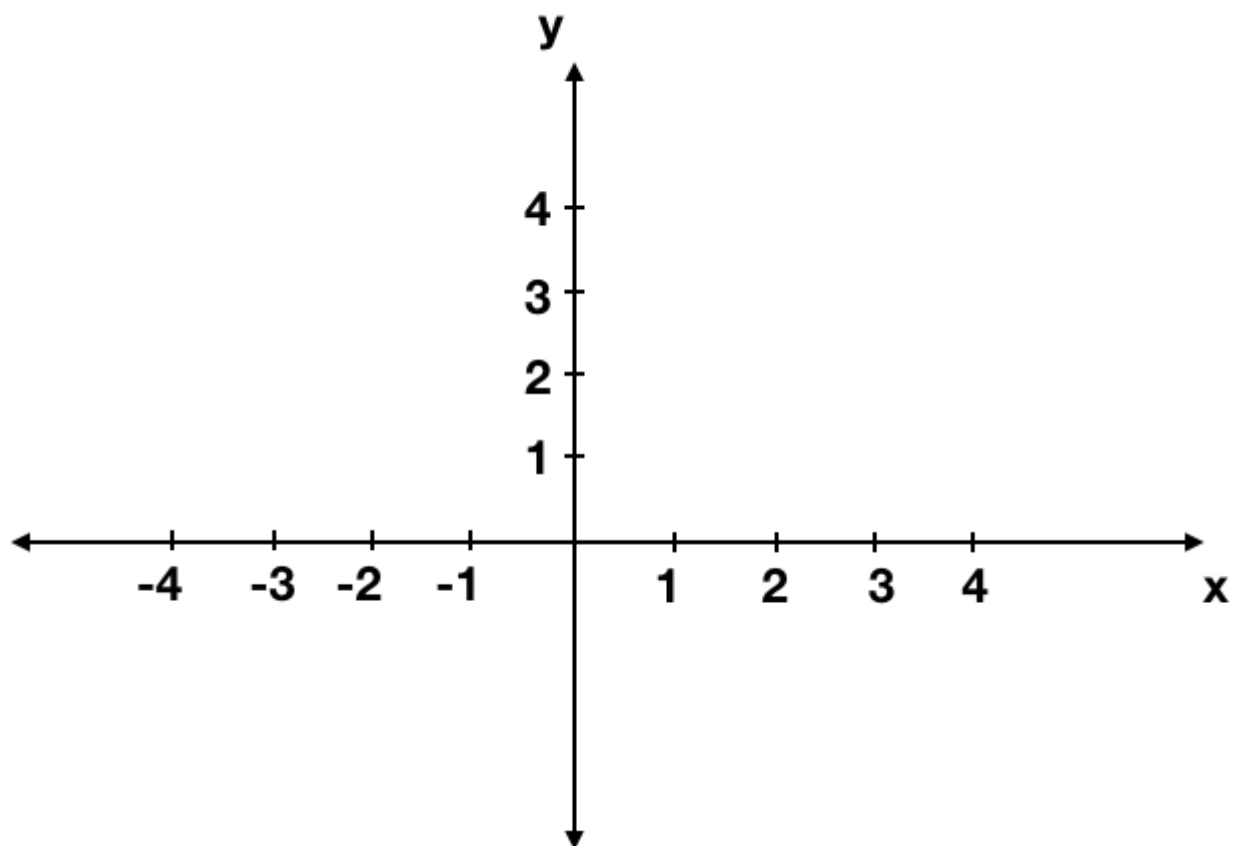


The goal of this problem is to run a linear perceptron algorithm. Assume that you have three training samples:

1. Sample $\mathbf{x}_1 = [1, 3]^T$ from Class 1 ($y_1 = 1$)
2. Sample $\mathbf{x}_2 = [3, 2]^T$ from Class 2 ($y_2 = -1$)
3. Sample $\mathbf{x}_3 = [4, 1]^T$ from Class 2 ($y_3 = -1$)

The linear perceptron is initialized with a line with corresponding weight $\mathbf{w}(\mathbf{0}) = [-\frac{1}{3}, 1]^T$. In the following, for the sake of simplicity, you will assume that all lines of the perceptron intersect point $(0, 0)$, therefore **you do not have to include any intercept w_0 or x_0 in the following calculations.**



(1) Plot \mathbf{x}_1 , \mathbf{x}_2 , and \mathbf{x}_3 in the given 2D space. Find and plot the line corresponding to weight $\mathbf{w}(\mathbf{0})$.

(2) Using the rule $\text{sign}(\mathbf{w}(\mathbf{t})^T \mathbf{x}_n)$, please indicate in which class are samples \mathbf{x}_1 , \mathbf{x}_2 , and \mathbf{x}_3 classified using the weight $\mathbf{w}(\mathbf{0})$. Which samples are not correctly classified based on this rule? **Note:** You have to compute the inner product $\mathbf{w}(\mathbf{0})^T \mathbf{x}_n$, $n = 1, 2, 3$, and see if it is greater or less than 0.

(3) Using the weight update rule from the linear perceptron algorithm, please find the value of the new weight $\mathbf{w}(\mathbf{1})$. Find and plot the new line corresponding to weight $\mathbf{w}(\mathbf{1})$ in the 2D space.

Note: The update rule is $\mathbf{w}(\mathbf{t} + \mathbf{1}) = \mathbf{w}(\mathbf{t}) + y_s \mathbf{x}_s$, where \mathbf{x}_s and $y_s \in \{-1, 1\}$ is the feature and class label of misclassified sample s .

(4) Using the rule $\text{sign}(\mathbf{w}(\mathbf{t})^T \mathbf{x}_n)$, please indicate in which class are samples \mathbf{x}_1 , \mathbf{x}_2 , and \mathbf{x}_3 classified using the weight $\mathbf{w}(\mathbf{1})$. Which samples are not correctly classified based on this rule?

Note: You have to compute the inner product $\mathbf{w}(\mathbf{1})^T \mathbf{x}_n$, $n = 1, 2, 3$, and see if it is greater or less than 0.

(5) Using the weight update rule from the linear perceptron algorithm, please find the value of the new weight $\mathbf{w}(\mathbf{2})$. Find and plot the new line corresponding to weight $\mathbf{w}(\mathbf{2})$ in the 2D space. How many samples are correctly classified now?

Note: The update rule is $\mathbf{w}(\mathbf{t} + \mathbf{1}) = \mathbf{w}(\mathbf{t}) + y_s \mathbf{x}_s$, where \mathbf{x}_s and $y_s \in \{-1, 1\}$ is the feature and class label of misclassified sample s .