



# Higher-Order Programming: The Essence of Functional Programming

- Higher-Order programming is defined as
  - *Programming with functions as arguments to other functions or functions as return values from functions.*



# Higher-Order Programming: A Cornerstone of Functional Programming

- It is a natural outgrowth from the lambda calculus where
  - a) lambda expressions can be passed to other lambda expressions, and
  - b) new lambda expressions can be computed by lambda expressions
- E.g.

a)  $(\lambda y. y \ 1)(\lambda x. x + 1) \Rightarrow 2$

b)  $(\lambda x. (\lambda y. x + y)) \ 1 \ 1 \Rightarrow 2$



# Modifying Behavior of a Function

- We can use this to write generic functions which we can then make specific by passing in desired behavior via a function.
- Note: this is NOT programming with generics
  - Generics are generic with respect to types
  - Higher-order functions are generic with respect to **behavior!**



# Modifying Behavior of a Function

- The 'filter' function is generic with regards to the ordering predicate

```
function filter
  with ([],_,_) do
    []
  with ([e|rest],pivot,fcmp) do
    [e] + filter (rest,pivot,fcmp)
    if fcmp (e,pivot) ←
    else filter (rest,pivot,fcmp)
  end
```

If e is kept or discarded depends on the passed in function – the filter function has a generic filtering capability which is made specific by the passed in predicate.



# Modifying Behavior of a Function - QuickSort

```
function filter
  with (e, [], fcmp) do
    []
  with (e, [a|rest], fcmp) do
    [a] + filter(e, rest, fcmp)
    if fcmp(a, e)
    else filter(e, rest, fcmp)
  end
end
```

```
function qsort
  with [] do -- empty list
    []
  with [pivot|rest] do -- head-tail operator
    let less = filter(pivot, rest, lambda with (x, y) do x < y).
    let more = filter(pivot, rest, lambda with (x, y) do x >= y).
    qsort less + [pivot] + qsort more.
  end
end
```



# Modifying Behavior of a Function

- The Asteroid sort module is another example,  
sort @sort (p:%function, l:%list)

```
1 load system sort.  
2  
3 let sl = sort @sort(  
4   (lambda with (x,y) do true if x<y else false),  
5   [10,5,110,50]).  
6  
7 assert (sl == [5,10,50,110]).
```



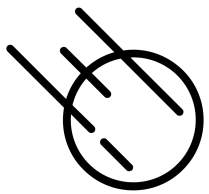
```
1 load system sort.  
2  
3 let sl = sort @sort(  
4   (lambda with (x,y) do true if x>y else false),  
5   [10,5,110,50]).  
6  
7 assert (sl == [110,50,10,5]).
```





# Dispatch Tables

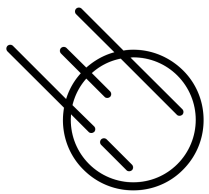
- We can also associate behavior with appropriate keys in a dispatch table.
- We can then dispatch (lookup) desired behavior given specific keys.
- Example: A generic 'calculate' function that takes two values and a key symbol and then performs the appropriate computation.



# Dispatch Tables

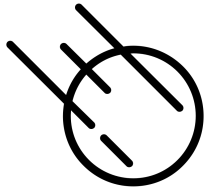
```
1  load system hash.
2
3  let dispatch_table = hash @hash ().
4
5  dispatch_table @insert [
6    | ("+", lambda with (a,b) do a + b),
7    | ("-", lambda with (a,b) do a - b),
8    | ("*", lambda with (a,b) do a * b),
9    | ("/", lambda with (a,b) do a / b)
10 ]
11
12 function calculate with (operator,a,b) do
13   | dispatch_table @get operator (a,b)
14 end
15
16 -- Example usage
17 assert (calculate("+", 3, 5) == 8)
18 assert (calculate("-", 7, 2) == 5)
19 assert (calculate("*", 2, 4) == 8)
20 assert (calculate("/", 10, 2) == 5)
```





# Map & Reduce

- The map and reduce functions are functions that take a function and apply the given function to a list..
- Both functions are higher-order functions that come straight out of the functional programming tradition.



# Map

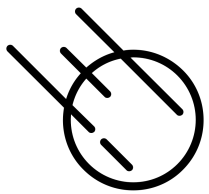
- Below is Asteroid code that explains the behavior of the map function.
- Beware that map is not required to apply the function *f* in the sequential manner shown here
  - For example, it is free to exploit threads to apply the function *f* in parallel to the elements of the list.

```
list @map f

-- is equivalent to --

let r = [].
for e in list do
|   r @append(f e).
end
let list = r.
```

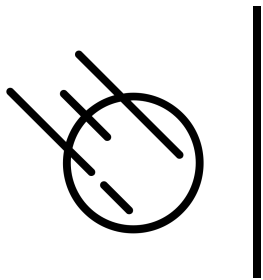
The function argument to *f* must be of the same type as the list elements



# Map

- One interesting application of map is the transformation of a simple list constructor into any kind of list
  - Here we compute a list of alternating 1's and -1's.

```
1  load system math.  
2  
3  let a = [1 to 10] @map(lambda with x do math @mod (x,2))  
4  | | | | | @map(lambda with x do 1 if x else -1).  
5  |  
6  assert (a == [1,-1,1,-1,1,-1,1,-1,1,-1]).
```



# Map

- Most modern languages support some form of 'map' since it is such a powerful programming tool.

Python

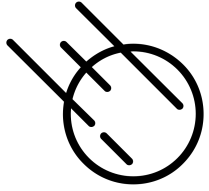
```
1  l = [x for x in range(1,10+1)]
2  it = map(lambda x : x % 2, l)
3  a = list(map(lambda x : 1 if x else -1, it))
4
5  assert(a == [1,-1,1,-1,1,-1,1,-1,1,-1])
```

In012/map.py

Rust

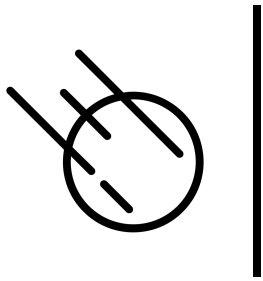
```
1  use std::vec::Vec;
2
3  fn main() {
4      let a : Vec<i32> = vec![1,2,3,4,5,6,7,8,9,10]
5          .iter()
6          .map(|x| x % 2)
7          .map(|x| if x == 0 { -1 } else { 1 })
8          .collect();
9
10     assert_eq!(a, vec![1, -1, 1, -1, 1, -1, 1, -1, 1, -1]);
11 }
```

In012/map.rs



# Reduce

- Whereas 'map' applies a function to a list producing another list, the 'reduce' function applies a function to a list so that the list gets *reduced* to a single value.
  - In functional languages this is often called 'fold' – folding the list into a single value



# Reduce

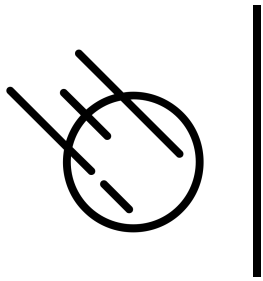
- For example, the reduce function lets us sum the elements of a list without a loop



In012/reduce.ast

```
1  let value = [1,2,3] @reduce (lambda with (x,y) do x+y).  
2  
3  assert(value == 6).
```

The argument of the reduce function must be a pair where each component of the pair is of the element type of the list.

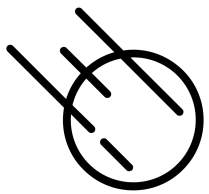


# Reduce

- The reduce function gives us an interesting way to implement the factorial of an integer

In012/fact.ast

```
1  -- factorial with reduce
2  function fact with x do
3    | [1 to x] @reduce (lambda with (i,j) do i*j)
4  end
5
6  assert (fact 3 == 6).
```

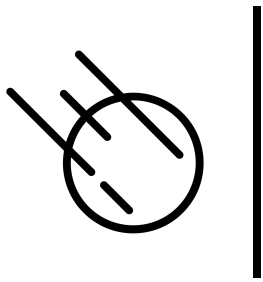


# Reduce

- The Asteroid code below illustrates the behavior of the 'reduce' function
  - Notice the function application to a pair of values!
  - The first value of the pair acts like an accumulator containing the partially reduced value at each function application

```
1  list @ reduce f
2
3  -- is equivalent to --
4
5  let value = list@0.
6  for i in range(len(list)) do
7  |   |   let value = f (value, l@i). ←
8  end
9  -- value has now the reduced value of the list
```





# Reduce

Python

```
1  from functools import reduce
2
3  value = [1, 2, 3]
4  result = reduce(lambda x, y: x + y, value)
5
6  assert result == 6
```

In012/reduce.py

Rust

```
1  fn main() {
2      let value: i32 = (1..10).reduce(|acc, e| acc + e).unwrap();
3      assert_eq!(value, 45);
4  }
```

In012/reduce.rs