Project

December 9, 2017

1 Elliptic equation in 1D

We use the finite difference method to solve the following equation

$$
\begin{aligned}
&-\nabla \cdot (a(x)\nabla u(x)) + \nabla \cdot (b(x)u(x)) + c(x)u(x) = f(x) \quad \forall x \in [0,1] \\
&u(0) = u(1) = 0
\end{aligned}
$$

2 Elliptic equation in 2D

We use the finite difference method to solve the following equation in $\Omega = [0,1] \times [0,1]$

$$
\begin{aligned}
&-\nabla \cdot (a(x)\nabla u(x)) + \nabla \cdot (b(x)u(x)) + c(x)u(x) = f(x) \quad \forall x \in \Omega \\
&u = 0 \text{ on } \partial\Omega
\end{aligned}
$$

3 Heat equation in 1D

We use the finite difference method to solve the following equation

$$
\begin{aligned}
&\frac{\partial u}{\partial t}(x,t) = \nabla \cdot (a\nabla u(x,t)) - \nabla \cdot (bu(x,t)) + f(x) \quad \forall (x,t) \in [0,1] \times [0,T] \\
&u(0,t) = g_0(t),\ u(1,t) = g_1(t) \quad \forall t \in [0,T] \\
&u(x,0) = u_0(x) \quad \forall x \in [0,1]
\end{aligned}
$$

Compare the results when changing values of constants a, b such as: $\frac{a}{b} = 0.1, 0.5, 1, 10, 100, ...$

4 Heat equation in 1D

We use the finite difference method to solve the following equation

$$
\begin{aligned}
&\frac{\partial u}{\partial t}(x,t) = \nabla \cdot (a(x,t)\nabla u(x,t)) - \nabla \cdot (b(x,t)u(x,t)) + f(x) \quad \forall (x,t) \in [0,1] \times [0,T] \\
&u(0,t) = g_0(t),\ u(1,t) = g_1(t) \quad \forall t \in [0,T] \\
&u(x,0) = u_0(x) \quad \forall x \in [0,1]
\end{aligned}
$$

Compare the results when changing values of constants a, b such as: $\frac{max|a|}{max|b|} = 0.1, 0.5, 1, 10, 100, ...$

5 Wave equation

We use the finite difference method to solve the wave equation

$$\frac{\partial^2 u}{\partial t^2}(x,t) = \frac{1}{c^2}\frac{\partial^2 u}{\partial x^2}(x,t) \quad \forall (x,t) \in [0,1] \times [0,T]$$
$$u(x,0) = u_0(x)(= e^{(\frac{-x^2}{\sigma})}) \;\; \forall x \in [0,1] \text{ and } u_t(x,0) = 0$$

6 Convection Equation

We use the finite difference method to solve the following equation

$$\frac{\partial u}{\partial t}(x,t) + \nabla \cdot (au(x,t)) = 0 \;\; \forall (x,t) \in [0,1] \times [0,T]$$
$$u(x,0) = u_0(x) \;\; \forall x \in [0,1]$$

Find some algorithms of finite volume discretization to implement and compare them. Show some results when u_0 is continuous and not continuous.

7 Stoke equations

We use the finite difference method to solve the following equation in $\Omega = [0,1] \times [0,1]$

$$\begin{aligned} -\Delta U(x) + \nabla p(x) &= f(x) \;\; \forall x \in \Omega \\ \nabla \cdot U(x) &= 0 \\ U &= 0 \text{ on } \partial\Omega \\ \int_\Omega p(x)dx &= 0 \end{aligned}$$