Functional Programming

- Functional programming is a **declarative programming paradigm** where programs are constructed by applying and composing functions.
- Function definitions are **expressions that map values to other values**, rather than a sequence of imperative statements which update the running state of a program.

Functional Programming

Everything is a Value!

- …including functions!
- This sets functional programming apart from imperative programming where statements like loops and conditionals do not represent values but change of an explicit machine state

Lambda Calculus

- Let’s explore this using the lambda calculus before we commit to any particular language.
- Recall that in the lambda calculus we construct functions as lambda expressions and these functions can be applied to values, e.g.

$$\overbrace{(\lambda x.\, x+1)\ 1}^{\text{Function application}} \Rightarrow x+1\overbrace{[x \leftarrow 1]}^{\text{Substitution}} \Rightarrow 1+1 \Rightarrow 2$$

Lambda Calculus

- Functions can be input values to other functions!

$$(\lambda y. y\ 1)\overbrace{(\lambda x. x + 1)}^{\text{Function as value}} \Rightarrow y\ 1[y \leftarrow (\lambda x. x + 1)]$$
$$\Rightarrow (\lambda x. x + 1)\ 1 \Rightarrow 2$$

Lambda Calculus

- Functions as return values from functions
  - That is, functions computing new functions!

$$\left(\lambda x. \overbrace{(\lambda y. x + y)}^{\text{Function as return value}}\right) 1\, 1 \Rightarrow (\lambda y. x + y)\, 1[x \leftarrow 1]$$

$$\Rightarrow (\lambda y. 1 + y)\, 1 \Rightarrow 1 + y[y \leftarrow 1] \Rightarrow 1 + 1 \Rightarrow 2$$

Functional Programming

- Functional programming is declarative in that the programs deal more with the **what** rather than the **how**.
- One way to think about this is: in declarative programming we “declare” **what to do for each input configuration**.
- This is in stark contrast to imperative programming where we describe **how to solve the whole problem** in one go without subdivision.

```
-- imperative solution
function len with list do
    let remaining_list = list.
    let cnt = 0.
    repeat
        let [_|remaining_list] = remaining_list.
        let cnt = cnt + 1.
    until remaining_list is [].
end

let q = [ 1 to 10].
assert (len q == 10).
```

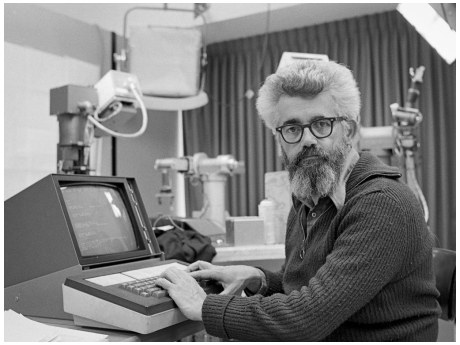
“The How”

```
-- declarative solution
function len
    with [] do
        0
    with [_|remaining_list] do
        1 + len remaining_list
end

let q = [ 1 to 10].
assert (len q == 10).
```

“The What”

Lisp



Dr John McCarthy, computer scientist,
1927 – 2011.

- Lisp was developed by John McCarthy in the late 1950's early 60's to solve problems in AI.
- It is the oldest functional programming language.
- Its syntax has been inspired by the lambda calculus.
- It introduced novel features such as recursion and garbage collection.
- It is still in use today as Common Lisp (ANSI compliant).
- Modern descendants: Scheme, Racket, Clojure

https://en.wikipedia.org/wiki/Lisp_%28programming_language%29

Lisp

$(\lambda x. x + 1)\ 1 \ \Rightarrow 2$

```
Welcome to GNU CLISP 2.49 (2010-07-07) <http://clisp.cons.org/>

Copyright (c) Bruno Haible, Michael Stoll 1992, 1993
Copyright (c) Bruno Haible, Marcus Daniels 1994-1997
Copyright (c) Bruno Haible, Pierpaolo Bernardi, Sam Steingold 1998
Copyright (c) Bruno Haible, Sam Steingold 1999-2000
Copyright (c) Sam Steingold, Bruno Haible 2001-2010

Type :h and hit Enter for context help.

[1]> ((lambda (x) (+ x 1)) 1)
2
[2]> (defun inc (x) (+ x 1))
INC
[3]> (inc 1)
2
[4]>
```

$(\lambda y. y\ 1)(\lambda x. x + 1) \Rightarrow 2$

```
[1]>  ((lambda (y) (apply y '(1))) (lambda (x) (+ x 1)))
2
[2]>
```

$(\lambda x. (\lambda y. x + y))\ 1\ 1 \Rightarrow 2$

```
[1]> (apply (apply (lambda (x) (lambda (y) (+ x y))) '(1)) '(1))
2
[2]>
```

ML

Robin Milner, computer scientist
1934 – 2010.

- Robin Milner designed ML as the implementation language for his proof assistant LCF (Logic for Computable Functions) in the 1970's.
- It can be considered the first modern functional programming language,
  - Statically type checked
  - A syntax that is easily recognized by today's developers
  - Very influential, virtually every modern functional programming language can trace its ancestry back to ML
- It is also one of the few high-level programming languages with a full mathematical specification.
- Dialects of ML in wide use today: SMLNJ, Ocaml, F#

https://smlnj.org/ and https://en.wikipedia.org/wiki/ML_(programming_language)

ML

$(\lambda x. x + 1)\ 1 \ \Rightarrow 2$

```
Standard ML of New Jersey (64-bit) v110.95 [built: Sun Nov 06 00:04:31 2022]
- (fn x => x + 1) 1;
val it = 2 : int
-
```

$(\lambda y. y\ 1)(\lambda x. x + 1) \Rightarrow 2$

```
- (fn y => y 1)(fn x => x+1);
val it = 2 : int
-
```

$(\lambda x. (\lambda y. x + y))\ 1\ 1 \Rightarrow 2$

```
- (fn x => (fn y => x+y)) 1 1;
val it = 2 : int
-
```

```
Standard ML of New Jersey (64-bit) v110.95 [built: Sun Nov 06 00:04:31 2022]
- fun inc x = x+1;
val inc = fn : int -> int
- inc 1;
val it = 2 : int
-
```

Function as Values: Another Look

A Type is a Set of Values.

- If we view functions as values, then they have to belong to a type.
- We can use ML's type system to compute the function types,

```
- fun inc x = x+1;
val inc = fn : int -> int
```

```
- (fn x => 2*x);
val it = fn : int -> int
-
```

```
- fun fold (x,y) = x+y;
val fold = fn : int * int -> int
-
```

Function as Values: Another Look

- In the previous slide we saw that we have at least two different types

Function Types

$int \rightarrow int$

$int * int \rightarrow int$

```
- (fn x => 2*x);
val it = fn : int -> int
-
```

```
- fun inc x = x+1;
val inc = fn : int -> int
```

"All functions that map integers to integers"

```
- fun fold (x,y) = x+y;
val fold = fn : int * int -> int
-
```

"All functions that map pairs of integers to integers"

Function as Values: Another Look

- Since we now have function types we can declare variables of that type,

ML

```
- val x:int->int = (fn x => x+1);
val x = fn : int -> int
- 
```

Rust

```
fn main() {
    let x: fn(i32) -> i32 = |x| x + 1;
}
```

Function as Values: Another Look

- Every function belongs to a particular function type.
- We can view a function as a value in the set of all values of a particular type.
- This particularly visible in statically typed languages like ML and Rust.
  - But it is also supported in dynamically typed languages like Python and Asteroid.
  - In Asteroid, all functions are members of the type ‘function’.

```
Asteroid Version 1.1.4
(c) University of Rhode Island
Type "asteroid -h" for help
Press CTRL-D to exit
ast> load system type.
ast> type @gettype (lambda with x do x+1).
function
ast> let x:%function = (lambda with x do x+1).
ast> x
(function ...)
ast> 
```

Reading

- Please read Chapter I in the following paper,

  lutzhamel.github.io/CSC493/docs/intro-fp-barendregt.pdf