Distributionally Robust Contingency-Constrained Unit Commitment

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***Abstract*—This paper proposes a distributionally robust optimization approach for the contingency-constrained unit commitment problem. In our approach, we consider a case where the true probability distribution of contingencies is ambiguous, i.e., difficult to accurately estimate. Instead of assigning a (fixed) probability estimate for each contingency scenario, we consider a set of contingency probability distributions (termed the ambiguity set) based on the $N-k$ security criterion and moment information. Our approach considers all possible distributions in the ambiguity set, and is hence distributionally robust. Meanwhile, as this approach utilizes moment information, it can benefit from available data and become less conservative than the robust optimization approaches. We derive an equivalent reformulation and study a Benders' decomposition algorithm for solving the model. Furthermore, we extend the model to incorporate wind power uncertainty. The case studies on a 6-Bus system and the IEEE 118-Bus system demonstrate that the proposed approach provides less conservative unit commitment decisions as compared with the robust optimization approach.**

***Index Terms*—Distributional robustness, $N-k$ security criterion, unit commitment.**

NOMENCLATURE

A. Sets

$\mathcal{T}$	Index set of time periods.
$\mathcal{N}$	Index set of all buses.
$\mathcal{E}$	Index set of all transmission lines.
$\mathcal{E}_{(n,\cdot)}$	Index set of transmission lines from bus n.
$\mathcal{E}_{(\cdot,n)}$	Index set of transmission lines to bus n.
$\mathcal{G}$	Index set of all generators.
$\mathcal{G}_n$	Index set of generators at bus n.
$\mathcal{H}$	Index set of all system components, $\mathcal{H} = \mathcal{G} \cup \mathcal{E}$.

B. Parameters

NL_i	No-load cost for generator i.
SU_i	Start-up cost for generator i.
SD_i	Shut-down cost for generator i.
UT_i	Minimum up-time for generator i.
DT_i	Minimum down-time for generator i.
LB_i	Minimal power output if generator i is on.
UB_i	Maximal power output if generator i is on.
RU_i	Ramp-up limit for generator i.
$\overline{\mathrm{RU}}_i$	Start-up ramp-up limit for generator i.
RD_i	Ramp-down limit for generator i.
$\overline{\mathrm{RD}}_i$	Shut-down ramp-down limit for generator i.
β_i^ℓ	Unit cost of electricity generation from generator i in the ℓth piece of its approximate fuel cost function.
γ_i^ℓ	Constant term in the ℓth piece of the approximate fuel cost function of generator i.
K_{mn}	Capacity of transmission line (m,n).
$\bar{\theta}_n$	Maximal value of the phase angle at bus n.
$\underline{\theta}_n$	Minimal value of the phase angle at bus n.
X_{mn}	Reactance of transmission line (m,n).
M_{mn}	Sufficiently large constant number to linearize the power flow equation of transmission line (m,n).
$F_i(\cdot)$	Fuel cost function of generator i.
D_{nt}	Load at bus n in time period t.
W_{nt}	Wind power output at bus n in period t.
P_{nt}^+	Unit penalty cost for load shedding at bus n in period t.
P_{nt}^-	Unit penalty for over-generation at bus n in period t.
p_n	Failure probability of bus n.
$p_{m,n}$	Failure probability of transmission line (m,n).
n_H	Number of geographical zones.

C. First-Stage Variables

y_{it}	Binary decision variable to indicate the on/off status of generator i in period t. "1" if it is on and "0" otherwise.
u_{it}	Binary decision variable to indicate if generator i starts up at the beginning of period t. "1" if it starts up and "0" otherwise.
v_{it}	Binary decision variable to indicate if generator i shuts down at the beginning of period t. "1" if it shuts down and "0" otherwise.
τ, α	Dual variables of the distributionally robust formulation.

D. Second-Stage Variables

x_{it}	Amount of electricity generated by generator i in period t.
f_{mn}^t	Power flow on transmission line (m,n) in period t.
θ_{nt}	Phase angle at bus n in period t.
q_{nt}^+	Amount of load shedding at bus n in period t.
q_{nt}^-	Amount of over-generation at bus n in period t.

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$\lambda, \eta, \delta, \kappa$ Dual variables of the second-stage formulation.

ζ, π Auxiliary variables for linearizing the bilinear terms.

E. Random Parameters

z_i Binary variable indicating the contingency status of generator i. "0" if it is under contingency and "1" otherwise.

z_{mn} Binary variable indicating the contingency status of transmission line (m, n). "0" if it is under contingency and "1" otherwise.

I. Introduction

SECURITY criterion is a critical component for maintaining power system reliability in daily operations. As extreme weather (e.g., heat waves and snowstorms) increases in both frequency and intensity, contingencies in power grids can become more often and severe. Meanwhile, the growing penetration of renewable energy (e.g., wind power) increases the instability of power grids. One of the most well-known security criteria in industrial practice is the $N-1$ criterion ([1], [2]): a power system with N components (including transmission lines and generators) needs to continue meeting the electricity loads when any single component suffers a contingency. This criterion can be generalized to the $N-k$ case (e.g., [3], [4]), which allows the simultaneous failure of any k or fewer components. Based on $N-1$ and/or $N-k$ criteria, many research works have considered reliability in power system planning and operational models, e.g., transmission expansion planning [5], [6], transmission switching [2], [7], and electricity market clearing [8]–[10]. An important stream of research, to which this paper attempts to contribute, incorporates security criteria in the unit commitment problem and is called the contingency-constrained unit commitment (CCUC). A CCUC model aims to optimally adjust pre-contingency generation schedules so that the power grid remains reliable when contingencies take place (e.g., [9], [11]–[16]).

A key component of the CCUC models is to identify contingencies. The classical approach is to set up a set of credible contingencies based on power engineering expertise and industrial practices, and ensure the system reliability in all credible contingencies. For example, [9] optimizes energy and reserve scheduling by considering all contingency scenarios in a pre-specified credible set. [17] identifies a small subset of credible contingencies (called "umbrella contingencies") that can lead to a similar level of security as compared to considering all credible contingencies. More recently, [18] proposes a co-optimization framework for energy and ancillary services, and utilizes a Benders' decomposition approach to address the contingency constraints.

Another important stream of research focus on stochastic CCUC models that incorporate the probability of contingencies. Different from the classical approach based on credible sets, stochastic CCUC can optimize the expected value of social welfare that covers all credible contingencies and their likelihood of occurring. For example, [11], [12] consider a stochastic security-constrained market clearing model and estimate the probability of outages based on reliability theory and historical failure rates. [13] studies a stochastic CCUC model under significant wind integration, and couples wind power, load, and contingency in each scenario. [14] shows that incorporating reserves in stochastic CCUC models can enhance both system cost-effectiveness and reliability. [19] considers the possibility of voluntary load reductions in better response to contingencies. In addition to the consideration of expected social welfare, [8] studies the probability of loss-of-loads and the expected amount of loads not served in contingencies. More recently, [20] formulates a chance constraint in the stochastic CCUC model to ensure low probability of loss-of-loads. Although effective and extensively studied, the stochastic CCUC model still faces challenges. For example, the model can become harder to solve as the number of credible contingencies increases. In particular, this computational challenge is significant when considering the $N-k$ security criterion with exponentially many contingencies.

To overcome this challenge, robust optimization models, i.e., robust CCUC approaches are proposed to find the most critical contingencies via solving an optimization problem. Under the $N-k$ security criterion, [15], [21] propose first studies of finding the worst-case generator outages, [16] applies a RO model to compute the worst-case loss-of-load considering generator outages, and [22] extends the RO approach in [15] to incorporate both generator and transmission line outages. On the one hand, a unique feature of the robust CCUC approaches is that they can automatically identify the contingencies that have large impacts on the power system. On the other hand, they often raise concerns of over-conservatism, i.e., sacrificing the average cost-effectiveness in exchange of better cost-effectiveness in the worst-case scenarios. Indeed, robust CCUC approaches focus on the "worst-case" contingency while ignoring its probability of occurring. This could lead to an unnecessarily high average cost if the worst-case contingency is unlikely to occur in practice.

In this paper, we propose a distributionally robust optimization (DRO) approach for solving the CCUC problem, termed the DR-CCUC approach. This approach considers the probability of contingency occurring and the corresponding expected total costs and hence is less conservative than the robust CCUC approach. Meanwhile, DR-CCUC identifies critical contingencies via solving optimization problems, and hence can handle exponentially many contingencies as in the $N-k$ security criterion. More precisely, we construct an ambiguity set of contingency probability distributions that match the $N-k$ security criterion and the available moment information (e.g., component failure probabilities, failure probability confidence intervals, etc). By exploiting more information on the contingency probability, the decision makers are able to discard certain probability distributions and obtain a tighter ambiguity set. Accordingly, the corresponding DR-CCUC approach becomes less conservative. Then, by optimizing the day-ahead UC, we minimize the worst-case expected total costs with respect to all probability distributions in the ambiguity set. In contrast, if we do not exploit any information on the contingency probability and only incorporate the $N-k$ security criterion in the ambiguity set, then DR-CCUC reduces to the robust CCUC approach. DRO approaches have been proposed and studied from the perspective of optimization methodology (see, e.g., [23]–[26]), and recently have also been used in addressing the power system problems under uncertainties, such as the UC problems (e.g., [27]–[29]), optimal power flow (OPF) problems (e.g., [30], [31]), and congestion management problems (e.g., [32], [33]). In our paper, we propose a DRO approach for the UC problem with uncertain component failures under the general $N-k$ security criteria. Our main contributions include:

1) The proposed DR-CCUC approach can handle exponentially many contingencies as in the $N-k$ security criterion.
2) We derive an equivalent reformulation of the DR-CCUC model that facilitates decomposition algorithms and computationally efficient commercial software (e.g., CPLEX).
3) Based on a 6-bus and the IEEE 118-bus test instances, we demonstrate that the DR-CCUC approach performs less conservative than the robust CCUC approach. That is, in the out-of-sample simulations, the UC decision obtained from the DR-CCUC approach leads to a lower average total cost than the one obtained from the robust CCUC approach.

We organize the remainder of this paper as follows. In Section II, we formulate the DR-CCUC model and describe the ambiguity set of the contingency probability distributions. In Section III, we derive a Benders' decomposition framework to solve the DR-CCUC model. In Section IV, we demonstrate this approach on the 6-bus and the IEEE 118-bus system to verify the effectiveness and conservatism of the proposed approach. Finally, we conclude this paper in Section V.

II. Mathematical Formulation

We present the two-stage DR-CCUC model as follows.

$$\min_{y,u,v} \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{G}}(\mathrm{NL}_i y_{it} + \mathrm{SU}_i u_{it} + \mathrm{SD}_i v_{it}) + \sup_{\mathbb{P}\in\mathcal{D}} E_{\mathbb{P}}[Q(y,u,v,z)] \tag{1a}$$

$$\text{s.t.} \quad -y_{i(t-1)} + y_{it} - y_{\mathrm{ik}} \le 0, \quad \forall t\in\mathcal{T}, \forall i\in\mathcal{G}, 1\le k-(t-1)\le \mathrm{UT}_i, \tag{1b}$$

$$y_{i(t-1)} - y_{it} + y_{\mathrm{ik}} \le 1, \quad \forall t\in\mathcal{T}, \forall i\in\mathcal{G}, 1\le k-(t-1)\le \mathrm{DT}_i, \tag{1c}$$

$$-y_{i(t-1)} + y_{it} - u_{it} \le 0, \quad \forall t\in\mathcal{T}, \forall i\in\mathcal{G}, \tag{1d}$$

$$y_{i(t-1)} - y_{it} - v_{it} \le 0, \quad \forall t\in\mathcal{T}, \forall i\in\mathcal{G}, \tag{1e}$$

$$y_{it}, u_{it}, v_{it} \in \{0,1\}, \quad \forall t\in\mathcal{T}, \forall i\in\mathcal{G}, \tag{1f}$$

where $Q(y,u,v,z)$ represents the worst-case expected operating cost for given on/off status y_{it}, start-up u_{it}, shut-down v_{it}, and realized contingencies z_i and z_{mn}, and equals to the optimal objective value of the following problem

$$\min_{x,q,\theta,f} \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{G}} F_i(x_{it}) + \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}}(P_n^+ q_{nt}^+ + P_n^- q_{nt}^-) \tag{2a}$$

$$\text{s.t.} \quad \mathrm{LB}_i y_{it} z_i \le x_{it} \le \mathrm{UB}_i y_{it} z_i, \quad \forall t\in\mathcal{T}, \forall i\in\mathcal{G}, \tag{2b}$$

$$-z_{mn}K_{mn} \le f_{mn}^t \le z_{mn}K_{mn}, \quad \forall (m,n)\in\mathcal{E}, \forall t\in\mathcal{T}, \tag{2c}$$

$$\underline{\theta}_n \le \theta_{nt} \le \bar{\theta}_n, \quad \forall t\in\mathcal{T}, \forall n\in\mathcal{N}, \tag{2e}$$

$$x_{it} - x_{i(t-1)} \le \overline{\mathrm{RU}}_i u_{it} + \mathrm{RU}_i y_{i(t-1)}, \quad \forall t\in\mathcal{T}, \forall i\in\mathcal{G}, \tag{2f}$$

$$x_{i(t-1)} - x_{it} \le \overline{\mathrm{RD}}_i v_{it} + \mathrm{RD}_i y_{it}, \quad \forall t\in\mathcal{T}, \forall i\in\mathcal{G}, \tag{2g}$$

$$(\theta_{mt} - \theta_{nt})/X_{mn} - f_{mn}^t + (1-z_{mn})M_{mn} \ge 0, \quad \forall (m,n)\in\mathcal{E}, \forall t\in\mathcal{T}, \tag{2h}$$

$$(\theta_{mt} - \theta_{nt})/X_{mn} - f_{mn}^t - (1-z_{mn})M_{mn} \le 0, \quad \forall (m,n)\in\mathcal{E}, \forall t\in\mathcal{T}, \tag{2i}$$

$$\sum_{i\in\mathcal{G}_n} x_{it} + \sum_{m\in\mathcal{E}_{(\cdot,n)}} f_{mn}^t - \sum_{m\in\mathcal{E}_{(n,\cdot)}} f_{nm}^t + q_{nt}^+ - q_{nt}^- = D_{nt} - W_{nt}, \quad \forall t\in\mathcal{T}, \forall n\in\mathcal{N}. \tag{2j}$$

In formulation (1)–(2) described above, the objective function (1a) seeks to minimize the worst-case expected total cost including the scheduling cost (i.e., costs for start-up, shut-down, and keeping generators on) and the operating cost $Q(y,u,v,z)$ including the fuel cost and the penalty for load shedding and over-generation. Note that the penalty term $P_n^- q_{nt}^-$ is mainly to facilitate the mathematical modeling, and q_{nt}^- equals to zero at optimality in all instances we test in Section IV. As compared to the robust CCUC models ([15], [22]) that hedge against contingency realizations $\{z_i, z_{mn}\}$, formulation (1)–(2) hedges against their probability distribution $\mathbb{P}$. As a consequence, the proposed approach is distributionally robust and can be less conservative than the robust CCUC approaches. In formulation (1), constraints (1b) (respectively, (1c)) describe the generator minimum up-time (respectively, minimum down-time) restriction, and constraints (1d) (respectively, (1e)) describe the generator start-up (respectively, shut-down) operations. In formulation (2), $F_i(x_{it})$ represents the fuel cost of generator i in period t for generating x_{it} amount of electricity. We use piece-wise linear functions to approximate the fuel cost such that

$$F_i(x_{it}) \ge \gamma_i^\ell y_{it} + \beta_i^\ell x_{it}, \quad \forall \ell = 1,\ldots,L, \tag{3}$$

where L represents the number of pieces. Constraints (2b)–(2d) describe the bounds for generation amounts, transmission flow amounts, and phase angles, respectively. Constraints (2e) and (2f) describe the ramping restrictions on the generation amounts, including the start-up and shut-down ramping restrictions. Constraints (2g) and (2h) describe the power flow based on the dc approximation, i.e., $z_{mn}(\theta_{mt} - \theta_{nt})/X_{mn} = f_{mn}^t$, and linearize this equation based on a sufficiently large constant M_{mn}. For example, we can set $M_{mn} := \max\{(\bar{\theta}_n - \underline{\theta}_m)/X_{mn}, (\bar{\theta}_m - \underline{\theta}_n)/X_{mn}\}$. Finally, constraints (2i) represent the balance between generation and load at each bus.

A. Abstract Formulation

For notation brevity, we present formulation (1)–(2) in a compact form as follows:

$$\min_{y} \quad a^\top y + \sup_{\mathbb{P} \in \mathcal{D}} E_{\mathbb{P}}[Q(y, z)] \tag{4a}$$

$$\text{s.t.} \quad Ay \leq b, \ y \in \{0, 1\}^{3|\mathcal{G}||\mathcal{T}|}, \tag{4b}$$

where

$$Q(y, z) = \min_{x} \quad c^\top x \tag{5a}$$

$$\text{s.t.} \quad Bx + Cy \geq d, \tag{5b}$$

$$Ex + Fz \geq f, \tag{5c}$$

$$Gx \geq g, \tag{5d}$$

$$x \geq D(z)y, \tag{5e}$$

where $z = [z_i, z_{mn}]^\top$ represents the (random) contingency scenarios, y represents the first-stage decision variables (y, u, v), x represents the second-stage decision variables (x, q, θ, f), constraint (4b) represents constraints (1b)–(1f), constraint (5b) represents constraints (2e)–(2f), constraint (5c) represents constraints (2c) and (2g)–(2h), constraint (5d) represents constraints (2d) and (2i), and constraint (5e) represents constraints (2b). In constraint (5e), $D(z)$ represents a $(2|\mathcal{T}||\mathcal{G}|) \times (3|\mathcal{T}||\mathcal{G}|)$ matrix such that $[D(z)]_{\text{jj}} = \text{LB}_i z_i$ and $[D(z)]_{(j+|\mathcal{T}||\mathcal{G}|),j} = -\text{UB}_i z_i$ if $i = j \bmod |\mathcal{G}|$ for all $j = 1, \ldots, |\mathcal{T}||\mathcal{G}|$, while all the other components of $D(z)$ equal to zero. Meanwhile, vector $y \in \mathbb{R}^{3|\mathcal{T}||\mathcal{G}|}$ is such that $y = [y_{11}, \ldots, y_{|\mathcal{G}|,1}, y_{12}, \ldots, y_{|\mathcal{G}|,2}, \ldots, y_{|\mathcal{G}|,|\mathcal{T}|}, u^\top, v^\top]^\top$.

B. Ambiguity Set Construction

We consider an ambiguity set (denoted as $\mathcal{D}$) consisting of contingency probability distributions that match the $N - k$ security criterion and the available moment information. More precisely,

$$\mathcal{D} := \{\mathbb{P} \in \mathcal{M}_+(\Omega) : \ \mathbb{E}_{\mathbb{P}}[Sz] \leq \bar{\mu}\}, \tag{6}$$

where $\bar{\mu}$ represents a vector of estimated mean values, matrix S represents pre-specified coefficients, and $\mathcal{M}_+(\Omega)$ represents the set of all probability distributions on a sigma-algebra of Ω, the support of random vector z. We designate Ω based on the $N - k$ contingency criterion, i.e.,

$$\Omega := \left\{ z \in \mathbb{R}^{|\mathcal{H}|} : \sum_{i \in \mathcal{H}} z_i \geq N - k, z_i \in \{0, 1\}, \forall i \in \mathcal{H} \right\}. \tag{7}$$

Note that we adopt a support of contingencies where no more than k components can simultaneously fail. Conditioned on Ω, the ambiguity set $\mathcal{D}$ can be interpreted as containing all conditional probability distributions that match the moment information in (6). In practice, the value of k can be selected by the power system operators or calibrated based on historical data so that the probability of having more than k simultaneous failures is very low. Nonetheless, if k is set to be N, then Ω contains all possible contingencies. In the above definition of $\mathcal{D}$, we incorporate moment information of contingency z by accordingly designating matrix S and mean estimates $\bar{\mu}$. Although presented in a simple form (6), $\mathcal{D}$ can cover moment information that is often available in practice or can be inferred from historical data. As the proposed model utilizes the moment information of contingencies, instead of only the support Ω as in the robust CCUC approaches, our approach can lead to less conservative UC decisions. We illustrate (6) in the following three examples.

1. **(Marginal Contingency Probability)** The component failure probabilities (denoted as p) can often be obtained from historical data (see, e.g., [11], [12]). We can incorporate the marginal contingency probabilities by designating $S = [I, -I]^\top \in \mathbb{R}^{2|\mathcal{H}| \times |\mathcal{H}|}$ and $\bar{\mu} = [e - p, p - e]^\top \in \mathbb{R}^{2|\mathcal{H}|}$, where $e \in \mathbb{R}^{2|\mathcal{H}|}$ represents a vector of all ones. In this case, the ambiguity set $\mathcal{D}$ can be recast as
$$\mathcal{D} = \{\mathbb{P} \in \mathcal{M}_+(\Omega) : \ \mathbb{E}_{\mathbb{P}}[1 - z_i] = p_i, \ \forall i \in \mathcal{G},$$
$$\mathbb{E}_{\mathbb{P}}[1 - z_{mn}] = p_{mn}, \ \forall (m, n) \in \mathcal{E}\},$$
where p_i and p_{mn} represent the failure probability estimates of generator i and transmission line (m, n), respectively.
2. **(Contingency Probability Confidence Interval)** When the failure probabilities are close to zero, confidence intervals (denoted as $[p^{\text{L}}, p^{\text{U}}]$) are often more reliable than point estimates. We can incorporate the confidence intervals by designating $S = [-I, I]^\top \in \mathbb{R}^{2|\mathcal{H}| \times |\mathcal{H}|}$ and $\bar{\mu} = [p^{\text{U}} - e, e - p^{\text{L}}]^\top \in \mathbb{R}^{2|\mathcal{H}|}$. In this case, the ambiguity set $\mathcal{D}$ can be recast as
$$\mathcal{D} = \{\mathbb{P} \in \mathcal{M}_+(\Omega) : \ p_i^{\text{L}} \leq \mathbb{E}_{\mathbb{P}}[1 - z_i] \leq p_i^{\text{U}}, \ \forall i \in \mathcal{G},$$
$$p_{mn}^{\text{L}} \leq \mathbb{E}_{\mathbb{P}}[1 - z_{mn}] \leq p_{mn}^{\text{U}}, \ \forall (m, n) \in \mathcal{E}\},$$
where $[p_i^{\text{L}}, p_i^{\text{U}}]$ and $[p_{mn}^{\text{L}}, p_{mn}^{\text{U}}]$ represent the confidence intervals of $\mathbb{E}_{\mathbb{P}}[1 - z_i]$ and $\mathbb{E}_{\mathbb{P}}[1 - z_{mn}]$, respectively.
3. **(Zone Contingency Rate)** With insufficient historical failure data, the zone contingency rates, describing the likelihood of contingencies taking place in geographical neighborhoods, can often be obtained based on system operators' domain knowledge. Suppose that the set of components $\mathcal{H}$ is geographically partitioned into n_H zones, i.e., $\mathcal{H} = \mathcal{H}_1 \cup \mathcal{H}_2 \cup \cdots \cup \mathcal{H}_{n_H}$, we can incorporate the zone contingency rates by designating matrix $S \in \mathbb{R}^{n_H \times |\mathcal{H}|}$ such that for all $i = 1, \ldots, n_H$, row $S_i = [-\mathbb{1}_{1 \in \mathcal{H}_i}, -\mathbb{1}_{2 \in \mathcal{H}_i}, \ldots, -\mathbb{1}_{|\mathcal{H}| \in \mathcal{H}_i}]$ indicates all components within zone $\mathcal{H}_i$, where $\mathbb{1}_{h \in \mathcal{H}_i} = 1$ if $h \in \mathcal{H}_i$ and $\mathbb{1}_{h \in \mathcal{H}_i} = 0$ otherwise for all $h = 1, \ldots, |\mathcal{H}|$. In this case, the ambiguity set $\mathcal{D}$ can be recast as
$$\mathcal{D} = \left\{ \mathbb{P} \in \mathcal{M}_+(\Omega) : \mathbb{E}_{\mathbb{P}} \left[\sum_{h \in \mathcal{H}_i} (1 - z_h) \right] \right.$$
$$\left. \leq s_i, \ \forall i = 1, \ldots, n_H \right\},$$
where s_i represents an upper bound of the expected number of contingencies in zone $\mathcal{H}_i$.

III. Solution Methodology

In this section, we derive a solution approach for solving the DR-CCUC model (4)–(5). First, we note that the linear program

(5) (i.e., formulation (2)) pertaining to $Q(y,z)$ is always feasible due to the load-shedding q^+ and over-generation curtailment q^-. Indeed, for any given UC $(\hat{y},\hat{u},\hat{v})$ and contingency $\hat{z}$, solution $(\hat{x},\hat{q},\hat{\theta},\hat{f})$ is feasible to formulation (5) with $\hat{x}_{it} = \text{LB}_i \hat{y}_{it}\hat{z}_i$, $\hat{f}^t_{mn} = 0$, $\hat{\theta}_{nt} = 0$, $\hat{q}^+_{nt} = \max\{D_{nt} - W_{nt} - \sum_{i\in\mathcal{G}_n} \hat{x}_{it}, 0\}$, and $\hat{q}^-_{nt} = \max\{\sum_{i\in\mathcal{G}_n} \hat{x}_{it} - D_{nt} + W_{nt}, 0\}$. Second, we rewrite the worst-case expectation $\sup_{\mathbb{P}\in\mathcal{D}} \mathbb{E}_{\mathbb{P}}[Q(y,z)]$ as an optimization problem

$$\sup_{\mathbb{P}\in\mathcal{D}} E_{\mathbb{P}}[Q(y,z)] = \max_{\mathbb{P}} \int_\Omega Q(y,z)\mathbb{P}(dz) \tag{8a}$$

$$\text{(S1)}\quad \text{s.t.} \int_\Omega Sz\,\mathbb{P}(dz) \le \bar{\mu}. \tag{8b}$$

$$\int_\Omega \mathbb{P}(dz) = 1, \tag{8c}$$

where constraints (8b)–(8c) represent $\mathbb{P}\in\mathcal{D}$. Note that if we disregard moment constraint (8b), i.e., if the ambiguity set only incorporates the $N-k$ security criterion, then formulation (S1) assigns probability 1 to the worst-case scenario in Ω. In that case, DR-CCUC reduces to the robust CCUC approach. Based on standard duality techniques (see, e.g., [34]), $\sup_{\mathbb{P}\in\mathcal{D}} \mathbb{E}_{\mathbb{P}}[Q(y,z)]$ equals to the optimal objective value of the following dual formulation

$$\min_{\tau\ge 0,\alpha} \ \bar{\mu}^\top \tau + \alpha,$$

$$\text{(D1)}\quad \text{s.t.}\ \tau^\top Sz + \alpha \ge Q(y,z),\ \ \forall z\in\Omega,$$

where dual variables τ and α are associated with constraints (8b) and (8c), respectively. We substitute this dual formulation for $\mathbb{E}_{\mathbb{P}}[Q(y,z)]$ into formulation (4)–(5) to obtain a reformulation as follows:

$$\min_{y,\tau\ge 0,\alpha} \ a^\top y + \bar{\mu}^\top\tau + \alpha \tag{9a}$$

$$\text{s.t.}\ (4b), \tag{9b}$$

$$\alpha \ge Q(y,z) - \tau^\top Sz,\ \ \forall z\in\Omega. \tag{9c}$$

Based on reformulation (9a)–(9c), we employ a Benders' decomposition (BD) framework for solving the DR-CCUC model. For any $z\in\Omega$, we call inequality (9c) a Benders' cut pertaining to z. In our BD framework, we start with a relaxation of reformulation (9a)–(9c) by incorporating only a subset of Benders cuts (9c). The BD framework then iteratively adds in more Benders cuts and obtains stronger relaxations till the stopping criterion is satisfied. We summarize the BD framework as follows.

0. Input: lower bound LB $:= -\infty$, upper bound UB $:= +\infty$, optimality gap tolerance ϵ, iteration number limit L, set of Benders' cuts $\texttt{CUT} := \emptyset$.
1. For $\ell = 1,\ldots,L$, repeat the following steps:
   (a) Solve the master problem

$$\min_{y,\tau\ge 0,\alpha} \ a^\top y + \bar{\mu}^\top\tau + \alpha \tag{10a}$$

$$\text{s.t.}\ (4b), \tag{10b}$$

   with the current set of Benders' cuts in CUT as additional constraints. Record optimal solutions $(y^\ell,\tau^\ell,\alpha^\ell)$, and set LB equal to the optimal objective function value.

   (b) Solve the separation problem

$$\max_{z\in\Omega} \Big\{ Q(y^\ell,z) - \big(\tau^\ell\big)^\top Sz \Big\}. \tag{11}$$

   Record optimal solution z^ℓ and the optimal objective function value V^ℓ, and set UB equal to LB $-\,\alpha^\ell + V^\ell$.

   c) If $|\text{UB} - \text{LB}|/\text{LB} < \epsilon$ or $\alpha^\ell \ge V^\ell$, then return and output y^ℓ as an optimal solution; otherwise, go to the next step.

   d) Add a Benders' cut $\alpha \ge Q(y,z^\ell) - \tau^\top Sz^\ell$ into set CUT.

Second, we elaborate Steps 1.(b) and 1.(d) in the BD framework. In the remainder of this section, we derive Benders' cut (9c) in both dual and primal forms.

A. Dual Benders' Cuts

We first take the dual of linear program (5) to yield

$$Q(y,z) = \max_{\lambda,\eta,\delta,\kappa} \ (d - Cy)^\top\lambda + (f - Fz)^\top\eta + g^\top\delta + y^\top D(z)^\top\kappa \tag{12a}$$

$$\text{s.t.}\ B^\top\lambda + E^\top\eta + G^\top\delta + \kappa \le c, \tag{12b}$$

$$\lambda,\eta,\delta,\kappa \ge 0, \tag{12c}$$

where dual variables λ, η, δ, and κ are associated with primal constraints (5b)–(5e), respectively. It follows that the separation problem (11) can be recast as

$$\max_{z,\lambda,\eta,\delta,\kappa} \ \big(d - Cy^\ell\big)^\top\lambda + (f - Fz)^\top\eta + g^\top\delta + \big(y^\ell\big)^\top D(z)^\top\kappa - \big(\tau^\ell\big)^\top Sz \tag{13a}$$

$$\text{s.t.}\ (12b)-(12c) \tag{13b}$$

$$\sum_{i\in\mathcal{H}} z_i \ge N - k,\ z_i\in\{0,1\},\ \forall i\in\mathcal{H}. \tag{13c}$$

For fixed y^ℓ and τ^ℓ, the objective function (13a) contains bilinear terms $\eta^\top Fz$ and $D(z)^\top\kappa$ between binary variable z and the dual variables. We can linearize these terms by using the McCormick inequalities:

$$\zeta_{ij} \ \ge\ F_{ij}\eta_i - M(1 - z_j),\ \ \zeta_{ij} \ge -Mz_j,\ \ \forall i,j \tag{14a}$$

$$\pi^+_{it} \ \ge\ \kappa^+_{it}\text{UB}_i - M(1 - z_i),\ \ \pi^+_{ij} \ge -Mz_i,\ \ \forall i,t \tag{14b}$$

$$\pi^-_{it} \ \le\ \kappa^-_{it}\text{LB}_i + M(1 - z_i),\ \ \pi^-_{ij} \le Mz_i,\ \ \forall i,t. \tag{14c}$$

where ζ_{ij} represents $\eta_i F_{ij} z_j$, π^+_{it} represents $\kappa^+_{it}\text{UB}_i z_i$, π^-_{it} represents $\kappa^-_{it}\text{LB}_i z_i$, and M represents a sufficiently large constant. Note that the linearization (14a)–(14c) are exact because $z_i\in\{0,1\}$ for all $i\in\mathcal{H}$ and so formulation (13a)–(13c) is equivalent to a mixed-integer linear program (MILP) that facilitates the off-the-shelf software (e.g., CPLEX). For notation brevity, we let $\zeta := \eta^\top Fz$ and $\pi := D(z)\kappa$, and present the obtained MILP as follows:

$$\max \ \left(d - Cy^\ell\right)^\top \lambda + f^\top \eta - \zeta + g^\top \delta + \left(y^\ell\right)^\top \pi - \left(\tau^\ell\right)^\top Sz \quad (15a)$$

$$\text{s.t.} \ (12b)-(12c), (13c), (14a)-(14c). \quad (15b)$$

In the dual Benders' cuts, we present $Q(y, z)$ in its dual form in constraint (9c). Accordingly, we present Steps 1.(b) and 1.(d) in the BD framework as follows:

1.(b-Dual) Solve the separation problem (15a)–(15b). Record optimal solution $(z^\ell, \lambda^\ell, \eta^\ell, \delta^\ell, \kappa^\ell)$ and the optimal objective function value V^ℓ, and set UB equal to LB $- \alpha^\ell + V^\ell$.

1.(d-Dual) Add a Benders' cut

$$\alpha \ \geq \ \left(D(z^\ell)\kappa^\ell - C^\top \lambda^\ell\right)^\top y + d^\top \lambda^\ell + (f - Fz^\ell)^\top \eta^\ell + g^\top \delta^\ell - (Sz^\ell)^\top \tau$$

into set CUT.

B. Primal Benders' Cuts

The primal Benders' cuts present $Q(y, z)$ based on its primal formulation (5). Accordingly, in the ℓth iteration of the BD framework, adding a primal Benders' cut generates a new set of second-stage decision variables x^ℓ pertaining to z^ℓ, the optimal solution to the separation problem (11). We present Steps 1.(b) and 1.(d) in the BD framework as follows:

1.(b-Primal) Solve the separation problem (15a)–(15b). Record optimal solution (z^ℓ) and the optimal objective function value V^ℓ, and set UB equal to LB $- \alpha^\ell + V^\ell$.

1.(d-Primal) Add a Benders' cut

$$\begin{aligned} \alpha \ &\geq \ c^\top x^\ell - \left(Sz^\ell\right)^\top \tau \\ Bx^\ell + Cy \ &\geq \ d, \\ Ex^\ell + Fz^\ell \ &\geq \ f, \\ Gx^\ell \ &\geq \ g, \\ x^\ell \ &\geq \ D(z^\ell)y \end{aligned}$$

into set CUT.

In the case studies reported in Section IV, we add both primal and dual Benders' cuts in Step 1.(d) to accelerate the BD framework.

IV. Case Study

In this section, we perform case studies on a 6-bus system and the IEEE 118-bus system to test the proposed DR-CCUC approach. We compare the DR-CCUC with the classical robust CCUC approach on both systems. All the experiments are implemented in the C++ language with CPLEX 12.1 on a computer workstation with 4 Intel Cores and 8 GB RAM. The time interval for all the experiments is 1 hour.

A. 6-Bus System

The 6-bus system is composed of three thermal generators, six loads, and eight transmission lines. The layout of the system is shown in Fig. 1 and the characteristics of the buses, the loads, and the transmission lines are displayed in Tables I–IV. Note that we approximate the quadratic fuel cost curves displayed in Table III by using piece-wise linear functions as described in (3). For this system, we consider the $N-2$ security criterion (i.e., we set $k = 2$ in the definition of Ω in (7)) and set the unit penalty costs $P_n^+ = P_n^- = \$1500$ per MWh [22]. First, we study the performance of DR-CCUC under various system-wide contingency rates. To this end, we set $\mathcal{D}_1 = \{\mathbb{P} \in \mathcal{M}_+(\Omega) : \mathbb{E}_\mathbb{P}[\sum_{h \in \mathcal{H}} z_h] \geq N - m\}$, where m represents the system-wide mean number of contingencies and varies from 0.1 to 2. Note that throughout this section, $N-k$ security criterion applies to both DR-CCUC and the robust CCUC approach and parameter m is only used to obtain solutions with DR-CCUC. Note that the likelihood of contingency grows as the value of m increases, and the DR-CCUC model reduces

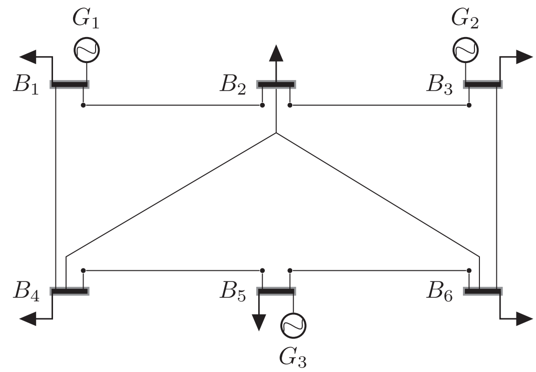

Fig. 1. A Modified 6-bus system.

TABLE I
BUS DATA

Bus	Unit	Hourly Load (MW)
B_1	G_1	20
B_2	-	20
B_3	G_2	20
B_4	-	20
B_5	G_3	20
B_6	-	20

TABLE II
GENERATOR DATA

Unit	Lower (MW)	Upper (MW)	Min. down (h)	Min. up (h)	Ramp (MW/h)
G_1	50	100	2	3	50
G_2	100	150	3	4	75
G_3	20	50	2	1	25

TABLE III
FUEL DATA, $F_i(x_{it}) = $ FUEL PRICE $\times \left(cx_{it}^2 + bx_{it} + a\right)$

Unit	a (MBtu)	b (MBtu/MWh)	c (MBtu/MW^{2}h)	Start-Up Fuel (MBtu)	Fuel Price ($/MBtu)
G_1	50	6.0	0.0004	100	1.2469
G_2	40	5.5	0.0001	300	1.2461
G_3	60	4.5	0.0050	50	1.2462

TABLE IV
TRANSMISSION LINE DATA

Line	From	To	Reactance	Flow Limit (MW)
L_1	B_1	B_2	0.189	100
L_2	B_1	B_4	0.163	100
L_3	B_2	B_3	0.247	200
L_4	B_2	B_4	0.190	100
L_4	B_2	B_6	0.185	100
L_5	B_3	B_6	0.137	150
L_6	B_4	B_5	0.152	200
L_7	B_5	B_6	0.155	100

TABLE V
COMPUTATIONAL RESULTS FOR THE 6-BUS SYSTEM WITH SYSTEM-WIDE CONTINGENCY RATES

m	OU	CTG	ETC ($)	SIM ($)
0.1	G_1, G_2	Buses 1, 3	159445	176384
RO	G_1, G_2, G_3	Buses 1, 3	763457	575265
0.5	G_1, G_2	Buses 1, 3	353348	176384
RO	G_1, G_2, G_3	Buses 1, 3	763457	575265
1	G_1, G_2	Buses 1, 3	595728	176384
RO	G_1, G_2, G_3	Buses 1, 3	763457	575265
1.5	G_1, G_2, G_3	Buses 1, 3	700789	503905
RO	G_1, G_2, G_3	Buses 1, 3	763457	575265
2	G_1, G_2, G_3	Buses 1, 3	763457	575265
RO	G_1, G_2, G_3	Buses 1, 3	763457	575265

to the robust CCUC model when $m = 2$ because of the $N - 2$ security criterion. In Table V, we report the optimal online units (OU), the failed components pertaining to the worst-case contingency distributions (CTG), the worst-case expected total costs (ETC), and the simulated costs (SIM). In particular, we conduct an out-of-sample simulation to test the optimal solutions of the DR-CCUC and the robust CCUC approaches, respectively. In this simulation, we assume that each component failure follows a Bernoulli distribution independently with identical component failure rate 0.01 for each generator and 0.001 for each transmission line [35], and we randomly generate 5000 contingency scenarios. Then, we obtain the simulated costs by averaging the total costs in each scenario. From Table V, we observe that the worst-case contingency distribution always involves buses 1 and 3 (see CTG). This indicates that buses 1 and 3 are most vulnerable in the 6-bus system. Meanwhile, we observe that units G_1 and G_2 are online during all time periods. When $m \leq 1$, G_3 is offline during all time periods. When $m > 1$, G_3 is online during period 1, offline during periods 2–3, and online again starting from period 4. This indicates that unit G_3 serves as a reserve of the system, and can become online if the contingency rate is sufficiently high. In addition, by comparing the results of DR-CCUC and those of the robust CCUC, we observe that DR-CCUC is less conservative than the robust CCUC according to SIM (saving up to 70% when m is low) and the online units (one less online unit).

Second, we fix $m = 1$ and study the performance of DR-CCUC under various individual contingency probability. To this end, we consider $\mathcal{D}_2 = \mathcal{D}_1 \cap \{\mathbb{P} \in \mathcal{M}_+(\Omega) : \mathbb{E}_\mathbb{P}[1 - z_{h^*}] \leq p_{h^*}\}$, where h^* is a selected component and p_{h^*} represents its failure probability. The out-of-sample simulation also follows

TABLE VI
COMPUTATIONAL RESULTS FOR THE 6-BUS SYSTEM WITH INDIVIDUAL CONTINGENCY PROBABILITIES

Component (h^*)	p_{h^*}	OU	CTG	ETC ($)	SIM ($)
Bus 1	0.1	G_1, G_2	Buses 1, 3	456981	168325
	RO	G_1, G_2, G_3	Buses 1, 3	763457	549551
	0.3	G_1, G_2	Buses 1, 3	526355	219631
	RO	G_1, G_2, G_3	Buses 1, 3	763457	562885
	0.5	G_1, G_2	Buses 1, 3	595728	341733
	RO	G_1, G_2, G_3	Buses 1, 3	763457	562997
Bus 3	0.1	G_1, G_2	Buses 1, 3	466109	152060
	RO	G_1, G_2, G_3	Buses 1, 3	763457	566186
	0.3	G_1, G_2	Buses 1, 3	530919	236917
	RO	G_1, G_2, G_3	Buses 1, 3	763457	593023
	0.5	G_1, G_2	Buses 1, 3	595728	254534
	RO	G_1, G_2, G_3	Buses 1, 3	763457	607037
Others	0.1	G_1, G_2	Buses 1, 3	595728	145918
	RO	G_1, G_2, G_3	Buses 1, 3	763457	545553
	0.3	G_1, G_2	Buses 1, 3	595728	139519
	RO	G_1, G_2, G_3	Buses 1, 3	763457	483034
	0.5	G_1, G_2	Buses 1, 3	595728	142453
	RO	G_1, G_2, G_3	Buses 1, 3	763457	442517

this assumption. That is, we assume that the failure of the selected component h^* follows a Bernoulli distribution independently with component failure rate p_{h^*} and the total expected number of failures of other components is $m - p_{h^*}$. We report the performance of DR-CCUC in Table VI. From this table, we observe that for all instances, the simulated costs of DR-CCUC are significantly smaller than the ones of RO. It indicates that the consideration of probability information of DR-CCUC can significantly reduce the cost, as compared with RO, which totally disregards such information. Also, the worst-case contingency distribution only involves buses 1 and 3, and the simulated costs increases as p_{h^*} increases for these two buses, which indicates that for the critical components, e.g., buses 1 and 3, the information of component failures is important to reduce the costs. For other non-critical components, with the increment of their failure rate, the total expected failures of critical components is reduced, which yields to a reduced simulated cost of RO. In addition, the worst-case expected total costs increase as p_{B_1} and p_{B_3} grow and become stable when $p_{B_1}, p_{B_3} > 0.5$. This indicates that, when $p_{B_1}, p_{B_3} > 0.5$, the worst-case contingency distribution assigns $\mathbb{P}\{z_{B_1} = 1\} = \mathbb{P}\{z_{B_3} = 1\} = 0.5$, which tightly satisfies the system-wide contingency rate constraint $\mathbb{E}_\mathbb{P}[\sum_{h\in\mathcal{H}} z_h] \geq N - m$ (i.e., $\mathbb{E}_\mathbb{P}[\sum_{h\in\mathcal{H}} z_h] = N - m$) but strictly satisfies the individual contingency probability constraint $\mathbb{E}_\mathbb{P}[1 - z_{h^*}] \leq p_{h^*}$ (i.e., $\mathbb{E}_\mathbb{P}[1 - z_{h^*}] < p_{h^*}$). As this worst-case distribution remains the same when $p_{B_1}, p_{B_3} > 0.5$, it makes sense that the worst-case expected total costs do not change.

Third, we fix $m = 1$ and study the performance of DR-CCUC under various zone contingency rates. To this end, we consider $\mathcal{D}_3 = \mathcal{D}_1 \cap \{\mathbb{P} \in \mathcal{M}_+(\Omega) : \mathbb{E}_\mathbb{P}[\sum_{h\in\mathcal{H}^*} (1 - z_h)] \leq s\}$, where $\mathcal{H}^*$ represents a selected zone and s represents its contingency rate. In this simulation, we follow the same assumption of the system-wide contingency rate test, that is, we assume that each component failure follows a Bernoulli distribution independently with identical component failure rate 0.01 for each generator and 0.001 for each transmission line. We report the performance of DR-CCUC in Table VII. From this table, we

TABLE VII
COMPUTATIONAL RESULTS FOR THE 6-BUS SYSTEM WITH ZONE CONTINGENCY RATES

Zone	s	OU	CTG	ETC ($)	SIM ($)
(Bus 1, 3)	0	G_1, G_2	Lines L_2, L_5	324310	176384
	0.5	G_1, G_2	Buses 1, 3	460019	176384
	1	G_1, G_2	Buses 1, 3	595728	176384
(Bus 1, 5)	0	G_1, G_2	Bus 3	422294	176384
	0.5	G_1, G_2	Buses 1, 3	595728	176384
	1	G_1, G_2	Buses 1, 3	595728	176384
(Lines L_2, L_4)	0	G_1, G_2	Buses 1, 3	595728	176384
	0.5	G_1, G_2	Buses 1, 3	595728	176384
	1	G_1, G_2	Buses 1, 3	595728	176384

TABLE VIII
COMPUTATIONAL RESULTS FOR THE 6-BUS SYSTEM WITH DIFFERENT SECURITY CRITERION

k	s	OU	CTG	ETC ($)	SIM ($)
1	DR-CCUC	G_1, G_2	Bus 3	142101	176384
	Robust	G_1, G_2	Bus 3	431081	176384
2	DR-CCUC	G_1, G_2	Buses 1, 3	159445	176384
	Robust	G_1, G_2, G_3	Buses 1, 3	763457	575365
3	DR-CCUC	G_1, G_2	Buses 1, 3	159445	176384
	Robust	None	Buses 1, 3, 5	1.08×10^6	1.08×10^6

observe that the simulated costs remain stable with various $\mathcal{H}^*$ and s, which indicates that system-wide contingency rate mainly decides the out-of-sample performance. Also, we observe that the worst-case expected total costs are the same when (i) $\mathcal{H}^* = \{B_1, B_3\}$ and $s = 1$, (ii) $\mathcal{H}^* = \{B_1, B_5\}$ and $s \geq 0.5$, and (iii) $\mathcal{H}^* = \{L_2, L_4\}$ and $s \in [0, 1]$. In contrast, the worst-case expected total costs become different in all other settings in Table VII. This confirms that the worst-case contingency distribution assigns $\mathbb{P}\{z_{B_1} = 1\} = \mathbb{P}\{z_{B_3} = 1\} = 0.5$ as long as $\mathbb{P} \in \mathcal{D}_3$, because $\mathbb{P} \in \mathcal{D}_3$ in settings (i)–(iii) and $\mathbb{P} \notin \mathcal{D}_3$ in all other settings.

Now, we show the system performances of both DR-CCUC and robust CCUC approaches under different security criteria, i.e., different value of k. We set $m = 0.1$ and report the online units, the failed components, the worst expected costs, and the simulated costs for both two approaches in Table VIII. When $k = 1$, although the worst-case cost obtained by DR-CCUC is less than the one of the robust CCUC, the UC decisions obtained by two approaches are the same. When $k = 2$, the UC decisions remain the same for DR-CCUC approach, however, for the robust CCUC approach, one more bus is put online. This indicates that with the consideration of moment information of contingencies, the conservativeness level of DR-CCUC remains unchanged even with more severe worst-case contingency scenarios (i.e., more components can fail simultaneously). In contrast, the conservativeness level of the robust CCUC increases as k increases. The case when $k = 3$ further verifies this observation. The UC decisions for the DR-CCUC approach still remain unchanged. In contrast, contingencies happen at all buses for the robust CCUC approach, and so no buses are put online and load shedding occurs at all buses.

TABLE IX
COMPUTATIONAL RESULTS FOR THE 118-BUS SYSTEM WITH TOTAL EXPECTED CONTINGENCIES

m	ETC ($)	SIM ($)	CPU Time (s)
0.1	1.1372×10^7	1.2003×10^7	982
RO	1.1381×10^7	1.2876×10^7	1054
0.5	1.1376×10^7	1.2003×10^7	947
RO	1.1381×10^7	1.2876×10^7	1054
0.9	1.1380×10^7	1.2243×10^7	997
RO	1.1381×10^7	1.2876×10^7	1054

B. 118-Bus System

We extend our study to the IEEE 118-bus system that contains 118 buses, 33 generators, and 186 transmission lines. We first consider the $N - 1$ security criterion and study the performance of DR-CCUC under various system-wide contingency rates. The unit penalty costs P_n^+ and P_n^- are set to be \$1500 per MWh. In the out-of-sample simulation, we assume that each component failure follows an independent Bernoulli distribution with identical component failure rate 0.01 for each generator and 0.001 for each transmission line [35], and we randomly generate 5000 contingency scenarios. We report the worst-case expected total costs and the simulated costs, together with the CPU time spent for solving each instance in Table IX. From this table, we observe that the CPU time of all instances are within 1800s. To increase the scalability of the proposed BD algorithm and solve large-sized instances, parallel computing techniques can be employed to solve the master problem and/or separation problem on a high-performance computing platform (see, e.g., [36]). Meanwhile, we observe that the solutions of DR-CCUC performs less conservative in the out-of-sample simulation than those of robust CCUC based on the simulated costs. This confirms our observations from the test instances based on the 6-bus system.

V. CONCLUSION

In this paper, we proposed a DR-CCUC approach that simultaneously considers the $N - k$ security criterion and the contingency probability distribution. Like the robust CCUC, the proposed approach can handle exponentially many contingency scenarios. Meanwhile, DR-CCUC could reduce the conservativeness by utilizing the moment information of contingencies (e.g., component failure probability, failure probability confidence intervals, etc). Furthermore, we derived a BD framework for solving the proposed model based on primal and dual Benders' cuts. Finally, the case studies based on the 6-bus system and the IEEE 118-bus system verified the reduced conservativeness of the proposed approach. By testing the proposed DR-CCUC approach, we find that incorporating moment information of contingencies can improve the out-of-sample performance of the UC decisions. Promising future research directions include (i) applying the BD algorithm on high-performance computing platforms to solve large-sized instances, (ii) extending the formulation to the ac power flow, (iii) incorporating the combined cycle plant heat rate characteristics with forbidden power areas, and (iv) exploring ambiguity sets based on the probability of having k contingencies, i.e., $\mathbb{P}\{\sum_{h\in\mathcal{H}}(1-z_h)=k\}$, for all $k=1,\ldots,|\mathcal{H}|$.

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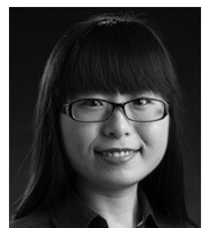

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