Homework #13

Instructions: Do your work on your own paper and give only the numerical answers in eCampus. Give your answers rounded to **two digits to the right of the decimal**.

This homework is due after Spring Break; namely on March 16.

1. The following matrix is a generator for a Markov process. Complete its entries.

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$$G = \begin{bmatrix} -10 & 3 & 0 & 7 \\ 5 & -12 & 4 & 3 \\ 1 & 0 & -6 & 5 \\ 3 & 2 & 9 & -14 \end{bmatrix}$$

2. Let Y be a Markov process with state space {a,b,c,d} and an imbedded Markov chain having a Markov matrix given by

$$P = \begin{bmatrix} 0.0 & 0.1 & 0.2 & 0.7 \\ 0.0 & 0.0 & 0.4 & 0.6 \\ 0.8 & 0.1 & 0.0 & 0.1 \\ 1.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \qquad G = \begin{bmatrix} -0.5 & 0.05 & 0.1 & 0.35 \\ 0.0 & -0.2 & 0.08 & 0.12 \\ 1.6 & 0.2 & -2.0 & 0.2 \\ 1.0 & 0.0 & 0.0 & -1 \end{bmatrix}$$

The mean sojourn times in states a, b, c, and d, are 2, 5, 0.5, and 1, respectively.

a. Determine the generator matrix for this Markov process.

b. What is the $\lim_{t\to\infty} P\{Y(t) = a\}$? (Use the G matrix, not the P matrix.)

$$\begin{aligned} -0.5p(a) + \quad + 1.6p(c) + p(d) &= 0 \\ 0.95p(a) - 0.2p(b) + 0.2p(c) &= 0 \\ 0.1p(a) + 0.08p(b) - 2p(c) &= 0 \\ p(a) + p(b) + p(c) + p(d) &= 1 \end{aligned}$$

solving this 4 × 4 system yields p = (0.5634, 0.1761, 0.0352 , 0.2254)

c. Let r = (10, 25, 30, 50) be a reward vector and determine

$\lim_{t\to\infty} E[\int_0^t r(Y(t))ds\ ] / t$

multiplying p × r yields 22.35915 or 22.36