

Homework 8: due Thursday, April 15, 2021, 11:59 pm CDT

*Professor: Tiandong Wang**Name:**UIN:***Instructions:**

- Whether you write out the solution by hand or in a text document, be sure that they are neat, legible and in order (even if you choose to solve them in different order). We highly recommend that you write your solutions in **LaTeX** and print them to a **PDF file**.
- Write/Type your name, UIN at the top of the first page. Otherwise, your submission will not be graded.
- Either scan or print your solutions to a **PDF file** under 15MB in size. It must be in a single file, not separate files for separate pages. Do not take a photo of each page and then paste them into a document - this will make your file too big and the results will generally not be very readable anyway.
- All students should login to their eCampus account to upload your file. You must do this by **11:59 pm U.S. Central time**, on the due date. You can make multiple submissions, but only the last submission will be graded.
- Write down all of your problem-solving process and cite any resources you have used in addition to lecture notes and the textbook.
- It is prohibited to share or distribute the content in this document.

1. 8.51 in C&B.
2. 9.13 in C&B.
3. (Sequential Hypothesis Testing) Recall the discussion on sequential hypothesis testing in Q3, Homework 7. Suppose that $\{X_i : 1 \leq i \leq n\}$ are iid $N(\mu, 1)$ and we would like to test between

$$H_0 : \mu = 0 \quad \text{and} \quad H_1 : \mu = \mu_1 > 0.$$

- (a) Suppose $p_n(X_1, \dots, X_n) = 1 - \Phi(\sqrt{n}\bar{X})$, where $\Phi(\cdot)$ is the cdf for a standard normal r.v.. Consider the test that we reject H_0 if $p_n(X_1, \dots, X_n) \leq \alpha$. Compute the type I and type II error for this test.
- (b) Suppose we ran the following sequential test:
 - For each epoch $l \in \{0, \dots, L-1\}$:
 - Collect 10 new data points $\{X_{10l+1}, \dots, X_{10l+10}\}$, sampled iid from $N(\mu, 1)$.
 - If for some l , $p_{10}(X_{10l+1}, \dots, X_{10l+10}) \leq \alpha$, then stop and output “reject”.
 - If finish all L epochs without rejecting, then “accept”.

Compute the type I and type II error of this test. What is the expected total number of samples collected?

Optional exercises from C&B: 8.49, 9.7, 9.17.