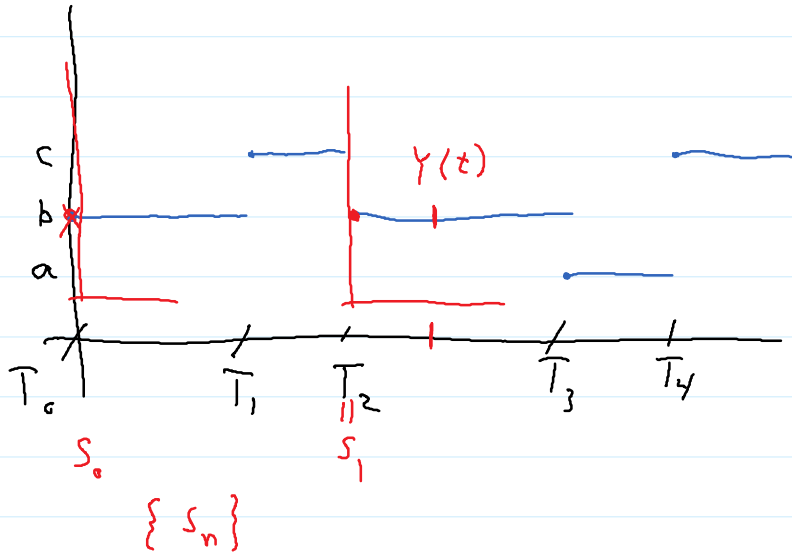
# Markov Processes

Context: $\{Y(t)\}$ is a Markov process with finite state space and no instantaneous states

Saturday, March 21, 2020 11:14 AM




- $Y(t) \rightarrow$ state at time $t$

$$P_t(i,j) = P\{Y(t)=j \mid Y(0)=i\}$$

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$$P = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} \qquad tr(P) = 0.6 + 0.8 = 1.4$$

$$\lambda_1 = 1 \qquad \lambda_2 = 0.4$$

$$v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix}$$

$\pi P = \pi$

$\sum \pi(j) = 1$

$\pi = \left(\frac{1}{3}, \frac{2}{3}\right)$

$$\lim_{k \to \infty} P^k = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

rate of convergence is proportional to $(0.4)^k$

$$P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.2 & 0.5 & 0.3 \\ 0 & 0.4 & 0.6 \end{bmatrix} \qquad P^2 = \begin{bmatrix} 0.04+0.06 & ? & ? \\ ? & 0.06+0.25+0.12 & ? \\ ? & ? & 0.12+0.36 \end{bmatrix}$$

$tr(P) = 0.2 + 0.5 + 0.6 = 1.3$

$tr(P^2) = 1.01$

$\lambda_1 = 1$

$1 + \lambda_2 + \lambda_3 = 1.3$

$\lambda_2 = 0.3 - \lambda_3$

$1 + \lambda_2^2 + \lambda_3^2 = 1.01$

quadratic

$\lambda_3 = 0.15 \pm i\sqrt{0.0175}$

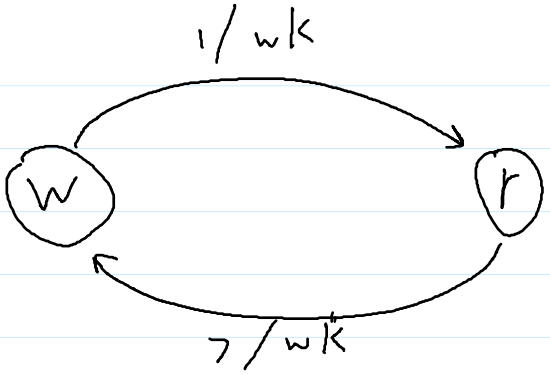
$\lambda_2 = 0.15 - i\sqrt{0.0175}$

$\lambda_3 = 0.15 + i\sqrt{0.0175}$

$|\lambda_2| = \sqrt{0.15^2 + 0.0175}$

$= 0.2$

$(0.2)^k$




$$G = \begin{array}{c} \\ w \\ r \end{array} \begin{array}{c} \begin{array}{cc} w & r \end{array} \\ \begin{bmatrix} -1 & 1 \\ 7 & -7 \end{bmatrix} \end{array}$$

$\lambda_1 = 0 \qquad \lambda_2 = -8$

$-1\,p_w + 7\,p_r = 0 \Rightarrow p_w = 7\,p_r$

$p_w + p_r = 1 \Rightarrow 8\,p_r = 1 \Rightarrow p = \left(\frac{7}{8}, \frac{1}{8}\right)$

$$P\left\{ Y\left(\tfrac{1}{7}\right) = w \;\middle|\; Y(0) = w \right\} = e^{G \times \frac{1}{7}}(w, w)$$

$v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 1 \\ -7 \end{pmatrix}$