Homework #19

Due by 7AM, Monday, April 27

***Instructions*:** Do your work on your own paper and give only the numerical answers in eCampus. Some questions in eCampus will be multiple choice. For numerical questions, give your answers rounded to the **nearest integer value,** except as instructed otherwise.

Using the maintenance problem given in the example, we wish to maximize long-run average profits. Profits are determined by revenue minus costs, where revenue is a function of the state but not of the decision and is given by the function $\mathbf{r} = (900, 400, 450, 750)^T$. The costs and transition probabilities are as presented in the example.

1. What is the optimum policy?
2. What is the long-run average profit using the optimum policy? (Round to the nearest integer.)

$\mathbf{a}_0 = (1,1,1,1) \Rightarrow \varphi^* = 408.18$ and $\mathbf{h} = (0, -95.84, 7.16, 176.42)$

$a_1(a) = \text{maxarg}\{408.18, 408.69\}$; $a_1(b) = \text{maxarg}\{-95.84, -27.09\}$
$a_1(c) = \text{maxarg}\{7.16, -0.80\}$; $a_1(d) = \text{maxarg}\{176.42, 99.60\}$
Thus, $\mathbf{a}_1 = (2, 2, 1, 1) \Rightarrow \varphi^* = 423.47$ and $\mathbf{h} = (0, -447.60, -422.46, -242.48)$

$a_2(a) = \text{maxarg}\{412.24, 423.47\}$; $a_2(b) = \text{maxarg}\{-98.50, -24.13\}$
$a_2(c) = \text{maxarg}\{1.01, 10.48\}$; $a_2(d) = \text{maxarg}\{180.99, 105.24\}$
Thus, $\mathbf{a}_2 = (2, 2, 2, 1) \Rightarrow \varphi^* = 424.32$ and $\mathbf{h} = (0, -23.32, 10.47, 180.89)$

$a_3(a) = \text{maxarg}\{417.40, 424.32\}$; $a_3(b) = \text{maxarg}\{-94.48, -23.32\}$
$a_3(c) = \text{maxarg}\{2.07, 10.47\}$; $a_3(d) = \text{maxarg}\{180.89, 105.24\}$
Thus, $\mathbf{a}_3 = (2, 2, 2, 1) \Rightarrow$ STOP with $\varphi^* = 424.32$

Note that if you start with the Markov matrix, P, and profit vector, f, associated with the action function $\mathbf{a}_3$, you should be able to determine the long-run steady-state probabilities, $\boldsymbol{\pi}$, and then calculate the long-run average profit by $\boldsymbol{\pi}\mathbf{f}$. This should result in $\boldsymbol{\pi}\mathbf{f} = 424.32$.