Final Exam Overview

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Tips for Final

To enhance **readability** of your answers:

- Try to avoid pencils; Black pen is the easiest to read.
- Write big; Scrawls are painful to read.
- Mark your response to every part of the questions clearly.
- Check your scanned copy before submission (you could try mobile apps like Adobe Scan).

Time management:

- You'll have 165 mins for the final, as the original duration was 2 hrs. With an additional 45 mins, you should be able to complete scanning, checking and uploading.
- Check out the cover page of the final exam on eCampus.
- Calculators are not necessary for the exam.

Your Feedback Matters!

- Please fill out the course evaluation forms via

  `https://yourvoice.tamu.edu/Menu/Student-Course-Evaluations/AEFIS`

- AEFIS closes on May 2.
- Current response rate: 73%. Target rate: 70%.

Data reduction

- Sufficient statistic
- How to find a sufficient statistic
  - **(Neyman-Fisher) Factorization theorem. Theorem 6.2.6**
  - Exponential family (Theorem 6.2.10)
  - Transformation
- Minimal Sufficient Statistics
  - Theorem 6.2.13
  - Exponential family
- Complete Statistics
  - Definition?
  - Exponential family

Methods for finding MLEs

- Look at a graph of the likelihood function.
- Calculus method: Take partial derivative of log-likelihood with respect to each parameter and solve the likelihood equations.
- Invariance property of MLEs: MLE of a function of θ is the function applied to MLE of θ (Theorem 7.2.10)

Important Concepts in Bayes Estimation

- prior distribution of θ: $\theta \sim \pi(\theta)$. Conjugate priors: The posterior is in the same family as the prior regardless of the data observed.
- posterior distribution of θ: $\pi(\theta|\mathbf{x}) = f(\mathbf{x}|\theta)\pi(\theta)/m(\mathbf{x})$
- marginal distribution of $\mathbf{x}$: $m(\mathbf{x}) = \int f(\mathbf{x}|\theta)\pi(\theta)d\theta$
- posterior mean of θ: $E(\theta|\mathbf{X}) = \int \theta\pi(\theta|\mathbf{x})d\theta$ (Bayes estimator of θ)

Methods of Evaluating Estimators

- Mean squared error: MSE equals estimator variance plus estimator squared bias.
- Best Unbiased Estimators (UMVUE) (Definition 7.3.7)
- **Cramér-Rao Lower Bound** (Theorem 7.3.9)
- Score Function and Fisher Information (one parameter/multiple parameters), exponential family. (Lemma 7.3.11)
- Attainment of C-R Bound (Corollary 7.3.15), exponential family.
- Sufficiency and UMVUE: Rao-Blackwell theorem (Theorem 7.3.17 and Theorem 7.3.23)

Loss Function Optimality

- Loss function
- Risk function
- Bayesian risk function
- Bayes rule (squared loss and absolute loss)

Hypothesis testing

- Power function
- Type I, II error
- Size/Level of a test
- LRT; Rejection regions; p-values.
- Critical value for a given size α/Sample Size Calculation

- Unbiased test
- Uniformly most powerful test(UMP level α test)
  - Neyman-Pearson lemma (simple test).
  - Neyman-Pearson based on sufficient statistic (simple test).
  - Neyman-Pearson lemma for randomized tests.
  - Karlin-Rubin Theorem for MLR family (composite test). Examples of MLR family: exponential with non decreasing $w(\theta)$.
- Multiple hypothesis testing.

How to construct confidence intervals

Method 1: By inverting the acceptance region of tests;
Method 2: Using pivotal quantities.
$Q(\mathbf{X}, \theta)$ is independent of θ

- How to find pivotal quantity?
- Find a,b such that $P_\theta(a \leq Q(X, \theta) \leq b) = 1 - \alpha$
- Pivoting the CDF of sufficient statistic

- Consistency: definition, how to check consistency? (MLE is consistent)
- Asymptotic normality (CLT, Delta method)
- Asymptotic efficiency, i.e., the asymptotic variance of T_n achieves the Cramer-Rao lower bound). Review: how to calculate Cramér-Rao lower bound?
- Asymptotic relative efficiency. Definition?
- Large sample hypothesis tests; Wilks' theorem; Contingency tables.

- Thanks a lot for your support throughout the semester, and
- Good luck with everything in the future!!!