$\bar{c}\Gamma c$ CONTRACTIONS

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I want to calculate

$$\frac{1}{\sqrt{\Lambda_3}}\sum_{\vec{y}}\left\langle\left(\bar{c}(y)\Gamma c(y)\right)\left(\bar{c}(x)\Gamma c(x)\right)\right\rangle$$

source	sink
$x = (x_0, \vec{x})$;	$y = (y_0, \vec{y})$

zero momentum projection at the sink (but results hold for any momentum)

Λ_3 is the spatial dimension

I group the contractions into two types:

"connected"

$$\frac{1}{\sqrt{\Lambda_3}}\sum_{\vec{y}}\left\langle\left(\bar{c}(y)\Gamma c(y)\right)\left(\bar{c}(x)\Gamma c(x)\right)\right\rangle$$

"disconnected"

$$\frac{1}{\sqrt{\Lambda_3}}\sum_{\vec{y}}\left\langle\left(\bar{c}(y)\Gamma c(y)\right)\left(\bar{c}(x)\Gamma c(x)\right)\right\rangle$$

“CONNECTED” CONTRACTIONS

greek letters = spinor
roman letters = color

What *hadspec* calculates . . .

$$\frac{1}{\sqrt{\Lambda_3}}\sum_{\vec{y}}\langle(\bar{c}(y)\Gamma c(y))\,(\bar{c}(x)\Gamma c(x))\rangle = \frac{\Gamma_{\alpha\beta}\Gamma_{\gamma\delta}}{\sqrt{\Lambda_3}}\sum_{\vec{y}}\left\langle c_a^\alpha(y)\bar{c}_a^\beta(y)c_b^\gamma(x)\bar{c}_b^\delta(x)\right\rangle$$

$$= \frac{\Gamma_{\alpha\beta}\Gamma_{\gamma\delta}}{\sqrt{\Lambda_3}}\sum_{\vec{y}}\left\langle \mathbb{C}_{ab}^{\beta\gamma}(y,x)\left(\delta_{\alpha\delta}\delta_{ab}\delta_{xy} - \mathbb{C}_{ba}^{\delta\alpha}(x,y)\right)\right\rangle$$

$$= \quad \vdots \qquad \vdots$$

$$= \underbrace{\frac{1}{\sqrt{\Lambda_3}}\langle\mathrm{tr}\left(\mathbb{C}(x,x)\right)\rangle}_{\text{This is just a number—it has no time dependence (actually it's related to the chiral condensate). Hadspec ignores this term.}} - \underbrace{\frac{1}{\sqrt{\Lambda_3}}\sum_{\vec{y}}\left\langle\mathrm{tr}\left(\Gamma\gamma_5\mathbb{C}^\dagger(y,x)\gamma_5\Gamma\mathbb{C}(y,x)\right)\right\rangle}_{\text{This is what hadspec calculates.}}$$

$\langle\cdots\rangle$ in these expressions denote ensemble average

Note: $\Gamma^2 = 1$ and $\mathrm{tr}(\cdots)$ denotes trace over spin and color

Hadspec ignores this minus sign as well.

"DISCONNECTED" CONTRACTIONS

What causes loss of sleep (or hair?) . . .

$$\frac{1}{\sqrt{\Lambda_3}}\sum_{\vec{y}}\langle(\underbrace{\bar{c}(y)\Gamma c(y)})\,(\underbrace{\bar{c}(x)\Gamma c(x)})\rangle = \frac{\Gamma_{\alpha\beta}\Gamma_{\gamma\delta}}{\sqrt{\Lambda_3}}\sum_{\vec{y}}\left\langle c_a^\alpha\underbrace{(y)\bar{c}_a^\beta}(y)c_b^\gamma\underbrace{(x)\bar{c}_b^\delta}(x)\right\rangle$$

$$= \frac{\Gamma_{\alpha\beta}\Gamma_{\gamma\delta}}{\sqrt{\Lambda_3}}\sum_{\vec{y}}\left\langle\left(\delta_{\alpha\beta}-\mathbb{C}_{aa}^{\beta\alpha}(y,y)\right)\left(\delta_{\gamma\delta}-\mathbb{C}_{bb}^{\delta\gamma}(x,x)\right)\right\rangle$$

$$= \quad \vdots \quad \vdots$$

Note: $\mathrm{tr}(\Gamma)=0$
so all terms in
this line vanish

$$\longrightarrow = \sqrt{\Lambda_3}\,\mathrm{tr}\,(\Gamma)\,\mathrm{tr}\,(\Gamma) - \sqrt{\Lambda_3}\,\mathrm{tr}\,(\Gamma)\,\langle\mathrm{tr}\,(\mathbb{C}(x,x)\Gamma)\rangle - \frac{\mathrm{tr}\,(\Gamma)}{\sqrt{\Lambda_3}}\sum_{\vec{y}}\langle\mathrm{tr}\,(\mathbb{C}(y,y)\Gamma)\rangle$$

$$+\frac{1}{\sqrt{\Lambda_3}}\underbrace{\sum_{\vec{y}}\langle\mathrm{tr}\,(\mathbb{C}(y,y)\Gamma)\,\mathrm{tr}\,(\mathbb{C}(x,x)\Gamma)\rangle}_{\text{This is the only term that survives}}$$

COMBINING "CONNECTED" AND "DISCONNECTED"

I only combine terms that have a temporal dependence. . .

"disconnected" "connected"

$$\frac{1}{\sqrt{\Lambda_3}}\sum_{\vec{y}}\left\langle\left(\bar{c}(y)\Gamma c(y)\right)\left(\bar{c}(x)\Gamma c(x)\right)\right\rangle=\frac{1}{\sqrt{\Lambda_3}}\sum_{\vec{y}}\left\langle \underbrace{\mathrm{tr}\left(\mathbb{C}(y,y)\Gamma\right)\mathrm{tr}\left(\mathbb{C}(x,x)\Gamma\right)}_{\text{What's missing.}}-\underbrace{\mathrm{tr}\left(\Gamma\gamma_5\mathbb{C}^\dagger(y,x)\gamma_5\Gamma\mathbb{C}(y,x)\right)}_{\text{What we've calculated}}\right\rangle$$

Regarding the "disconnected" term. For the *non-interacting* case (i.e. unit gauge), this term has NO time dependence. It is just a number that is the same for every timeslice y_0. But for the *interacting* case, there is a time dependence that comes from the ensemble average over the links. But my suspicion is that this time dependence is very mild, and so its contribution to the effective mass is small. I guess this is what all the literature points at.

So most likely we are "ok". But maybe it would be a good idea to try to estimate the contribution of the "disconnected" term? Do we have the ability to do this? I guess we would need stochastic sources in spin, color, and spatial positions.

This is the only term that survives