Rank(A[n], k)

time

\\ return the k-th smallest in A[n]

1. if (n ≤ 50)

   brute-force, return — $O(1)$

2. divide A[n] into groups of 5 elements;

   and find the median for each group. — $O(n)$

3. $m^* = \text{Rank}(\text{group-medians}, \frac{n}{10})$ — $T(\frac{n}{5})$

   " median of group medians

4. partition A[n] into $A_{\le m^*}$ and $A_{>m^*}$ — $O(n)$

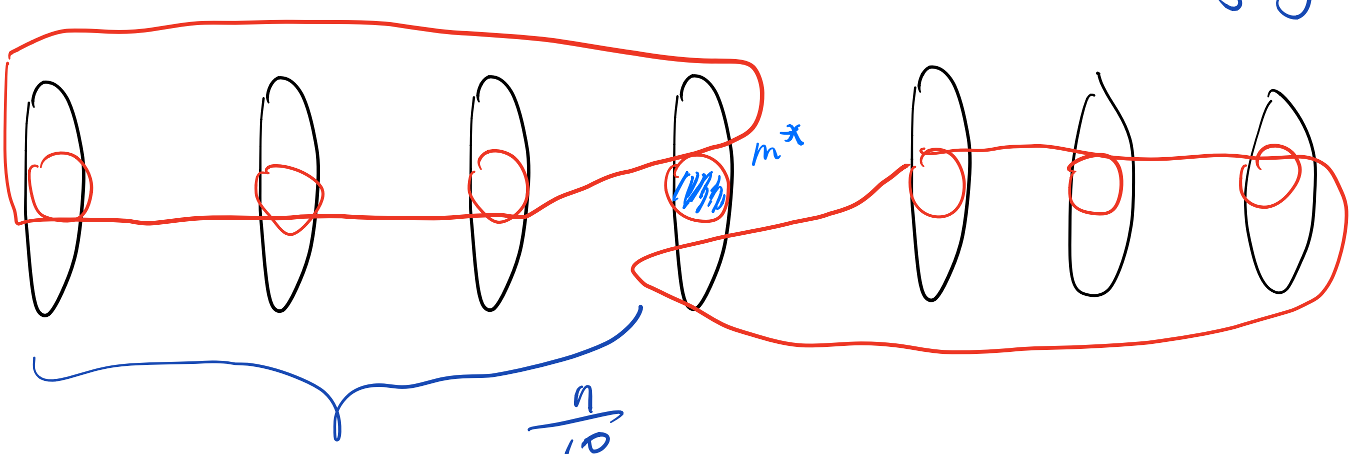
5. if ($|A_{\le m^*}| \ge k$)

   return Rank($A_{\le m^*}$, k)

   $|A_{\le m^*}|, |A_{>m^*}| \le \frac{7n}{10}$

   else return Rank($A_{>m^*}$, $k - |A_{\le m^*}|$) — $T(\frac{7n}{10})$

$\frac{n}{5}$ groups



$m^* \geq \frac{3n}{10}$ elements

$\leq \frac{3n}{10}$ elements.

assume the alg takes time $T(n)$

$T(n) = O(n) + T(\frac{n}{5}) + T(\frac{7n}{10}) \leq Cn + 10 \cdot C \cdot \frac{n}{5} + 10C\frac{7n}{10}$

$= 10cn$

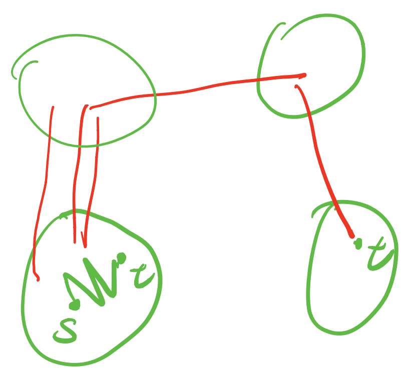
$= O(n)$

$n \to \frac{n}{5} + \frac{7n}{10} = \frac{9n}{10}$

$T(n) \leq Cn + T(\frac{n}{5}) + T(\frac{7n}{10})$

$T(n) \leq C$ for $n \leq 50$

guess $T(n) \leq 10cn$



Max-bandwidth Path

time

1. remove the smallest half of edges. $O(m)$

2. if [s and t are still in the same piece] $O(m)$
   recursively work on the piece (with $\le \frac{m}{2}$ edge)

3. else. shrink each piece into a single vertex
   connect the new vertices by smaller edges.
   (if there are parallel, remove all except the one with max bandwidth.) (m)

4. recursively work on the new graph (with $\le \frac{m}{2}$ edge)

$$T(m) \le Cm + T\left(\frac{m}{2}\right)$$

$$T(m) \le 2 \cdot C \cdot m$$

Input $(a_1, a_2 \ldots, a_n)$

$$\Downarrow$$

$$((a_1, 1), (a_2, 2), \ldots, (a_n, n))$$