Homework #7

1. Review carefully the logic used in deriving the distribution for L and E[L]. After reviewing the derivation, close your notes from class and re-derive both expressions. Assume you have a friend who doesn't understand the derivation and as you derive it, explain the various steps. (The derivation of the distribution is very similar to many more derivations we will do in renewal theory.)

2. Consider the pedestrian delay problem for the case when the arrival times for cars is a Poisson process with mean rate of 2 per minute. Assume you need a 30 second break to cross the street. What is the expected time (in minutes) that you will have to wait before being able to cross the street? (My answer was between 0.3 and 0.4 minutes.)

3. Simulate the process thus giving yourself some confidence in your answer. (Use any software package you want for simulation: Excel, Simio, MatLab, etc.)

Distribution for time between cars: $\varphi(t) = 1 - \exp(-2t)$ for $t >= 0$. The distribution for the transient inter-renewal times is

$$F(t) = \varphi(t) = 1 - e^{-2t} \text{ for } t <= 0.5 \text{ minutes, and } F(t) = \varphi(0.5) = 1 - e^{-1} \text{ for } t >= 0.5 \text{ minutes.}$$

$$\int (F(\infty) - F(t))\, dt = \int (1 - e^{-1} - 1 + e^{-2t})\, dt = \int (e^{-2t} - e^{-1})\, dt = 0.5 - e^{-1} = 0.13212$$

Limits of integration on first integral are 0 to infinity. Limits for the next two integrals are 0 to ½ since $F(\infty) - F(t)$ is zero for $t > 0.5$.

Therefore, $E[L] = 0.13212 / e^{-1} =$ 0.3591.