### Source Language:

```
A ::= n
      add(A,A)
      sub(A,A)
      mult(A,A)
B ::= true
      false
      eq(A,A)
      le(A,A)
      not(B)
      and(B,B)
      or(B,B)
C ::= skip
      assign(x,A)
      seq(C,C)
      if(B,C,C)
      whiledo(B,C)
```

Semantics are the same compared to our initial simple imperative language; arithmetic expressions:

```
(C, ) -->> C :-
                                   % constants
   int(C).!.
(X,State) -->> Val :-
                                   % variables
   atom(X).
   lookup(X,State,Val),!.
(add(A.B),State) -->> Val :-
                                   % addition
   (A,State) -->> ValA,
   (B,State) -->> ValB,
   Val xis ValA + ValB.!.
(sub(A,B),State) -->> Val :-
                                   % subtraction
   (A.State) -->> ValA.
   (B.State) -->> ValB.
   Val xis ValA - ValB,!.
(mult(A.B),State) -->> Val :-
                                  % multiplication
   (A,State) -->> ValA,
   (B,State) -->> ValB,
   Val xis ValA * ValB.!.
```

#### Boolean expressions:

```
(true, ) -->> true :- !.
                                       % constants
(false,_) -->> false :- !.
                                       % constants
(eg(A.B),State) -->> Val :-
                                       % equality
   (A,State) -->> ValA,
   (B.State) -->> ValB.
   Val xis (ValA =:= ValB),!.
(le(A,B),State) -->> Val :-
                                       % le
   (A.State) -->> ValA.
   (B.State) -->> ValB.
   Val xis (ValA =< ValB),!.
(not(A).State) -->> Val :-
                                       % not
   (A,State) -->> ValA,
   Val xis (not ValA).!.
(and(A,B),State) -->> Val :-
                                       % and
   (A,State) -->> ValA,
   (B.State) -->> ValB.
   Val xis (ValA and ValB),!.
                                       % or
(or(A.B).State) -->> Val :-
   (A,State) -->> ValA,
   (B,State) -->> ValB,
   Val xis (ValA or ValB).!.
```

#### Statements:

```
(skip,State) -->> State :- !.
                                      % skip
(assign(X,A),State) -->> OState :-
                                      % assignment
    (A,State) -->> ValA,
   put(X.ValA.State.OState).!.
(seq(C0,C1),State) -->> OState :-
                                      % composition, seq
    (CO.State) -->> SO.
    (C1,S0) -->> OState.!.
(if(B,CO,_),State) -->> OState :-
                                     % if
    (B.State) -->> true.
   (CO.State) -->> OState,!.
(if(B, ,C1),State) -->> OState :-
                                     % if
    (B.State) -->> false.
    (C1,State) -->> OState,!.
(whiledo(B,_),State) -->> OState :- % while
    (B,State) -->> false,
   State=OState,!.
(whiledo(B,C),State) -->> OState :- % while
    (B,State) -->> true,
    (C.State) -->> SC.
    (whiledo(B,C),SC) -->> OState,!.
```

### Target Language:

```
prog ::= [ cmseq ]
                   1 []
cmseq ::= cm | cm , cmseq
cm ::= push(V)
      add
      sub
      mult.
      and
      or
      neg
      eq
      le
      pop(x)
     label(L)
     jmp(L)
      jmpt(L)
      jmpf(L)
      stop
V ::= x | n | true | false
L ::= <alpha string>
```

A state in our machine is a term of arity two where the first component is an integer stack used for expression evaluation and the second component is a binding environment for variables:

```
'(Stack, Environment)'
```

Flow of control instructions: perhaps the most surprising part of the semantics for our target language is the notion of a *continuation*. A continuation is a way to model the address space of the target machine so that we can perform *jumps* to labels.

```
% the predicate '(+Syntax,+Continuation,+State) -->> -State' computes
% the semantic value for each syntactic structure
([],_,State) -->> State :- !.
                                    % an empty instruction sequence is a noop
([stop|], .State) -->> State :- !. % the 'stop' instruction ignores the rest of the program
([imp(L)| ].Cont.State) -->> OState :-
    afindlabel(L.Cont.JT).
    (JT, Cont, State) -->> OState,!.
([impt()|P].Cont.([false|Stk].Env)) -->> OState :-
        (P.Cont.(Stk,Env)) -->> OState,!.
([jmpt(L)|_],Cont,([true|Stk],Env)) -->> OState :-
         afindlabel(L,Cont,JT),(JT,Cont,(Stk,Env)) -->> OState.!.
([impf(L)| ].Cont.([false|Stk].Env)) -->> OState :-
         afindlabel(L.Cont.JT),(JT.Cont.(Stk.Env)) -->> OState.!.
([impf()|P].Cont.([true|Stk].Env)) -->> OState :-
        (P.Cont.(Stk.Env)) -->> OState.!.
```

A continuation is a copy of the original program and we use it to look up jump targets:

```
% the predicate 'afindlabel(+Label,+Continuation,-JumpTarget)'
% looks up a label definition in the continutation and returns its associate code.
:- dynamic afindlabel/3.
afindlabel(L,[label(L)|P],[label(L)|P]).
afindlabel(L,[_|P],JT) :-
    afindlabel(L,P,JT).
afindlabel(_,[],_) :-
    writeln('ERROR: label not found.'),!,fail.
```

### Computational instructions:

```
%%% computational instructions
([Instr|P],Cont,State) -->> OState :- % interpret an instruction sequence.
    (Instr.Cont.State) -->> IState.
    (P,Cont,IState) -->> OState,!.
(push(C),_,(Stk,Env)) -->> ([C|Stk],Env) :- % constants
   int(C),!.
(push(X), .(Stk.Env)) -->> ([ValX|Stk].Env) :- % variables
    atom(X).
    alookup(X,Env,ValX),!.
(pop(X),_,([ValA|Stk],Env)) -->> (Stk,OEnv) :- % store
    aput(X, ValA, Env, OEnv), !.
(add,_,([ValB,ValA|Stk],Env)) -->> ([Val|Stk],Env) :- % addition
    Val xis ValA + ValB,!.
(and,_,([ValB,ValA|Stk],Env)) -->> ([Val|Stk],Env) :- % and
   Val xis (ValA and ValB), !.
(neg,_,([ValA|Stk],Env)) -->> ([Val|Stk],Env) :- % not
   Val xis (not ValA),!.
(label(), .State) -->> State :- !.
```

Interpreting the target language in its model,

```
?- ['target.pl'].
% xis.pl compiled 0.00 sec, 6,824 bytes
% target.pl compiled 0.00 sec, 14,600 bytes
true.
?- assert(program([push(1),push(2),add])).
true.
?- program(P), (P,P,([],e)) -->> S.
P = [push(1), push(2), add],
S = ([3], e).
?-
```

The P in red is the continuation.