STAT 611 600: Theory of Inference	**Spring 2021**

Homework 1: due Thursday, January 28, 2021, 11:59 pm CDT

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Instructions:

- Whether you write out the solution by hand or in a text document, be sure that they are neat, legible and in order (even if you choose to solve them in different order). We highly recommend that you write your solutions in **LaTeX** and print them to a **PDF file**.

- Write/Type your name, UIN at the top of the first page. Otherwise, your submission will not be graded.

- Either scan or print your solutions to a **PDF file** under 15MB in size. It must be in a single file, not separate files for separate pages. Do not take a photo of each page and then paste them into a document - this will make your file too big and the results will generally not be very readable anyway.

- All students should login to their eCampus account to upload your file. You must do this by **11:59 pm U.S. Central time**, on the due date. You can make multiple submissions, but only the last submission will be graded.

- Write down all of your problem-solving process and cite any resources you have used in addition to lecture notes and the textbook.

- It is prohibited to share or distribute the content in this document.

Optional exercises are not compulsory and will not be graded. They are just for practice, and no solutions will be provided.

1. Exercise 6.2 in C&B.

2. Exercise 6.7 in C&B.

3. In survival analysis, the observed data often happens to be right-censored. In other words, for n iid observations, $X_1, \ldots, X_n$, generated from some distribution with pdf $f(x; \theta)$, the right-censored data at level $c > 0$ becomes
$$Y_i := \min\{X_i, c\}, \qquad i = 1, \ldots, n,$$
where $c > 0$ is a known constant.

   Suppose that $X_1, \ldots, X_n$ are iid exponentially distributed with pdf $f_X(x; \lambda) = \lambda e^{-\lambda x}$, $x \geq 0$. Then $Y_1, \ldots, Y_n$ are said to be iid observations from the *censored exponential distribution*.

   (a) Find the pdf of Y_1, i.e. $f(y; \lambda, c)$. (Hint: You may start with finding the cdf of Y_1.)

   (b) Based on the results in part (a), show that the censored exponential distribution belongs to the exponential family and find its natural parametrization. Is this exponential family curved or full-rank?

   (c) Find the sufficient statistics.

4. **Optional exercises**: Exercises 6.3, 6.5, 6.16 (a)&(b) in C&B.