Homework #15

Let {Xn,Tn} be a Markov renewal process with state space {a, b} and semi-Markov kernel Q given as

$$Q(t) = \begin{bmatrix} 0.6(1 - e^{-5t}) & 0.4 - 0.4e^{-2t} \\ 0.5 - 0.2e^{-3t} - 0.3e^{-5t} & 0.5 - 0.5e^{-2t} - te^{-2t} \end{bmatrix}$$

where t represents *days*.

a. If the process is currently in state a, what is the probability that the next jump will be back to itself (i.e., state a)?

Both parts a and part b require you determine $P(i, j) = \lim_{t\to\infty}Q(i, j, t)$. The answer to part a is P(a, a) = 0.6.

b. If the process is currently in state b, what is the probability that the next jump will be to state a? P(b,a) = 0.5.

c. Given that the process has just made a jump to state a, what is the probability that the next jump will occur within six hours given that the next jump will be a return to state a?

The next three parts involve the matrix G(t), where G(i, j, t) = Q(i, j, t)/P(i, j)

$$G(t) = \begin{bmatrix} 1 - e^{-5t} & 1 - e^{-2t} \\ 1 - 0.4e^{-3t} - 0.6e^{-5t} & 1 - e^{-2t} - 2te^{-2t} \end{bmatrix}$$

The answer to part c is G(a, a, 0.25) = 0.7135

d. Given that the process has just made a jump to state a, what is the probability that the next jump will occur within six hours given that the next jump will be to state b?

G(a, b, 0.25) = 0.3935. **Note**: this cannot be a Markov process because the sojourn time depends on which state the imbedded Markov chain will jump to. Of course when the process is in state b, the sojourn times are not even exponential.

e. Given that the process starts in state a, then next moves to state b, and then back to state a, what is the probability that both initial sojourn times in state a and state b were less than six hours?

G(a, b, 0.25) × G(b, a, 0.25) = 0.393469 × 0.639151 = 0.2515