Homework #16

Due by 7AM, Monday, April 13

Let $\{X_n, T_n\}$ be a Markov renewal process with state space {a, b} and semi-Markov kernel Q given as

$$Q(t) = \begin{array}{cc} 0.6(1 - e^{-5t}) & 0.4 - 0.4e^{-2t} \\ 0.5 - 0.2e^{-3t} - 0.3e^{-5t} & 0.5 - 0.5e^{-2t} - te^{-2t} \end{array}$$

where t represents *days*.

a. What is the average time, *in hours*, between visits to state a?
b. What is the average time, *in hours*, between visits to state b?
c. Find the $\lim_{n\to\infty} P_i\{X_n = a\}$.
d. Find the $\lim_{n\to\infty} P_i\{X_n = b\}$.
e. Find the $\lim_{t\to\infty} P_i\{Y(t) = a\}$.
f. Find the $\lim_{t\to\infty} P_i\{Y(t) = b\}$.

First step is to determine the ν vector by solving $0.6\nu(a) + 0.5\nu(b) = \nu(a)$ which yields $\nu=(1.0,0.8)$. We next need the mean sojourn times
$\mu(a) = \int_0^\infty 24 \times [0.6\, e^{-5t} + 0.4\, e^{-2t}]\, dt = 7.68$ and
$\mu(b) = \int_0^\infty 24 \times [0.2\, e^{-3t} + 0.3\, e^{-5t} + 0.5\, e^{-2t} + t\, e^{-2t}]\, dt = 15.04$; thus,
$\nu\,\mu = 1\times 7.68 + 0.8\times 15.04 = 19.712$.

a. Avg. time between visits to a = 19.712/1 = 19.71 hrs
b. Avg. time between visits to b = 19.712/0.8 = 24.64 hrs
c. $\pi(a) = \nu(a) / (\nu(a)+ \nu(b)) = 0.56$
d. $\pi(b) = \nu(b) / (\nu(a)+ \nu(b)) = 0.44$
e. $\lim_{t\to\infty} P_i\{Y(t) = a\} = \nu(a) \times \mu(a) / (\nu\,\mu) = 1\times 7.68 / 19.712 = 0.39$
f. $\lim_{t\to\infty} P_i\{Y(t) = b\} = \nu(b) \times \mu(b) / (\nu\,\mu) = 0.8\times 15.04 / 19.712 = 0.61$