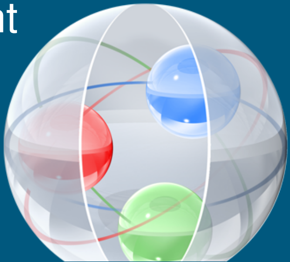
UNIVERSITÄT BONN

JÜLICH FORSCHUNGSZENTRUM

SU(3) Interpolating operators

@ SU(3) flavor degenerate point

October 30, 2018 | Thomas Luu, IAS-4



Stating the problem

What are the energies of the [6] and [$\bar{3}$] systems at the SU(3) point?

$$(c\ [\bar{3}])\otimes[8]=c([15]\oplus[6]\oplus[\bar{3}])$$

Tensor framework

Here I use tensor formalism given in Georgi's book (`Lie Algebra in Particle Physics`, chapter 10)[1]. Lower tensors ([3]), for example, are defined as

$$
\begin{aligned}
|1/2, \sqrt{3}/6\rangle &\equiv |_1\rangle \\
|-1/2, \sqrt{3}/6\rangle &\equiv |_2\rangle \\
|0, -1/\sqrt{3}\rangle &\equiv |_3\rangle
\end{aligned}
$$

General lower tensor transforms as

$$T_a|_i\rangle = |_j\rangle\,[T_a]^j_i$$

One has similar definitions for upper tensors ($[\bar{3}]$)

[1] Thanks Christoph for pointing this out to me!

General tensor

A general tensor is defined as

$$\left|{}^{i_1\cdots i_m}_{j_1\cdots j_n}\right\rangle = \left|{}_{j_1}\right\rangle \cdots \left|{}_{j_n}\right\rangle \otimes \left|{}^{i_1}\right\rangle \cdots \left|{}^{i_m}\right\rangle$$

and transforms as

$$\begin{aligned}
[T_a] \left|{}^{i_1\cdots i_m}_{j_1\cdots j_n}\right\rangle = & \\
& \sum_{\ell=1}^{\infty} \left|{}^{i_1\cdots i_m}_{j_1\cdots j_{\ell-1},k,j_{\ell+1}\cdots j_n}\right\rangle [T_a]^k_{j_\ell} \\
& - \sum_{\ell=1}^{\infty} \left|{}^{i_1\cdots i_{\ell-1},k,i_{\ell+1}\cdots i_m}_{j_1\cdots j_n}\right\rangle [T_a]^{i_\ell}_k
\end{aligned}$$

General vectors in this tensor space

A general vector in this space is

$$|v\rangle = |^{i_1 \cdots i_m}_{j_1 \cdots j_n}\rangle v^{j_1 \cdots j_n}_{i_1 \cdots i_m}$$

This defines the tensor *component*. Tensor *products* of components are

$$[u \otimes v]^{i_1 \cdot i_n \; i'_1 \cdots i'_p}_{j_1 \cdots j_m j'_1 \cdots j'_q} = u^{i_1 \cdots i_n}_{j_1 \cdots j_m} v^{i'_1 \cdots i'_p}_{j'_1 \cdots j'_q}$$

We can build up our SU(3) basis states from such tensor component products, and their transformation properties are dictated by T_a.
To get the states to fall into definite (n, m) irreps, must *project*!

Projection operators

Will only show relevant projection operators[2].

$$P[\bar{3}]^{ij}_{k} = \frac{1}{8}\left(3\delta^{i}_{k}u^{\ell}v^{j}_{\ell} - \delta^{j}_{k}u^{\ell}v^{i}_{\ell}\right)$$

$$P[6]^{ij}_{k} = -\frac{1}{4}\epsilon^{ij\ell}\left(\epsilon_{\ell mn}u^{m}v^{n}_{k} + \epsilon_{kmn}u^{m}v^{n}_{\ell}\right)$$

[2] Have tested $P[1]$, $P[3]$, $P[8]$, and $P[15]$.

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$[\bar{3}]$

state	components	T_a^2	I_z	Y
1	$\|1\rangle\|\bar{1}\bar{3}\rangle\sqrt{\frac{3}{8}} - \|1\rangle\|\bar{3}\bar{1}\rangle\frac{1}{2\sqrt{6}} + \|2\rangle\|\bar{2}\bar{3}\rangle\sqrt{\frac{3}{8}} - \|2\rangle\|\bar{3}\bar{2}\rangle\frac{1}{2\sqrt{6}} + \|3\rangle\|\bar{3}\bar{3}\rangle\sqrt{\frac{1}{6}}$	$\frac{4}{3}$	0	$+\frac{2}{3}$
2	$\|1\rangle\|\bar{1}\bar{2}\rangle\sqrt{\frac{3}{8}} - \|1\rangle\|\bar{2}\bar{1}\rangle\frac{1}{2\sqrt{6}} + \|2\rangle\|\bar{2}\bar{2}\rangle\sqrt{\frac{1}{6}} + \|3\rangle\|\bar{3}\bar{2}\rangle\sqrt{\frac{3}{8}} - \|3\rangle\|\bar{2}\bar{3}\rangle\frac{1}{2\sqrt{6}}$	$\frac{4}{3}$	$+\frac{1}{2}$	$-\frac{1}{3}$
3	$\|1\rangle\|\bar{1}\bar{1}\rangle\sqrt{\frac{1}{6}} + \|2\rangle\|\bar{2}\bar{1}\rangle\sqrt{\frac{3}{8}} - \|2\rangle\|\bar{1}\bar{2}\rangle\frac{1}{2\sqrt{6}} + \|3\rangle\|\bar{3}\bar{1}\rangle\sqrt{\frac{3}{8}} - \|3\rangle\|\bar{1}\bar{3}\rangle\frac{1}{2\sqrt{6}}$	$\frac{4}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$

These states have *disconnected* diagrams!!

Show me more money!

[6]

state	components	T_a^2	I_z	Y
1	$-\lvert 1\rangle\lvert\bar{1}\bar{2}\rangle\frac{1}{2}+\lvert 1\rangle\lvert\bar{2}\bar{1}\rangle\frac{1}{2}-\lvert 3\rangle\lvert\bar{2}\bar{3}\rangle\frac{1}{2}+\lvert 3\rangle\lvert\bar{3}\bar{2}\rangle\frac{1}{2}$	$\frac{10}{3}$	$+\frac{1}{2}$	$-\frac{1}{3}$
2	$\lvert 2\rangle\lvert\bar{1}\bar{2}\rangle\frac{1}{2}-\lvert 2\rangle\lvert\bar{2}\bar{1}\rangle\frac{1}{2}-\lvert 3\rangle\lvert\bar{1}\bar{3}\rangle\frac{1}{2}+\lvert 3\rangle\lvert\bar{3}\bar{1}\rangle\frac{1}{2}$	$\frac{10}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$
3	$\lvert 1\rangle\lvert\bar{1}\bar{3}\rangle\frac{1}{2}-\lvert 1\rangle\lvert\bar{3}\bar{1}\rangle\frac{1}{2}-\lvert 2\rangle\lvert\bar{2}\bar{3}\rangle\frac{1}{2}+\lvert 2\rangle\lvert\bar{3}\bar{2}\rangle\frac{1}{2}$	$\frac{10}{3}$	0	$+\frac{2}{3}$
4	$\lvert 2\rangle\lvert\bar{3}\bar{1}\rangle\frac{1}{\sqrt{2}}-\lvert 2\rangle\lvert\bar{1}\bar{3}\rangle\frac{1}{\sqrt{2}}$	$\frac{10}{3}$	-1	$+\frac{2}{3}$
5	$\lvert 1\rangle\lvert\bar{3}\bar{2}\rangle\frac{1}{\sqrt{2}}-\lvert 1\rangle\lvert\bar{2}\bar{3}\rangle\frac{1}{\sqrt{2}}$	$\frac{10}{3}$	$+1$	$+\frac{2}{3}$
6	$\lvert 3\rangle\lvert\bar{2}\bar{1}\rangle\frac{1}{\sqrt{2}}-\lvert 3\rangle\lvert\bar{1}\bar{2}\rangle\frac{1}{\sqrt{2}}$	$\frac{10}{3}$	0	$-\frac{4}{3}$

These states have *NO* disconnected diagrams (I believe)!!

Show me all the money!

[15]

state	components	T_a^2	I_z	Y
1	$\lvert 3\rangle\lvert\bar{3}\bar{3}\rangle\frac{1}{\sqrt{3}} - \lvert 1\rangle\lvert\bar{1}\bar{3}\rangle\frac{1}{\sqrt{3}} - \lvert 1\rangle\lvert\bar{3}\bar{1}\rangle\frac{1}{\sqrt{3}}$	$\frac{16}{3}$	0	$+\frac{2}{3}$
2	$-\lvert 2\rangle\lvert\bar{1}\bar{2}\rangle\frac{1}{2} - \lvert 2\rangle\lvert\bar{2}\bar{1}\rangle\frac{1}{2} + \lvert 3\rangle\lvert\bar{2}\bar{3}\rangle\frac{1}{2} + \lvert 3\rangle\lvert\bar{3}\bar{2}\rangle\frac{1}{2}$	$\frac{16}{3}$	$+\frac{1}{2}$	$-\frac{1}{3}$
3	$-\lvert 1\rangle\lvert\bar{1}\bar{1}\rangle\frac{1}{\sqrt{3}} + \lvert 3\rangle\lvert\bar{1}\bar{3}\rangle\frac{1}{\sqrt{3}} + \lvert 3\rangle\lvert\bar{3}\bar{1}\rangle\frac{1}{\sqrt{3}}$	$\frac{16}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$
4	$\lvert 2\rangle\lvert\bar{3}\bar{3}\rangle$	$\frac{16}{3}$	$-\frac{1}{2}$	$+\frac{5}{3}$
5	$\lvert 1\rangle\lvert\bar{3}\bar{3}\rangle$	$\frac{16}{3}$	$+\frac{1}{2}$	$+\frac{5}{3}$
6	$\lvert 2\rangle\lvert\bar{1}\bar{3}\rangle\frac{1}{\sqrt{2}} + \lvert 2\rangle\lvert\bar{3}\bar{1}\rangle\frac{1}{\sqrt{2}}$	$\frac{16}{3}$	-1	$+\frac{2}{3}$
7	$\lvert 2\rangle\lvert\bar{2}\bar{3}\rangle\frac{1}{2} + \lvert 2\rangle\lvert\bar{3}\bar{2}\rangle\frac{1}{2} - \lvert 1\rangle\lvert\bar{1}\bar{3}\rangle\frac{1}{2} - \lvert 1\rangle\lvert\bar{3}\bar{1}\rangle\frac{1}{2}$	$\frac{16}{3}$	0	$+\frac{2}{3}$
8	$\lvert 1\rangle\lvert\bar{2}\bar{3}\rangle\frac{1}{\sqrt{2}} + \lvert 1\rangle\lvert\bar{3}\bar{2}\rangle\frac{1}{\sqrt{2}}$	$\frac{16}{3}$	-1	$+\frac{2}{3}$
9	$\lvert 3\rangle\lvert\bar{1}\bar{1}\rangle$	$\frac{16}{3}$	-1	$-\frac{4}{3}$
10	$\lvert 3\rangle\lvert\bar{1}\bar{2}\rangle\frac{1}{\sqrt{2}} + \lvert 3\rangle\lvert\bar{2}\bar{1}\rangle\frac{1}{\sqrt{2}}$	$\frac{16}{3}$	0	$-\frac{4}{3}$
12	$\lvert 3\rangle\lvert\bar{2}\bar{2}\rangle$	$\frac{16}{3}$	$+1$	$-\frac{4}{3}$
12	$\lvert 2\rangle\lvert\bar{1}\bar{1}\rangle$	$\frac{16}{3}$	$-\frac{3}{2}$	$-\frac{1}{3}$
13	$-\lvert 1\rangle\lvert\bar{1}\bar{1}\rangle\frac{1}{\sqrt{3}} + \lvert 2\rangle\lvert\bar{1}\bar{2}\rangle\frac{1}{\sqrt{3}} + \lvert 2\rangle\lvert\bar{2}\bar{1}\rangle\frac{1}{\sqrt{3}}$	$\frac{16}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$
14	$-\lvert 1\rangle\lvert\bar{1}\bar{2}\rangle\frac{1}{\sqrt{3}} - \lvert 1\rangle\lvert\bar{2}\bar{1}\rangle\frac{1}{\sqrt{3}} + \lvert 2\rangle\lvert\bar{2}\bar{2}\rangle\frac{1}{\sqrt{3}}$	$\frac{16}{3}$	$+\frac{1}{2}$	$-\frac{1}{3}$
15	$\lvert 1\rangle\lvert\bar{2}\bar{2}\rangle$	$\frac{16}{3}$	$+\frac{3}{2}$	$-\frac{1}{3}$

These states have *NO* disconnected diagrams (I believe)!!

Inserting spinors

First make following replacements

$$
\begin{aligned}
1 &\to u \\
2 &\to d \\
3 &\to s
\end{aligned}
$$

Now pair one anti-quark with the charm quark, sandwiching them between a Γ matrix, and the remaining $q\bar{q}$ pair also sandwiches a Γ matrix. For example, the 5^{th} state in the [6] representation is[3],

$$
O^5_{[6]} = \frac{1}{\sqrt{2}} \left(\left(c\Gamma\bar{s} \right) \left(u\Gamma\bar{d} \right) - \left(c\Gamma\bar{d} \right) \left(u\Gamma\bar{s} \right) \right) \ .
$$

With some states, I now contract

$$
\langle O\bar{O} \rangle \ .
$$

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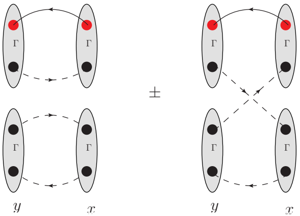
[3] Here I ignore an overall minus sign.

Contractions[4] for the [15] and [6]



$$[15]: \quad \mathrm{Tr}\left[\Gamma\gamma_5\mathcal{S}^{\dagger}_{y;x}\gamma_5\Gamma\mathcal{S}_{y;x}\right]\mathrm{Tr}\left[\Gamma\gamma_5\mathcal{S}^{\dagger}_{y;x}\gamma_5\Gamma\mathcal{C}_{y;x}\right] - \mathrm{Tr}\left[\Gamma\gamma_5\mathcal{S}^{\dagger}_{y;x}\gamma_5\Gamma\mathcal{S}_{y;x}\Gamma\gamma_5\mathcal{S}^{\dagger}_{y;x}\gamma_5\Gamma\mathcal{C}_{y;x}\right]$$

$$[6]: \quad \mathrm{Tr}\left[\Gamma\gamma_5\mathcal{S}^{\dagger}_{y;x}\gamma_5\Gamma\mathcal{S}_{y;x}\right]\mathrm{Tr}\left[\Gamma\gamma_5\mathcal{S}^{\dagger}_{y;x}\gamma_5\Gamma\mathcal{C}_{y;x}\right] + \mathrm{Tr}\left[\Gamma\gamma_5\mathcal{S}^{\dagger}_{y;x}\gamma_5\Gamma\mathcal{S}_{y;x}\Gamma\gamma_5\mathcal{S}^{\dagger}_{y;x}\gamma_5\Gamma\mathcal{C}_{y;x}\right]$$

$\mathcal{C}_{y;x}$ = charm quark propagator

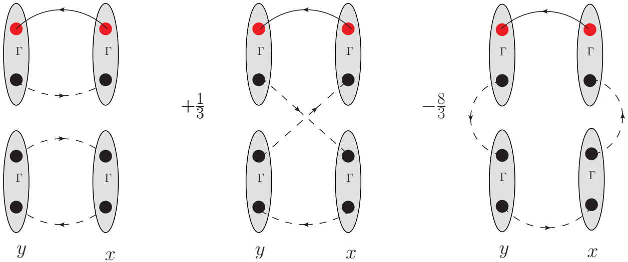
$\mathcal{S}_{y;x}$ = light quark propagator (SU(3) point)

$\Gamma = \gamma_5 \implies$ pseudoscalar

$\Gamma = \gamma_i \implies$ vector

[4] Performed using Jan-Lukas' `form` script.

Contractions[5] for the $[\bar{3}]$



$$
\begin{aligned}
[\bar{3}]: \quad & \mathrm{Tr}\left[\Gamma\gamma_5\mathcal{S}^\dagger_{y;x}\gamma_5\Gamma\mathcal{S}_{y;x}\right]\mathrm{Tr}\left[\Gamma\gamma_5\mathcal{S}^\dagger_{y;x}\gamma_5\Gamma\mathcal{C}_{y;x}\right] \\
& + \frac{1}{3}\mathrm{Tr}\left[\Gamma\gamma_5\mathcal{S}^\dagger_{y;x}\gamma_5\Gamma\gamma_5\mathcal{S}^\dagger_{y;x}\gamma_5\Gamma\mathcal{S}_{y;x}\Gamma\mathcal{C}_{y;x}\right] \\
& - \frac{8}{3}\mathrm{Tr}\left[\Gamma\mathcal{S}_{x;x}\Gamma\gamma_5\mathcal{S}^\dagger_{y;x}\gamma_5\Gamma{\color{red}\mathcal{S}_{y;y}}\Gamma\mathcal{C}_{y;x}\right]
\end{aligned}
$$

[5] Performed using Jan-Lukas' `form` script.