

Homework #14

Due by 7AM, Wednesday, April 1

Instructions: Do your work on your own paper and give only the numerical answers in eCampus. Give your answers rounded to **four digits to the right of the decimal**, except for part e round to three digits.

Consider a small machine shop. Customers arrive to the shop according to Poisson process with mean rate of three per day. Because the shop is small, they only work on one job at a time and if they stay busy they can average two completed jobs per day. If two jobs are in the system and a customer arrives with a job, that customer is sent to another shop; thus, there are at most two jobs in the shop at any time. Let $Z(t)$ be a process with state space $\{0,1,2\}$ where $Z(t)$ is the number of jobs in the shop at time t and assume that the time units for t are in days.

- Notice that $\{Z(t)\}$ is a Markov process. Form the generator matrix for this process. What is $G(0,1)$?
- What is $G(1,1)$?
- One eigenvalue of G is a number between -2 and -3. What is its value rounded to four digits to the right of the decimal.
- What is $\lim_{t \rightarrow \infty} P\{Z(t) = 0 \mid Z(0) = 0\}$ rounded to four digits to the right of the decimal?
- Given that the shop was empty at the start of the day, what is the probability that it will be empty at noon on the first day? Assume each day is from 8AM to 4PM so that noon is $t=0.5$. Round your answer to three digits to the right of the decimal.