In Class Problem

March 30, 2020

Consider a machine repair problem. The life time of an electronic component in a machine is according to an exponential random variable with mean one week. The component is difficult to replace and it takes, on the average, one day to replace the component although the time to replace it is actually according to an exponential distribution. (Assume the company that uses the machine is open 24/7.) Let $Z(t)$ be a process with state space $\{w, r\}$ so that $Z(t) = w$ if the component is working at time t and $Z(t) = r$ if the component is being replaced at time t. Let weeks be the time units.

(**Note**: the process $\{Z(t)\}$ is also a regenerative process so there is a choice in which method to use in analyzing this machine-replacement process. Today, we practice our knowledge of Markov processes.)

1. Notice that $\{Z(t)\}$ is a Markov process. Form the generator matrix for this process.
2. What is $\lim_{t\to\infty} P\{Z(t) = w \mid Z(0) = w\}$?
3. Given that the component is working at the beginning of today, what is the probability that it will be working at the beginning of tomorrow?
4. Given that the component is working at the beginning of today, what is the probability that it will be working at the beginning of next week?