Practical Assignment 2: Finite Difference Method

Deadline 10/11/2017

Let $\Omega = (0,1) \times (0,1) \subset \mathbb{R}^2$ and $f \in L^2(\Omega)$.

$$-\Delta u(x,y) = f(x,y) \quad \text{in } \Omega \tag{0.1}$$

1. Dirichlet boundary condition

a. Solve equation (0.1) with uniform mesh subject to a Dirichlet boundary condition:

$$u(x,y) = 0 \qquad \forall (x,y) \in \partial\Omega$$

b. Solve equation (0.1) with uniform mesh subject to a Dirichlet boundary condition:

$$u(x,y) = g(x,y) \qquad \forall (x,y) \in \partial\Omega$$

c. Solve equation (0.1) subject to a Dirichlet boundary condition:

$$u(x,y) = g(x,y) \qquad \forall (x,y) \in \partial\Omega$$

with following mesh: $\{x_i\}_{i \in [0,N_x]}$ with $x_i = ih$, $h = \frac{1}{N_x}$ and $\{y_j\}_{j \in [0,N_y]}$ with $y_i = jk$, $k = \frac{1}{N_y}$. Noting that there exit positive constants α, β such that

$$\alpha \leq \frac{h}{k} \leq \beta$$

2. Dirichlet-Neumann boundary condition

Solve equation (0.1) with uniform mesh subject to a Dirichlet Neumann boundary condition:

$$u(x,0) = g_1(x),\ u(0,y) = g_2(y),\ u(x,1) = g_3(x),\ \frac{\partial u}{\partial x}(1,y) = g_4(y).$$

3. Neumann boundary condition

Solve equation (0.1) with condition $\displaystyle\int_\Omega f(x,y)dxdy = 0$ and with uniform mesh subject to a Neumann boundary condition:

$$\nabla u \cdot \vec{\mathbf{n}}(x,y) = 0 \qquad \forall (x,y) \in \partial\Omega$$

where $\vec{\mathbf{n}}$ is unit normal vector to boundary $\partial\Omega$.