STAT 611 600: Theory of Inference **Spring 2021**

Homework 4: due Thursday, March 11, 2021, 11:59 pm CDT

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Instructions:

- Whether you write out the solution by hand or in a text document, be sure that they are neat, legible and in order (even if you choose to solve them in different order). We highly recommend that you write your solutions in **LaTeX** and print them to a **PDF file**.
- Write/Type your name, UIN at the top of the first page. Otherwise, your submission will not be graded.
- Either scan or print your solutions to a **PDF file** under 15MB in size. It must be in a single file, not separate files for separate pages. Do not take a photo of each page and then paste them into a document - this will make your file too big and the results will generally not be very readable anyway.
- All students should login to their eCampus account to upload your file. You must do this by **11:59 pm U.S. Central time**, on the due date. You can make multiple submissions, but only the last submission will be graded.
- Write down all of your problem-solving process and cite any resources you have used in addition to lecture notes and the textbook.
- It is prohibited to share or distribute the content in this document.

1. Ex 7.57 in C&B. (To get your best unbiased estimator, you may need to consider several different cases.)

2. Ex 7.58 in C&B.
   Additional part: Repeat (b) and (c) provided that you now have $X_1, \ldots, X_n$, n iid observations. Simplify your expression for the best unbiased estimator.

3. (UMVUE may be inadmissible) Let X and Y be iid exponential random variables with mean $1/\theta$ or equivalently with density

$$f(t;\theta) = \begin{cases} \theta e^{-\theta t}, & t \geq 0; \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find the UMVUE for the mean $1/\theta$. Justify your answer using the attainment theorem for the UMVUE.

   (b) Show that for all θ, the geometric mean $(XY)^{1/2}$ has smaller MSE than the UMVUE.

4. **Optional exercises**: 7.37, 7.46, 7.49 in C&B.