Prelim 2: Practice Exam

1. Let $f(x;\theta)$ be the logistic location pdf

$$f(x \mid \theta) = \frac{e^{x-\theta}}{(1+e^{x-\theta})^2}, \quad -\infty < x < \infty; -\infty < \theta < \infty$$

Based on one observation X, find the size α test of $H_0 : \theta = 0$ versus $H_1 : \theta > 0$.

2. Let $X_1, \cdots, X_n$ be n independent observations from $N(\mu, 1)$.

- (a) Find the LRT for $H_0 : \mu \leq 2$ vs $H_1 : \mu > 2$ at $\alpha = 0.05$
- (b) Calculate the power function for the LRT test. Is the power function increasing or decreasing? Justify your answer.
- (c) How large should n be so that the test in (a) has power 0.9 for $\mu = 2.5$
- (d) If we took 9 samples and observed $\bar{x} = 2.5$. Compute the p-value based on this data for the test in (a). Will you reject H_0 at the level $\alpha = 0.05$ based on this p-value? Explain your answer.

  (You may need the following information:) $z_{0.1} = 1.28, z_{0.05} = 1.645, z_{0.025} = 1.960, z_{0.01} = 2.33, z_{0.005} = 2.58$.

3. Let $X_1, \ldots, X_n$ be sampled iid from Poisson(λ). Assume λ is sampled from the Gamma(α, β) prior and let the loss function be

$$L(\lambda, \hat{\lambda}) = \lambda(\lambda - \hat{\lambda})^2.$$

Calculate the Bayes optimal estimator $\hat{\lambda}_{\text{Bayes}}$ for λ that minimizes the expected loss.

4. Let $X_1, \cdots, X_n$ be independent random variables, and $X_i \sim$ Exponential($scale = \lambda$), i.e., $f(x) = \dfrac{1}{\lambda}\exp(-x/\lambda)$, for $x \geq 0$

(a) Find the UMVUE of λ.

(b) Calculate the mean squared error of the UMVUE.

(c) Find the Fisher information $I(\lambda)$ for n observations. Is the CRLB attainable?

5. Review the examples in class and go over suggested optional exercises in each homework assignments.