## Phase Space Diagrams

May 10, 2020

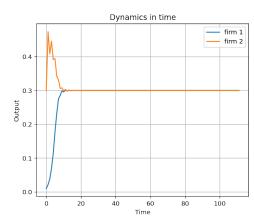
```
[2]: import numpy as np
     import matplotlib.pyplot as plt
     %matplotlib inline
     import sympy as sm
[3]: def myopic(q1,q2, alpha1,alpha2, a,b,c1,c2, n):
         xs=[q1]
         ys=[q2]
         for i in range(n):
             xs.append(xs[i] + alpha1*xs[i]*(a-c1-2*b*xs[i]-b*ys[i]))
             ys.append(ys[i] + alpha2*ys[i]*(a-c2-2*b*ys[i]-b*xs[i]))
         return np.array([xs, ys])
[4]: def myop_map(x,y, alpha1,alpha2, a,b,c1,c2, n):
         return x + alpha1*x*(a-c1-2*b*x-b*y), y + alpha2*y*(a-c2-2*b*y-b*x)
[3]: iv1
            = 0.01
     iv2
            = 0.3
     alpha1 = 2
     alpha2 = 2
           = 1.9
     b
            = 1
     c1
            = 1
           = 1
     c2
     n=111
     time
           = np.linspace(0, 100, 1001)
     q1s
            = myopic(iv1,iv2,
                                  alpha1,alpha2,
                                                    a,b,c1,c2,
                                                                  n)[0]
            = myopic(iv1,iv2,
                                  alpha1,alpha2,
                                                                  n)[1]
     q2s
                                                    a,b,c1,c2,
     fig = plt.figure(figsize=(15,5))
     fig.subplots_adjust(wspace = 0.5, hspace = 0.3)
     ax1 = fig.add_subplot(1,2,1)
     ax2 = fig.add_subplot(1,2,2)
```

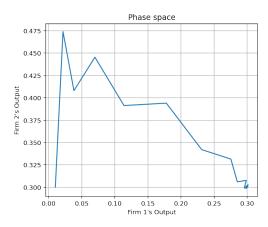
```
ax1.plot(q1s, label='firm 1')
ax1.plot(q2s, label='firm 2')
ax1.set_title("Dynamics in time")
ax1.set_xlabel("Time")
ax1.set_ylabel("Output")
ax1.grid()
ax1.legend(loc='best')

ax2.plot(q1s, q2s)
ax2.set_xlabel("Firm 1's Output")
ax2.set_ylabel("Firm 2's Output")
ax2.set_title("Phase space")
ax2.grid()
print("iv1 = ", iv1, ", iv2 = ", iv2, ", alpha1 = ", alpha1, ", alpha2 = ", uhealpha2, ", a = ", a, ", b = ", b, ", c1 = ", c1, ", c2 = ", c2)
```

iv1 = 0.01, iv2 = 0.3, alpha1 = 2, alpha2 = 2, a = 1.9, b = 1, c1 = 1, c2 = 1

[3]:





```
[13]: fig2 = plt.figure(figsize=(10,7))
    ax4 = fig2.add_subplot(1,1,1)

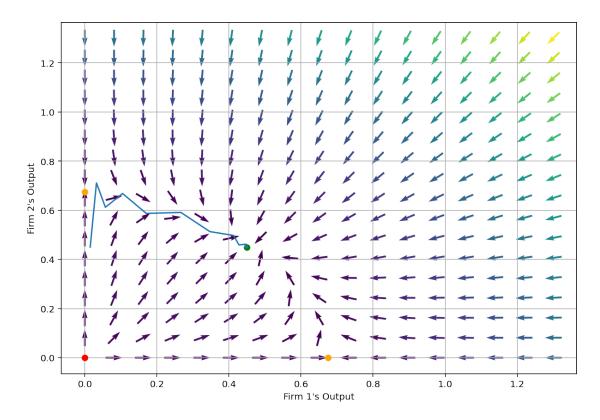
alpha1 = 2
    alpha2 = 2
    a = 1.9
    b = 1
    c1 = 1
    c2 = 1

x = np.linspace(0, 1.3, 17)
y = np.linspace(0, 1.3, 17)
X1 , Y1 = np.meshgrid(x, y)
```

```
DX1, DY1 = myop_map(X1, Y1, alpha1,alpha2, a,b,c1,c2, n)
M = (np.hypot(DX1, DY1))
M[M == 0] = 1.
DX1 /= M
DY1 /= M
ax4.plot(0,
                                                 color='red', marker='o')
                           0,
ax4.plot(1.5*(a-c1)/(2*b), 0,
                                                 color='orange', marker='o')
                           1.5*(a-c1)/(2*b),
                                                 color='orange', marker='o')
ax4.plot(0,
ax4.plot(1.5*(a-c1)/(3*b), 1.5*(a-c1)/(3*b),
                                                 color='green', marker='o')
       = 1.5 * myopic(iv1,iv2,
                                   alpha1,alpha2,
                                                      a,b,c1,c2,
                                                                    n)[0]
                                                                    n)[1]
q2s
      = 1.5 * myopic(iv1,iv2,
                                   alpha1,alpha2,
                                                      a,b,c1,c2,
ax4.plot(q1s, q2s)
ax4.quiver(X1, Y1, DX1, DY1, M, pivot='mid')
ax4.grid()
ax4.set_xlabel("Firm 1's Output")
ax4.set_ylabel("Firm 2's Output")
```

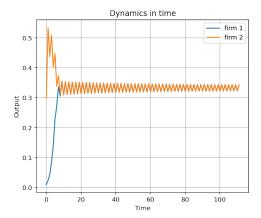
## [13]: Text(0, 0.5, "Firm 2's Output")

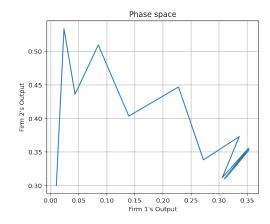
## [13]:



```
[14]: iv1
            = 0.01
            = 0.3
      iv2
      alpha1 = 2
      alpha2 = 2
             = 2
             = 1
      c1
             = 1
      c2
             = 1
      n=111
      time
             = np.linspace(0, 100, 1001)
             = myopic(iv1,iv2,
                                   alpha1,alpha2,
                                                     a,b,c1,c2,
                                                                    n)[0]
      q1s
      q2s
             = myopic(iv1,iv2,
                                   alpha1,alpha2,
                                                      a,b,c1,c2,
                                                                    n)[1]
      fig = plt.figure(figsize=(15,5))
      fig.subplots_adjust(wspace = 0.5, hspace = 0.3)
      ax1 = fig.add_subplot(1,2,1)
      ax2 = fig.add_subplot(1,2,2)
      ax1.plot(q1s, label='firm 1')
      ax1.plot(q2s, label='firm 2')
      ax1.set_title("Dynamics in time")
      ax1.set xlabel("Time")
      ax1.set_ylabel("Output")
      ax1.grid()
      ax1.legend(loc='best')
      ax2.plot(q1s, q2s)
      ax2.set_xlabel("Firm 1's Output")
      ax2.set_ylabel("Firm 2's Output")
      ax2.set_title("Phase space")
      ax2.grid()
      print("iv1 = ", iv1, ", iv2 = ", iv2, ", alpha1 = ", alpha1, ", alpha2 = ", u
       \rightarrowalpha2, ", a = ", a, ", b = ", b, ", c1 = ", c1, ", c2 = ", c2)
```

iv1 = 0.01, iv2 = 0.3, alpha1 = 2, alpha2 = 2, a = 2, b = 1, c1 = 1, c2 = 1



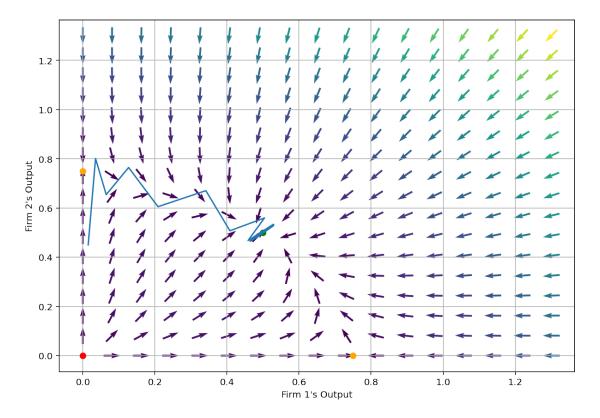


```
[16]: fig2 = plt.figure(figsize=(10,7))
     ax4 = fig2.add_subplot(1,1,1)
     alpha1 = 2
     alpha2 = 2
            = 2
            = 1
            = 1
     c1
     c2
            = 1
     x = np.linspace(0, 1.3, 17)
     y = np.linspace(0, 1.3, 17)
     X1 , Y1 = np.meshgrid(x, y)
     DX1, DY1 = myop_map(X1, Y1, alpha1,alpha2, a,b,c1,c2, n)
     M = (np.hypot(DX1, DY1))
     M[M == 0] = 1.
     DX1 /= M
     DY1 /= M
     ax4.plot(0,
                                                     color='red', marker='o')
     ax4.plot(1.5*(a-c1)/(2*b), 0,
                                                     color='orange', marker='o')
     ax4.plot(0,
                                1.5*(a-c1)/(2*b),
                                                     color='orange', marker='o')
     ax4.plot(1.5*(a-c1)/(3*b), 1.5*(a-c1)/(3*b),
                                                     color='green', marker='o')
     q1s = 1.5 * myopic(iv1,iv2, alpha1,alpha2,
                                                          a,b,c1,c2,
                                                                        n)[0]
     q2s = 1.5 * myopic(iv1,iv2,
                                      alpha1,alpha2,
                                                          a,b,c1,c2,
                                                                       n)[1]
     ax4.plot(q1s, q2s)
     ax4.quiver(X1, Y1, DX1, DY1, M, pivot='mid')
     ax4.grid()
     ax4.set_xlabel("Firm 1's Output")
```

```
ax4.set_ylabel("Firm 2's Output")
```

## [16]: Text(0, 0.5, "Firm 2's Output")

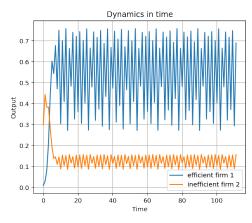
[16]:

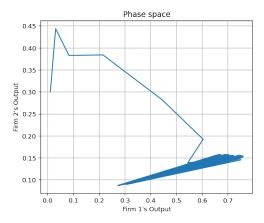


```
[9]: iv1
            = 0.01
     iv2
            = 0.3
     alpha1 = 2
     alpha2 = 2
            = 2.1
            = 1
            = 0.75
     c1
     c2
            = 1.25
     n=111
            = np.linspace(0, 100, 1001)
     time
            = myopic(iv1,iv2,
                                   alpha1,alpha2,
     q1s
                                                     a,b,c1,c2,
                                                                    n)[0]
            = myopic(iv1,iv2,
                                   alpha1,alpha2,
                                                                    n)[1]
     q2s
                                                     a,b,c1,c2,
     fig = plt.figure(figsize=(15,5))
     fig.subplots_adjust(wspace = 0.5, hspace = 0.3)
     ax1 = fig.add_subplot(1,2,1)
```

iv1=0.01 , iv2=0.3 , alpha1=2 , alpha2=2 , a=2.1 , b=1 , c1=0.75 , c2=1.25

[9]:



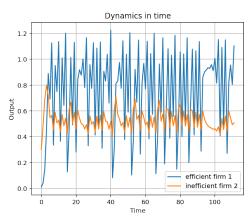


```
[8]: iv1
            = 0.01
            = 0.3
     iv2
     alpha1 = 1.5
     alpha2 = 0.5
            = 3.1
     a
     b
            = 1
     c1
            = 0.75
            = 1.25
     c2
     n=111
            = np.linspace(0, 100, 1001)
     time
```

```
q1s
       = myopic(iv1,iv2,
                              alpha1,alpha2,
                                                a,b,c1,c2,
                                                               n)[0]
       = myopic(iv1,iv2,
                              alpha1,alpha2,
                                                a,b,c1,c2,
                                                               n)[1]
q2s
fig = plt.figure(figsize=(15,5))
fig.subplots_adjust(wspace = 0.5, hspace = 0.3)
ax1 = fig.add_subplot(1,2,1)
ax2 = fig.add_subplot(1,2,2)
ax1.plot(q1s, label='efficient firm 1')
ax1.plot(q2s, label='inefficient firm 2')
ax1.set title("Dynamics in time")
ax1.set_xlabel("Time")
ax1.set_ylabel("Output")
ax1.grid()
ax1.legend(loc='best')
ax2.plot(q1s, q2s)
ax2.set_xlabel("Firm 1's Output")
ax2.set_ylabel("Firm 2's Output")
ax2.set_title("Phase space")
ax2.grid()
print("iv1 = ", iv1, ", iv2 = ", iv2, ", alpha1 = ", alpha1, ", alpha2 = ", |
\hookrightarrowalpha2, ", a = ", a, ", b = ", b, ", c1 = ", c1, ", c2 = ", c2)
```

iv1=0.01 , iv2=0.3 , alpha1=1.5 , alpha2=0.5 , a=3.1 , b=1 , c1=0.75 , c2=1.25

[8]:

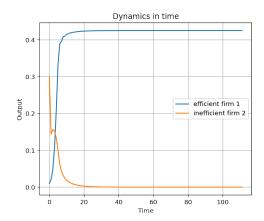


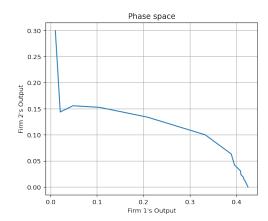
```
[19]: iv1 = 0.01
iv2 = 0.3
alpha1 = 2
alpha2 = 2
a = 1.6
b = 1
```

```
c1
       = 0.75
c2
       = 1.25
n=111
time
       = np.linspace(0, 100, 1001)
       = myopic(iv1,iv2,
                             alpha1,alpha2,
                                                              n)[0]
q1s
                                                a,b,c1,c2,
       = myopic(iv1,iv2,
                             alpha1,alpha2,
                                                              n)[1]
q2s
                                                a,b,c1,c2,
fig = plt.figure(figsize=(15,5))
fig.subplots_adjust(wspace = 0.5, hspace = 0.3)
ax1 = fig.add_subplot(1,2,1)
ax2 = fig.add_subplot(1,2,2)
ax1.plot(q1s, label='efficient firm 1')
ax1.plot(q2s, label='inefficient firm 2')
ax1.set_title("Dynamics in time")
ax1.set_xlabel("Time")
ax1.set_ylabel("Output")
ax1.grid()
ax1.legend(loc='best')
ax2.plot(q1s, q2s)
ax2.set xlabel("Firm 1's Output")
ax2.set_ylabel("Firm 2's Output")
ax2.set_title("Phase space")
ax2.grid()
print("iv1 = ", iv1, ", iv2 = ", iv2, ", alpha1 = ", alpha1, ", alpha2 = ", |
\rightarrowalpha2, ", a = ", a, ", b = ", b, ", c1 = ", c1, ", c2 = ", c2)
```

iv1 = 0.01, iv2 = 0.3, alpha1 = 2, alpha2 = 2, a = 1.6, b = 1, c1 = 0.75, c2 = 1.25

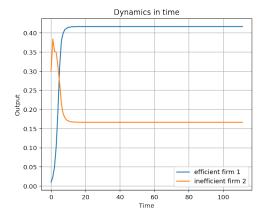
[19]:

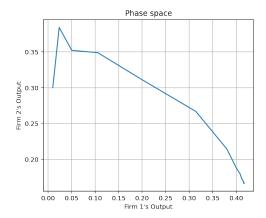




```
[9]: iv1
           = 0.01
     iv2
           = 0.3
     alpha1 = 2
     alpha2 = 2
           = 2
            = 1
     c1
           = 1
     c2
           = 1.25
     n=111
     time
           = np.linspace(0, 100, 1001)
           = myopic(iv1,iv2,
                                  alpha1,alpha2,
                                                    a,b,c1,c2,
     q1s
                                                                  n)[0]
           = myopic(iv1,iv2,
                                  alpha1,alpha2,
                                                    a,b,c1,c2,
                                                                  n)[1]
     q2s
     fig = plt.figure(figsize=(15,5))
     fig.subplots_adjust(wspace = 0.5, hspace = 0.3)
     ax1 = fig.add_subplot(1,2,1)
     ax2 = fig.add_subplot(1,2,2)
     ax1.plot(q1s, label='efficient firm 1')
     ax1.plot(q2s, label='inefficient firm 2')
     ax1.set_title("Dynamics in time")
     ax1.set xlabel("Time")
     ax1.set_ylabel("Output")
     ax1.grid()
     ax1.legend(loc='best')
     ax2.plot(q1s, q2s)
     ax2.set_xlabel("Firm 1's Output")
     ax2.set_ylabel("Firm 2's Output")
     ax2.set_title("Phase space")
     ax2.grid()
     print("iv1 = ", iv1, ", iv2 = ", iv2, ", alpha1 = ", alpha1, ", alpha2 = ", u
      \rightarrowalpha2, ", a = ", a, ", b = ", b, ", c1 = ", c1, ", c2 = ", c2)
    iv1 = 0.01, iv2 = 0.3, alpha1 = 2, alpha2 = 2, a = 2, b = 1, c1 = 1
```

```
iv1 = 0.01, iv2 = 0.3, alpha1 = 2, alpha2 = 2, a = 2, b = 1, c1 = 1, c2 = 1.25
```





```
[10]: fig3 = plt.figure(figsize=(8,6))
      ax4 = fig3.add_subplot(1,1,1)
      alpha1 = 2
      alpha2 = 2
             = 2
      c1
             = 1
     c2
             = 1.
      x = np.linspace(0, 1.3, 17)
      y = np.linspace(0, 1.3, 17)
      X1 , Y1 = np.meshgrid(x, y)
      DX1, DY1 = myop_map(X1, Y1, alpha1,alpha2, a,b,c1,c2, n)
      M = (np.hypot(DX1, DY1))
     M[M == 0] = 1.
      DX1 /= M
      DY1 /= M
      ax4.plot(0,
                                                        color='red', marker='o')
                                 0,
                                                        color='red', marker='o')
      ax4.plot(0.75,
                                 0.25,
      ax4.quiver(X1, Y1, DX1, DY1, M, pivot='mid')
      ax4.grid()
      ax4.set_xlabel("Firm 1's Output")
      ax4.set_ylabel("Firm 2's Output")
```

[10]: Text(0, 0.5, "Firm 2's Output")

[10]:

