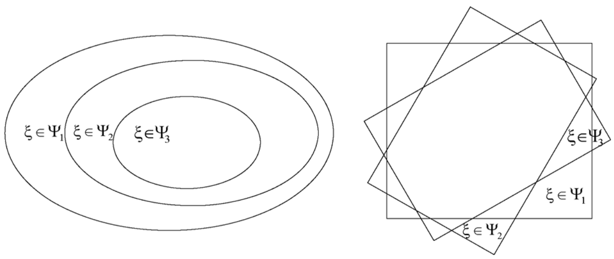

Fig. 1. Deterministic sets representing uncertainties.

then becomes a standard robust optimization under a given uncertainty set $\Psi(\boldsymbol{\sigma})$ as shown in the formulation (5)–(8). There are multiple uncertainty sets, Ψ_1, Ψ_2, Ψ_3, and so on that are associated with different probability (as shown in Fig. 1) so that we can obtain the expectation cost of different worst cases in the outer problem of the second stage by solving the model (5)–(8):

$$\min \boldsymbol{c}^T\boldsymbol{x} + E_{\mathrm{P}(\boldsymbol{\sigma})}[\max_{\boldsymbol{\xi}\in\Psi(\boldsymbol{\sigma})} \min_{\boldsymbol{y}\in\chi(\boldsymbol{\xi})} \boldsymbol{q}(\boldsymbol{\xi})^T\boldsymbol{y}] \tag{5}$$

$$\text{s.t.} \quad \boldsymbol{Fx} \leq \boldsymbol{g} \tag{6}$$

$$\boldsymbol{x} \in \mathbf{X} = span\{\boldsymbol{R}^{m_1} \times \{0,1\}^{n_1}\} \tag{7}$$

$$\chi(\boldsymbol{\xi}) = \{\boldsymbol{Gy} \leq \boldsymbol{d}(\boldsymbol{\xi}) - \boldsymbol{H}(\boldsymbol{\xi})\boldsymbol{x},\ \boldsymbol{y} \in \mathbf{Y} = span\{\boldsymbol{R}^{m_2}\}\}. \tag{8}$$

The solution of the proposed stochastic robust model uses a mathematical decomposition algorithm. The master problem and subproblem are solved iteratively. The worst cases are selected through solving feasibility and optimality subproblems, and then the new constraints and the worst dispatch cost are added into master problem. The subproblems are a max-min bilevel programming problem. After taking the dual of the inner problem, the subproblem becomes a bilinear programming problem. We transform the bilinear program into a mixed-integer linear program after introducing binary variables. The iterative process between the master problem and subproblems stops when the optimality gap is less than a given tolerance. The whole process is an adaption of the method described [2].

III. Case Studies

The proposed model can be used to deal with different power system optimization problems, such as unit commitment, transmission switching, capacity expansion, and so on. In this letter, we demonstrate the effectiveness of the method by unit commitment cases. With the presence of uncertainty, the unit commitment decision needs to be made before the uncertainty is revealed; however, the power generation can be determined after the uncertain parameters are observed, as a recourse. The load shift factor method described in [3] will be considered and used to reduce computational time. The program is coded in AMPL, which formulates the problem and sends it to the solver CPLEX 12.2. All case studies are solved on a computer with an Intel i7 core and 8 GB of memory.

We apply the model into a modified IEEE 118-bus system that includes 3 areas, 54 generators, and 186 transmission lines. The penetration level of wind power generation is 20%. The forecasted load for the three time periods are 3546.35 MW, 3595.67 MW, and 3733 MW. We assume the interval uncertainty set around the forecast values as follows: $(+/-)$ 3% of the total load for each time period, and the interval uncertainty sets around the forecast wind power generation are shown in Table I for the each one of three wind farms. The probabilities for different interval uncertainty sets are 20%, 30%, and 50%, respectively.

We run deterministic unit commitment, robust unit commitment, and the proposed new stochastic robust programming-based unit commitment. If there is only one uncertainty set,

TABLE I
Uncertainty Sets of Wind Power Generation for the Proposed Stochastic Robust Model

Uncertainty Set	Upper Bound and Lower Bound of Variation		
	Hour 1	Hour 2	Hour 3
1	+20%, -20%	+15%, -25%	+25%, -15%
2	+15%, -25%	+25%, -15%	+20%,-20%
3	+25%, -15%	+20%, -20%	+15%, -25%

TABLE II
Unit Commitment of Different Models

Cases	Number of committed units		
	Hour 1	Hour 2	Hour 3
Deterministic	16	17	17
Robust	26	27	27
Stochastic robust	26	27	27

the stochastic robust model becomes the robust model. In the case of the robust unit commitment, we assume $(+/-)$ 25% of the wind power generation for each one of the three wind farms during each time period as the interval uncertainty sets. We solve the model to the optimality. The number of committed units is shown in Table II. The worst cost of the robust unit commitment is \$146 392. The expected cost of the different worst cases of the new stochastic robust model is \$144 512. In computational experiment, the robust model has a higher optimal objective value because the proposed model lies between the robust and stochastic optimization and it is less conservative than the robust.

We also consider 24-h period stochastic robust scheduling problems with three uncertainty sets and study their computational performance. The wall clock time for running the model is 5582 s.

IV. Conclusion

We propose a stochastic robust programming model for power system optimization. Compared with traditional distributionally robust programming models, the new one has a stochastic minmax structure in which the inner problem of the second stage obtains the worst cases and the outer problem of the second stage optimizes the expected cost of different worst-case scenarios. The new stochastic robust optimization can also be regarded as the extension of stochastic programming and robust optimization; in fact, it is a bridge between the two methodologies to deal with the uncertainty. The case study shows the availability of the proposed model and the proposed algorithm's effectiveness in solving the model.

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