Phase Space Diagrams

May 10, 2020

[2]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
import sympy as sm
```

[3]:
```
def myopic(q1,q2, alpha1,alpha2, a,b,c1,c2, n):
    xs=[q1]
    ys=[q2]
    for i in range(n):
        xs.append(xs[i] + alpha1*xs[i]*(a-c1-2*b*xs[i]-b*ys[i]))
        ys.append(ys[i] + alpha2*ys[i]*(a-c2-2*b*ys[i]-b*xs[i]))
    return np.array([xs, ys])
```

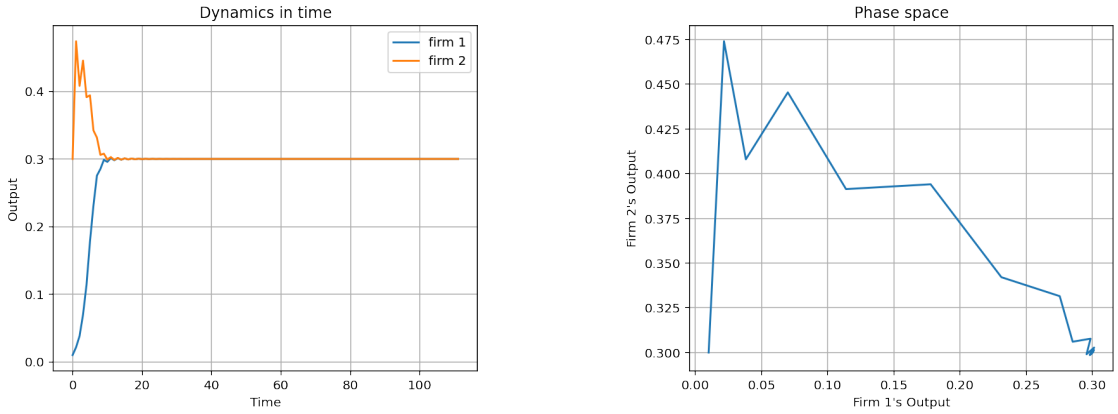
[4]:
```
def myop_map(x,y, alpha1,alpha2, a,b,c1,c2, n):

    return x + alpha1*x*(a-c1-2*b*x-b*y),  y + alpha2*y*(a-c2-2*b*y-b*x)
```

[3]:
```
iv1    = 0.01
iv2    = 0.3
alpha1 = 2
alpha2 = 2
a      = 1.9
b      = 1
c1     = 1
c2     = 1
n=111

time   = np.linspace(0, 100, 1001)

q1s    = myopic(iv1,iv2,     alpha1,alpha2,    a,b,c1,c2,    n)[0]
q2s    = myopic(iv1,iv2,     alpha1,alpha2,    a,b,c1,c2,    n)[1]

fig = plt.figure(figsize=(15,5))
fig.subplots_adjust(wspace = 0.5, hspace = 0.3)
ax1 = fig.add_subplot(1,2,1)
ax2 = fig.add_subplot(1,2,2)
```

```
ax1.plot(q1s, label='firm 1')
ax1.plot(q2s, label='firm 2')
ax1.set_title("Dynamics in time")
ax1.set_xlabel("Time")
ax1.set_ylabel("Output")
ax1.grid()
ax1.legend(loc='best')

ax2.plot(q1s, q2s)
ax2.set_xlabel("Firm 1's Output")
ax2.set_ylabel("Firm 2's Output")
ax2.set_title("Phase space")
ax2.grid()
print("iv1 = ",  iv1, ", iv2 = ", iv2, ", alpha1 = ", alpha1, ", alpha2 = ",␣
 ↪alpha2, ", a = ", a, ", b = ", b, ", c1 = ", c1, ", c2 = ", c2)
```

```
iv1 =  0.01 , iv2 =  0.3 , alpha1 =  2 , alpha2 =  2 , a =  1.9 , b =  1 , c1 =
1 , c2 =  1
```

[3]:



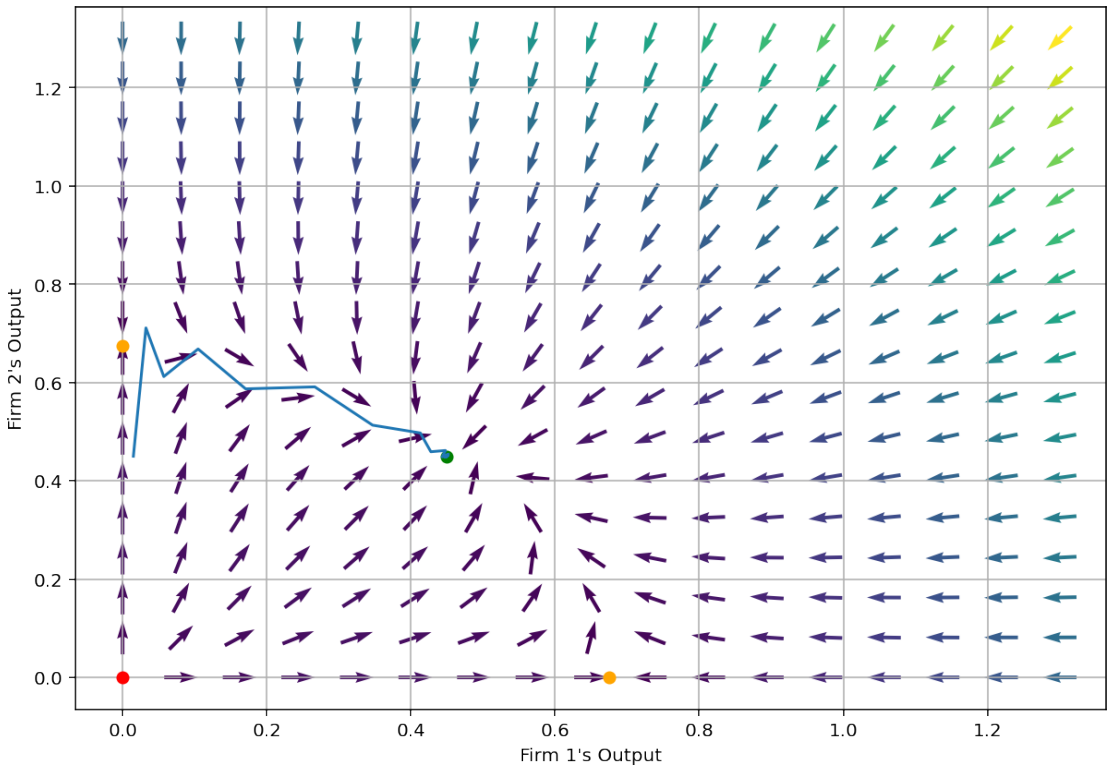
[13]:

```
fig2 = plt.figure(figsize=(10,7))
ax4 = fig2.add_subplot(1,1,1)

alpha1 = 2
alpha2 = 2
a      = 1.9
b      = 1
c1     = 1
c2     = 1

x = np.linspace(0, 1.3, 17)
y = np.linspace(0, 1.3, 17)

X1 , Y1  = np.meshgrid(x, y)
```

```
DX1, DY1 = myop_map(X1, Y1, alpha1,alpha2, a,b,c1,c2, n)
M = (np.hypot(DX1, DY1))
M[ M == 0] = 1.
DX1 /= M
DY1 /= M

ax4.plot(0,                 0,                   color='red', marker='o')
ax4.plot(1.5*(a-c1)/(2*b), 0,                    color='orange', marker='o')
ax4.plot(0,                 1.5*(a-c1)/(2*b),    color='orange', marker='o')
ax4.plot(1.5*(a-c1)/(3*b), 1.5*(a-c1)/(3*b),    color='green', marker='o')

q1s    = 1.5 * myopic(iv1,iv2,     alpha1,alpha2,    a,b,c1,c2,    n)[0]
q2s    = 1.5 * myopic(iv1,iv2,     alpha1,alpha2,    a,b,c1,c2,    n)[1]
ax4.plot(q1s, q2s)

ax4.quiver(X1, Y1, DX1, DY1, M, pivot='mid')
ax4.grid()
ax4.set_xlabel("Firm 1's Output")
ax4.set_ylabel("Firm 2's Output")
```

[13]: Text(0, 0.5, "Firm 2's Output")

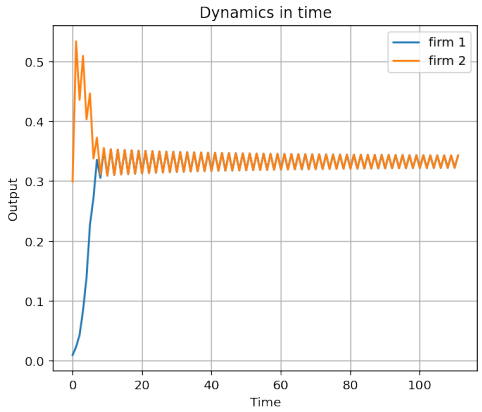
[13]:

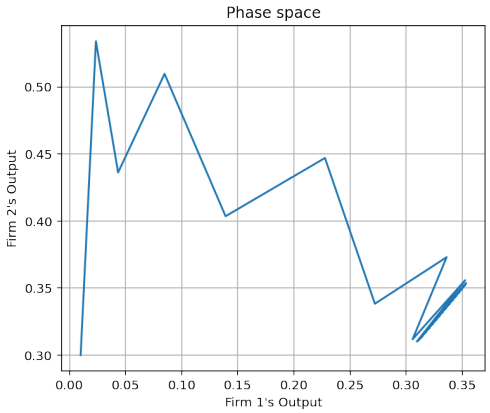


[14]:
```
iv1    = 0.01
iv2    = 0.3
alpha1 = 2
alpha2 = 2
a      = 2
b      = 1
c1     = 1
c2     = 1
n=111

time   = np.linspace(0, 100, 1001)

q1s    = myopic(iv1,iv2,     alpha1,alpha2,    a,b,c1,c2,    n)[0]
q2s    = myopic(iv1,iv2,     alpha1,alpha2,    a,b,c1,c2,    n)[1]

fig = plt.figure(figsize=(15,5))
fig.subplots_adjust(wspace = 0.5, hspace = 0.3)
ax1 = fig.add_subplot(1,2,1)
ax2 = fig.add_subplot(1,2,2)

ax1.plot(q1s, label='firm 1')
ax1.plot(q2s, label='firm 2')
ax1.set_title("Dynamics in time")
ax1.set_xlabel("Time")
ax1.set_ylabel("Output")
ax1.grid()
ax1.legend(loc='best')

ax2.plot(q1s, q2s)
ax2.set_xlabel("Firm 1's Output")
ax2.set_ylabel("Firm 2's Output")
ax2.set_title("Phase space")
ax2.grid()
print("iv1 = ",  iv1, ", iv2 = ", iv2, ", alpha1 = ", alpha1, ", alpha2 = ",␣
 ↪alpha2, ", a = ", a, ", b = ", b, ", c1 = ", c1, ", c2 = ", c2)
```

```
iv1 =  0.01 , iv2 =  0.3 , alpha1 =  2 , alpha2 =  2 , a =  2 , b =  1 , c1 =  1
, c2 =  1
```

[14]:



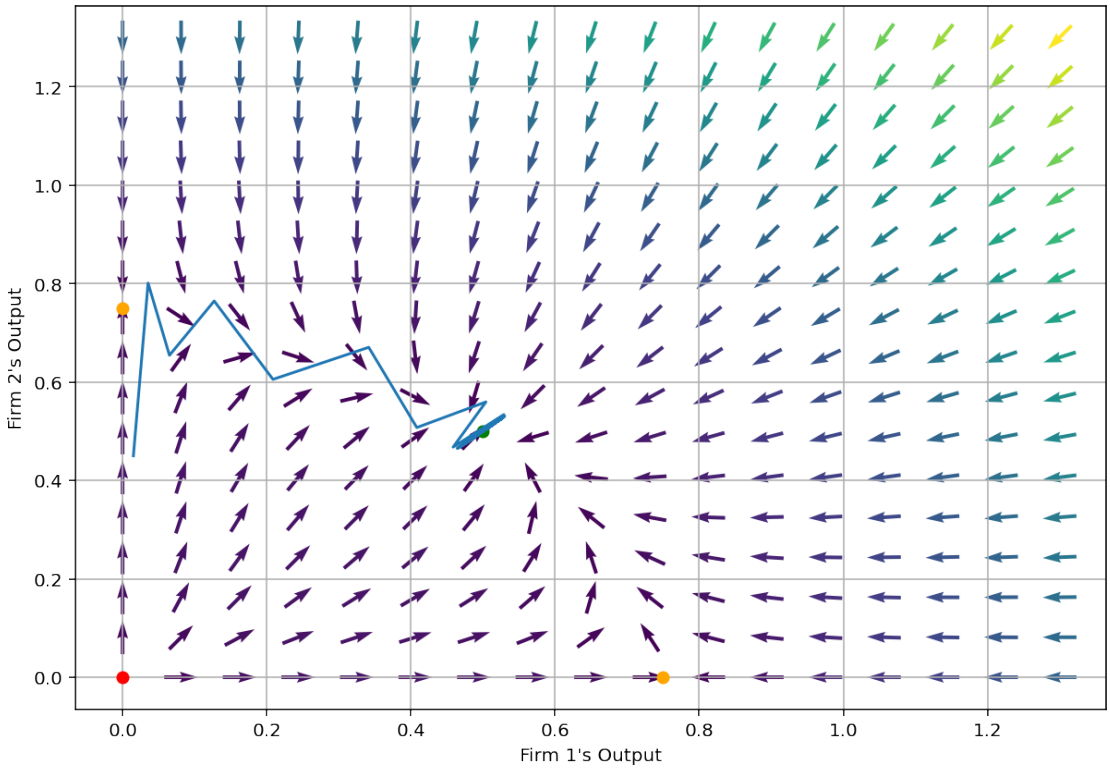


[16]:
```
fig2 = plt.figure(figsize=(10,7))
ax4 = fig2.add_subplot(1,1,1)

alpha1 = 2
alpha2 = 2
a      = 2
b      = 1
c1     = 1
c2     = 1

x = np.linspace(0, 1.3, 17)
y = np.linspace(0, 1.3, 17)

X1 , Y1  = np.meshgrid(x, y)
DX1, DY1 = myop_map(X1, Y1, alpha1,alpha2, a,b,c1,c2, n)
M = (np.hypot(DX1, DY1))
M[ M == 0] = 1.
DX1 /= M
DY1 /= M

ax4.plot(0,                 0,                  color='red', marker='o')
ax4.plot(1.5*(a-c1)/(2*b), 0,                  color='orange', marker='o')
ax4.plot(0,                 1.5*(a-c1)/(2*b),  color='orange', marker='o')
ax4.plot(1.5*(a-c1)/(3*b), 1.5*(a-c1)/(3*b),  color='green', marker='o')

q1s    = 1.5 * myopic(iv1,iv2,    alpha1,alpha2,    a,b,c1,c2,    n)[0]
q2s    = 1.5 * myopic(iv1,iv2,    alpha1,alpha2,    a,b,c1,c2,    n)[1]
ax4.plot(q1s, q2s)

ax4.quiver(X1, Y1, DX1, DY1, M, pivot='mid')
ax4.grid()
ax4.set_xlabel("Firm 1's Output")
```

```
ax4.set_ylabel("Firm 2's Output")
```

[16]:
```
Text(0, 0.5, "Firm 2's Output")
```

[16]:

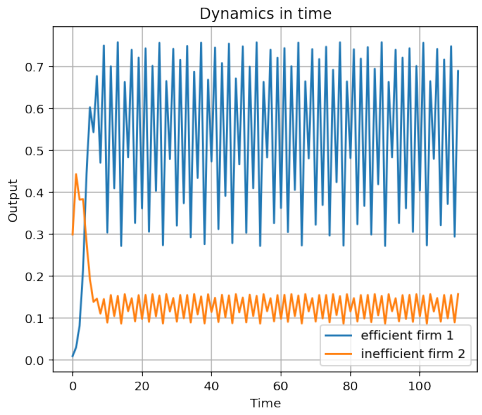


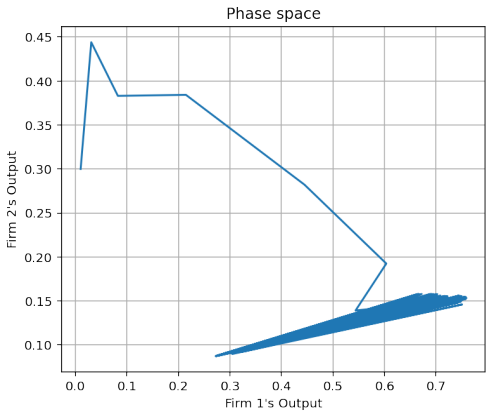
[9]:
```
iv1    = 0.01
iv2    = 0.3
alpha1 = 2
alpha2 = 2
a      = 2.1
b      = 1
c1     = 0.75
c2     = 1.25
n=111

time   = np.linspace(0, 100, 1001)

q1s    = myopic(iv1,iv2,     alpha1,alpha2,    a,b,c1,c2,    n)[0]
q2s    = myopic(iv1,iv2,     alpha1,alpha2,    a,b,c1,c2,    n)[1]

fig = plt.figure(figsize=(15,5))
fig.subplots_adjust(wspace = 0.5, hspace = 0.3)
ax1 = fig.add_subplot(1,2,1)
```

```
ax2 = fig.add_subplot(1,2,2)

ax1.plot(q1s, label='efficient firm 1')
ax1.plot(q2s, label='inefficient firm 2')
ax1.set_title("Dynamics in time")
ax1.set_xlabel("Time")
ax1.set_ylabel("Output")
ax1.grid()
ax1.legend(loc='best')

ax2.plot(q1s, q2s)
ax2.set_xlabel("Firm 1's Output")
ax2.set_ylabel("Firm 2's Output")
ax2.set_title("Phase space")
ax2.grid()
print("iv1 = ",  iv1, ", iv2 = ", iv2, ", alpha1 = ", alpha1, ", alpha2 = ",␣
 ↪alpha2, ", a = ", a, ", b = ", b, ", c1 = ", c1, ", c2 = ", c2)
```

```
iv1 =  0.01 , iv2 =  0.3 , alpha1 =  2 , alpha2 =  2 , a =  2.1 , b =  1 , c1 =
0.75 , c2 =  1.25
```

[9]:





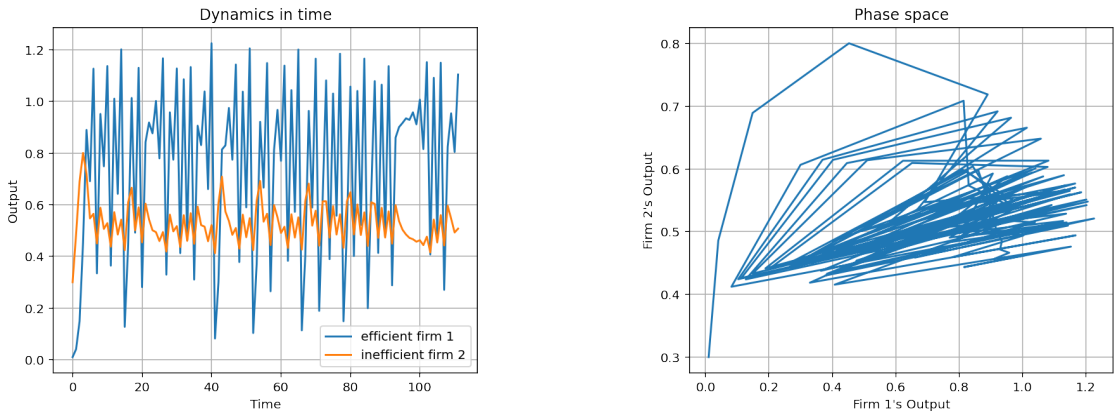
[8]:

```
iv1    = 0.01
iv2    = 0.3
alpha1 = 1.5
alpha2 = 0.5
a      = 3.1
b      = 1
c1     = 0.75
c2     = 1.25
n=111

time   = np.linspace(0, 100, 1001)
```

```
q1s    = myopic(iv1,iv2,     alpha1,alpha2,    a,b,c1,c2,    n)[0]
q2s    = myopic(iv1,iv2,     alpha1,alpha2,    a,b,c1,c2,    n)[1]

fig = plt.figure(figsize=(15,5))
fig.subplots_adjust(wspace = 0.5, hspace = 0.3)
ax1 = fig.add_subplot(1,2,1)
ax2 = fig.add_subplot(1,2,2)

ax1.plot(q1s, label='efficient firm 1')
ax1.plot(q2s, label='inefficient firm 2')
ax1.set_title("Dynamics in time")
ax1.set_xlabel("Time")
ax1.set_ylabel("Output")
ax1.grid()
ax1.legend(loc='best')

ax2.plot(q1s, q2s)
ax2.set_xlabel("Firm 1's Output")
ax2.set_ylabel("Firm 2's Output")
ax2.set_title("Phase space")
ax2.grid()
print("iv1 = ",  iv1, ", iv2 = ", iv2, ", alpha1 = ", alpha1, ", alpha2 = ",␣
 ↪alpha2, ", a = ", a, ", b = ", b, ", c1 = ", c1, ", c2 = ", c2)
```

```
iv1 =  0.01 , iv2 =  0.3 , alpha1 =  1.5 , alpha2 =  0.5 , a =  3.1 , b =  1 ,
c1 =  0.75 , c2 =  1.25
```

[8]:



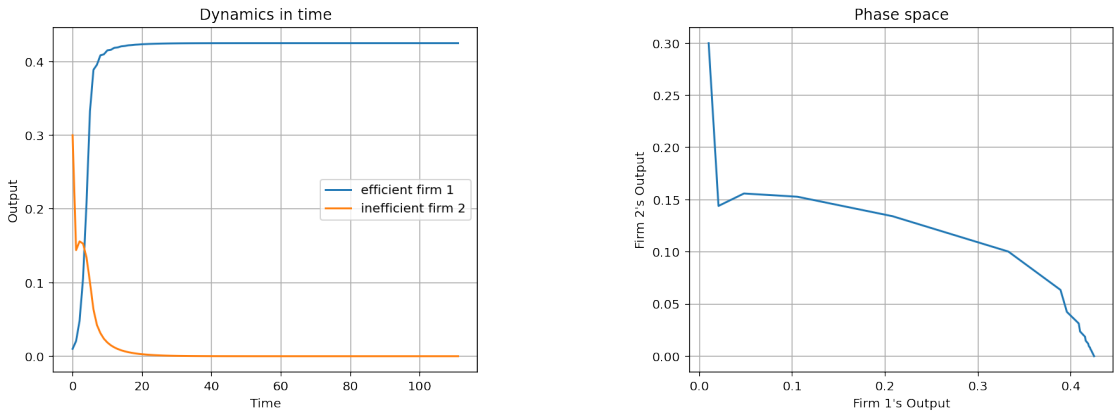
[19]:

```
iv1    = 0.01
iv2    = 0.3
alpha1 = 2
alpha2 = 2
a      = 1.6
b      = 1
```

```
c1      = 0.75
c2      = 1.25
n=111

time    = np.linspace(0, 100, 1001)

q1s     = myopic(iv1,iv2,      alpha1,alpha2,    a,b,c1,c2,    n)[0]
q2s     = myopic(iv1,iv2,      alpha1,alpha2,    a,b,c1,c2,    n)[1]

fig = plt.figure(figsize=(15,5))
fig.subplots_adjust(wspace = 0.5, hspace = 0.3)
ax1 = fig.add_subplot(1,2,1)
ax2 = fig.add_subplot(1,2,2)

ax1.plot(q1s, label='efficient firm 1')
ax1.plot(q2s, label='inefficient firm 2')
ax1.set_title("Dynamics in time")
ax1.set_xlabel("Time")
ax1.set_ylabel("Output")
ax1.grid()
ax1.legend(loc='best')

ax2.plot(q1s, q2s)
ax2.set_xlabel("Firm 1's Output")
ax2.set_ylabel("Firm 2's Output")
ax2.set_title("Phase space")
ax2.grid()
print("iv1 = ",  iv1, ", iv2 = ", iv2, ", alpha1 = ", alpha1, ", alpha2 = ",␣
 ↪alpha2, ", a = ", a, ", b = ", b, ", c1 = ", c1, ", c2 = ", c2)
```

```
iv1 =  0.01 , iv2 =  0.3 , alpha1 =  2 , alpha2 =  2 , a =  1.6 , b =  1 , c1 =
0.75 , c2 =  1.25
```

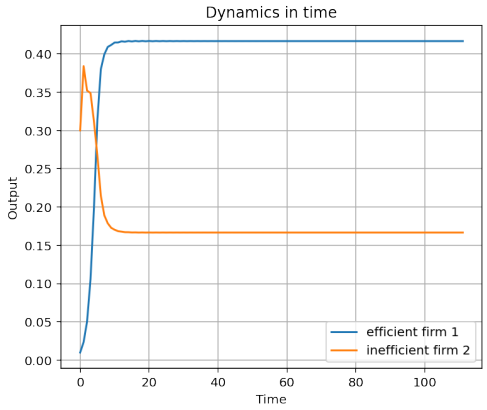
[19]:

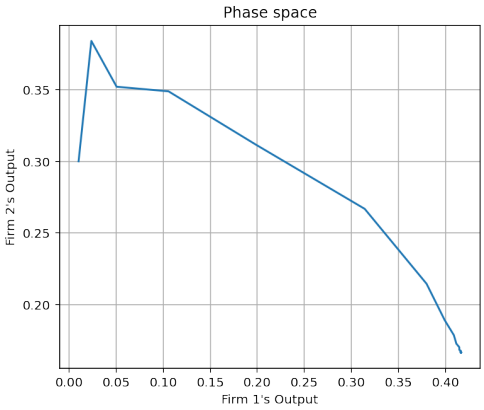


[9]:
```
iv1    = 0.01
iv2    = 0.3
alpha1 = 2
alpha2 = 2
a      = 2
b      = 1
c1     = 1
c2     = 1.25
n=111

time   = np.linspace(0, 100, 1001)

q1s    = myopic(iv1,iv2,     alpha1,alpha2,    a,b,c1,c2,    n)[0]
q2s    = myopic(iv1,iv2,     alpha1,alpha2,    a,b,c1,c2,    n)[1]

fig = plt.figure(figsize=(15,5))
fig.subplots_adjust(wspace = 0.5, hspace = 0.3)
ax1 = fig.add_subplot(1,2,1)
ax2 = fig.add_subplot(1,2,2)

ax1.plot(q1s, label='efficient firm 1')
ax1.plot(q2s, label='inefficient firm 2')
ax1.set_title("Dynamics in time")
ax1.set_xlabel("Time")
ax1.set_ylabel("Output")
ax1.grid()
ax1.legend(loc='best')

ax2.plot(q1s, q2s)
ax2.set_xlabel("Firm 1's Output")
ax2.set_ylabel("Firm 2's Output")
ax2.set_title("Phase space")
ax2.grid()
print("iv1 = ",  iv1, ", iv2 = ", iv2, ", alpha1 = ", alpha1, ", alpha2 = ",␣
 ↪alpha2, ", a = ", a, ", b = ", b, ", c1 = ", c1, ", c2 = ", c2)
```

```
iv1 =  0.01 , iv2 =  0.3 , alpha1 =  2 , alpha2 =  2 , a =  2 , b =  1 , c1 =  1
, c2 =  1.25
```

[9]:

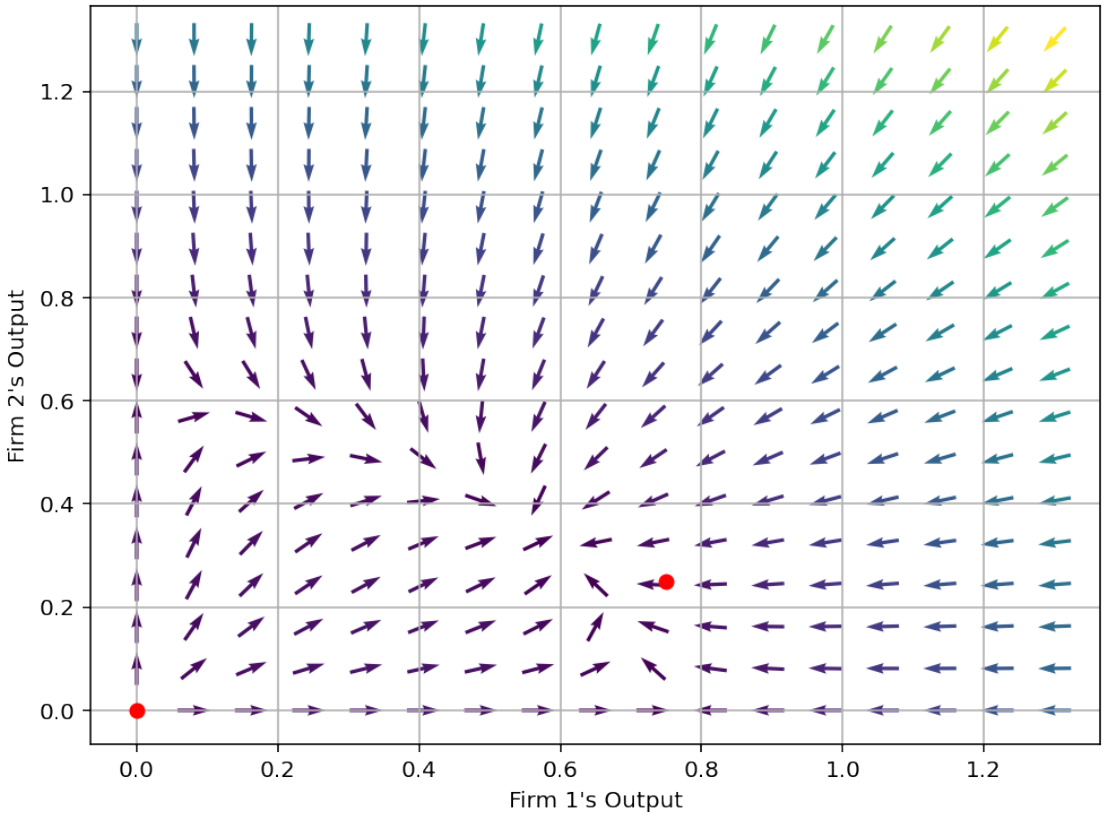




[10]:
```
fig3 = plt.figure(figsize=(8,6))
ax4 = fig3.add_subplot(1,1,1)

alpha1 = 2
alpha2 = 2
a      = 2
b      = 1
c1     = 1
c2     = 1.

x = np.linspace(0, 1.3, 17)
y = np.linspace(0, 1.3, 17)

X1 , Y1  = np.meshgrid(x, y)
DX1, DY1 = myop_map(X1, Y1, alpha1,alpha2, a,b,c1,c2, n)
M = (np.hypot(DX1, DY1))
M[ M == 0] = 1.
DX1 /= M
DY1 /= M

ax4.plot(0,                0,                    color='red', marker='o')
ax4.plot(0.75,             0.25,                 color='red', marker='o')

ax4.quiver(X1, Y1, DX1, DY1, M, pivot='mid')
ax4.grid()
ax4.set_xlabel("Firm 1's Output")
ax4.set_ylabel("Firm 2's Output")
```

[10]: Text(0, 0.5, "Firm 2's Output")

[10]:



[0]:

[0]: