STAT 611 600: Theory of Inference **Spring 2021**

Homework 3: due Thursday, February 11, 2021, 11:59 pm CDT

Professor: Tiandong Wang *Name:* *UIN:*

Instructions:

- Whether you write out the solution by hand or in a text document, be sure that they are neat, legible and in order (even if you choose to solve them in different order). We highly recommend that you write your solutions in **LaTeX** and print them to a **PDF file**.
- Write/Type your name, UIN at the top of the first page. Otherwise, your submission will not be graded.
- Either scan or print your solutions to a **PDF file** under 15MB in size. It must be in a single file, not separate files for separate pages. Do not take a photo of each page and then paste them into a document - this will make your file too big and the results will generally not be very readable anyway.
- All students should login to their eCampus account to upload your file. You must do this by **11:59 pm U.S. Central time**, on the due date. You can make multiple submissions, but only the last submission will be graded.
- Write down all of your problem-solving process and cite any resources you have used in addition to lecture notes and the textbook.
- It is prohibited to share or distribute the content in this document.

1. Ex 7.10 in C&B.
2. Ex 7.11 in C&B.
3. The article The Load-Life Relationship for M50 Bearings with Silicon Nitride Ceramic Balls (Lubraication Engr., 1984: 153159) reports the accompanying data on bearing load life (million revs.) for bearings tested at a 6.45 kN load.

   47.1 126.0 289.0 68.1 146.6 289.0 68.1 90.8 229.0 240.0 367.0 385.9 103.6 106.0 240.0 278.0 392.0 505.0 115.0 278.0

   This data is given in LoadLife.csv.

   a. Construct a normal probability plot. Is normality plausible?
   b. Construct a Weibull probability plot. The pdf of a Weibull random variable is:

$$f(x;a,b) = \frac{a}{b}\left(\frac{x}{b}\right)^{a-1} e^{-(x/b)^a}, \qquad x \geq 0,$$

   where the two parameters are (shape, scale) $= (a, b)$. Is the Weibull distribution family plausible? Method of moments may be hard in this case since the Weibull mean and variance involve the Gamma function. But the distribution function is explicit so can compute the quartiles. Try the following (somewhat crude) method for getting reasonable estimates.

      i. For fixed (a, b), find the quartile of order p, $F^{\leftarrow}(p; a, b)$ for Weibull with (shape, scale) $= (a, b)$. This can be found explicitly.
      ii. Find the sample median and 3rd quartile of the data set. (Running `summary(data)` will yield this information in R).
      iii. Equate

$$\begin{aligned} F^{\leftarrow}(1/2; a, b) &= \text{ Data Median} \\ F^{\leftarrow}(3/4; a, b) &= \text{ Data 3rd Quartile.} \end{aligned}$$

      iv. Solve for a, b from the two equations in 2 unknowns to get estimates $(\hat{a}, \hat{b})$. You have effectively equated theoretical median and 3rd quartile to the sample versions to solve for the parameters.
      v. Make a Weibull QQ plot using $(\hat{a}, \hat{b})$. Does it look worse than the normal QQ plot?

4. **Optional exercises**: 7.1, 7.2 & 7.6 in C& B.