Reading and assignment Week 1

May 2021

1 Reading

The content for this week can be read from book Introduction to theory of computation by Michael Sipser, Chapter 1 (you can find the book online if you wish, though I'll share the chapter)

2 Assignment

You have to solve all the problems(possibly), the design problem(must) and atleast one bonus problem. Ask if some problem is not clear.

Problems: 1.1 , 1.2; 1.11; 1.15; 1.38; 1.45; 1.51; 1.56

Design Problem: You have to design a simple automata that recognizes the following language. $\mathcal{L} = \{\langle x, y, z\rangle \mid x + y = z\}$ here $\langle x, y, z\rangle$ is a tuple of three non-negative integers x,y and z, $0 \leq x, y \leq n$ for some fixed integer n and the language is the set of all tuples such that x + y = z. But language is a set of strings, right?, yes and you have to come up with such a mapping of tuples to strings (maybe you know one.. or many), then we have a valid language. This technique is called encoding. With just a simple alphabet(binary alphabet, say) and proper encoding, you can represent any general object and decision problem as strings and a Language of strings in the alphabet, respectively. It says that you don't have to worry for what alphabet to use, a single alphabet is sufficient and indeed we never go beyond bits 0 and 1. Hope you get this feel.

For the solution: clearly state the alphabet you are going to use, encoding of the numbers in the alphabet, then encoding of the tuples, thereby giving an idea of what the strings of the language will be. Then give the description of the automata. The statements should be rigorous enough to clearly and precisely express your ideas.

Bonus: 1.52,

Ramsey's theorem. Let $\mathcal{G}$ be a graph. A clique in $\mathcal{G}$ is a subgraph in which every two nodes are connected by an edge. An anti-clique, also called an independent set, is a subgraph in which every two nodes are not connected by an edge. Show that every graph with n nodes contains either a clique or an anti-clique with at least $\frac{1}{2}log_2 n$ nodes.

{ Read about Graphs if you don't know what graphs are, I'll give you the reading. }

You can read more about Ramsey's numbers and an interesting statement by Erdos:

"Suppose aliens invade the earth and threaten to obliterate it in a year's time unless human beings can find the Ramsey number $\mathcal{R}(5,5)$. We could marshal the world's best minds and fastest computers, and within a year we could probably calculate the value. If the aliens demanded the Ramsey number $\mathcal{R}(6,6)$, however, we would have no choice but to launch a preemptive attack."