Practice Final, STAT 611

Name:

1. Let $X_1, \cdots, X_n$ be independent identically distributed random variables with pdf

$$f(x) = \frac{1}{\lambda} \exp\left[-\left(1 + \frac{1}{\lambda}\right) \log(x)\right]$$

   where $\lambda > 0$ and $x \geq 1$

   (a) Find the maximum likelihood estimator of λ.

   (b) What is the maximum likelihood estimator of λ^8? Explain your answer.

2. Let $X_1, \ldots, X_n$ be independent identically distributed from a $\mathrm{N}\left(\mu, \sigma^2\right)$ population, where σ^2 is known. Let $\overline{X}$ be the sample mean.

   (a) Calculate the MSE of the maximum likelihood estimator of μ. Does this estimator attain the Cramer-Rao lower bound?

   (b) Find the UMVUE of μ^2.

3. Let $X_1, \ldots, X_n$ be iid random variables from the Pareto(a, b) distribution. The PDF is $f_X(x) = ba^b x^{-b-1}$ for $x > a$ and $f_X(x) = 0$ for $x \leq a$. The parameters a, b satisfy $a > 0, b > 2$.

   (a) Find the moment estimator for a, b. Provide brief derivation.

   (b) Is your estimator strongly consistent? Briefly justify your answers.

   (c) Find the MLE for a, b. Provide brief derivation.

   (d) Is your estimator strongly consistent? Briefly justify your answer.

4. Let $X_1, \cdots, X_n$ be independent identically distributed random variables from a $N(\mu, \sigma^2)$ distribution where the variance σ^2 is known. We want to test $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$

   (a) Derive the likelihood ratio test

   (b) Let λ be the likelihood ratio. Show that $-2 \log \lambda$ is a function of $(\bar{X} - \mu_0)$

   (c) Derive the distribution for $-2 \log \lambda$.

5. Let $X \sim$ Binomial(n, p), where the positive integer n is large and $0 < p < 1$.

(a) Find the asymptotic distribution of X/n.

(b) Find the asymptotic distribution of $(X/n)^2$.

6. Let $X_1, \ldots, X_n$ be a random sample from a uniform $(0, \theta)$ distribution. Let $Y = \max(X_1, X_2, \ldots, X_n)$.

   (a) Find the pdf of Y/θ .

   (b) Find a pivotal quantity and use it to construct a $(1-\alpha)\%$ confidence interval for θ.

7. Let $X_1, \ldots, X_n$ be a random sample from a location-exponential family with density

$$f(x;\theta) = \exp^{-(x-\theta)}, \quad \text{if}, \quad x \geq \theta, -\infty < \theta < \infty$$

   and CDF,

$$F(x;\theta) = 1 - \exp^{-(x-\theta)}, \quad \text{if}, \quad x \geq \theta, -\infty < \theta < \infty$$

   (a) Write down the likelihood function of θ. (Pay attention to the range/support of θ)

   (b) Derive the likelihood ratio test and calculate the power function for the test

$$H_0 : \theta \leq \theta_0 \quad \text{versus} \quad H_1 : \theta > \theta_0$$

   (c) Construct a $100(1-\alpha)\%$ confidence interval of θ.

8. Review exercises on other topics which are not covered in the previous 7 questions.