Homework #10

***Instructions**:* This is a derivation so hand in a *neat* copy of your derivation at the start of class on Wednesday, Feb 26.

Write on one side only. Put your name in the top right on the first page and the top left on the back side of the last page. You receive 5 points for simply trying. There will also be a deduction of 2 points if you don't follow instructions (write on one side only, name on front of first page and back of last page.)

Interval Reliability. Consider an item installed at time 0. When it fails, it is replaced by an identical item; when that item fails, it in turn is replaced by a new item; and so on. Suppose the lifetime of successive items are i.i.d. random variables $U_1, U_2, \ldots$ with distribution function given by $\varphi(.)$. Let the time to replace the items be denoted by i.i.d. random variables $V_1, V_2, \ldots$ with distribution function given by $\psi(.)$. Further assume that the lifetimes and replacement times are independent.

Let $f_x(t)$ be the probability that the item is in working condition at time t and will remain in working condition at least x more units of time.

a. Obtain an expression for $f_x(t)$.

P{working at t, will continue to work for x more time units, and $S_1>t$} = $P\{U_1 > t+x\} = 1 - \varphi(t+x)$

Therefore, $f_x(t) = 1 - \varphi(t+x) + \int_{[0,t]} F(ds) f_x(t-s) = 1 - \varphi(t+x) + \int_{[0,t]} m(ds)\,[1 - \varphi(t+x-s)]$

Where $F = \varphi * \psi$ and $m = \Sigma F_n$

b. Obtain an expression for $\lim_{t\to\infty} f_x(t)$. Let $a=E[U_1]$ and $b=E[V_1]$.

$\lim_{t\to\infty} f_x(t) = (1/(a+b)) \int_0^\infty [1-\varphi(t+x)]dt = (1/(a+b)) \int_x^\infty [1-\varphi(u)]du$