**Homework #2**

Class assignment given Jan 15, 2020

***Round all answers to three digits to the right of the decimal.***

1. Let {Xn; n=1, 2, …} by an i.i.d. stochastic process, with Xn representing profit from customer n. Let N be a Poisson random variable independent of {Xn} representing the number of customers per day. The random variable X1 has a gamma distribution with shape parameter =4 and ~~shape~~ scale parameter =25.

Let profit be denoted by S; in other words, S = X1 + … + XN

* 1. Give E[X1]. Mean of gamma =  = 100
  2. Give E[X12]. Variance of gamma = 2=2500. E[X12]=var(X1) + E[X1]2=12,500.
  3. If the average number of customers per day is 5, what is the mean profit per day?

E[S] = 5\*100 = 500

* 1. If the average number of customers per day is 5, what is the coefficient of variation of the profit per day? First note that mean and variance for Poisson random variable are equal.

Var(S) = E[N] var(X1) + var(N) E[X1]2 = 62,500 🡺 c.v. = SQRT(62,500)/500=0.5

* 1. If the average number of customers per day is 50, what is the mean profit per day?

E[S] = 50\*100 = 5000

* 1. If the average number of customers per day is 50, what is the coefficient of variation of the profit per day?

Var(S) = E[N] var(X1) + var(N) E[X1]2 = 625,000 🡺 c.v. = SQRT(625,000)/5000=0.158

Note: The coefficient of variation is standard deviation divided by the mean.

1. Let X have distribution function given by F and let Y have a distribution function given by G. Assume that X and Y nonnegative random variables mutually independent of each other. Finally, let Z = X – Y have distribution function H. Using the fact that E[ P{ X – Y <= z | Y} ] = H(z) for real value z, obtain an expression for H(**.**).

Use your expression to calculate the value of

* 1. P{X – Y <= 0.5}
  2. P{X – Y <= -0.5}

Where X and Y are both uniformly (continuous) distributed between 0 and 1

First note that P{X – Y <= z | Y=y} = P{X <= y+z} = F(y+z); therefore,

E[ P{ X – Y <= z | Y} ] = ∫ F(y+z) G(dy), where the limits of integration are 0 and 1.

Observe that dG(y) = dy for 0<=y<=1; otherwise dG(y) = 0.

Also, F(t)=0 for t<0, =t for 0<= t <1, and =1 for t >=1.

1. For P{X – Y <= 0.5}=\int\_{y=0}^{0.5} F(y+0.5)dy + \int\_{y=0.5}^{1} F(y+0.5) dy

= \int\_{y=0}^{0.5} (y+0.5) dy + \int\_{y=0.5}^{1} (1) dy = 7/8 = 0.875

1. For P{X – Y <= –0.5}=\int\_{y=0}^{0.5} F(y-0.5)dy + \int\_{y=0.5}^{1} F(y-0.5) dy

= 0 + \int\_{y=0.5}^{1} (y-0.5) dy = 1/8 = 0.125