Name:____________________________________
Full name

Test Two

Open Book, Open Notes

You may use the internet, including http://integral-table.com, if necessary

***Instructions**:* This test represents your independent work. You may not talk to anyone else about this test until after you have turned it in. Answer the questions neatly so that I can read your work. Please return a scan or a photo of the test to me at richf@tamu.edu after insuring that the scan or photo will be readable. The test must be sent to me before 10:00 AM; thus, you have 2 hours for the test. (Every minute past 10:00 will cost five points.) If you have any questions regarding the meaning of a problem, make what you feel is an appropriate assumption and write down your assumption. You will then be graded on the correctness of the problem based on your assumption and the appropriateness of the assumption. Since I cannot see what you enter into your calculator, it is important to show all your work on the test. Guesses do not count, so if your work is not present, I'll assume a lucky guess which deserves no credit. Answers to the parts of Problem 1 should be rounded and accurate to three digits to the right of the decimal. Put a box around your answers to insure I see your results.

1. Consider a renewal process where the distribution function for the inter-renewal time is given by

$$F(t) = 1 - \exp(-3t) - 3t\exp(-3t) \qquad \text{for } t \geq 0.$$

Using the fact that the Laplace-Stieltjes transform of the distribution function, F, is given by $(3/(3+s))^2$, it is possible to show that the renewal function is given by

$$m(t) = 1.5t - 0.25\times(1 - \exp(-6t)) \qquad \text{for } t \geq 0.$$

(I give the Laplace transform in case you are curious as to the type of distribution that F is. From the Laplace transform you should see that F is an Erlang type-2 distribution. This was FYI only; you do not need to know anything about the Laplace-Stieltjes transform to work the problem.)

Assume that the time units for t is in terms of *days*. In addition, assume each day has three 8-hour shifts with a shift starting at midnight. Call the shift starting at midnight Shift 1. Answer the following questions with respect to this renewal process.

a. What is the expected number of renewals during Shift 1 on the initial day?

b. What is the long-run expected number of renewals per shift? Hint: consider Blackwell's Thm.

c. Time t=0 refers to midnight. Assume it is now at the beginning of the first shift change, i.e., it is the now 8AM, the beginning of Shift 2 on the initial day.

i. What is the expected length of time, in hours, until the next renewal?

ii. Using a 12-hour clock (i.e., using AM/PM) give the expected time at which the next renewal is expected to occur, rounded to the nearest minute.

d. What is the long-run probability that there will be at least one renewal during a given shift? Another way to ask the same question is as follows: assume we are at the some point in time in the "far" future, what is the probability that the time until the next renewal is less than or equal to eight hours from now?

2. Passengers arrive to a train station according to a Poisson process having rate λ. A train (which is large enough to pick up all waiting passengers) arrives at the train station in accordance to a recurrent renewal process with an aperiodic inter-arrival distribution F. At a train arrival time, all passengers waiting will get onto the train. Let X(t) denote the number of passengers waiting at the station at time t. Assume at time 0, a train just left and thus the train station is empty.

a. What type of process is $\{X(t);\, t \geq 0\}$?

b. Obtain an expression for $P\{X(t) = k\}$ for $t \geq 0$ and $k \geq 0$.

c. Obtain an expression for $\lim_{t\to\infty} P\{X(t) = k\}$ for $k \geq 0$.