K.8

PHP

Peubah Acak Diskret

- bisa dicacah, bilangan bulat
  contoh: banyak orang di kelas.

1. Peubah acak Bernoulli

ex:

① dadu dilempar 1 kali.

$P(B) = \frac{2}{6} = \frac{1}{3}$ ~ Berhasil

$P(G) = \frac{4}{6} = \frac{2}{3}$ ~ gagal

$P(X=x) \begin{cases} x=0 , & 2/3 \\ x=1 , & 1/3 . \end{cases}$

0 selainnya

- $E(x) = p = \frac{1}{3}$

$V(x) = p(1-p) = \frac{1}{3}(1-\frac{1}{3}) = \frac{2}{9}$

2. Peubah acak Binomial.

- Kejadian bernoulli, yang diulang ulang. Jika antar kejadian tidak saling mempengaruhi 2 peluang sukses sama.

$$\boxed{P(X=x) = \binom{n}{x} p^x \cdot q^{n-x}}$$

- $E(x) = np \quad V(x) = 1 - p$

ex:

① 10 soal, setiap soal ada 4 pilihan

a. $P(x=5) = \binom{10}{5} \frac{1}{4}^5 \cdot \left(\frac{3}{4}\right)^5$

$= 0,1615146.$

b. $P(x \geq 3) = 1 - P(x=2) - P(x=1) - P(x=0)$

$= 1 - \left[\binom{10}{0}\frac{1}{4}^0 \frac{3}{4}^{10} + \binom{10}{1}\frac{1}{4}^1\left(\frac{3}{4}\right)^9 + \binom{10}{2}\frac{1}{4}^2\frac{3}{4}^8\right]$

$= 0,763912$

3. Peubah Acak Geometrik

1. Berapa banyak percobaan sampai terjadi 1 sukses.

$$\boxed{P(X=x) = (1-p)^{x-1} p} \quad x = 1,2,\dots$$

$E(x) = \frac{1}{p} \quad V(x) = \frac{1-p}{p^2}$

contoh:

① ada 100 produksi, 1 cacat. peluang setelah 5 butir diperiksa baru menemukan cacat? $p = \frac{1}{100} = 0{,}01.$

- $P(X=5) = (1-0{,}01)^4 \cdot 0{,}01$

$= (0{,}99)^4 \cdot 0{,}01$

$= 0{,}0096$

- $E(x) = \frac{1}{p} = 100 \quad V(x) = \frac{0{,}99}{0{,}0001} = 9900$

$\sigma = \sqrt{9900}$

✓ sifat peubah acak geometrik

$$\boxed{P(X=x+n \mid x > n) = P(X=x)}$$

contoh:

$P(X=15 \mid x > 10) = P(X=5)$

↳ $P(X=15)$ dg syarat $x > 10.$

Soal:

(1.) 3 orang ibu melahirkan bayi tunggal. Peluang semua bayi laki-laki?

$= \binom{3}{3} \frac{1}{2}^3 \left(\frac{1}{2}\right)^0 = \frac{1}{8} = 0{,}125.$

② Peluang suatu komponen lulus uji tertentu 2/3. mengikuti distribusi binomial. Peluang bahwa 3 dari 6 komponen yg diuji berikutnya lolos uji adalah:

$$P(x=3) = \binom{6}{3} \frac{2}{3}^3 \frac{1}{3}^3$$

$$= \frac{160}{729}$$

3. 5 orang melamar kerja, peluang diterima 0,4.

$E(x) = np = 5 \times 0{,}4 = 2$

PHP.

Kg.

Peubah Acak Diskret

4. Peubah Acak Binom Negatif.

↳ Banyaknya percobaan sampai memperoleh sukses sebanyak "k" kali.

Fungsi massa peluang

$$P(x) = \binom{x-1}{k-1} (1-p)^{x-k} p^k.$$

untuk $k=1$ → geometrik

- Peubah binom negatif = penjumlahan dari peubah geometrik.

k = Banyak kejadian sukses

$$E(x) = \frac{k}{p}$$

$$V(x) = \frac{k(1-p)}{p^2}$$

contoh:

1. ada 3 server, peluang kegagalan server = 0,0005.

a. $E(x) = \frac{k}{p} = \frac{3}{0{,}0005} = 6000$

	Bernoulli	Binom	Geometrik
Definisi x	Hanya ada peluang sukses atau gagal.	Kejadian Bernoulli yg berulang	Banyak per cobaan sampai terjadi 1 sukses.
Rentang nilai x	$x = 0,1$	$0,1,2,\dots,n$	$1,2,\dots$
$P(x)$	$p^x(1-p)^{1-x}$	$\binom{n}{x} p^x q^{n-x}$	$(1-p)^{x-1} \cdot p$
$E(x)$	p	np	$1/p$
$var(x)$	$p(1-p)$	$np(1-p)$	$1-p/p^2$

b. Peluang ketiga server gagal dalam 5 permintaan.

$$p(x=5) = \binom{x-1}{k-1} (1-p)^{x-k} p^k$$

$$= \binom{5-1}{3-1} (1-0{,}0005)^{5-3} \cdot 0{,}0005^3$$

$$= \dots$$

5. Peubah Acak Hipergeometrik

↳ Angka keberhasilan dalam contoh berukuran n yang diambil dari populasi berukuran N.

$$p(x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

N = populasi
n = sampel
$x = 0,1,\dots,n$.

contoh:

1. 

diambil 3 bola → n = 3.

$$P(1m) = \frac{\binom{6}{1}\binom{4}{2}}{\binom{10}{3}}$$

k = 6
x = 1

2. x = banyaknya spare part yang diperoleh dari pemasok lokal.

N = 300 k = 100 n = 4

a. $P(x=4) = \frac{\binom{100}{4}\binom{200}{0}}{\binom{300}{4}}$

Lanjutan soal: PHP.

b. $P(x \geq 2) = P(x=2) + P(x=3) + P(x=4)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - \left[\frac{\binom{100}{0}\binom{200}{4}}{\binom{300}{4}} + \frac{\binom{100}{1}\binom{200}{3}}{\binom{300}{4}} \right]$

$= 0{,}408$

c. $P(x \geq 1) = 1 - P(x=0)$

$= 1 - \frac{\binom{100}{0}\binom{200}{4}}{\binom{300}{4}} = 0{,}804.$ //

(3.) $N = 496$

x = banyaknya kokarn

k = banyaknya kokarn dari 496 paket.

$P(x=4) = \frac{\binom{k}{4}\binom{496-k}{0}}{\binom{496}{4}}$

$P(x=2 \mid x=4) = \frac{\binom{k-4}{2}\binom{492-k}{0}}{\binom{492}{2}}$

K.10 PHP.

Peubah Acak Poisson

$P(X=x) = \frac{\lambda^x \cdot e^{-\lambda}}{x!}$, $x = 0, 1, 2, \ldots$

$\boxed{E(x) = var(x) = \lambda = np.}$

Def:

Kejadian yg jarang terjadi, jarang dr. temukan 2 kejadian yg berturut-turut.

contoh:

Kecelakaan lalu lintas, banjir, gempa, dll

contoh:

1. diketahui pengguna jasa internet suatu provider membuat akun baru dg rata² 10 akun per hari.

a. Berapakah peluang bahwa akan ada lebih dari 8 akun baru yg dibuat hari ini?

x = banyaknya akun baru (per hari)

$P(x > 8) = 1 - P(x \leq 8)$

$= 1 - \sum_{x=0}^{8} \frac{10^x \cdot e^{-10}}{x!}$

b. Berapakah peluang bahwa akan ada lebih dari 16 akun baru yg dibuat dalam 2 hari ini?

Y = banyaknya akun baru (per 2 hari).

$E(Y) = E(2x) = 2E(x) = 2 \cdot 10 = 20.$

$P(Y > 16) = 1 - P(Y \leq 16)$

$= 1 - \sum_{x=0}^{16} \frac{20^y \cdot e^{-20}}{y!}$

$= 1 - 0{,}221$

$= 0{,}779.$

2. kita bisa invest \$1000 dalam 3 cara:

a. Bunga 10 %. 1 tahun → \$1100. $E(x) = 1100$

b. 2/3 → \$1000 $E(x) = 666{,}67$ [dipilih.]

1/3 → \$1200 $E(x) = 400$ +

$1066{,}67$

c. 5%. → \$10.000

95 %. → \$0 $E(x) = 500$

K-11 PHP.

Peubah Acak Kontinu

↳ peubah acak dengan ruang contoh yg terdiri dari suatu selang (interval) atau gabungan dari beberapa selang.

✓ Fungsi Kepekatan Peluang.

syarat :

a. $f(x) \geq 0$

b. $\int_{-\infty}^{\infty} f(x)\, dx = 1$

c. $P(a \leq x \leq b) = \int_a^b f(x)\, dx$

contoh :

① $f(x) = \begin{cases} 2/x^3, & \text{untuk } x \geq 1 \\ 0, & \text{untuk } x \text{ lainnya} \end{cases}$

• $f(x) \geq 0$ terpenuhi u/ $-\infty \leq x \leq +\infty$

• $\int_{-\infty}^{+\infty} f(x)\, dx = \int_{-\infty}^{1} 0\, dx + \int_1^{\infty} \frac{2}{x^3}\, dx$

$= -\frac{1}{x^2}\Big]_1^{\infty}$

$= 0 - (-1) = 1$ //

∴ memenuhi syarat fkp.

② $f(x) = \begin{cases} 0{,}025x + 0{,}15, & 2 < x < 6 \\ 0, & \text{untuk } x \text{ lainnya.} \end{cases}$

$\int_{-\infty}^{\infty} f(x)\, dx = \int_{-\infty}^{2} 0\, dx + \int_2^6 (0{,}025x + 0{,}15)\, dx$

$+ \int_6^{\infty} 0\, dx.$

$= \frac{0{,}025}{2} x^2 + 0{,}15x \Big]_2^6$

$= 1{,}35 - 0{,}35$

$= 1$

∴ fkp terpenuhi. //

✓ Sebaran Peluang P.a Kontinu.

$$\boxed{\begin{aligned} F_X(x) &= P(-\infty \leq f(X < x)) \\ F_X(x) &= \int_{-\infty}^{x} f(x)\, dx \end{aligned}}$$

✓ Fungsi Sebaran Kumulatif.

$$0 \leq F_X(x) \leq 1$$

Jika $a > b$ maka $F_X(a) \geq F_X(b)$

↳ monoton tidak turun.

$\boxed{\lim_{x \to -\infty} F_X(x) = 0}$ $\boxed{\lim_{x \to +\infty} F_X(x) = 1}$

dan contoh no. 2 tadi :

③ $F_X(x) = \begin{cases} 0, & x \leq 2 \\ \int f(x)\, dx, & 2 < x < 6 \\ 1, & x \geq 6 \end{cases}$

$0{,}0125x^2 + 0{,}15x - 0{,}35$ u/ $2 < x < 6$.

$\int_2^x f(t)\, dt = \int_2^x (0{,}025t + 0{,}15)\, dt$

$= \left[\frac{0{,}025}{2} t^2 + 0{,}15t\right]_2^x$

$= \left(\frac{25}{2000} x^2 + 0{,}15x\right) - \frac{350}{1000}$ //

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Peubah Acak Kontinu.

Nilai Harapan p.a Kontinu.

$$\boxed{E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx.}$$

dari soal no.2:

(4.) $E(x) = \int_{2}^{6} x\,(0{,}025x + 0{,}15)\, dx$

$= \int_{2}^{6} (0{,}025x^2 + 0{,}15x)\, dx$

$= \left[\frac{0{,}025}{3}x^3 + \frac{0{,}15}{2}x^2\right|_2^6$

$= \left(\frac{0{,}025}{3}\cdot 6^3 + \frac{0{,}15}{2}\cdot 6^2\right) - \left(\frac{0{,}025}{3}\cdot 2^3 + \frac{0{,}15}{2}\cdot 2^2\right)$

$= 4{,}133$

Ragam p.a Kontinu.

$$\boxed{Ex^2 = \int_{-\infty}^{\infty} x^2 f_x(x)\, dx.}$$

$\text{Var}\, x = Ex^2 - (E(x))^2$

dari soal no.2:

(5) $E(x^2) = \int_{2}^{6} x^2 (0{,}025x + 0{,}15)\, dx$

$= \int_{2}^{6} (0{,}025x^3 + 0{,}15x^2)\, dx$

$= \left[\frac{0{,}025}{4}x^3 + \frac{0{,}15}{3}x^3\right]_2^6$

$= 18{,}4$

$V(x) = 18{,}4 - (4{,}133)^2$

$= 1.3183$ //

Distribusi Seragam

fkp:

$$f(x) = \begin{cases} \frac{1}{b-a}, & \text{untuk } a \le x \le b \\ 0, & \text{untuk } x \text{ lainnya} \end{cases}$$

f.kumulatif:

$$F_x(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \le x < b \\ 1, & x \ge b \end{cases}$$

$\boxed{E(x) = \frac{a+b}{2}}$ $\boxed{V(x) = \frac{(b-a)^2}{12}}$

contoh:

1. suatu fungsi sebaran seragam yg didefinisikan pada selang (0,5). peluang ?

a. $P(x<3)$?

$$f_x(x) = \begin{cases} \frac{1}{5-0} = \frac{1}{5}, & 0 \le x \le 5 \\ 0, & x \text{ lainnya} \end{cases}$$

$P(x<3) = \int_{0}^{3} \frac{1}{5}\, dx$

$= \frac{1}{5}x \Big]_0^3 = \frac{3}{5}$

b. $P(x>3)$?

$= 1 - P(x \le 3)$

$= 1 - P(x<3)$

$= \frac{2}{5}$

atau: $\int_{3}^{5} \frac{1}{5}\, dx = \frac{2}{5}$

c. $P(1 < x < 2)$?

$$= \int_1^2 \frac{1}{5}\,dx = \frac{1}{5}$$

2. Kereta api tiba di suatu stasiun pada setiap selang waktu 15 menit mulai pukul 07.00, artinya jadwal tiba kereta api di stasiun itu adalah 07.00, 07.15, 07.30, 07.45, dst. Jika seorang calon penumpang sampai di stasiun itu pada jam yg tersebar seragam antara 07.00 dan 07.30, hitunglah peluang bahwa ia harus menunggu kereta api dg waktu:

a. < 5 menit

b. > 10 menit.

Jawab:

x = waktu kedatangan seorang calon penumpang. → (0,30)

$$f_x(x) = \begin{cases} \frac{1}{30-0} = \frac{1}{30}, & \text{untuk } 0 < x < 30 \\ 0, & x \text{ lainnya.} \end{cases}$$

a. P (penumpang menunggu < 5 menit)

$$= P(10 < x < 15) + P(25 < x < 30)$$

$$= \int_{10}^{15} \frac{1}{30}\,dx + \int_{25}^{30} \frac{1}{30}\,dx.$$

$$= \frac{5}{30} + \frac{5}{30} = \frac{1}{3}$$ //

b. $\boxed{\text{tiba di stasiun pukul 07.00-07.05 atau pukul 07.15-07.20.}}$

P (penumpang menunggu > 10 menit)

$$= P(0 < x < 5) + P(15 < x < 20)$$

$$= \int_0^5 \frac{1}{30}\,dx + \int_{15}^{20} \frac{1}{30}\,dx$$

$$= \frac{5}{30} + \frac{5}{30} = \frac{1}{3}$$ //

✓ Distribusi Eksponensial

- Banyak kejadian (yg jarang terjadi) ↳ Poisson.
- Waktu antar kejadian → eksponensial.

$$\boxed{f_x = \lambda e^{-\lambda x}, \ x > 0}$$

$$F_x = \int_0^x \lambda e^{-\lambda x}\,dt$$

$$\boxed{F_x = 1 - e^{-\lambda x}}$$

$$\boxed{E(x) = \frac{1}{\lambda} \qquad V(x) = \frac{1}{\lambda^2}}$$

contoh:

1. lamanya waktu menelepon dimisalkan mengikuti distribusi eksponensial dg parameter $\lambda = 0{,}1$. Jika seseorang datang ke telepon umum sebelum anda, dapatkan peluang bahwa anda akan menunggu untuk

K.12 PHP.

lanjutan soal

menggunakan telepon umum :

a. lebih dari 10 menit

b. antara 10 - 20 menit.

Jawab :

$$f_X(x) = \begin{cases} 0{,}1 e^{-0{,}1x}, & x > 0 \\ 0, & x \text{ lainnya.} \end{cases}$$

x = waktu menelepon

a. $P(X > 10) = \int_{10}^{\infty} 0{,}1 e^{-0{,}1x} \, dx.$

$= -e^{-0{,}1x} \Big]_{10}^{\infty}$

$= -e^{-0{,}1(\infty)} + e^{-0{,}1(10)}$

$= 0 + e^{-1}$

$= \frac{1}{e}$

b. $P(10 < X < 20) = \int_{10}^{20} 0{,}1 e^{-0{,}1x} \, dx$

$= -e^{-0{,}1x} \Big]_{10}^{20}$

$= -e^{-0{,}1(20)} + e^{-0{,}1(10)}$

$= e^{-1} - e^{-2}$

$= 0{,}2333$ //

Aplikasi Distribusi Eksponensial

- Failure Rate & Reliability

$$\boxed{f(t) = \begin{cases} 0{,}5 e^{-0{,}5t} & , t \geq 0 \\ 0 & \text{selainnya.} \end{cases}}$$

λ = failure rate

$$\boxed{rel(t) = P(T > t) = e^{-0{,}5t}}$$

contoh !

Ilustrasi 1.

X = waktu s.d. mesin rusak

a $E(X) = 10 \rightarrow E(X) = \frac{1}{\lambda}$

$10 = \frac{1}{\lambda}$

$\lambda = \frac{1}{10} = 0{,}1$

b. $Rel(t) = 1 - F(t) = 1 - (1 - e^{-\lambda t})$

$= e^{-\lambda t}$

$= e^{-0{,}1t}$.

c. $P(X > 12) = Rel(12)$

$= e^{-0{,}1(12)}$

$= e^{-1{,}2}$ //

Distribusi Gamma (α, β)

K.B. PHP.

$X \sim gamma\ (\alpha, \beta)$

↓ kejadian Poisson ke-α

$\beta = \frac{1}{\lambda} \rightarrow$ rata² kejadian Poisson

$$\boxed{X \sim gamma\ (1, \tfrac{1}{\lambda}) \rightarrow X \sim exp\ (\lambda)}$$

- **Fungsi Gamma**

$$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y}\, dy$$

$$\boxed{f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, \quad x > 0}$$

$$\boxed{E(X) = \frac{\alpha}{\lambda} \qquad V(X) = \frac{\alpha}{\lambda^2}}$$

catatan:

$$\Gamma(1) = \int_0^\infty e^{-y}\, dy = 1$$

$$\Gamma(\alpha) = (\alpha-1)\,\Gamma(\alpha-1)$$

$$= (\alpha-1)!$$

Distribusi Normal.

- simetrik
- fkp :

$$\boxed{f(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}$$

- mean, median, & modus berada dalam 1 titik.

- $P(a \le x \le b) = \int_a^b f(x)\, dx$

$= F(b) - F(a).$

- $P(\mu - \sigma < X < \mu + \sigma) = 0{,}683$
- $P(\mu - 2\sigma < X < \mu + 2\sigma) = 0{,}954.$

$$\boxed{\begin{aligned} &X \sim normal\ (\mu, \sigma^2) \\ &z = \frac{x-\mu}{\sigma} \\ &z \sim normal\ (0, 1) \end{aligned}}$$

contoh:

① dik: x = nilai ujian tulis cpns.

$X \sim N(56, 9) \rightarrow \sigma^2 = 9$

$\sigma = 3.$

$P(x > 80) = P\left(z > \frac{80-56}{3}\right)$

$= P(z > 8) \approx 0$

K.13. PHP.

Peubah Acak Ganda.

- Diskret.

fungsi peluang bersama :

$$\boxed{p_{Y_1 Y_2}(y_1, y_2) = P(Y_1 = y_1, Y_2 = y_2)}$$

Syarat :

1. $p_{Y_1, Y_2}(y_1, y_2) \geq 0$
2. $\Sigma\, p_{Y_1, Y_2}(y_1, y_2) = 1$

- Kontinu.

fkp bagi (x, y)

$$\boxed{f_{x,y}(x, y)}$$

fkp marginal bagi x :

$$\boxed{f_x(x) = \int_y f(x,y)\, dy.}$$

- fmp marginal bagi x.

$$P(X = x) = \Sigma_y\, P(X = x, Y = y)$$

- fmp marginal bagi y.

$$P(Y = y) = \Sigma_x\, P(X = x, Y = y)$$

contoh:

① $f_{Y_1 Y_2}(y_1, y_2) = \begin{cases} c y_1 y_2, & 10 \leq y_1 < 20, \\ & 0 < y_2 < 3. \\ 0, & \end{cases}$

a. tentukan c

Jawab:

$$\int_{10}^{20}\int_0^3 c y_1 y_2\, dy_2\, dy_1 = 1$$

$$\int_{10}^{20} \left[\tfrac{1}{2} c y_1 y_2^2\right]_0^3 dy_1 = 1$$

$$\int_{10}^{20} \frac{9}{2} c y_1\, dy_1 = 1$$

$$\frac{9}{2} \cdot \frac{1}{2} c y_1^2 \Big]_{10}^{20} = 1$$

$$\frac{9}{4} c\, (400 - 100) = 1$$

$$\frac{2700}{4} c = 1$$

$$675 c = 1$$

$$c = 1/675 \quad //$$

b. $P(Y_1 > 15,\ Y_2 < 1)$?

$$f(y_1, y_2) = \frac{1}{675} y_1 y_2, \quad 10 < y_1 < 20, \quad 0 < y_2 < 3.$$

$$= \int_{15}^{20}\int_0^1 \frac{1}{675} y_1 y_2\, dy_2\, dy_1$$

$$= \frac{1}{675} \int_{15}^{20} y_1 \tfrac{1}{2} y_2^2 \Big]_0^1 dy_1$$

$$= \frac{1}{675} \int_{15}^{20} y_1 \cdot \tfrac{1}{2}\, dy_1$$

$$= \frac{1}{675}\ \tfrac{1}{2} \cdot \tfrac{1}{2} y_1^2 \Big]_{15}^{20}$$

$$= \frac{1}{1350} \left(200 - \frac{225}{2}\right)$$

$$\approx 0{,}065 \quad //$$

c. $P\left(Y_2 > \frac{Y_1}{5}\right)$

$$= \int_{10}^{20}\int_{y_1/5}^{3} \frac{1}{675}\, y_1 y_2 \, dy_2\, dy_1$$

atau

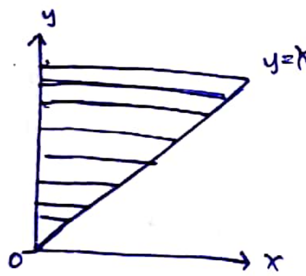
$$= \int_{0}^{3}\int_{10}^{5y_2} \frac{1}{675}\, y_1 y_2 \, dy_1\, dy_2.$$

(2) $f(x,y) = \begin{cases} 2e^{-x}e^{-2y} & , 0<x<\infty \\ & \;\; 0<y<\infty \\ 0 & \end{cases}$

a. $P(X>1, Y<1)$

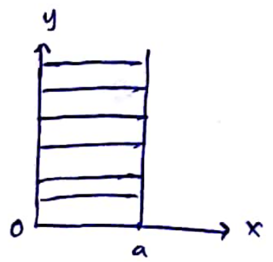
$$= \int_{1}^{\infty}\int_{0}^{1} 2e^{-x}e^{-2y}\, dy\, dx.$$

b. $P(X<Y)$



$$= \int_{0}^{\infty}\int_{0}^{y} 2e^{-x}e^{-2y}\, dx\, dy$$

c. $P(X<a)$



$$= \int_{0}^{\infty}\int_{0}^{a} 2e^{-x}e^{-2y}\, dx\, dy.$$

✓ fkp bersama 2 peubah acak yg saling bebas.

$$\boxed{f(x,y) = f_X(x)\, f_Y(y)}$$

contoh:

(1) X = waktu kedatangan laki²

Y = waktu kedatangan perempuan

$X \sim \text{unif}(0,60)$, $Y \sim \text{unif}(0,60)$ } X & Y saling bebas.

$$f(x,y) = f(x)\cdot f(y) = \frac{1}{60}\cdot\frac{1}{60} = \frac{1}{3600}, \text{ untuk } 0<x<60,\; 0<y<60$$

soal:

(1) $f(y_1,y_2) = 6y_1$, u/ $0<y_1<y_2<1$

- fkp marginal bagi y_1

$$f_{y_1}(y_1) = \int_{y_1}^{1} 6y_1\, dy_2 = 6y_1y_2 \Big]_{y_1}^{1} = 6y_1 - 6y_1^2, \text{ u/ } 0<y_1<1$$

- fkp marginal bagi y_2.

$$f_{y_2}(y_2) = \int_{0}^{y_2} 6y_1\, dy_1 = 3y_1^2 \Big]_{0}^{y_2} = 3y_2^2 \;//$$

K-14

PHP.

lanjutan soal:

apakah Y_1 dan Y_2 saling bebas?

$\rightarrow f(y_1) \cdot f(y_2)$

$= (6y_1 - 6y_1)^2 \; 3y_2^2$

$= 18 y_1 y_2^2 - 18 y_1^2 y_2^2 \neq f(y_1, y_2)$

$\therefore$ y_1 dan y_2 tdk saling bebas.

- Sebaran Peluang Bersyarat

diskret.

$P_{X|Y=y}(x|y) = P(X=x \mid Y=y)$

$$\boxed{\frac{P(X=x, Y=y)}{P(Y=y)} = \frac{P(x,y)}{P_Y(y)}}$$

kontinu.

$$f_{X|Y=y}(x|y) = \frac{f(x,y)}{f_Y(y)}$$

- Nilai Harapan.

misal fkp bersama $\rightarrow f(x,y)$

$$E(XY) = \int_{\forall x} \int_{\forall y} xy \, f(x,y) \, dy \, dx.$$

$$E(X) = \int_{\forall x} x \, f(x) \, dx$$

$$E(Y) = \int_{\forall y} y \, f(y) \, dy$$

- Peragam / covariance.

$$\boxed{cov(x,y) = E(XY) - E(X)E(Y).}$$

- Korelasi

$$\boxed{corr(x,y) = \frac{cov(x,y)}{\sqrt{var(x)}\sqrt{var(y)}}}$$

dengan : $var(x) = E(x^2) - (E(x))^2$

$var(y) = E(y^2) - (E(y))^2$

R-8

PHP.

Peubah Acak Diskret.

1. Sebaran Bernoulli

1 kejadian memiliki 2 kemungkinan
- sukses
- gagal.

$$P(X=x) \begin{cases} p^x (1-p)^{1-x}, & x=0,1 \\ 0, & \text{lainnya.} \end{cases}$$

$$\boxed{E(X) = p \quad V(X) = p(1-p)}$$

2. Binomial.

n kejadian Bernoulli, antar kejadian saling bebas

$$\boxed{P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}}$$

contoh:

1. $n = 25$

$P(\text{cacat}) = 0{,}01$

$P(X=x)$

$P(X>3) = 1 - P(X \leq 3)$

$= 1 - [P(X=3) + P(X=2) + P(X=1) + P(X=0)]$

$= \ldots$

$P(X=3) = \binom{25}{3} (0{,}01)^3 (0{,}99)^{22}$

$= \ldots$

3. Geometrik

Banyaknya tindakan yg dilakukan sampai diperoleh 1 keberhasilan.

$$\boxed{P(X=x) = p(1-p)^{x-1}, \quad x \geq 1}$$

$E(X) = \frac{1}{p} \quad V(X) = \frac{1-p}{p^2}$

soal:

1. dalam suatu pengendalian mutu, diket. rata" 1 cacat dari 500 item. berapa peluang bahwa teknisi telah melakukan sebanyak 25 kali pengecekan sebelum akhirnya ditemukan 1 cacat.

2. Peluang seseorang sembuh dari operasi jantung 0,7. bila dari 10 orang menjalani operasi jantung: peluang?
   a. 5 orang sembuh
   b. min 3 orang sembuh
   c. antara 3 - 8 sembuh.

Jawab:

1. $P(X=25) = (1-0{,}002)^{24} \cdot 0{,}002$

$= 0{,}0019$

2. a. $P(X=5) = \binom{10}{5} 0{,}7^5 \; 0{,}3^5$

$=$

b. $P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - [P(X=2) + P(X=1) + P(X=0)]$

$= 1 - [$

$P(X=2) = \binom{10}{2} 0{,}7^2 \cdot 0{,}3^8 =$

$P(X=1) = \binom{10}{1} 0{,}7^1 \cdot 0{,}3^9$

$P(X=0) = \binom{10}{0} 0{,}7^0 \cdot 0{,}3^{10}$

c. $P(3 \leq X \leq 8) = P(X \leq 8) - P(X < 3)$

$= \sum_{i=0}^{8} \binom{10}{i} 0{,}7^i \, 0{,}3^{10-i} - P(X<3)$

Kg PHP.

4. Binomial Negatif. → Penjumlahan sebaran geometrik

$X \sim$ Binom negatif (k, p)

X = banyaknya tindakan yg dilakukan

k = Banyaknya kejadian sukses.

p = peluang sukses.

$$FMP = \binom{x-1}{k-1} (1-p)^{x-k} p^{x}$$

$$E(X) = E(X_1 + \dots + X_k) = \frac{k}{p}$$

↓

kejadian geometrik

$$V(X) = V(X_1 + \dots + X_k) = \frac{k(1-p)}{p^2}$$

5. Hipergeometrik

kasus: contoh acak berukuran n.

diambil secara acak tanpa pemulihan.

$$fmp = f(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

$$\mu = E(X) = np$$

$$V(X) = np(1-p)\left(\frac{N-n}{N-1}\right)$$

contoh:

1. Suatu kotak berisi 40 suku cadang dikatakan memenuhi syarat penerimaan tentu tidak lebih dari 3 yg cacat. Cara pengambilan contoh acak ialah dg memilih 5 suku cadang secara acak dari dalamnya & menolak kotak bila ada yg cacat. Berapa peluang memperoleh 1 cacat dalam c. acak berukuran 5. Bila kotak memiliki 3 yg cacat.

Jawab:

x = banyak cacat

n = banyak contoh acak

M = banyak cacat dalam 1 kotak

$$P(X=x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

$$P(X=1) = \frac{\binom{3}{1}\binom{40-3}{5-1}}{\binom{40}{5}}$$

$$= \frac{\binom{3}{1}\binom{37}{4}}{\binom{40}{5}} = \dots$$

2. The drilling records for an oil company suggest that the probability the company will hit oil in productive quantities at a certain off share location is 0,3 suppose the company plans to drill a series of looking for three successful wells.

a, what is the probability that the third success will be achieved, with 8 well drilled

$$= \binom{8-1}{3-1} (0,3)^3 (0,7)^5$$

b. $$E(X) = \frac{k}{p} = \frac{3}{0,3} = 10$$

$$V(X) = \frac{k(1-p)}{p^2} = \frac{3(0,7)}{(0,3)^2} = \frac{70}{3} \approx 23,3$$

R.10 PHP.

Poisson

$$p(X=x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!} \qquad \lambda = \mu = np$$

soal:

1. rata** hitung (μ) pada tahun 2012 = 2,5 orang albino per 175 populasi. Jumlah sampel yg diambil 525 orang. Berapa peluang [redacted] dari sampel tidak ada yg albino.

Jawab:

x = kejadian albino.

175 populasi → $\mu = 2,5$

525 populasi → $\mu = 7,5$

$$p(X=0) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!} = \frac{e^{-7,5} \cdot (7,5)^{0}}{0!}$$

$$= e^{-7,5}$$

$$= 0,00055.$$

$p \cdot$ albino $= 1 - p$ (tdk ada albino)

$= 1 - 0,00055$

$= 0,99.$

2. misal gempa bumi terjadi di wilayah barat Amerika serikat sesuai dg asumsi** 1,2, dan 3 dg $\underline{\lambda = 2}$ dan sebagai satuan waktu diambil $\underline{1 \text{ minggu}}$ (artinya, gempa bumi terjadi mengikuti ke 3 asumsi di atas pada laju 2 kali per minggu). Tentukan:

a. Paling sedikit 3 gempa bumi selama 2 minggu mendatang.

b. fungsi sebaran peluang waktu mulai sekarang sampai gempa bumi berikutnya.

Jawab:

$G(2)$ = kejadian gempa dalam waktu 2 minggu

a. $P(G(2) \geq 3) = 1 - P(G(2)=0) - P(G(2)=1) - P(G(2)=2)$

$$= 1 - \frac{e^{-4} \cdot 4^{0}}{0!} - \frac{e^{-4} \cdot 4^{1}}{1!} - \frac{e^{-4} \cdot 4^{2}}{2!}$$

$$= 1 - 0,0183 - 0,0733 - 0,1465$$

$$= 0,7619 \; //$$

b. x = lama waktu gempa bumi dalam minggu

$$P(x > t) = P(G(t) = 0) = \frac{e^{-\lambda t} \, \lambda t^{x}}{x!}$$

$$= \frac{e^{-\lambda t} \cdot 1}{1}$$

$$= e^{-\lambda t}.$$

$$F_x(t) = 1 - P(x > t)$$

$$= 1 - P(G(t) = 0)$$

$$= 1 - e^{-\lambda t}.$$

3. rata^{2} seorang sekretaris baru melakukan kesalahan mengetik perhalaman. Berapa peluang bahwa pada halaman berikut:

a. Ia tidak ada kesalahan.

b. tidak lebih dari 3 kesalahan.

$n = 200$, $p = 0{,}01$.

Jawab:

a. x = kejadian melakukan kesalahan.

$\lambda = n \cdot p = 200 \cdot 0{,}01 = 2$.

$$P(x=0) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!} = \frac{e^{-2} \cdot 2^{0}}{0!}$$

$$= e^{-2}$$

b. $P(x \le 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$$= e^{-2} + e^{-2} \cdot 2 + \frac{e^{-2} \cdot 2^{2}}{2!} + \frac{e^{-2} \cdot 2^{3}}{3!}$$

$$= e^{-2} + 2e^{-2} + 2e^{-2} + \frac{4}{3} e^{-2}$$

$$= 6\frac{1}{3} e^{-2} \;//$$

R.11.

Peubah Acak Kontinu.

↳ PA dengan ruang contoh berupa selang atau gabungan beberapa selang

✓ Fungsi Kepekatan Peluang

- $f(x) \ge 0$
- $\int_{-\infty}^{\infty} f(x) = 1$
- $P(a \le x \le b) = \int_{a}^{b} f(x)\, dx$.

Ilustrasi

$f(x) = \frac{2}{x^3}$, $x \ge 2$

$$\int_{-\infty}^{\infty} f(x)\, dx = \int_{-\infty}^{2} 0\, dx + \int_{2}^{\infty} \frac{2}{x^3}\, dx$$

$$= -\frac{1}{x^2} \Big]_{2}^{+\infty}$$

$$= 0 - \left(-\frac{1}{4}\right) = \frac{1}{4}$$

✓ Sebaran Peluang Kontinu / Fungsi sebaran Kumulatif

$$\boxed{\begin{aligned} F(x) &= P(-\infty \le f(x \le x)) \\ &= \int_{-\infty}^{x} f(x)\, dx. \end{aligned}}$$

$0 \le F_x(x) \le 1$

Jika $a > b$ maka $F_x(a) \ge F_x(b)$

↳ monoton tidak turun.

$\lim_{x \to -\infty} F_x(x) = 0$	$\lim_{x \to +\infty} F_x(x) = 1$

R.11

PHP.

Peubah Acak Kontinu

ilustrasi

x dengan p.a fkp

$$f(x) = \begin{cases} 2/x^3 & , x \geq 2 \\ 0 & , x \text{ lainnya.} \end{cases}$$

$$F(x) = \begin{cases} 0 & , u/ \ x < 2 \\ \frac{1}{4} - \frac{1}{x^2} & , x \geq 2 \end{cases}$$

- Nilai Harapan

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

- Ragam

$$V(x) = E(x^2) - (E(x))^2$$

$$\boxed{E(x^2) = \int_{-\infty}^{+\infty} x^2 \cdot f(x)\, dx}$$

contoh:

1. misal x adalah p.a kontinu dg fkp

$$f_x(x) = \begin{cases} cx^2 & , \text{ jika } 1 < x < 2 \\ 0 & , \text{ jika } x \text{ lainnya} \end{cases}$$

a. tentukan nilai c

b. fungsi sebaran kumulatif.

Jawab:

$$\int_{-\infty}^{1} 0\, dx + c \int_{1}^{2} x^2\, dx + \int_{2}^{\infty} 0\, dx = 1$$

$$c \cdot \frac{1}{3} x^3 \Big]_1^2 = 1$$

$$c\left[\frac{8}{3} - \frac{1}{3}\right] = 1$$

$$\frac{7}{3} c = 1$$

$$c = 3/7 \quad //$$

(b) Fungsi kumulatif

$$f(x) = \begin{cases} 3/7\, x^2 & , 1 < x < 2 \\ 0 & , x \text{ lainnya} \end{cases}$$

$$F(x) = \begin{cases} 0 & , x \leq 1 \\ \frac{x^3}{7} - \frac{1}{7} & , 1 < x < 2 \\ 1 & , x \geq 2 \end{cases}$$

$$\boxed{\begin{aligned} \int_1^x \frac{3}{7} x^2\, dx &= \frac{3}{7} \int_1^x x^2\, dx \\ &= \frac{\cancel{3}}{7} \cdot \frac{1}{\cancel{3}} x^3 \Big]_1^x \\ &= \frac{x^3}{7} - \frac{1}{7} \quad // \end{aligned}}$$

R.12

PHP.

Beberapa Fungsi P. Kontinu

1. Distribusi Seragam

Peubah acak x dikatakan mempunyai fkp seragam pada interval (α, β)

$$\text{fkp} = f(x) \begin{cases} \frac{1}{b-a} & , u/ \ a \leq x \leq b \\ 0 & , u/ \ x \text{ lainnya} \end{cases}$$

f kumulatif:

$$F_x(x) = \begin{cases} 0 & , x < a \\ \frac{x-a}{b-a} & , a \leq x \leq b. \end{cases}$$

- nilai harapan

$$\boxed{E(x) = \frac{a+b}{2}}$$

- varian / ragam

$$\boxed{V(x) = \frac{(b-a)^2}{12}}$$

2. Distribusi Eksponensial.

- untuk memodelkan waktu.
- selang waktu antar kejadian
- contoh: waktu tunggu
  waktu di antara panggilan telepon.

fkp : $f(x) \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$

f. kumulatif :

$$F(x) = \int_0^x f(x)\, dx$$

$$= \int_0^x \lambda e^{-\lambda x}$$

$$= 1 - e^{-\lambda x}$$

$$\boxed{E(x) = \frac{1}{\lambda}} \quad \boxed{V(x) = \frac{1}{\lambda^2}}$$

3. Distribusi Normal.

fkp : $f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Peluang : $p(a \leq x \leq b) = \int_a^b f(x)\, dx = F(b) - F(a)$

Peubah acak (x) dg mean (μ) dan ragam (σ^2) menyebar normal dituliskan sebagai berikut $x \sim N(\mu, \sigma^2)$

4. Sebaran Gamma.

fkp : $f(x) \begin{cases} \frac{1}{\Gamma(\alpha)\beta^\alpha} \cdot x^{\alpha-1} \cdot e^{-x/\beta} & , x \geq 0 \\ 0, & x < 0 \end{cases}$

dengan $\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} \cdot e^{-y}\, dy$.

misal : $\alpha = 1$, α merupakan bil. bulat positif.

$$\Gamma(1) = \int_0^\infty e^{-y}\, dy = 1$$

maka $\Gamma(\alpha) = (\alpha - 1)!$

contoh :

1. $f(x) = \begin{cases} a(1-x)^3 & , 0 < x < 3 \\ 0 & , x \text{ selainnya} \end{cases}$

tentukan nilai a!

$$\int_{-\infty}^{\infty} f(x)\, dx = 1 \quad \rightarrow u = 1 - x$$

$$du = -dx$$

$$dx = -du$$

$$-\int a u^3\, du = -\frac{a}{4} u^4 + c$$

$$= -\frac{a}{4}(1-x)^4 + c$$

$$= \int_0^3 a(1-x)^3\, dx = -\frac{a}{4}(1-x)^4 \Big]_0^3$$

$$= -\frac{a}{4}(-2)^4 + \frac{a}{4}(1)^4$$

$$1 = -\frac{a}{4} \cdot 16 + \frac{a}{4}$$

$$-\frac{15}{4} a = 1 \rightarrow \boxed{a = -4/15}$$

contoh soal :

2. $f_X(x) = \begin{cases} 0{,}5x - 0{,}5 & , 1 \leq x < 2 \\ 0{,}5 & , 2 \leq x < 3 \\ 2 - 0{,}5x & , 3 \leq x < 4 \end{cases}$

$F_X(x)$, $E(X)$, $V(X)$?

$-\infty \quad x<1 \quad 1 \quad 2 \quad 3 \quad x>3 \quad \infty$

$1 \leq x < 2$ $\quad$ $2 \leq x < 3$

- $x < 1$

$F_X(x) = \int_{-\infty}^{x} f(x)\,dx = 0$

- $1 \leq x < 2$

$F_X(x) = \int_{-\infty}^{x} f(x)\,dx$

$= \int_{-\infty}^{1} f(x)\,dx + \int_{1}^{x} f(x)\,dx$

$= 0 + \int_{1}^{x} (0{,}5x - 0{,}5)\,dx$

$= \int_{1}^{x} (0{,}5x - 0{,}5)\,dx$

$= 0{,}25x^2 - 0{,}5x \Big]_1^x$

$= 0{,}25x^2 - 0{,}5x + 0{,}25.$

- $2 \leq x < 3$

$F_X(x) = \int_{-\infty}^{x} f(x)\,dx$

$= \int_{-\infty}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx + \int_{2}^{x} f(x)\,dx$

$= 0 + 0{,}25 + \int_{2}^{x} 0{,}5\,dx$

$= 0{,}25 + 0{,}5x \Big]_2^x$

$= 0{,}25 + 0{,}5x - 1$

$= 0{,}5x - 0{,}75$

- $3 \leq x < 4$

$F_X(x) = \underbrace{\int_{-\infty}^{1} f(x)\,dx + \ldots}_{} + \int_{3}^{x} f(x)\,dx$

$= 0{,}75 + \int_{3}^{x} (2 - 0{,}5x)\,dx$

$= 2x - 0{,}25x^2 \Big]_3^x$

$= 2x - 0{,}25x^2 - (6 - 0{,}25(9)) + 0{,}75$

$= 2x - 0{,}25x^2 - 3{,}75 + 0{,}75$

$= 2x - 0{,}25x^2 - 3$ //

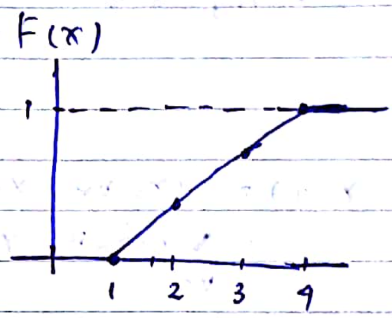
- $x \geq 4$

$F_X(x) = \underbrace{\int_{-\infty}^{1} f(x)\,dx + \ldots}_{1+0} + \int_{4}^{x} f(x)\,dx$

$= 1 + 0$

$= 1$

$F_X(x) = \begin{cases} 0 & ; x < 1 \\ 0{,}25x^2 - 0{,}5x + 0{,}25 & ; 1 \leq x < 2 \\ 0{,}5 - 0{,}75 & ; 2 \leq x < 3 \\ 2x - 0{,}25x^2 - 3 & ; 3 \leq x < 4 \\ 1 & ; x \geq 4 \end{cases}$



$E(X) = \int_{-\infty}^{\infty} x\, f(x)\,dx$

$= \int_{1}^{2} x(0{,}5x - 0{,}5)\,dx + \int_{2}^{3} x(0{,}5)\,dx + \int_{3}^{4} x(2 - 0{,}5x)\,dx$

$= \ldots$

$V(X) = E(X^2) - (E(X))^2$

$E(X^2) = \int_{1}^{2} x^2(0{,}5x - 0{,}5)\,dx + \int_{2}^{3} x^2(0{,}5)\,dx + \int_{3}^{4} x^2(2 - 0{,}5x)\,dx$

$= \ldots$

R.13.

No

Date

"Peubah Acak Ganda."

Peluang Marginal.

diket: 1. D = $P(X=x, Y=y)$

2. k = $f_{x,y}(x,y)$

a. Bagi x.

1. D $P(X=x) = P(X=x, Y=\forall y)$

$= \sum_{\forall y} P(X=x, Y=y);\ x \in D_x.$

2. k: $f_x(x) = f_{x,y}(x, \forall y)$

$= \int_{\forall y} f_{x,y}(x,y)\,dy,$

$x \in D_x.$

b. Bagi Y

1) D : $P(Y=y) = P(X=\forall x, Y=y)$

$= \sum_{\forall x} P(X=\forall x, Y=y),$

$y \in D_y.$

2. K: $f_x(y) = f_{x,y}(\forall x, y)$

$= \int_{\forall x} f_{x,y}(x,y)\,dx;$

$y \in D_y.$

Peluang Bersyarat.

a. Pada $Y = Y_0$

1. D: $P(X=x \mid Y=y_0) = \dfrac{P(X=x, Y=y_0)}{P(Y=y_0)}$

$= \dfrac{P(X=x, Y=y_0)}{\sum_{\forall x} P(X=x, Y=y_0)}$

2. k: $f_{x|y}(x|y_0) = \dfrac{f_{x,y}(x,y_0)}{f_y(y_0)}$

$= \dfrac{f_{x,y}(x,y_0)}{\int_{\forall x} f_{x,y}(x,y_0)\,dx}.$

b. Pada $X = X_0$

1. D: $P(Y=y \mid X=x_0) = \dfrac{P(X=x_0, Y=y)}{P(X=x_0)}$

$= \dfrac{P(X=x_0, Y=y)}{\sum_{\forall y} P(X=x_0, Y=y)}$

2. k: $f_{y|x}(y|x_0) = \dfrac{f_{x,y}(x_0,y)}{f_x(x_0)}$

$= \dfrac{f_{x,y}(x_0,y)}{\int_{\forall y} f_{x,y}(x_0,y)\,dy}.$

Nilai Harapan.

1. $E(XY) = \sum_{\forall x}\sum_{\forall y} xy\ P(X=x, Y=y)$

$\iint_{\forall y\, \forall x} xy\, f_{x,y}(x,y)\,dx\,dy.$

2. $E(ax+by) = aE(x) + bE(y)$

3. $E(ax-by) = aE(x) - bE(y)$

$$\boxed{\begin{aligned} E(ax) &= aE(x) \\ E(a) &= a \end{aligned}}$$

Ragam Peragam / Variance Covariance.

1. $Cov(x,y) = E[(x-E(x))(Y-E(y))]$

$= E(xy) - E(x)E(y)$

2. $Var(ax+by) = a^2 Var(x) + b^2 Var(y) + 2ab\ Cov(x,y)$

3. $Var(ax-by) = a^2 Var(x) + b^2 Var(y) - 2ab\ Cov(x,y)$

- $E(xy) = E(x)E(y)$, Jika x & y saling bebas.
- $Var(x) = E[(x-E(x))^2]$

$= E(x^2) - [E(x)]^2$

- $Var(ax) = a^2 Var(x)$
- $Var(a) = 0$

$$\boxed{\begin{aligned} &\cdot\ Var(x) = \sigma_x^2 \\ &\cdot\ Cov(x,y) = \sigma_{xy} \end{aligned}}$$

Korelasi

$Corr(x,y) = \rho_{x,y} = r_{x,y}$

$= \dfrac{Cov(x,y)}{\sqrt{Var(x)\,Var(y)}}$

$\sqrt{Var(x)} = \sqrt{\sigma^2 x} = \sigma x = S_x.$

R.14 soal: php.

(3.) x dan y pa diskret menyebar bersama dg fmp

$$P_{x,y}(x,y) = \begin{cases} c(x^2+y), & x=0,1,2 \\ & y=2,3 \\ 0, & \text{lainnya.} \end{cases}$$

a. tentukan nilai c

b. tentukan fmp marginal x dan y.

c. tentukan $E(xy)$

d. apakah x dan y saling bebas.

e. tentukan sebaran x dg syarat $y=2$

f. tentukan nilai $E(x \mid y=2)$.

Jawab:

a.

x/y	2	3	total	
0	2c	3c	5c	5/25
1	3c	4c	7c	7/25
2	6c	7c	13c	13/25.
total	11c = 11/25	14c = 14/25	25c.	

$$P_{x,y}(x,y) = \begin{cases} 1/25\,(x^2+y), & x=0,1,2 \\ & y=2,3. \\ 0, & \text{lainnya.} \end{cases}$$

b. fmp x.

$$P_x(x) = \begin{cases} 5/25, & x=0 \\ 7/25, & x=1 \\ 13/25, & x=2 \\ 0, & x \text{ lainnya} \end{cases}$$

$$P(X=x) = \begin{cases} \frac{2x^2+5}{25}; & x=0,1,2. \\ 0, & x \text{ lainnya.} \end{cases}$$

fmp y.

$$P_y(Y=y) = \begin{cases} 11/25, & y=2 \\ 14/25, & y=3. \\ 0, & y \text{ lainnya} \end{cases}$$

$$P(Y=y) = \begin{cases} \frac{3y+5}{25}; & y=2,3. \\ 0, & y \text{ lainnya.} \end{cases}$$

c. $E(XY) = 0.2 \cdot \frac{1}{25}(2) + 0 \cdot 3 \cdot \frac{1}{25}(3)$
$+ 1 \cdot 2 \cdot \frac{1}{25}(3) + 1 \cdot 3 \cdot \frac{1}{25} \cdot (4)$
$+ 2 \cdot 2 \cdot \frac{1}{25}(6) + 2 \cdot 3 \cdot \frac{1}{25} \cdot (7)$

$= 0 + 0 + \frac{6}{25} + \frac{12}{25} + \frac{24}{25} + \frac{42}{25}$

$= \frac{84}{25}$ //

d. x dan y saling bebas?

$P_{x,y}(x,y) = \frac{1}{25}(x^2+y)$

$P_x(x)\,P_y(y) = \frac{1}{25}(2x^2+5)\frac{1}{25}(3y+5)$

$= \frac{1}{625}(2x^2 + 10x^2 + 15y + 25)$

$P_{x,y}(x,y) \neq P_x(x)\,P_y(y)$

$\therefore$ x dan y tdk saling bebas.

e. $P(x \mid y=2)$?

$$P_{x|y}(x|y) = \frac{P_{xy}(xy)}{P_x(y)}$$

$$= \frac{\cancel{\frac{1}{25}}(x^2+y)}{\cancel{1/25}(3y+5)} = \frac{x^2+y}{3y+5}$$

$$P_{x|y=2} = \frac{x^2+2}{3(2)+5} = \frac{x^2+2}{11} \quad \Big] \; x = 0,1,2.$$

$$P_{x|y=2} = \begin{cases} 2/11 & , x=0 \\ 3/11 & , x=1 \\ 6/11 & , x=2 \\ 0 & , x \text{ lainnya.} \end{cases}$$

f $E(x|y=2) = \sum_{\forall x} x \cdot p(x|y=2)$

$$= 0\left(\frac{2}{11}\right) + 1\left(\frac{3}{11}\right) + 2\left(\frac{6}{11}\right)$$

$$= \frac{3}{11} + \frac{12}{11} = \frac{15}{11}$$

(4) fkp bersama

$$f(x,y) = \begin{cases} cxy & , \text{jika } 0 \le y \le x \le 4. \\ 0 & , \text{selainnya} \end{cases}$$

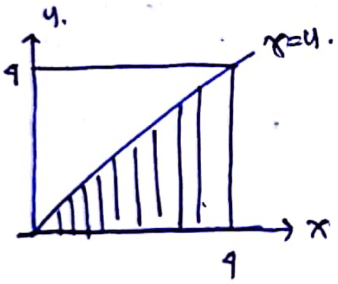
a. tent. nilai c

b. tent. $P(x+y \le 4)$

c. tent. sebaran marginal x dan y.

Jawab:

a



$$1 = \int_0^4 \int_y^4 cxy \; dx\, dy.$$

$$= \int_0^4 \left[c \cdot \frac{1}{2} x^2 y \right]_y^4 dy$$

$$= \int_0^4 \frac{1}{2} c\,(16y - y^3)\, dy$$

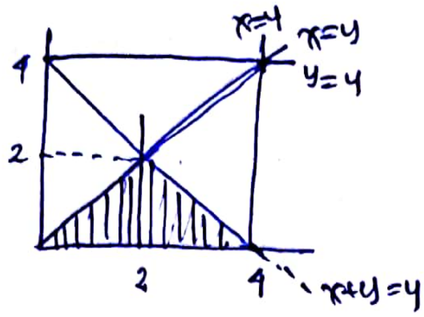
$$= \frac{1}{2} c \left[16 \cdot \frac{1}{2} y^2 - \frac{1}{4} y^4 \right]_0^4$$

$$= \frac{1}{2} c \left[2 \cdot 4^3 - 4^3 \right]$$

$$= \frac{1}{2} c \left[4^3 \right]$$

$$1 = \frac{1}{2} c\,(64) \rightarrow 1 = 32c \quad \boxed{c = \frac{1}{32}}$$

b. $P(x+y \le 4)$?



$x+y \le 4$

$x \le 4-y$

$$P = \int_0^2 \int_y^{4-y} \frac{1}{32} \cdot xy \; dx\, dy$$

$$= \frac{1}{32} \int_0^2 \frac{1}{2} x^2 y \Big]_y^{4-y} dy$$

$$= \frac{1}{32} \int_0^2 \left[\frac{1}{2}(4-y)^2 y - \frac{1}{2} y^3 \right] dy$$

$$= \frac{1}{32} \int_0^2 \left[\frac{1}{2} y^3 - 4y^2 + 8y - \frac{1}{3} y^3 \right] dy$$

$$= \frac{1}{32} \left[\frac{1}{8} y^4 - \frac{4}{3} y^3 + 4y^2 - \frac{1}{12} y^4 \right]_0^2$$

$$= \frac{1}{32} \left[2 - \frac{32}{3} + 16 - \frac{16}{2} \right]$$

$$= \frac{1}{32} \left[\frac{24 - 128 + 152 - 16}{12} \right]$$

$$= \frac{1}{32} \left[\frac{32}{12} \right] = \frac{1}{12} \;//$$

c $f_X(x) = \int_0^x \frac{1}{32} xy \, dy$

$$= \frac{1}{32} \cdot \frac{1}{2} xy^2 \Big]_0^x$$

$$= \frac{1}{64} \cdot x^3 \quad ; 0 \le x \le 4$$

$f_Y(y) = \int_y^4 \frac{1}{32} xy \, dx$

$$= \frac{1}{32} \cdot \frac{1}{2} x^2 y \Big]_y^4$$

$$= \frac{1}{64} y\,(4^2 - y^2)$$

$$= \frac{1}{4} y - \frac{y^3}{64} \quad ; 0 \le y \le 4 \;//$$

R.13. contoh: php.

1. misal x dan y p.a diskret menyebar bersama dg fungsi peluang bersama.

$$P_{x,y}(x,y) = \begin{cases} c(x^2+y^2), & u/\ x=0,1,2 \\ & y=1,2 \\ 0 & x \& y \text{ lain.} \end{cases}$$

a. tentukan nilai c.

b. tentukan f marginal x dan y.

c. tentukan nilai harapan dari xy atau $E(xy)$.

Jawab:

a.

x / y	1	2	Total
0	c	4c	$5c = \frac{5}{25}$
1	2c	5c	$7c = 7/25$
2	5c	8c	$13c = 13/25$
total.	$8c = \frac{8}{25}$	$17c = \frac{17}{25}$	25c.

$$\boxed{\begin{aligned} 25c &= 1 \\ c &= \frac{1}{25} \end{aligned}}$$

$$P_{x,y}(x,y) = \begin{cases} 1/25\,(x^2+y^2) & ;\ x=0,1,2 \\ & \ \ y=1,2 \\ 0 & ;\ \text{lainnya.} \end{cases}$$

b. f. marginal.

$$P_x(x) = \begin{cases} 5/25, & x=0 \\ 7/25, & x=1 \\ 13/25, & x=2 \\ 0, & \text{lainnya} \end{cases}$$

$$P_y(y) = \begin{cases} 8/25, & y=1 \\ 17/25, & y=2 \\ 0, & y \text{ lainnya.} \end{cases}$$

c. $E(xy) = \sum_{\forall x} \sum_{\forall y} xy\ p(x,y)$

$= (x=0)(y=1)\,P(x=0,y=1) + (x=0)(y=2)\,P(x=0,y=2) + \dots + (x=2)(y=2)\,P(x=2,y=2)$

$= 0(1)\left(\frac{1}{25}\right) + 0(2)\cdot\frac{4}{25} + 1(1)\cdot\frac{2}{25} + 1(2)\cdot\frac{5}{25} + 2(1)\cdot\frac{5}{25} + 2(2)\cdot\frac{8}{25}.$

$= 0 + 0 + \frac{2}{25} + \frac{10}{25} + \frac{10}{25} + \frac{32}{25}$

$= \frac{54}{25}$ //

d. $\text{corr}(x,y) = \dfrac{\text{cov}(x,y)}{\sqrt{\text{var}(x)\text{var}(y)}}$

$\text{cov}(x,y) = E(xy) - E(x)E(y)$

$\text{var}(x) = E(x^2) - (E(x))^2$

$\text{var}(y) = E(y^2) - (E(y))^2$

Jawab:

$E(x) = \sum_{\forall x} x\,p(x) = 0\cdot\frac{5}{25} + 1\cdot\frac{7}{25} + 2\cdot\frac{13}{25}$

$= 0 + \frac{7}{25} + \frac{26}{25} = \frac{33}{25}.$

$E(x^2) = \sum x^2 p(x) = 0 + \frac{7}{25} + \frac{52}{25} = \frac{59}{25}.$

$\text{var}(x) = E(x^2) - (E(x))^2 = \frac{59}{25} - \left(\frac{33}{25}\right)^2 = \frac{386}{625}.$

$E(y) = 1\cdot\frac{8}{25} + 2\cdot\frac{17}{25} = \frac{42}{25}$

$E(y^2) = 1^2\cdot\frac{8}{25} + 2^2\cdot\frac{17}{25} = \frac{76}{25}$

$\text{var}(y) = \frac{76}{25} - \left(\frac{42}{25}\right)^2 = \frac{136}{625}.$

$$\operatorname{cov}(x,y) = \frac{54}{25} - \frac{33}{25}\left(\frac{42}{25}\right) = \frac{-36}{625}$$

$$\operatorname{corr}(x,y) = \frac{-36/625}{\sqrt{\frac{386}{625}\left(\frac{136}{625}\right)}} = -0,157.$$

2 misal x dan y dua p.a kontinu yg menyebar bersama dg fkp.

$$f_{xy}(x,y) = \begin{cases} cxy, & \text{jika } 0 \le x \le y \le 2 \\ 0, & x \text{ lainnya.} \end{cases}$$

$x=0$, $x=y$, $y=2$, $y=0$, $x=2$.

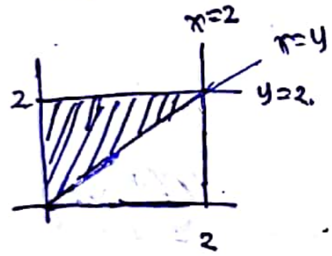
a. tentukan nilai c.

b. tentukan peluang marginal x dan marginal y.

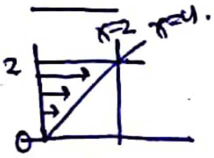
c. tentukan $E(xy)$.

Jawab:

a.

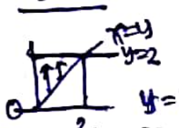


dx dy



$x=0$ ke $x=y$.
$y=0$ ke $y=2$.

dy dx



$y=x$ ke $y=2$.
$x=0$ ke $x=2$.

$$f_{x,y}(x,y) = \int_0^2\int_0^y cxy \, dx\,dy$$

$$1 = \int_0^2 \tfrac{1}{2} c x^2 y \Big]_0^y dy.$$

$$1 = \int_0^2 \tfrac{1}{2} c y^3 \, dy$$

$$1 = \tfrac{1}{2}\cdot\tfrac{1}{4} c y^4 \Big]_0^2$$

$$\tfrac{1}{8} c (2^4 - 0^4) = 1$$

$$2c = 1$$

$$\boxed{c = \tfrac{1}{2}}$$

b. $$f_x(x) = \int_x^2 \tfrac{1}{2} xy \, dy = \tfrac{1}{2}\cdot\tfrac{1}{2} xy^2 \Big]_x^2$$

$$= \tfrac{1}{4} xy^2 \Big]_x^2 = \tfrac{1}{4} x (2^2 - x^2)$$

$$= x - \tfrac{1}{4} x^3$$

$$f_x(x) = \begin{cases} x - \frac{1}{4} x^2, & 0 \le x \le 2, \\ 0, & x \text{ lainnya.} \end{cases}$$

$$f_x(y) = \int_0^y \tfrac{1}{2} xy \, dx$$

$$= \tfrac{1}{2}\cdot\tfrac{1}{2} \cdot x^2 y \Big]_0^y$$

$$= \tfrac{1}{4} y \cdot (y^2 - 0^2) = \tfrac{1}{4} y^3.$$

$$f_y(y) = \begin{cases} 1/4\, y^3, & 0 \le y \le 2 \\ 0, & y \text{ lainnya.} \end{cases}$$

c. $$E(xy) = \int_0^2\int_0^y xy\left(\tfrac{1}{2} xy\right) dx\,dy.$$

$$= \int_0^2\int_0^y \tfrac{1}{2} x^2 y^2 \, dx\, dy.$$

$$= \int_0^2 \tfrac{1}{2}\cdot\tfrac{1}{3} x^3 y^2 \Big]_0^y dy$$

$$= \tfrac{1}{6} \int_0^2 y^2 (y^3 - 0^3)\, dy.$$

$$= \tfrac{1}{6} \int_0^2 y^5 \, dy$$

$$= \tfrac{1}{6}\cdot\tfrac{1}{6} y^6 \Big]_0^2$$

$$= \tfrac{1}{36} (2^6 - 0^6)$$

$$= \frac{64}{36}$$

$$= 16/9 \quad //$$

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