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Distributionally robust fuzzy project portfolio optimization problem with interactive returns

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ABSTRACT

Effective project selection and staff assignment strategies directly impact organizational profitability. Based on critical value optimization criterion, this paper discusses how uncertainty and interaction impact the project portfolio return and staff allocation. Since the exact possibility distributions of uncertain parameters in practical project portfolio problems are often unavailable, we adopt variable parametric possibility distributions to characterize uncertain model parameters. Furthermore, this paper develops a novel parametric credibilistic optimization method for project portfolio selection problem. According to the structural characteristics of variable parametric possibility distributions, we derive the equivalent analytical expressions of credibility constraints, and turn the original credibilistic project portfolio model into its equivalent nonlinear mixed-integer programming models. To show the advantages of the proposed parametric credibilistic optimization method, some numerical experiments are conducted by setting various values of distribution parameters. The computational results support our arguments by comparing with the optimization method under fixed possibility distributions.

1. Introduction

In the highly competitive economic circumstance, project portfolio selection is a crucial part of managerial strategy for organizations. In general, the project portfolio selection problem involves two key modeling aspects: selecting a suitable subset of projects as a portfolio and assigning the available human resource efficiently. A decision maker should incorporate these two aspects to produce a more robust and realistic formulation for project portfolio selection.

The concept of portfolio origins from financial investment theory. Following the basic thought, some researchers introduce the theoretical foundations into project management field. Although many similar research aspects exist in the financial and project portfolio theory, several authors have identified the differences between them. For instance, Casault et al. [6] mentioned that the project portfolios were different from financial portfolios because projects have no market price despite financial assets; Gutjahr and Froeschl [17] pointed out that investment in projects to be carried out by the own personnel of a firm also required a careful consideration of the available human resources.

The interaction among projects is a critical issue to project portfolio management. The previous theory on project portfolio decision typically assumed that each project was independent. In reality, due to meeting the same consumer need or requiring the same development resources, the return from a project depends not only on its properties but also on the other projects included in the same portfolio. Numerous previous studies about this issue have existed in the literature [9,20,32,33]. However, it has been noted that many of these works merely focused on the enhancement effect of synergy [8,21], and its counterpart cannibalization is an important aspect that has been ignored. The phenomenon of cannibalization often refers to a reduction in portfolio return as a result of the introduction of a new similar product by the similar producer. To address the unilateral influences of interaction, our aim in the present paper is to develop a project portfolio selection model considering both possible positive and negative impact on project portfolio management.

Another critical characteristic of the project portfolio problem is the high degree of uncertainties involved in the decision making process, such as uncertainties in the project return, human resource, and interaction among the projects. To cope with these uncertainties,

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probabilistic and fuzzy approaches have been proposed to capture the imprecision of model parameters by considering reasonable distributions. Ballestin and Leus [1] investigated resource-constrained project scheduling with stochastic activity durations, and developed a GRASP-heuristic to produce high-quality solutions. Fernande et al. [12] proposed a non-outranked ant colony optimization II method, which incorporated a fuzzy outranking preference model for optimizing project portfolio problem. Carlsson et al. [5] developed a methodology for valuing options on R&D projects, where future cash flows were estimated by trapezoidal fuzzy numbers.

In the literature, some recent researches addressed both uncertainty and interaction of project portfolio selection. For instance, Girotra et al. [15] empirically investigated the structure and significance of these portfolio-level project interactions, and explained the variance in the value of projects based on interactions with other projects in the firm's portfolio. Ghapanchi et al. [14] used data envelopment analysis to select the best portfolio of IS/IT projects while taking both project uncertainties and project interactions into consideration simultaneously. Gemici et al. [13] presented a multistage stochastic program to maximize expected operating income subject to risk, product interdependency, capacity, and resource allocation constraints.

However, as Hall et al. [18] pointed out, the full distributional information about the uncertain return in project portfolio problem is often unavailable. Specifically, when the imprecise parameter is affected by the noise of historic data or the ambiguity of expert's opinion, these approaches depending on the exact distribution will be invalid. Robust optimization [4] is a modeling methodology to process optimization problem, in which the uncertainty can be represented as deterministic variability in the value of the parameter. As a more recent approach to optimization, some authors have used robust optimization to deal with stochastic project portfolio problem. For example, Goh and Hall [16] considered projects with uncertain activity times that came from a partially specified distribution within a family of distributions; Hassanzadeh et al. [19] developed a multiobjective binary integer programming model for R&D project portfolio selection, where each imprecise coefficient belonged to an interval of real numbers without prior distribution details; Chen et al. [7] refined a framework for robust linear optimization by introducing a new uncertainty set that captured the asymmetry of the underlying random variables, and demonstrated the framework through an application of a project management problem.

In fuzzy decision systems, credibilistic optimization methods have been studied by many researchers. The interested reader may refer to the recent works [10,11,22–24,31,34], where the possibility distributions of uncertain model parameters are assumed to be known exactly. On the other hand, when the exact possibility distributions of uncertain model parameters are unavailable, some researchers studied credibilistic optimization methods based on distributionally robust model parameters [2,3,25,35], where the distributional robustness refers to the secondary possibility distributions of model parameters are uncertain instead of crisp values in the unit interval [0, 1]. In this study, we employ the interval-valued possibility distribution to describe uncertainty embedded in the secondary possibility distribution, and model the uncertain parameters in project portfolio problems as parametric interval-valued fuzzy variables [29]. More precisely, we take selection variable as the representative of a parametric interval-valued fuzzy variable. The possibility distribution function of a selection variable is variable and depends on spread and location distribution parameters, and it can run over the entire support of the secondary possibility distribution as the location distribution parameters vary their values. The proposed optimization model based on variable possibility distributions leads to robust parametric optimization method for our project portfolio selection problem.

This paper aims to discuss the project portfolio selection problem by a novel robust credibilistic optimization method and wants to gain more insights into project portfolio regarding project interaction. The main contributions of this paper can be summarized as follows.

- We employ interval-valued possibility distributions to characterize uncertain parameters in project portfolio selection problem. When the exact possibility distributions of the uncertain model parameters are difficult to be determined by historic data or the experiences of experts in advance, using our distributionally robust optimization method to model practical project portfolio selection problems can provide a decision maker a set of optimal solutions under various values of distribution parameters, which may facilitate the decision maker to make his informed decision for the project portfolio selection problem.
- Based on optimistic value optimization criterion, a new parametric credibilistic optimization model is built for project portfolio selection problem. The proposed optimization model shows that the portfolio objective of an organization is to maximize the optimistic value of the total portfolio returns under a prescribed credibility level instead of maximizing the expected total project portfolio return.
- In our project portfolio selection problem, we identify two interactions—synergy and cannibalization, and discuss the computational issue considering the additional returns among interactive projects.
- To facilitate the solution of the proposed credibilistic project portfolio selection model, we analyze the properties of optimistic value objective as well as credibility constraints, and turn the original project portfolio selection model into its equivalent mixed-integer programming models. As a result, conventional optimization softwares can be employed to solve our equivalent mixed-integer programming models, hence the original project portfolio selection problem.

The remainder of this paper is organized as follows. Section 2 reviews some basic concepts in fuzzy possibility theory. Section 3 illustrates the problem setting with the consideration of project interaction, and constructs a credibilistic project portfolio selection model based on optimistic value criterion. Section 4 derives the analytical expressions of optimistic value objective and credibility constraints. Section 5 presents the equivalent parametric mixed-integer programming of credibilistic project portfolio selection model. Section 6 conducts some numerical experiments to illustrate our new modeling idea and the efficiency of the proposed parametric programming approach. Section 7 gives our conclusions in this paper.

2. Preliminaries

In this section, we briefly recall some basic concepts in fuzzy possibility theory [29,30], which will be used to model our project portfolio selection problem.

Let $(\Gamma, \mathcal{P}(\Gamma), \tilde{\mathrm{P}}os)$ be a fuzzy possibility space [30]. A map $\tilde{\xi} = (\tilde{\xi}_1, \tilde{\xi}_2, \ldots, \tilde{\xi}_n) : \Gamma \mapsto \Re^n$ is called an n-ary type-2 (T2) fuzzy vector. When $n = 1$, the map $\tilde{\xi}$ is called a T2 fuzzy variable.

The secondary possibility distribution function $\tilde{\mu}_{\tilde{\xi}}(x)$ of ξ is defined as

$$\tilde{\mu}_{\tilde{\xi}}(x) = \tilde{P}os\{\gamma \in \Gamma \mid \tilde{\xi}(\gamma) = x\}, \quad x \in \Re^n,$$

and the type-2 possibility distribution function $\mu_{\tilde{\xi}}(x, u)$ of ξ is defined as

$$\mu_{\tilde{\xi}}(x, u) = \mathrm{Pos}\{\tilde{\mu}_{\tilde{\xi}}(x) = u\}, \quad (x, u) \in \Re^n \times J_x,$$

where $J_x \subset [0, 1]$ is the support of $\tilde{\mu}_{\tilde{\xi}}(x)$.

Particularly, if for any $x \in \Re$, $u \in J_x \subseteq [0, 1]$, the T2 possibility distribution function $\mu_{\tilde{\xi}}(x, u) = 1$, then $\tilde{\xi}$ is an interval T2 fuzzy variable [29].

Furthermore, if the secondary possibility distribution $\tilde{\mu}_{\tilde{\xi}}(x)$ is a subinterval $[\mu_{\xi^L}(x;\theta_l), \mu_{\xi^U}(x;\theta_r)]$ of [0, 1] with parameters $\theta_l, \theta_r \in [0, 1]$, then $\tilde{\xi}$ is called a parametric interval-valued fuzzy variable [29].

If $\theta_l = \theta_r = 0$, then the reduced secondary possibility distribution is called the principle possibility distribution of $\tilde{\xi}$, and the fuzzy variable associated with the principle possibility distribution is denoted by ξ^p.

Let $r_1 < r_2 \leq r_3 < r_4$ be real numbers. Then a map $\tilde{\xi}$ is called a parametric interval-valued trapezoidal fuzzy variable if its secondary possibility distribution $\tilde{\mu}_{\tilde{\xi}}(r)$ is the following subinterval

$$\left[\frac{r-r_1}{r_2-r_1} - \theta_l \min\left\{\frac{r-r_1}{r_2-r_1}, \frac{r_2-r}{r_2-r_1}\right\}, \frac{r-r_1}{r_2-r_1} + \theta_r \min\left\{\frac{r-r_1}{r_2-r_1}, \frac{r_2-r}{r_2-r_1}\right\}\right]$$

of [0, 1] for $r \in [r_1, r_2]$, the interval [1, 1] for $r \in [r_2, r_3]$, and the following subinterval

$$\left[\frac{r_4-r}{r_4-r_3} - \theta_l \min\left\{\frac{r_4-r}{r_4-r_3}, \frac{r-r_3}{r_4-r_3}\right\}, \frac{r_4-r}{r_4-r_3} + \theta_r \min\left\{\frac{r_4-r}{r_4-r_3}, \frac{r-r_3}{r_4-r_3}\right\}\right]$$

of [0, 1] for $r \in [r_3, r_4]$, where $\theta_l, \theta_r \in [0, 1]$ are two parameters characterizing the degree of uncertainty that $\tilde{\xi}$ takes on the value r. We denote the parametric interval-valued trapezoidal fuzzy variable $\tilde{\xi}$ with the above distribution by $[r_1, r_2, r_3, r_4;\theta_l, \theta_r]$. Particularly, if $r_2 = r_3$, then $\tilde{\xi}$ is called a parametric interval-valued triangular fuzzy variable.

In [29], a fuzzy variable ξ is defined as a λ selection variable of parametric interval-valued fuzzy variable $\tilde{\xi}$, if ξ has the following parametric possibility distribution function

$$\mu_{\xi}(r;\theta,\lambda) = (1-\lambda)\mu_{\xi^L}(r;\theta_l) + \lambda\mu_{\xi^U}(r;\theta_r), \quad \lambda \in [0, 1],$$

where $\theta = (\theta_l, \theta_r)$. By definition, there are two types of distribution parameters θ and λ embedded in the possibility distribution $\mu_{\xi}(r;\theta, \lambda)$, where θ is the spread parameter to characterize the uncertainty degree in the secondary possibility distribution of $\tilde{\xi}$, while λ is the location parameter to characterize the location of $\mu_{\xi}(r;\theta, \lambda)$ in the support of $\tilde{\xi}$.

By the definition of credibility measure [27], the credibility of event $\{\xi \leq t\}$ is computed by

$$\mathrm{Cr}\{\xi \leq t\} = \frac{1}{2}\left(1 + \sup_{r \leq t}\mu_{\xi}(r;\theta,\lambda) - \sup_{r>t}\mu_{\xi}(r;\theta,\lambda)\right),$$

where $\mu_{\xi}(r;\theta, \lambda)$ is the parametric possibility distribution of the λ selection variable ξ.

3. Project portfolio selection problem

The aim of our project portfolio selection problem is to select a subset as a portfolio from a large set of possible candidate projects considering the limitation of human resource and interaction between any two candidate projects.

3.1. Problem statement and notations

Suppose a set of candidate projects indexed by $i = 1, 2, \ldots, n$ to be selected, an organization wants to select the most suitable candidates to formulate a project portfolio. The decision on which candidate project is to be chosen is represented by a binary variable x_i. If project i is included in the portfolio, then the binary variable x_i equals 1, otherwise, x_i equals 0. Every selected project in the portfolio can provide a return, and denoted by $\tilde{\xi}$. Sine the gain resulting from project in the portfolio is subject to considerable uncertainty, the uncertain return $\tilde{\xi}$ is assumed to be an interval-valued fuzzy variable with variable parametric possibility distribution.

In addition, this paper considers the interactions among projects. More specifically, we discuss synergy and cannibalization interactions between different projects. Whenever both project i and project j are included in a portfolio, an additional return can be created by the two kinds of interactions. If the joint gain exceeds the sum of $\tilde{\xi}_i$ and $\tilde{\xi}_j$, then the positive interaction is called synergy. For instance, let us assume that we have two projects A and B that would return us \$20,000 and \$18,000, respectively. An synergy interaction of \$2000 between project A and B means that by including both projects in the same portfolio we could obtain an additional return \$2000 and gain \$40,000 rather than \$38,000. On the contrary, if the joint gain falls below the sum of $\tilde{\xi}_i$ and $\tilde{\xi}_j$, then the negative interaction is called cannibalization. We study this phenomenon by adding a corresponding term $\tilde{\xi}_{ij}$ that is the additional return from the joint return minus the sum of $\tilde{\xi}_i$ and $\tilde{\xi}_j$. Therefore, the term $\tilde{\xi}_{ij}$ $(1 \leq i < j \leq n)$ could be positive, negative or zero.

The execution of project needs sufficient and competent employees. The staff in an organization consists of the employees indexed by $k = 1, 2, \ldots, m$. The number of employees is assumed to be fixed during the entire project process, and the outsourcing opportunity is taken into account. Every employee is supposed to possess some capabilities, which could be skills, knowledge, ability, etc. in different fields. To measure the capabilities in a straightforward way, as in [17], we use the term *competence* and index competencies by $s = 1, 2, \ldots,$

l. The extent to which an employee k possesses the competence s is measured by a real number, named as *competence value* and denoted by $\rho_{ks} \in [0, 1]$. If $\rho_{ks} = 1$, the employee is fully qualified, while $0 < \rho_{ks} < 1$ means the employee is partially qualified. We denote the effort of project i that requires competence s by $\tilde{\eta}_{is}$, which is often measured by work time and incorporated with high uncertainty. So, in this paper, $\tilde{\eta}_{is}$ is also characterized by variable possibility distribution, and expressed as a parametric interval-valued fuzzy variable. Moreover, we introduce a decision variable y_{iks} to present the planned project work assigned to employee. The variable y_{iks} denotes the quantity of work time from project i assigned to employee k need to execute with competence s. Furthermore, we suppose π_k is the total capacity of the kth employee's work time during the entire project process.

For the sake of convenience, we use the following notations in our project portfolio selection problem.

Indices:

I : Set of candidate project, indexed by i or j;
K : Set of human resource, indexed by k;
S : Set of desired competence in the execution of project, indexed by s.

Constant parameters:

ρ_{ks} : The competence value of employee k in competence s;
π_k : The total work capacity of employee k.

Fuzzy parameters:

$\tilde{\xi}_i$: Interval-valued fuzzy return of project i;
$\tilde{\xi}_{ij}$: Interval-valued fuzzy additional return from the interaction of project i and project j;
$\tilde{\eta}_{is}$: Interval-valued fuzzy work time of project i in competence s.

Decision variables:

x_i : Binary variable indicating whether project i is selected or not;
x_j : Binary variable indicating whether project j is selected or not;
y_{iks} : The amount of work time if project i is assigned to employee k and executed with competence s.

3.2. An optimistic value-based project portfolio selection model

In this subsection, we will analyze the portfolio return and limitation of human resource in the entire project process, and build a novel credibilistic project portfolio selection model based on optimistic value optimization criterion.

While there are various aspects that can be taken as measures to evaluate a project portfolio, this paper is only concerned with portfolio return. The portfolio objective of the organization is to maximize the total portfolio returns generated in the project portfolio process. The portfolio returns are obtained not only from the returns of every selected project, but also from the interactions among projects. Therefore, the total portfolio returns are the cumulative sum of returns from individual projects and additional returns from interacting projects.

If $\tilde{\xi}_i$ is the parametric interval-valued fuzzy return from selected project i, then $\sum_i x_i\tilde{\xi}_i$ is the total return from individual projects. Meanwhile, if $\tilde{\xi}_{ij}$ is the parametric interval-valued fuzzy additional return from interactive projects, then $\sum_{i<j} x_i x_j \tilde{\xi}_{ij}$ is the sum of interactive returns between two projects. If we denote $x = \{x_i, x_j\}$ and $\xi = \{\tilde{\xi}_i, \tilde{\xi}_{ij}\}$, then the total portfolio return is represented as:

$$f(x, \xi) = \sum_i x_i\tilde{\xi}_i + \sum_{i<j} x_i x_j \tilde{\xi}_{ij}.$$

The portfolio return function $f(x, \xi)$ is a parametric interval-valued fuzzy variable, and its λ selection variable is denoted by

$$f(x, \xi) = \sum_i x_i\xi_i + \sum_{i<j} x_i x_j \xi_{ij}.$$

Assume that the decision maker wants to maximize the optimistic value of the project portfolio return with predetermined confidence levels. By introducing a variable $\bar{f}$, for any prescribed $0 < \alpha \leq 1$, the optimistic value of the total portfolio returns is denoted as $R_\alpha = \sup\{\bar{f} \mid \mathrm{Cr}\{f(x, \xi) \geq \bar{f}\} \geq \alpha\}$, where α reflects decision maker's attitude to risk during the project portfolio decision process.

As a result, the portfolio objective can be expressed as the following programming model:

$$\begin{array}{ll} \max & \bar{f} \\ \text{s.t.} & \mathrm{Cr}\{f(x, \xi) \geq \bar{f}\} \geq \alpha, \end{array} \tag{1}$$

where $\max \bar{f}$ means the maximal project portfolio return the decision maker can obtain at a credibility level α.

Since the cost of external human resource is usually higher than that of internal ones, it is reasonable to limit the amount of work time of selected projects $x_i\tilde{\eta}_{is}$ not more than the real work that employees can provide $\sum_k \rho_{sk} y_{isk}$. That is, for any $k = 1, 2, \ldots, m$, $s = 1, 2, \ldots, l$, one has $\sum_k \rho_{sk} y_{isk} \geq x_i\tilde{\eta}_{is}$. Letting η_{is} be the λ selection variable of $\tilde{\eta}_{is}$, then this expression can be rewritten as

$$\sum_k \rho_{sk} y_{iks} \geq x_i \eta_{is}.$$

If the decision maker wants to ensure that the credibility of this constraint holding is great than a prescribed level β_{is}, then for any i, s, we have the following credibility constraint:

$$\mathrm{Cr}\{\sum_k \rho_{sk} y_{iks} \geq x_i \eta_{is}\} \geq \beta_{is}.$$

In addition, we need a constraint of employee's work capacity, which means the work time of employee k is limited by his work capacity π_k, i.e., for any $k = 1, 2, \ldots, m$, one has

$$\sum_i \sum_s y_{iks} \le \pi_k.$$

And for each i, k and s, the variable $y_{iks} \ge 0$, and $x_i, x_j \in \{0, 1\}$.

Based on optimistic value optimization criterion [28], our parametric credibilistic programming model for project portfolio selection problem is formally built as follows:

$$\begin{aligned} \max \quad & \bar{f} \\ \text{s.t.} \quad & \mathrm{Cr}\{(\sum_i x_i\xi_i + \sum_{i<j} x_i x_j \xi_{ij}) \ge \bar{f}\} \ge \alpha \end{aligned} \tag{2}$$

$$\mathrm{Cr}\{\sum_k \rho_{sk} y_{iks} \ge x_i \eta_{is}\} \ge \beta_{is}, \quad \forall i, s \tag{3}$$

$$\sum_i \sum_s y_{iks} \le \pi_k, \quad \forall k \tag{4}$$

$$x_i \in \{0, 1\}, \quad \forall i \tag{5}$$

$$x_j \in \{0, 1\}, \quad \forall j \tag{6}$$

$$y_{iks} \ge 0, \quad \forall i, k, s. \tag{7}$$

In the above model, α and β_{is} are predetermined credibility levels. The objective function is to maximize the total portfolio return. Constraint (2) means that the total portfolio returns will be more than $\bar{f}$ with the confidence level α. Constraint (3) implies the selected projects can be done with the internal resource with the confidence level β_{is}. Constraint (4) ensures the work time of employee will not exceed the given work capacity. Note that the project return and work time are imprecise due to the complexity of the actual situation. Hence, the variables ξ_i, ξ_{ij} and η_{is} are the λ selection variables of the corresponding parametric interval-valued fuzzy variables.

4. Analysis of credibilistic optimization model

To solve our parametric credibilistic project portfolio selection model (2)–(7), it is necessary to analyze the properties of optimistic value function as well as credibility constraints. In this section, we will discuss how credibility constraints can be turned into their equivalent deterministic constraints.

The computational complexity of the constraints stems in the involved parametric interval-valued fuzzy variables, which make the constraints difficult to be dealt with. There are many types of parametric interval-valued fuzzy variables can be used in practical optimization problem. In this paper, we assume the uncertain return is represented as parametric interval-valued triangular fuzzy variable, and uncertain work time is represented as parametric interval-valued trapezoidal fuzzy variable.

In the following, we first discuss the analytical expression of credibility constraint (2) about the total project portfolio return.

Theorem 1. *Let uncertain return from project i and uncertain interaction return from project (i, j) be parametric interval-valued triangular fuzzy variable* $\tilde{\xi}_i = [r_1^i, r_2^i, r_3^i; \theta_l^i, \theta_r^i]$, $\tilde{\xi}_{ij} = [r_1^{ij}, r_2^{ij}, r_3^{ij}; \theta_l^{ij}, \theta_r^{ij}]$, *respectively, and* $x_i, x_j (i < j)$ *real numbers for* $i, j = 1, 2, \ldots, n$. *If the principle possibility distributions of* $\tilde{\xi}_i$ *and* $\tilde{\xi}_{ij}$ *are mutually independent, then credibility constraint (2) has the following equivalent representations.*

(i) If $\alpha \in (0, \frac{\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi + 1}{4}]$, *then constraint (2) is equivalent to the following deterministic constraint*

$$\frac{[1 + \lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi - 2\alpha](\sum_i x_i r_3^i + \sum_{i<j} x_i x_j r_3^{ij}) + 2\alpha(\sum_i x_i r_2^i + \sum_{i<j} x_i x_j r_2^{ij})}{\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi + 1} \ge \bar{f}.$$

(ii) If $\alpha \in (\frac{\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi + 1}{4}, \frac{1}{2}]$, *then constraint (2) is equivalent to the following deterministic constraint*

$$\frac{(2\alpha - 1)(\sum_i x_i r_3^i + \sum_{i<j} x_i x_j r_3^{ij}) + [\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi - 2\alpha](\sum_i x_i r_2^i + \sum_{i<j} x_i x_j r_2^{ij})}{\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi - 1} \ge \bar{f}.$$

(iii) If $\alpha \in (\frac{1}{2}, \frac{3 - \lambda\theta_r^\xi + (1-\lambda)\theta_l^\xi}{4}]$, *then constraint (2) is equivalent to the following deterministic constraint*

$$\frac{[2 - 2\alpha - \lambda\theta_r^\xi + (1-\lambda)\theta_l^\xi](\sum_i x_i r_2^i + \sum_{i<j} x_i x_j r_2^{ij}) + (2\alpha - 1)(\sum_i x_i r_1^i + \sum_{i<j} x_i x_j r_1^{ij})}{1 - \lambda\theta_r^\xi + (1-\lambda)\theta_l^\xi} \ge \bar{f}.$$

(iv) If $\alpha \in (\frac{3 - \lambda\theta_r^\xi + (1-\lambda)\theta_l^\xi}{4}, 1]$, *then constraint (2) is equivalent to the following deterministic constraint*

$$\frac{(2 - 2\alpha)(\sum_i x_i r_2^i + \sum_{i<j} x_i x_j r_2^{ij}) + [\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi + 2\alpha - 1](\sum_i x_i r_1^i + \sum_{i<j} x_i x_j r_1^{ij})}{1 + \lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi} \ge \bar{f}.$$

In the above expressions, the parameters $\theta_l^\xi = \max_{i,j}\{\theta_l^i, \theta_l^{ij}\}$ *and* $\theta_r^\xi = \min_{i,j}\{\theta_r^i, \theta_r^{ij}\}$.

Proof. We only prove assertions (i) and (ii), the rest can be proved similarly.

Let $\tilde{\xi}_i = [r_1^i, r_2^i, r_3^i; \theta_l^i, \theta_r^i]$ and $\tilde{\xi}_{ij} = [r_1^{ij}, r_2^{ij}, r_3^{ij}; \theta_l^{ij}, \theta_r^{ij}]$ be parametric interval-valued triangular fuzzy variables. Since the principle possibility distributions of $\tilde{\xi}_i$ and $\tilde{\xi}_{ij}$ are mutually independent in the sense of [26], their linear combination $f(x, \tilde{\xi}) = \sum_i x_i\tilde{\xi}_i + \sum_{i<j} x_i x_j \tilde{\xi}_{ij}$ is a parametric interval-valued triangular fuzzy variable [29], and denoted by $[r_1, r_2, r_3; \theta_l, \theta_r]$, where

$$\begin{aligned} r_1 &= \sum_i x_i r_1^i + \sum_{i<j} x_i x_j r_1^{ij}, \\ r_2 &= \sum_i x_i r_2^i + \sum_{i<j} x_i x_j r_2^{ij}, \\ r_3 &= \sum_i x_i r_3^i + \sum_{i<j} x_i x_j r_3^{ij} \end{aligned} \tag{8}$$

with parameters $\theta_l^\xi = \max_{i,j}\{\theta_l^i, \theta_l^{ij}\}$ and $\theta_r^\xi = \min_{i,j}\{\theta_r^i, \theta_r^{ij}\}$.

If $f(x, \xi)$ is the λ selection variable of $f(x, \tilde{\xi})$, then its parametric interval-valued possibility distribution $\mu_{f(x,\xi)}(r;\theta, \lambda)$ is

$$\mu_{f(x,\xi)}(r;\theta,\lambda) = \begin{cases} \dfrac{[1+\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi](r-r_1)}{r_2-r_1}, & r_1 < r \le \dfrac{r_1+r_2}{2} \\ \dfrac{[1-\lambda\theta_r^\xi + (1-\lambda)\theta_l^\xi]r + [\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi]r_2 - r_1}{r_2-r_1}, & \dfrac{r_1+r_2}{2} < r \le r_2 \\ \dfrac{[\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi - 1]r - [\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi]r_2 + r_3}{r_3-r_2}, & r_2 < r \le \dfrac{r_2+r_3}{2} \\ \dfrac{[1+\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi](r_3-r)}{r_3-r_2}, & \dfrac{r_2+r_3}{2} < r \le r_3 \end{cases}$$

where $\theta = (\theta_l^\xi, \theta_r^\xi)$.

For simplicity, denote $\boldsymbol{\xi} = f(x, \xi) = \sum_i x_i\xi_i + \sum_{i<j} x_i x_j \xi_{ij}$. If $\alpha \le 0.5$, then we have

$$\begin{aligned} &\mathrm{Cr}\{(\sum_i x_i\xi_i + \sum_{i<j} x_i x_j \xi_{ij}) \ge \bar{f}\} = \mathrm{Cr}\{\boldsymbol{\xi} \ge \bar{f}\} \\ &= \frac{1}{2}\{1 + \sup_{r\ge\bar{f}} \mu_{\boldsymbol{\xi}}(r;\theta,\lambda) - \sup_{r<\bar{f}} \mu_{\boldsymbol{\xi}}(r;\theta,\lambda)\} \\ &= \frac{1}{2}\sup_{r\ge\bar{f}} \mu_{\boldsymbol{\xi}}(r;\theta,\lambda). \end{aligned}$$

Thus, credibility constraint $\mathrm{Cr}\{\boldsymbol{\xi} \ge \bar{f}\} \ge \alpha$ is equivalent to $\sup_{r\ge\bar{f}} \mu_{\boldsymbol{\xi}}(r;\theta,\lambda) \ge 2\alpha$.

By the definition of optimistic value [28], if we denote

$$\boldsymbol{\xi}_{\sup}(\alpha) = \sup\{\bar{f} \mid \sup_{r\ge\bar{f}} \mu_{\boldsymbol{\xi}}(r;\theta,\lambda) \ge \alpha\}$$

for $\alpha \in (0, 1]$, then credibility constraint (2) has the following equivalent representation

$$\boldsymbol{\xi}_{\sup}(2\alpha) \ge \bar{f}. \tag{9}$$

Note that $\mu_{\boldsymbol{\xi}}((r_2+r_3)/2) = (1+\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi)/2$. If $0 < 2\alpha \le (1+\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi)/2$, i.e., $\alpha \in (0, (1+\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi)/4]$, then $\boldsymbol{\xi}_{\sup}(2\alpha)$ is the solution of following equation

$$\frac{[1+\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi](r_3-r)}{r_3-r_2} - 2\alpha = 0.$$

Solving the above equation, we have

$$\boldsymbol{\xi}_{\sup}(2\alpha) = \frac{[1+\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi]r_3 - 2\alpha(r_3-r_2)}{\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi + 1}.$$

On the other hand, $\mu_{\boldsymbol{\xi}}(r_2) = 1$. If $(1+\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi)/4 < 2\alpha \le 1$, i.e., $\alpha \in ((1+\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi)/8, 1/2]$, then $\boldsymbol{\xi}_{\sup}(2\alpha)$ is the solution of following equation

$$\frac{[\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi - 1]r - [\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi]r_2 + r_3}{r_3-r_2} - 2\alpha = 0.$$

Solving the above equation, we have

$$\boldsymbol{\xi}_{\sup}(2\alpha)=\frac{[\lambda\theta_r^{\xi}-(1-\lambda)\theta_l^{\xi}]r_2+2\alpha(r_3-r_2)-r_3}{\lambda\theta_r^{\xi}-(1-\lambda)\theta_l^{\xi}-1}.$$

Combining Eqs. (8) and (9), if $\alpha\in(0,\frac{\lambda\theta_r^{\xi}-(1-\lambda)\theta_l^{\xi}+1}{4}]$, then credibility constraint (2) is equivalent to

$$\frac{[1+\lambda\theta_r^{\xi}-(1-\lambda)\theta_l^{\xi}-2\alpha](\sum_i x_i r_3^i+\sum_{i<j}x_ix_jr_3^{ij})+2\alpha(\sum_i x_ir_2^i+\sum_{i<j}x_ix_jr_2^{ij})}{\lambda\theta_r^{\xi}-(1-\lambda)\theta_l^{\xi}+1}\ge\bar{f}.$$

If $\alpha\in(\frac{\lambda\theta_r^{\xi}-(1-\lambda)\theta_l^{\xi}+1}{4},\frac{1}{2}]$, then credibility constraint (2) is equivalent to

$$\frac{(2\alpha-1)(\sum_i x_ir_3^i+\sum_{i<j}x_ix_jr_3^{ij})+[\lambda\theta_r^{\xi}-(1-\lambda)\theta_l^{\xi}-2\alpha](\sum_i x_ir_2^i+\sum_{i<j}x_ix_jr_2^{ij})}{\lambda\theta_r^{\xi}-(1-\lambda)\theta_l^{\xi}-1}\ge\bar{f}.$$

The proof of theorem is complete. □

We next consider the limitation of internal human resource and derive the analytical expression of credibility constraint (3) about the internal resource.

Theorem 2. *Let uncertain work time of project i with competence s be parametric interval-valued trapezoidal fuzzy variable $\tilde{\eta}_{is}=[r_1^{is},\ r_2^{is},\ r_3^{is},\ r_4^{is};\theta_l^{is},\ \theta_r^{is}]$, and x_i real numbers for $i=1,2,\ldots,n$. If the principle possibility distributions of $\tilde{\eta}_{is}$ are mutually independent, then credibility constraint (3) has the following equivalent representations.*

(i) If $\beta_{is}\in(0,\frac{\lambda\theta_r^{\eta}-(1-\lambda)\theta_l^{\eta}+1}{4}]$, then constraint (3) is equivalent to the following deterministic constraint

$$\frac{2\beta_{is}x_ir_2^{is}+[\lambda\theta_r^{\eta}-(1-\lambda)\theta_l^{\eta}+1-2\beta_{is}]x_ir_1^{is}}{1+\lambda\theta_r^{\eta}-(1-\lambda)\theta_l^{\eta}}\le\sum_k\rho_{sk}y_{iks}.$$

(ii) If $\beta_{is}\in(\frac{\lambda\theta_r^{\eta}-(1-\lambda)\theta_l^{\eta}+1}{4},\frac{1}{2}]$, then constraint (3) is equivalent to the following deterministic constraint

$$\frac{[2\beta_{is}-\lambda\theta_r^{\eta}+(1-\lambda)\theta_l^{\eta}]x_ir_2^{is}+(1-2\beta_{is})x_ir_1^{is}}{1-\lambda\theta_r^{\eta}+(1-\lambda)\theta_l^{\eta}}\le\sum_k\rho_{sk}y_{iks}.$$

(iii) If $\beta_{is}\in(\frac{1}{2},\frac{3-\lambda\theta_r^{\eta}+(1-\lambda)\theta_l^{\eta}}{4}]$, then constraint (3) is equivalent to the following deterministic constraint

$$\frac{(1-2\beta_{is})x_ir_4^{is}+[\lambda\theta_r^{\eta}-(1-\lambda)\theta_l^{\eta}-2+2\beta_{is}]x_ir_3^{is}}{\lambda\theta_r^{\eta}-(1-\lambda)\theta_l^{\eta}-1}\le\sum_k\rho_{sk}y_{iks}.$$

(iv) If $\beta_{is}\in(\frac{3-\lambda\theta_r^{\eta}+(1-\lambda)\theta_l^{\eta}}{4},1]$, then constraint (3) is equivalent to the following deterministic constraint

$$\frac{[\lambda\theta_r^{\eta}-(1-\lambda)\theta_l^{\eta}-1+2\beta_{is}]x_ir_4^{is}+(2-2\beta_{is})x_ir_3^{is}}{1+\lambda\theta_r^{\eta}-(1-\lambda)\theta_l^{\eta}}\le\sum_k\rho_{sk}y_{iks}.$$

In the above expressions, the parameters $\theta_l^{\eta}=\max_{i,s}\{\theta_l^{is}\}$ and $\theta_r^{\eta}=\min_{i,s}\{\theta_r^{is}\}$.

Proof. We only prove assertions (*iii*) and (*iv*), the rest can be proved similarly.

Let $\tilde{\eta}_{is}=[r_1^{is},r_2^{is},r_3^{is},r_4^{is};\theta_l^{is},\theta_r^{is}]$ be parametric interval-valued trapezoidal fuzzy variables. Since the principle possibility distributions of $\tilde{\eta}_{is}$ are mutually independent in the sense of [26], $\tilde{\eta}=x_i\tilde{\eta}_{is}$ is also a parametric interval-valued trapezoidal fuzzy variable [29], denoted by $[r_1,r_2,r_3,r_4;\theta_l^{\eta},\theta_r^{\eta}]$, where

$$\begin{aligned}r_1&=x_ir_1^{is},\\ r_2&=x_ir_2^{is},\\ r_3&=x_ir_3^{is},\\ r_4&=x_ir_4^{is},\end{aligned}\tag{10}$$

with parameters $\theta_l^{\eta}=\max_{i,s}\{\theta_l^{is}\}$ and $\theta_r^{\eta}=\min_{i,s}\{\theta_r^{is}\}$.

If η is the λ selection variable of $\tilde{\eta}$, then its parametric interval-valued possibility distribution $\mu_\eta(r;\theta,\lambda)$ is

$$\mu_\eta(r;\theta,\lambda)=\begin{cases}\dfrac{[1+\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta](r-r_1)}{r_2-r_1}, & r_1<r\le\dfrac{r_1+r_2}{2}\\ \dfrac{[1-\lambda\theta_r^\eta+(1-\lambda)\theta_l^\eta]r+[\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta]r_2-r_1}{r_2-r_1}, & \dfrac{r_1+r_2}{2}<r\le r_2\\ 1, & r_2<r\le r_3\\ \dfrac{[\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta-1]r-[\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta]r_3+r_4}{r_4-r_3}, & r_3<r\le\dfrac{r_3+r_4}{2}\\ \dfrac{[1+\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta](r_4-r)}{r_4-r_3}, & \dfrac{r_3+r_4}{2}<r\le r_4\end{cases}$$

where $\theta=(\theta_l^\eta,\theta_r^\eta)$.

If $\beta_{is}>0.5$, then we have

$$\begin{aligned}&\mathrm{Cr}\{x_i\eta_{is}\le\sum_k\rho_{sk}y_{iks}\}=\mathrm{Cr}\{\eta\le\sum_k\rho_{sk}y_{iks}\}\\&=\frac{1}{2}\{1+\sup_{r\le t}\mu_\eta(r;\theta,\lambda)-\sup_{r>t}\mu_\eta(r;\theta,\lambda)\}\\&=1-\frac{1}{2}\sup_{r>t}\mu_\eta(r;\theta,\lambda),\end{aligned}$$

where $t=\sum_k\rho_{sk}y_{iks}$.

Thus, credibility constraint $\mathrm{Cr}\{x_i\eta_{is}\le\sum_k\rho_{sk}y_{iks}\}\ge\beta_{is}$ is equivalent to $\sup_{r>t}\mu_\eta(r;\theta,\lambda)\le 2-2\beta_{is}$.

If we denote

$$\eta_{\sup}(\beta)=\sup\{r\ \mid\ \sup_{r>t}\mu_\eta(r;\theta,\lambda)\le\beta\}$$

for $\beta\in(0,1]$, then credibility constraint (3) has the following equivalent representation

$$\eta_{\sup}(2-2\beta_{is})\le t. \tag{11}$$

Note that $\mu_\eta((r_3+r_4)/2)=(1+\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta)/2$. If $(1+\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta)/2<2-2\beta_{is}\le 1$, i.e., $\beta_{is}\in(1/2,(3-\lambda\theta_r^\eta+(1-\lambda)\theta_l^\eta)/4]$, then $\eta_{\sup}(2-2\beta_{is})$ is the solution of following equation

$$\frac{[\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta-1]r-[\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta]r_3+r_4}{r_4-r_3}-(2-2\beta_{is})=0.$$

Solving the above equation, we have

$$\eta_{\sup}(2-2\beta_{is})=\frac{(1-2\beta_{is})r_4+[\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta-2+2\beta_{is}]r_3}{\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta-1}.$$

On the other hand, $\mu_\eta(r_4)=0$. If $0<2-2\beta_{is}\le(1+\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta)/2$, i.e., $\beta_{is}\in((3-\lambda\theta_r^\eta+(1-\lambda)\theta_l^\eta)/4,1]$, then $\eta_{\sup}(2-2\beta_{is})$ is the solution of following equation

$$\frac{[1+\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta](r_4-r)}{r_4-r_3}-(2-2\beta_{is})=0.$$

Solving the above equation, we have

$$\eta_{\sup}(2-2\beta_{is})=\frac{[\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta-1+2\beta_{is}]r_4+(2-2\beta_{is})r_3}{1+\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta}.$$

Combining Eqs. (10) and (11), if $\beta_{is}\in(\frac{1}{2},\frac{3-\lambda\theta_r^\eta+(1-\lambda)\theta_l^\eta}{4}]$, credibility constraint (3) is equivalent to

$$\frac{(1-2\beta_{is})x_ir_4^{is}+[\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta-2+2\beta_{is}]x_ir_3^{is}}{\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta-1}\le\sum_k\rho_{sk}y_{iks}.$$

If $\beta_{is}\in(\frac{3-\lambda\theta_r^\eta+(1-\lambda)\theta_l^\eta}{4},1]$, then credibility constraint (3) is equivalent to

$$\frac{[\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta-1+2\beta_{is}]x_ir_4^{is}+(2-2\beta_{is})x_ir_3^{is}}{1+\lambda\theta_r^\eta-(1-\lambda)\theta_l^\eta}\le\sum_k\rho_{sk}y_{iks}.$$

The proof of theorem is complete. □

5. Equivalent parametric programming models

In this section, we will discuss how to turn credibilistic programming model (2)–(7) into its equivalent deterministic programming model.

Firstly, the parameter α is decomposed to the following four subregions:

$$I_\alpha^1 = \{0 < \alpha \le (\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi + 1)/4\},$$

$$I_\alpha^2 = \{(\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi + 1)/4 < \alpha \le 1/2\},$$

$$I_\alpha^3 = \{1/2 < \alpha \le (3 - \lambda\theta_r^\xi + (1-\lambda)\theta_l^\xi)/4\},$$

$$I_\alpha^4 = \{(3 - \lambda\theta_r^\xi + (1-\lambda)\theta_l^\xi)/4 < \alpha \le 1\}.$$

In addition, we introduce the following index sets:

$$U_{\beta_{is}}^1 = \{(i,s) \mid 0 < \beta_{is} \le (\lambda\theta_r^\eta - (1-\lambda)\theta_l^\eta + 1)/4\},$$

$$U_{\beta_{is}}^2 = \{(i,s) \mid (\lambda\theta_r^\eta - (1-\lambda)\theta_l^\eta + 1)/4 < \beta_{is} \le 1/2\},$$

$$U_{\beta_{is}}^3 = \{(i,s) \mid 1/2 < \beta_{is} \le (3 - \lambda\theta_r^\eta + (1-\lambda)\theta_l^\eta)/4\},$$

$$U_{\beta_{is}}^4 = \{(i,s) \mid (3 - \lambda\theta_r^\eta + (1-\lambda)\theta_l^\eta)/4 < \beta_{is} \le 1\}.$$

Based on notations above, we now turn credibilistic project portfolio selection model (2)–(7) into its equivalent parametric programming models.

In the case of $\alpha \in I_\alpha^1$ and $\beta_{is} \in U_{\beta_{is}}^1$, according to Theorems 1 and 2, the credibility constraint about total project portfolio returns

$$\mathrm{Cr}\{(\sum_i x_i\xi_i + \sum_{i<j} x_i x_j \xi_{ij}) \ge \bar{f}\} \ge \alpha$$

and the credibility constraint about internal resources

$$\mathrm{Cr}\{\sum_k \rho_{sk} y_{iks} \ge x_i\eta_{is}\} \ge \beta_{is}, \quad \forall i, s$$

have the following equivalent representations

$$\frac{[1 + \lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi - 2\alpha](\sum_i x_i r_3^i + \sum_{i<j} x_i x_j r_3^{ij}) + 2\alpha(\sum_i x_i r_2^i + \sum_{i<j} x_i x_j r_2^{ij})}{\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi + 1} \ge \bar{f} \tag{12}$$

and for any $i = 1, 2, \ldots, n$, $s = 1, 2, \ldots, l$

$$\frac{2\beta_{is} x_i r_2^{is} + [\lambda\theta_r^\eta - (1-\lambda)\theta_l^\eta + 1 - 2\beta_{is}] x_i r_1^{is}}{1 + \lambda\theta_r^\eta - (1-\lambda)\theta_l^\eta} \le \sum_k \rho_{sk} y_{iks}. \tag{13}$$

As a consequence, model (2)–(7) is equivalent to the following parametric programming model:

$$\begin{array}{ll} \max & \bar{f} \\ \text{s.t.} & \dfrac{[1 + \lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi - 2\alpha](\sum_i x_i r_3^i + \sum_{i<j} x_i x_j r_3^{ij}) + 2\alpha(\sum_i x_i r_2^i + \sum_{i<j} x_i x_j r_2^{ij})}{\lambda\theta_r^\xi - (1-\lambda)\theta_l^\xi + 1} \ge \bar{f} \\ & \dfrac{2\beta_{is} x_i r_2^{is} + [\lambda\theta_r^\eta - (1-\lambda)\theta_l^\eta + 1 - 2\beta_{is}] x_i r_1^{is}}{1 + \lambda\theta_r^\eta - (1-\lambda)\theta_l^\eta} \le \sum_k \rho_{sk} y_{iks}, \quad \forall i, s \\ & (4)-(7). \end{array} \tag{14}$$

For simplicity, if we denote the functions in the left sides of Eqs. (12) and (13) by $F_1(x;\theta^\xi, \lambda)$ and $P_1(x;\theta^\eta, \lambda)$ respectively, and write $\theta^\xi = (\theta_l^\xi, \theta_r^\xi)$ and $\theta^\eta = (\theta_l^\eta, \theta_r^\eta)$, then model (14) can be rewritten equivalently as

$$\begin{array}{ll} \max & \bar{f} \\ \text{s.t.} & F_1(x;\theta^\xi, \lambda) \ge \bar{f} \\ & P_1(x;\theta^\eta, \lambda) \le \sum_k \rho_{sk} y_{iks}, \quad \forall i, s \\ & (4)-(7). \end{array} \tag{15}$$

From the discussion above, it is easy to know that the equivalent programming model is determined by the values of confidence levels α and β_{is}, and the total number of equivalent programming models is $4^{n\times l+1}$. Using the aforementioned notations, the equivalent programming models of credibilistic project portfolio selection model (2)–(7) can be represented as the following general form:

$$\begin{array}{lll} \max & \bar{f} & \\ \text{s.t.} & F_t(x;\theta^{\xi},\lambda) \geq \bar{f}, & \alpha \in I_{\alpha}^t \\ & P_t(x;\theta^{\eta},\lambda) \leq \sum_k \rho_{sk} y_{iks}, & \beta_{is} \in U_{\beta_{is}}^t \\ & (4)-(7). & \end{array} \tag{16}$$

where $F_t(x;\theta^{\xi}, \lambda)$ and $P_t(x;\theta^{\eta}, \lambda)$ are defined in I_{α}^t and $U_{\beta_{is}}^t$ (t = 1, 2, 3, 4), respectively, and their analytical expressions can be obtained in the same way as $F_1(x;\theta^{\xi}, \lambda)$ and $P_1(x;\theta^{\eta}, \lambda)$ by applying Theorems 1 and 2.

Model (16) is a nonlinear mixed-integer programming, it can be solved by conventional optimization softwares such as Lingo. It is well-known that Lingo software is a comprehensive optimization tool designed to build and solve linear, nonlinear (convex or nonconvex), and integer optimization models. With the embedded branch-and-bound code, Lingo can solve our parametric programming model (16) fast and efficiently. This issue will be addressed in the next section.

6. Numerical experiments

6.1. Statement of problem

In this section, we provide a numerical example to illustrate an application of the proposed credibilistic optimization method. The example relates to a decision-making problem in a national electronic commerce competence center (ECCC). The ECCC is usually a public–private-partnership (as in Austria), or only private (as in China) enterprise that dedicates to applied research and development in new business models of electronic and mobile business. The ECCC application is a typical example of knowledge work. According to the ISCO 2008 standard [36], the competence classification needed to the ECCC is shown in Table 1.

Assume that the ECCC has 16 candidate projects, and wishes to select some projects as a portfolio. One department in the ECCC are assigned to complete this selected project work. As shown in Table 1, there are six employees in the department, and everyone is partial qualified with 9 competencies. We solve the equivalent parametric programming models obtained in Section 5, where the parameters embedded in the possibility distributions of uncertain return and work time are randomly generated, and the possibility distribution of uncertain model parameters are obtained by the following way. First, a set of projects is chosen from a real-life scenario. If the crisp return of project i is m_i, then the triplet of triangular coefficients can be expressed by $(a_i m_i, m_i, b_i m_i)$, where a_i is a reduction factor and b_i is an amplification factor. Similarly, we can represent the uncertain return $[a_i m_i, m_i, b_i m_i; \theta_l^i, \theta_r^i]$ after the values of θ_l^i and θ_r^i become known.

In our numerical experiments, the returns $\tilde{\xi}_i$ of all projects are normalized and generated randomly from the interval [0.8, 1]. The corresponding reduction factor a_i and amplification factor b_i are generated in interval [0.9, 1] and [1, 1.2], respectively. Meanwhile, $\tilde{\xi}_{ij} = [a_{ij}m_{ij}, m_{ij}, b_{ij}m_{ij}; \theta_l^{ij}, \theta_r^{ij}]$ is an additional return that may be positive, negative or zero. The parameters m_{ij}, a_{ij} and b_{ij} are generated randomly from the interval [−0.3, 0.4], [0.8, 0.9] and [1, 1.1], respectively. Similarly, we can represent the uncertain work time of project $\tilde{\eta}_{is} = [r_1^{is}, r_2^{is}, r_3^{is}, r_4^{is}; \theta_l^{is}, \theta_r^{is}]$ in the interval [7, 13]. The parameters θ_l^i, θ_r^i, θ_l^{ij}, θ_r^{ij}, θ_l^{is} and θ_r^{is} are generated randomly from the interval [0, 1], and denote $\theta_l^{\xi} = \max_{i,j}\{\theta_l^i, \theta_l^{ij}\}$, $\theta_r^{\xi} = \min_{i,j}\{\theta_r^i, \theta_r^{ij}\}$, $\theta_l^{\eta} = \max_{i,s}\{\theta_l^{is}\}$ and $\theta_r^{\eta} = \min_{i,s}\{\theta_r^{is}\}$. Finally, the competence values ρ_{ks} and π_k are generated randomly from [0, 1], and they are independent from each other.

6.2. Computational results and discussion

To examine the influence of the model parameters on the optimal decisions, we take various values of model parameters, and solve our optimization problems by software Lingo 9.0. All the experiments are performed on a personal computer with the following parameters: Intel(R)Core(TM) i5-3337U, 1.80 GHz CPU, 8.00 GB RAM, and Win 8 operating system. For each experiment, we report the optimal portfolio policy and maximal portfolio return.

For the sake of presentation, we denote the parameter λ in project return as λ_{ξ} and the parameter λ in human resource as λ_{η}. To ensure credibility constraints holding with high credibility levels, the values of levels α and β_{is} are generated randomly from the interval [0.7, 0.9]. We do our numerical experiments according to the following two cases:

Table 1
The classification of competencies needed for ECCC.

Number of sub-major group	Code	competence
21 Science and engineering professionals	2120	Mathematicians, actuaries and statisticians
25 Information and communications technology professionals	2511	System analysts
	2513	Web and multimedia developers
	2514	Applications programmers
	2519	Software and applications developers and analysts not elsewhere classified
	2521	Database designers and administrators
	2523	Computer network professionals
	2529	Database and network professionals not elsewhere classified
26 Legal, social and cultural professionals	2631	Economists

Case (I): parameters $\lambda_\xi = \lambda_\eta$ and $(\theta_l^\xi, \theta_r^\xi) = (\theta_l^\eta, \theta_r^\eta)$;

Case (II): parameters $\lambda_\xi \neq \lambda_\eta$ and $(\theta_l^\xi, \theta_r^\xi) \neq (\theta_l^\eta, \theta_r^\eta)$.

6.2.1. The influence of parameters λ_ξ and λ_η

We next identify the influence of parameters λ_ξ and λ_η on the optimal project portfolio decision.

First, we do the experiments in Case (I): $\lambda_\xi = \lambda_\eta$ and $(\theta_l^\xi, \theta_r^\xi) = (\theta_l^\eta, \theta_r^\eta)$. We generate randomly the values of parameters θ_l^i, θ_r^i, θ_l^{ij}, θ_r^{ij}, θ_l^{is} and θ_r^{is} from the interval [0, 1], and obtain $\theta_l^\xi = \theta_l^\eta = 0.9787483$, $\theta_r^\xi = \theta_r^\eta = 0.0731421$. The values of $\lambda_\xi = \lambda_\eta$ vary in the interval [0.785, 0.820]. The computational results in case (I) are reported in Table 2.

Second, we do the experiments in case (II): $\lambda_\xi \neq \lambda_\eta$ and $(\theta_l^\xi, \theta_r^\xi) \neq (\theta_l^\eta, \theta_r^\eta)$. In this case, the values of the parameters θ_l^i, θ_r^i, θ_l^{ij}, θ_r^{ij}, θ_l^{is} and θ_r^{is} are generated randomly from the interval [0, 1], and we obtain the values $(\theta_l^\xi, \theta_r^\xi) = (0.9787483, 0.0731421)$ and $(\theta_l^\eta, \theta_r^\eta) = (0.9335576, 0.1155300)$. The values of λ_ξ and λ_η vary in the interval [0.72, 0.77]. The computational results in case (II) are shown in Table 3.

In the following, we discuss the computational results under different cases.

Case (I): From Table 2, we observe that the optimal project portfolio policy depends heavily on the values of parameter λ. For instance, when $\lambda_\xi = \lambda_\eta = 0.786$, the organization should select the projects 2–4, 6, 9–16 as the optimal project portfolio. When $\lambda_\xi = \lambda_\eta = 0.798$, the selected projects are 3, 10, 12, 13 and 15. It is evident that the optimal project selection and portfolio return are distinctly different.

With various values of parameters λ_ξ and λ_η, the variation tendency of portfolio return is depicted in Fig. 1, from which we observe that when $\lambda_\xi = \lambda_\eta$, the maximal portfolio return decreases with respect to parameter λ_ξ.

Case (II): The value of parameter λ_ξ is not equal to λ_η and the value of $(\theta_l^\xi, \theta_r^\xi)$ is different from $(\theta_l^\eta, \theta_r^\eta)$. In this case, given $(\theta_l^\xi, \theta_r^\xi) = (0.9787483, 0.0731421)$ and $(\theta_l^\eta, \theta_r^\eta) = (0.9335576, 0.1155300)$, Table 3 summarizes the computational results when parameters λ_ξ and λ_η take various values in interval [0.72, 0.77]. From the computational results, we conclude that the optimal portfolio selection and maximal portfolio return depend heavily on the parameters λ_ξ and λ_η. For example, given $\lambda_\xi = 0.732$, $\lambda_\eta = 0.758$, the organization should select the projects 3, 6, 12, 13 and 15 as the optimal project portfolio. On the other hand, when $\lambda_\xi = 0.742$, $\lambda_\eta = 0.748$, the optimal project portfolio for the organization includes projects 1, 3, 4, 6, 9, 10, 12, 13, 15 and 16. The optimal values are also significantly different.

With various values of parameters λ_ξ and λ_η, the variation tendency of portfolio return in Case (II) is depicted in Fig. 2, from which we can see that the maximal portfolio return increases with respect to parameter λ_ξ and decreases with respect to parameter λ_η.

Table 2
The computational results in Case (I).

$\lambda_\xi = \lambda_\eta$	Optimal project portfolio	Maximal return
0.785	(0,1,1,1,0,1,1,0,1,1,1,1,1,1,1,1)	12.69488
0.786	(0,1,1,1,0,1,0,0,1,1,1,1,1,1,1,1)	11.89868
0.787	(1,1,1,1,0,1,1,0,1,1,0,1,1,0,1,1)	11.48585
0.788	(0,1,1,1,0,1,0,1,1,1,0,1,1,0,1,1)	10.78329
0.789	(0,1,1,1,0,1,0,1,0,1,0,1,1,0,1,1)	9.875594
0.790	(0,0,1,1,0,1,0,1,0,1,0,1,1,0,1,1)	9.073986
0.791	(1,1,0,1,0,1,0,0,1,1,0,1,1,0,1,0)	8.348961
0.792	(0,1,1,1,0,1,0,0,0,1,0,0,1,0,1,1)	7.960947
0.793	(0,0,1,0,0,1,0,0,0,1,0,1,1,0,1,1)	7.317056
0.794	(0,0,1,1,0,1,0,0,0,1,0,1,1,0,1,0)	6.997251
0.795	(0,1,0,1,0,1,0,0,0,1,0,1,1,0,1,0)	6.608497
0.796	(0,0,1,0,0,1,0,0,0,1,0,1,1,0,1,0)	6.166496
0.798	(0,0,1,0,0,0,0,0,0,1,0,1,1,0,1,0)	5.314636
0.800	(0,0,1,0,0,1,0,0,0,0,0,1,1,0,1,0)	4.695201
0.805	(0,0,1,0,0,0,0,0,0,1,0,0,1,0,0,0)	3.439077
0.818	(0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0)	1.837866
0.820	(0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0)	1.189594

Table 3
The computational results in Case (II).

λ_ξ	λ_η	Optimal project portfolio	Maximal return
0.720	0.770	(0,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0)	2.357819
0.725	0.765	(0,0,1,0,0,0,0,0,0,0,0,1,1,0,0,0)	3.347771
0.728	0.762	(0,0,1,0,0,1,0,0,0,0,0,1,0,0,1,0)	3.924534
0.730	0.760	(0,0,1,0,0,0,0,0,0,1,0,1,1,0,0,0)	4.430730
0.732	0.758	(0,0,1,0,0,1,0,0,0,0,0,1,1,0,1,0)	5.092111
0.735	0.755	(0,0,1,0,0,1,0,0,0,1,0,1,1,0,1,0)	6.174818
0.738	0.752	(0,0,1,0,0,1,0,0,0,1,0,1,1,0,1,1)	7.324979
0.740	0.750	(1,1,0,1,0,1,0,0,1,1,0,1,1,0,1,0)	8.364772
0.742	0.748	(1,0,1,1,0,1,0,0,1,1,0,1,1,0,1,1)	9.900827
0.745	0.745	(0,1,1,1,0,1,0,0,1,1,1,1,1,1,1,1)	11.91201
0.748	0.742	(1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1)	14.56668
0.750	0.740	(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)	15.49310

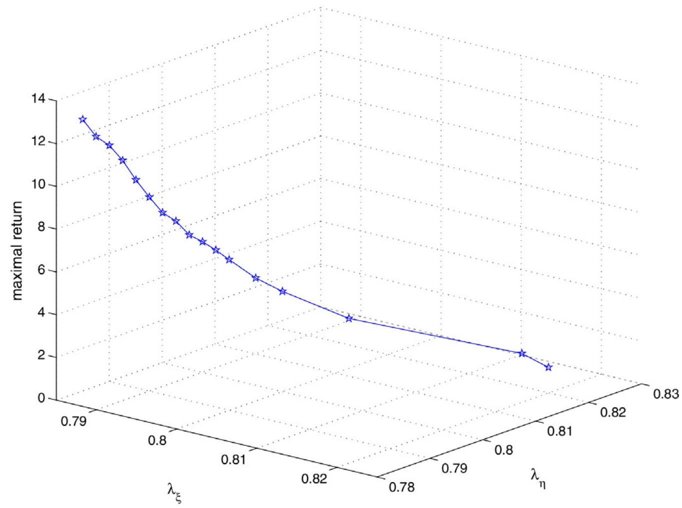

Fig. 1. The variation of maximal portfolio return about λ_ξ and λ_η in Case (I).

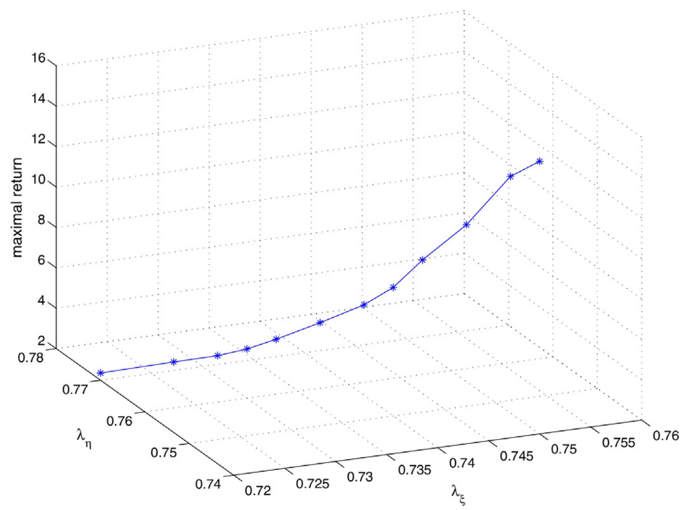

Fig. 2. The variation of maximal portfolio return about λ_ξ and λ_η in Case (II).

6.2.2. *The influence of parameter* θ_l *and* θ_r

To identify the impact of parameters $\theta_l = (\theta_l^\xi, \theta_l^\eta)$ and $\theta_r = (\theta_r^\xi, \theta_r^\eta)$ on optimal portfolio decision, we do additional experiments in cases (I) and (II) as parameters θ_l and θ_r vary their values.

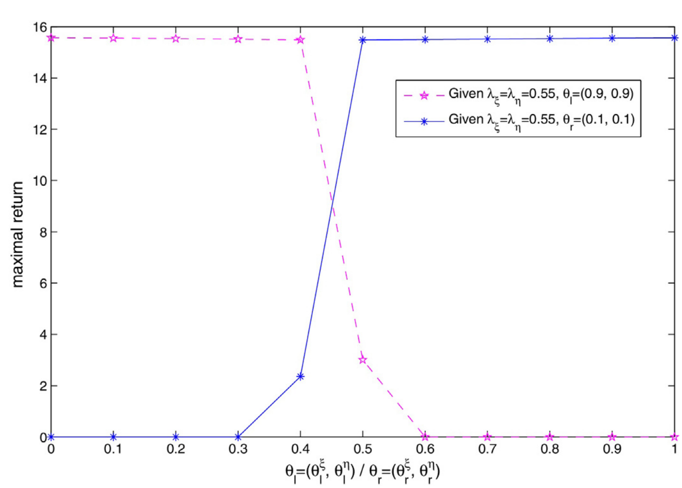

Fig. 3. The variation of maximal portfolio return about θ_l and θ_r in Case (I).

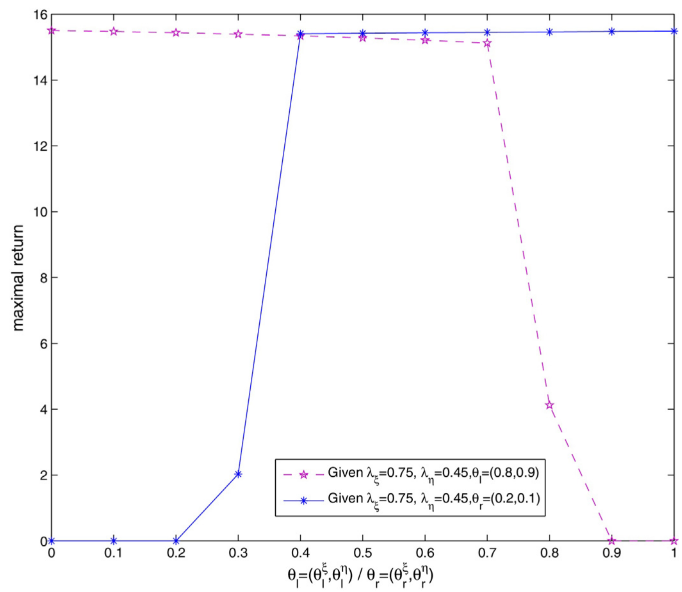

Fig. 4. The variation of maximal portfolio return about θ_l and θ_r in Case (II).

Case (I): Given $\lambda_\xi = \lambda_\eta = 0.55$, and $\theta_l = (\theta_l^\xi, \theta_l^\eta) = (0.9, 0.9)$, we perform some numerical experiments and depict the computational results in Fig. 3, from which we observe that the maximal portfolio return decreases as parameter θ_r increases from 0 to 1. That is, the maximal portfolio return is monotonic decreasing with respect to θ_r. On the other hand, given $\lambda_\xi = \lambda_\eta = 0.55$ and $\theta_r = (\theta_r^\xi, \theta_r^\eta) = (0.1, 0.1)$, the maximal portfolio return increases as parameter θ_l increases. That is, the maximal portfolio return is monotonic increasing with respect to θ_l.

Case (II): In this case, the parameter $\lambda_\xi \neq \lambda_\eta$ and $(\theta_l^\xi, \theta_r^\xi) \neq (\theta_l^\eta, \theta_r^\eta)$. If we set $\lambda_\xi = 0.75$, $\lambda_\eta = 0.45$, and $\theta_l = (\theta_l^\xi, \theta_l^\eta) = (0.8, 0.9)$, then we plot the computational results in Fig. 4, from which we find that the maximal portfolio return decreases with respect to parameter θ_r. On the other hand, if we set $\lambda_\xi = 0.75$, $\lambda_\eta = 0.45$, and $\theta_r = (\theta_r^\xi, \theta_r^\eta) = (0.2, 0.1)$, then the maximal portfolio return increases as parameter θ_l increases from 0 to 1, i.e., the maximal portfolio return is monotonic increasing with respect to θ_l.

7. Practical implications and conclusions

The project portfolio selection is a rather complex task to choose the suitable projects considering the interactions among projects and the decision criterion of project manager. The high uncertainty and complexity of project portfolio circumstance are forcing an organization to find better portfolio decisions via novel optimization methods.

The practical significance of this paper mainly includes the following three aspects:

(i) The proposed optimization framework gives an organization a new guidance on how to find the optimal project portfolio decision under robust uncertain environment. In this study, a distributionally robust credibilistic project portfolio programming model has been presented based on optimistic value criterion. The objective of the proposed model is to maximize the optimistic value of the total project portfolio returns, while considering uncertain return and interaction among projects, and assignment of human resource.
(ii) The interaction among projects is common in practice, and is one of the most important issues in practical project portfolio selection problem. Compared with the other optimization models, the proposed optimization model enables the decision maker to identify the impacts of synergy and cannibalization on uncertain project portfolio return.
(iii) The third significant aspect of this study is that the interval-valued possibility distribution is used to characterize the uncertain returns in practical project portfolio selection problems. When the exact distributions of uncertain model parameters are difficult to be determined by historic data or the experiences of experts in advance, the proposed novel approach is a good alternative in practice. Compared with other optimization methods under fixed possibility distributions, the proposed parametric optimization method can provide a decision maker a set of optimal solutions under various values of distribution parameters, which may facilitate the decision maker to make his informed decision for the project portfolio selection problem.

In conclusion, this study developed a robust credibilistic optimization method for the project portfolio selection problem. The uncertain parameters in our problem were modeled as interval-valued fuzzy variables with variable parametric possibility distributions. The proposed project portfolio selection optimization model considering interactions among projects was built based on optimistic value criterion. To solve the proposed project portfolio selection model, we derived the equivalent analytical expressions of credibility constraints, and turned the original optimization model to its equivalent nonlinear mixed-integer programming models. Finally, an application example was provided and some numerical experiments were performed to illustrate our new modeling ideas and the efficiency of the proposed credibilistic optimization method.

Future academical innovation directions might address the following topics. First, our current optimization model is based on optimistic value criterion, an extension of the model to other optimization criterion is possible. Second, the current model considers the total portfolio returns as a single objective function, a multi-objective optimization model that accounts for both return and risk management objectives may be helpful in practical project portfolio selection problems. Third, the present model finds the optimal project portfolio decision via parametric optimization method, the case to find robust optimal decision with respect to uncertain distribution set is an innovative direction for our future research.

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