

Type Systems

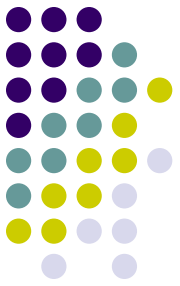


- As we saw previously, any programming language that has some complexity to it allows us to create syntactically correct statements that semantically do not make any sense:

```
declare z = function (x) return x+1;  
put z+1; // ???
```

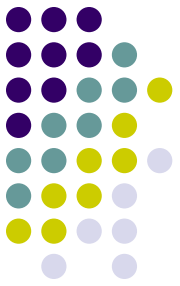
- The error in the expression can easily be caught by an interpreter or compiler by tagging the operands with *type names*: `z.{function} + i.{int}`
- Now it is simple for the language processor to find the problem: it is only allowed to apply addition to `{int}` terms, e.g., `j.{int} + i.{int}`

Type Systems



- A principled approach to tagging terms and expressions with type names is called a *type system*
- Every modern programming language has one
- We have
 - Implicit type systems - type systems where the system automatically recognizes the type of a variable or constant
 - Explicit type systems - type systems where the user has to explicitly declare the type of variables (and sometimes constants).

Why do we use type systems?



- Types allow the language system to assist the developer in writing *better programs*. *Type mismatches* in a program usually indicate some sort of *programming error*.
 - Static type checking – check the types of all statements and expressions at compile time.
 - Dynamic type checking – check the types at runtime.

Types



A Type is a Set of Values

Consider the statement:

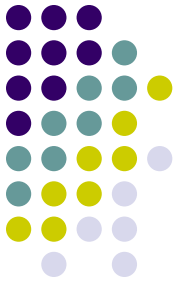
```
int n;
```

Here we declare `n` to be a variable of *type* `int`; what we mean, `n` can take on any value from the *set of all integer values*.

Also observe that the elements in a type share a common representation: each element is encoded in the same way (float, double, char, etc.)

Also, all elements of a type share the same operations the language supports for them.

Types



Def: A *type* is a set of values.

Def: A *primitive type* is a type a programmer can use but not define.

Def: A *constructed type* is a user-defined type.

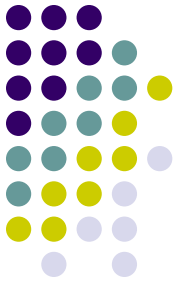
Example: Java, primitive type

float q;

type float \Rightarrow set of all
possible floating point values

} q is of type float, only
a value that is a member
of the set of all floating point
values can be assigned to q.

Types



Example: Java, constructed type

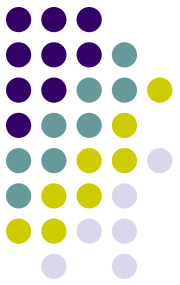
```
class Foobar { int i; String s; };
```

```
Foobar c = new Foobar();
```



Now the variable `c` only accepts values that are members of type `Foobar`;
☞ *object instantiations* of class `Foobar`; objects are the values of type `Foobar`..

Types



Example: C, constructed type

`int a[3];`

the variable `a` will accept values which are arrays of 3 integers.

e.g.: `int a[3] = {1,2,3};`
`int a[3] = {7,24,9}`

We will have more to say about this later on.



Subtypes

- We saw that the notion of a type as a set of values is a nice model for explaining variable declarations and object-oriented structures
- But it is also essential to developing the notion of a *subtype*



Subtypes

Def: a *subtype* is a *subset* of the elements of a type.

Example: Java

‘Short’ is a subtype of ‘int’, that is, all the values in set ‘short’ are also in set ‘int’: $\text{short} \subset \text{int}$

Example: Java

‘Float’ is a subtype of ‘double’ (all the values in set ‘float’ are also in set ‘double’): $\text{float} \subset \text{double}$

Observations:

- (1) converting a value of a subtype to a value of the supertype is called a *widening* type conversion. (safe)
- (2) converting a value of a supertype to a value of a subtype is called a *narrowing* type conversion. (not safe - information loss)



Subtypes

Consider this example in Java with an implicit *narrowing* conversion:

```
int i = 33000;  
short j = i;    //problematic, short is only 2 bytes, overflow!
```

On the other hand this example in Java with an implicit *widening* conversion has no problems:

```
short i = 20000;  
int j = i;
```

☞ Compilers/interpreters will often insert widening conversions but will flag errors when a supertype needs to be converted to a subtype.



Subtypes

- An important implication of subtypes in programming languages is the notion of *type hierarchies*
- Here the types of a language are ordered along the subtype relation, e.g. in Java
 - $\text{int} \subset \text{float} \subset \text{string}$



Type Equivalence

- I. Name Equivalence – two objects are of the same type of and only if they share the same *type name*.

Example: Java

```
Class Foobar {  
    int i;  
    float f;  
}
```

```
Class Goobar {  
    int i;  
    float f;  
}
```

```
Foobar o = new Goobar();
```

Error; even though the types look the same, their names are different, therefore, Java will raise an error.

☞ Java uses *name equivalence*



Type Equivalence

II. Structural Equivalence – two objects are of the same type if and only if they share the same *type structure*.

Example: ML

- type person = int * int * string * string;
- type mytuple = int * int * string * string;
- val joe:person = (38, 185, “married”, “pilot”):mytuple;

Think of this as:

```
class Person {  
  int age;  
  int weight;  
  String mstatus;  
  String profession;  
}
```

Even though the type names are different, ML correctly recognizes this statement.

☞ ML uses *structural equivalence*.



Polymorphism

- An interesting implication of type systems is *polymorphism*:
 - Function overloading
 - Subtype polymorphism

Def: A function is *polymorphic* if it has at least two possible types.

polymorphism \equiv comes from Greek meaning 'many forms'



Polymorphism

Function Overloading

Def: An *overloaded function* is one that has at least two definitions, all of different types.

Example: In Java the '+' operator is overloaded.

```
String s = "abc".{String} + "def".{String} ;
```

```
int i = 3.{int} + 5.{int} ;
```

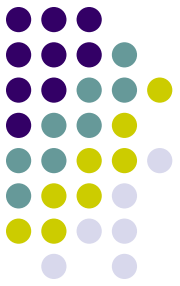
Polymorphism



Subtype Polymorphism

Def: A function exhibits *subtype polymorphism* if one or more of its formal parameters has subtypes.

Polymorphism



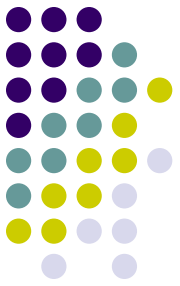
Example: Java

```
void g (double a) { ... }
```

<pre>int \subset double float \subset double short \subset double byte \subset double char \subset double</pre>	<pre>}</pre>	all legal types that can be passed to function 'g'.
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<pre>int i = 10; g(i);</pre>	<pre>}</pre>	Legal because of subtype polymorphism
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Polymorphism



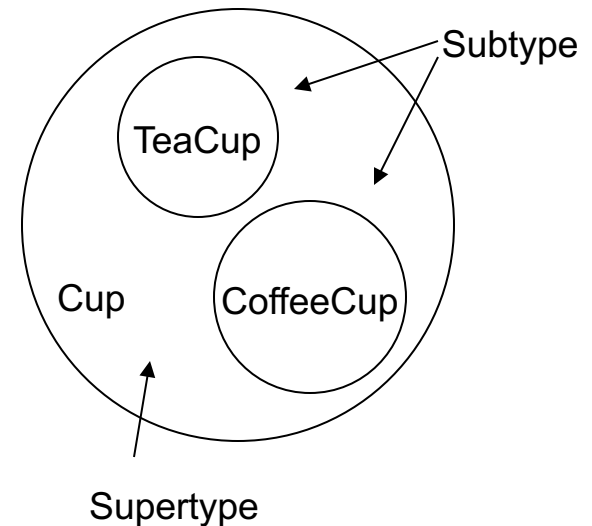
Example: Java

```
class Cup { ... };  
class CoffeeCup extends Cup { ... };  
class TeaCup extends Cup { ... };
```

```
void fill (Cup c) {...}
```

```
TeaCup t = new TeaCup();  
CoffeeCup k = new CoffeeCup();
```

```
fill(t);  
fill(k);
```

 } subtype polymorphism

```
TeaCup t = new TeaCup();
```

```
Cup c = t; ← widening type conversion: TeaCup → Cup
```

safe!