$a = (1, 1, 2, 1)$

$k=1$ $\quad \varphi^* + h(a) = 100 + 0.1\,h(a) + 0.3\,h(b) + 0.6\,h(c)$

$k=1$ $\quad \varphi^* + h(b) = 125 + 0.2\,h(b) + 0.5\,h(c) + 0.3\,h(d)$

$k=2$ $\quad \varphi^* + h(c) = 350 + 0.8\,h(a) + 0.2\,h(b)$

$k=1$ $\quad \varphi^* + h(d) = 506 + 0.8\,h(a) + 0.1\,h(b) + 0.1\,h(d)$

$h(a) = 0$

$\varphi^* = 219.24$ and $h = (0, 97.10, 150.18, 322.75)^T$

Let $i = a$, check $k=1$ and $k=2$

true $\quad \varphi^* + h(a) \le 100 + 0.1\,h(a) + 0.3\,h(b) + 0.6\,h(c)$

true $\quad \varphi^* + h(a) \le 300 + 0.6\,h(a) + 0.3\,h(b) + 0.1\,h(c)$

Let $i = b$, check $k=1$ and $k=2$

$\varphi^* + h(a)$

$\vdots$

If instead $h(b) = 0 \Rightarrow h(a) = -97.10$

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Assume there is a cost of c per step

Then total discounted is $V = \sum_{n=0}^{\infty} \alpha^n c$

$$\Rightarrow V = c \cdot \frac{1}{1-\alpha}$$

If V is total discounted cost

then equivalent cost per step $= (1-\alpha) V$

Step 1: $a_0 = (1, 1, 1, 1)$

Step 2: $f = (100, 125, 150, 500)$

$$P = \begin{bmatrix} 0.1 & 0.3 & 0.6 & 0 \\ 0 & 0.2 & 0.5 & 0.3 \\ 0 & 0.1 & 0.2 & 0.7 \\ 0.8 & 0.1 & 0 & 0.1 \end{bmatrix}$$

$\varphi + Ih = f + Ph$

$\varphi + (I - P)h = f$

$$I - P = \begin{bmatrix} 0.9 & -0.3 & -0.6 & 0 \\ 0 & 0.8 & -0.5 & -0.3 \\ -0.1 & -0.1 & 0.8 & -0.7 \\ -0.1 & -0.1 & 0 & 0.9 \end{bmatrix}$$

Step 3: Note $h(a) = 0$ $\therefore$ Find $\varphi, h(b), h(c), h(d)$ $\Rightarrow$

$h(a)$ $h(b)$ $h(c)$ $h(d)$

φ

$$\begin{aligned} \varphi &= 100 + 0.3h(b) + 0.6h(c) \\ \varphi + h(b) &= 125 + 0.2h(b) + 0.5h(c) + 0.3h(d) \\ \varphi + h(c) &= 150 + 0.1h(b) + 0.2h(c) + 0.7h(d) \\ \varphi + h(d) &= 500 + 0.1h(b) + 0.1h(d) \end{aligned}$$

$\Rightarrow \varphi = 232.86$ and $h = (0, 90.40, 176.23, 306.87)^T$

Let $k = 1$, determine $f_1 + P_1 \begin{pmatrix} 0 \\ 90.40 \\ 176.23 \\ 306.87 \end{pmatrix}$ and

Let $k = 2$, determine $f_2 + P_2 \begin{pmatrix} 0 \\ 90.40 \\ 176.23 \\ 306.87 \end{pmatrix}$

$\Rightarrow$ Compare $\begin{pmatrix} 232.86 \\ 323.26 \\ 409.09 \\ 539.73 \end{pmatrix}$ and $\begin{pmatrix} 344.74 \\ 367.01 \\ 368.08 \\ 609.04 \end{pmatrix}$

$q_1 = (1, 1, 2, 1)$

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