

# CSCE-629 Analysis of Algorithms

Fall 2019

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## Assignment # 6 (Due December 3, 2019)

1. A *vertex cover* in an undirected graph  $G$  is a set  $C$  of vertices in  $G$  such that every edge in  $G$  has at least one end in  $C$ . Consider the following two versions of the VERTEX-COVER problem:

VC-D: Given a graph  $G$  and an integer  $k$ , decide whether  $G$  contains a vertex cover of at most  $k$  vertices.

VC-O: Given a graph  $G$ , construct a minimum vertex cover for  $G$

Prove: VC-D is solvable in polynomial time if and only if VC-O is solvable in polynomial time.

2. Prove that the VC-D problem given in Question 1 is in  $\mathcal{NP}$ .

3. Using the fact that the INDEPENDENT SET problem is  $\mathcal{NP}$ -complete, prove that the following problem is  $\mathcal{NP}$ -complete:

CLIQUE: Given a graph  $G$  and an integer  $k$ , is there a set  $C$  of  $k$  vertices in  $G$  such that for every pair  $v$  and  $w$  of vertices in  $C$ ,  $v$  and  $w$  are adjacent in  $G$ ?

4. Prove: if the problem VC-O is solvable in polynomial time then  $\mathcal{P} = \mathcal{NP}$ . *Hint:* you may use the result in Question 1.