STAT 611-600: Theory of Inference		**Spring 2021**
	Prelim 2	
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Instructions:

- There are 3 questions in this exam.
- You have 90 mins to finish the exam **AND** upload your answers to eCampus.
- By **6:30 PM CST, April 1, 2020**, you must finish writing and uploading your answers. No late submission will be allowed.
- Please make sure your exam paper has: your name, your UIN.
- This exam is open-book and open-notes but **NO googling or other online resources**. Everything must be your own work.
- Please mark your answers **clearly**.
- The usual punishment for students caught cheating is an F* in the class. Cheating includes, but is not limited to, communicating in any form with any other student about the questions or answers on this exam before the solutions are posted.
- Please affirm the Aggie Code of Honor with your signature on the first page of your answer sheets: "An Aggie does not lie, cheat or steal, or tolerate those who do." ______________________

Problem 1 (40 pts) UMVUE

1. (15 pts) Recall Q1 from Prelim 1. Let $X_1, \dots, X_n$ be iid Uniform$[-\theta, \theta]$, where $\theta > 0$ is an unknown parameter. We have seen that
$$\max_{1 \le i \le n} |X_i|$$
is complete for θ. Find the UMVUE of $\theta + \theta^{-1}$.

2. Recall Q2 from Prelim 1. Let $X_1, \cdots, X_n$ be independent random variables, and $X_i \sim$ Poisson$(i\lambda^2)$, $\lambda > 0$. Consider the parametrization that $\eta = \lambda^2$.
   (a) (5 pts) Calculate the mean squared error (MSE) of the MLE for η.
   (b) (10 pts) Calculate the Cramér-Rao lower bound (CRLB) for the variance of the unbiased estimators of η.
   (c) (10 pts) Does the MLE for η attain the CRLB? Explain your answer using the attainment theorem for the CRLB.

Problem 2 (20 pts) Bayes Risk

Let $X_1, \dots, X_n$ be iid random variables from an exponential distribution with density function
$$f(x;\theta) = \theta e^{-\theta x}, \qquad x \ge 0.$$
Use a Gamma(c, λ) prior
$$f(\theta) = \frac{\lambda^c}{\Gamma(c)} \theta^{c-1} e^{-\lambda\theta}$$
to find an estimator of θ such that it minimizes the Bayes risk based on:

1. (10 pts) $L(\theta, a) = (\theta - a)^2$.
2. (10 pts) $L(\theta, a) = \mathbf{1}_{\{|\theta - a| > \epsilon\}}$, for some small $\epsilon > 0$.

Problem 3 (40 pts) Hypothesis Tests

1. (15 pts) Let $X_1, \dots, X_n$ be a random sample from the $Unif[\theta, \theta+1]$ distribution. To test
$$H_0 : \theta = 0 \qquad \text{v.s.} \qquad H_1 : \theta > 0,$$
use the test
$$\text{reject } H_0 \text{ if } X_{(1)} \ge k \quad \text{or} \quad X_{(n)} \ge 1,$$
where k is a constant, and $X_{(1)}, X_{(n)}$ are minimum and maximum of $X_1, \dots, X_n$, respectively. Determine k so that the test will have size α.

2. (25 pts) Let $X_1, \dots, X_n$ be iid random variables from an exponential distribution with density function
$$f(x;\theta) = \theta e^{-\theta x}, \qquad x \ge 0.$$

Assume we only have a single observation (i.e., $n = 1$) and $X_1 = 0.1$. Find a LRT of the hypothesis

$$H_0 : \theta \leq 1 \qquad \text{v.s.} \qquad H_1 : \theta > 1.$$

Calculate the power function and report whether or not to reject H_0 at level $\alpha = 0.05$. You may want to use the approximation:

$$-\log(1 - x) \approx x, \qquad \text{for } x \text{ small.}$$