# Homogenization & improvement in energy dissipation of nonlinear composites

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## ABSTRACT

Due to their high strength to weight and stiffness to weight ratio, there is a huge shift towards the composite materials from the conventional metals, but composites have poor damage resistance in the transverse direction. Undergoing impact loads, they can fail in wide variety of modes which severely reduces the structural integrity of the component. This paper deals with the homogenization of glass-fibers and epoxy composite with a material introduced as an inelastic inclusion. This nonlinearity is being modelled by kinematic hardening procedure and homogenization is done by one of the mean field homogenization technique known as Mori-Tanaka method. The homogenization process consider two phases, one is the matrix and another is the inelastic inclusion, thus glass-fibers and epoxy are two phases which can be considered as one phase and act as a matrix while homogenizing non-linear composite. Homogenization results have been compared to the matrix at volume fraction zero of the inelastic inclusions and to the inelastic material at volume fraction one. After homogenization, increase of the energy dissipation into the composite due to addition of inelastic material and effects onto the same by changing the properties of the matrix material have been discussed.

**Keywords:** Homogenization, composites, inclusions, energy dissipation.

## 1. INTRODUCTION

Composite materials have been used extensively in the recent years in many areas of industry. The main advantage of such materials are high strength to weight and stiffness to weight ratio, but when composite material undergo impact loads, they can fail in wide variety of modes which severely reduces the structural integrity of the component. The majority of impacts which composite undergo are in the transverse direction but due to lack of thickness-reinforcement, damage resistance in transverse direction is poor. In order to improve the impact resistance, reinforcing the fibers that can absorb the impact, by undergoing increment in the strains without proportional increase in the stress, can offer a good solution. It is well known that material dissipates most of its energy, when it goes under the plastic deformation. The amount of dissipation happening depends on the area under the hysteresis curve. If such a material is introduced into a composite material, non-linear behavior is observed. In the paper material which exhibits inelastic properties after certain yield stress has been introduced as an inclusion into the composites. Predicting the mechanical behavior of a non-linear heterogeneous materials require careful modeling strategies. Micromechanical approaches aim at predicting the overall (or 'homogenized') response of the material starting from a description of the microstructure and the knowledge of the behavior of the constitutive phases. Mean field homogenization methods provide semi-analytical estimates of the effective response based on simplified relationships between the microfields statistics[1].

Homogenization is a mechanics based modelling scheme that transforms a body of a heterogeneous material into a constitutively equivalent body of a homogenous material, where the total energy stored in the both system is approximately same. The macroscopic properties are determined by the homogenization process, which yields the effective stresses and strains acting on the effective homogenized sample material. The sample of material is often called as statistically representative volume element (RVE) [2, 3]. The RVE for a material point of continuum mass is a material volume which is statistically representative of the infinitesimal material neighborhood of that material point[3].

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Inside RVE, there are many inhomogeneities present, Out of many, say rth inhomogeneity is considered and magnified, whose stiffness is given by C and this stiffness tends to varies with the load applied. The field in the RVE nearby that inhomogeneity is given as $\hat{\varepsilon}, \hat{\sigma}, \hat{C}$, where $\hat{C}$ is the stiffness of nearby embedding medium. The overall relation between stress and strain is given by $\overline{\sigma} = \overline{C} : \overline{\varepsilon}$ .What should be the correct values of these hat fields, so that $\overline{C}$ represents average value of stiffness over the RVE. The techniques which utilize eigenstrains in the determination of the hat fields and average stiffness tensor are known as mean field homogenization techniques[5, 6].

Therefore, instead of dealing with the heterogeneous RVE, it is convenient and effective to consider an equivalent homogenous solid, which has uniform elasticity tensor C of the matrix material everywhere including the inhomogeneities and in these equivalent homogenous solids, inhomogeneities are termed as inclusions. These inclusions and matrix fields in equivalent homogenous solids have same strain and stress fields as actual heterogeneous solid under applied traction or displacement. Thus in order to account for the mismatch of the material properties in the heterogeneous material and equivalent homogenous material a suitable strain field known as eigenstrains $\varepsilon^*(x)$ is introduced[4] .

In the present work the problem considered is a composite RVE with two types of inhomogenities which have different elastic modulus, embedded into the epoxy medium. These constituents are glass-fibers and aluminum metal. The Al. behaves inelastically after reaching yield stress on the application of the loads. Hence the overall behavior of the RVE is non-linear. To account for this non-linearity, inelastic strain field ( $\varepsilon_{in}$ ), is considered in order to account for the inelastic strains which are generated into the materials undergoing plastic deformations.

The proposed solution of the problem described in above paragraph is done in four sections. The kinematic hardening non-linear constitutive model adopted for the aluminum inclusions and linear model for the matrix is described in the section two. In section three, the Mori-Tanaka method and how it takes into the account of non-linearity in the RVE has been described. In section four, homogenized stress-strain results when load is applied longitudinally have been described in which it is shown that these results give exact values of the stress and strain behavior of matrix at volume fraction zero and inelastic material at volume fraction one. Then the effects on the energy dissipation, advantages of adding inelastic material in the enhancement of energy dissipation in comparison to the linear composites have been discussed. What are the effects of changing matrix material onto energy dissipation have also been discussed.

## 2. DISCRETE 3D KINEMATIC HARDENING MODEL

The central problem in constitutive theory of elasto-plastic material, is given the total strain ( $\varepsilon_n$ ) at current step n, we need to find out the corresponding plastic strain ( $\varepsilon_{n+1}^{in}$ ) at next step[8]. The adopted model is a phenomenological model to describe the kinematic hardening behavior of the material[9]. The equations are explicit equations, and the problem is strain driven, where we subdivide time interval of interest [0,T] in sub increments and wish to solve the problem of getting plastic strains over the generic time interval [ $t_n, t_{n+1}$ ] with $t_{n+1} > t_n$ .

### 2.1 Time Discrete Model

The time discrete constitutive model for kinematic hardening is[9]:

$$s = 2\mu(e - e^p) \tag{1}$$

$$\beta = s - \alpha \tag{2}$$

$$e_{n+1}^p = e_n^p + \Delta\lambda n \tag{3}$$

$$n = \frac{\beta}{\|\beta\|} \tag{4}$$

$$\alpha_{n+1} = \alpha_n + \frac{2}{3} H_k (\Delta\lambda n) \tag{5}$$

$$F(\beta) \le 0; \Delta\lambda \ge 0 \ \& \ \Delta\lambda F(\beta) = 0 \tag{6}$$

### 2.2 The convex cutting plane solution algorithm

To apply convex cutting algorithm, eqns. need not be implicit as done in the return mapping algorithm, and thus application of this one, is far easier and cheaper in the computations than return mapping. The basis of this algorithm is Taylor series[8]. Following gives the description of the variables used.

Constitutive variables: [ $s, e, e^p, \beta, \alpha$ ]

Independent variables: [ $e, e^p, \alpha$ ]

Dependent variables: [ $s, \beta$ ]

Plastic variables: [ $e^p, \alpha$ ]

Initial values: [ $e_0, e_0^p, \alpha_0$ ]

Initial values are supposed to be known at the beginning of time.

The basic trick is first see if the material yields by updating the total strain and checking whether yield condition[8] $F(\beta_{n+1}) < 0$. If it is then update is not required. If ( $F(\beta_{n+1}) \ge 0$ ), it means material has yielded. Thus we expand the yield function at (n+1)$^{th}$ iteration as follows:

$$F_{n+1} = F_n + \frac{dF}{d\beta}_n \Delta\beta_n \tag{7}$$

$$\Delta\beta = \frac{\partial\beta}{\partial s}\left(\frac{ds}{de^p}\right)\Delta e^p + \frac{\partial\beta}{\partial\alpha}\left(\frac{d\alpha}{de^p}\right)\Delta e^p \tag{8}$$

For Plastic region, we consider yield function $F_{n+1} = 0$, and thus putting into eqn. (7), we can calculate $\Delta\lambda$ as follows:

$$\Delta\lambda = \frac{-F_n}{\frac{df}{d\beta}_n \left[\frac{\partial\beta}{\partial s}\left(\frac{ds}{de^p}\right) + \frac{\partial\beta}{\partial\alpha}\left(\frac{d\alpha}{de^p}\right)\right]_n \frac{\beta}{\|\beta\|}} \tag{9}$$

Now $\Delta e^p$ can be calculated from eqn. (3 & 4) and hence stress can be updated for the next step.

## 3. NONLINEAR HOMOGENIZATION

The overall behavior for a monotonic loading and unloading of an inelastic material as an inclusion in the glass-fiber epoxy composite can be derived by solving the non-linear homogenization problem[10, 11]. In the following one of the mean field homogenization technique known as Mori-Tanaka method has been employed.

### 3.1 Mori-Tanaka method

In the Mori-Tanaka method, we consider that inhomogenieties are distributed into the RVE, such that disturbance effect coming from other inhomogenities are communicated through strain and stress fields in its surrounding matrix material. Although the stress and strain fields are different from one location to another in the matrix, the average fields ( $\bar{\varepsilon}^0$ & $\bar{\sigma}^0$ ) represents good approximation of the embedding fields around the inhomogeneities. We can say that number of inhomogeneities are such in number that, if we take out or remove rth inhomogeneity, $C_r$ will be replaced by embedding medium which is considered to be uniform matrix with stiffness tensor $\overline{C}$, subjected to a uniform strain & stress fields $\overline{\varepsilon}^0$ & $\overline{\sigma}^0$.

The Mori-Tanaka Method used for the homogenization for the modified composite with assumption described in the above paragraph is based on the Eshelby equivalence principle[4]. The composite is subjected to an inelastic strain inside the inclusion and zero in the glass-fiber/epoxy as they are considered to be linear.

$E^{\Omega}$ is the stiffness tensor of the inclusion. $\varepsilon_d$ is defined as the disturbances with respect to the average stress and strain tensor, respectively due to the presence of the inclusion $\Omega$. By the introduction of the eigenstrains and the Eshelby equivalence principle, the classical consistency condition can be written down for an inelastic material as:

$$E^{\Omega}(\overline{\varepsilon}_0 + \varepsilon^d - \varepsilon_{in}) = E^M(\overline{\varepsilon}_0 + \varepsilon^d - \varepsilon_{in} - \varepsilon^*) \tag{10}$$

When we introduce a material which becomes inelastic after yield stress, the modulus of the material is no longer constant, and we have to consider the tangent modulus of the material which is given as $d\sigma/d\varepsilon$. Thus considering eqn. (10) the strain in the inclusion can be written as:

$$\varepsilon^{\Omega} = \overline{\varepsilon}_0 + \varepsilon^d = \varepsilon_{in} + A^{\Omega}\varepsilon^* \tag{11}$$

With

$$A^{\Omega} = (E^M - \frac{d\sigma}{d\varepsilon})^{-1}E^M \tag{12}$$

The disturbance strains $\varepsilon^d$ inside the RVE, is due to the presence of the inclusion, which introduces inelastic strains into the material. This disturbance is also caused due to eigenstrains which are introduced in order to homogenize the heterogeneous RVE. Thus a relation between disturbance and eigen strains given by Eshelby[4] is modified as:

$$\varepsilon^d = S^{\Omega}(\varepsilon^* + \varepsilon_{in}) \tag{13}$$

Where $S^{\Omega}$ is the 4th order Eshelby tensor and it is a function of the geometry of the inclusion. Substituting eqn. (12) into eqn. (13) and solving w.r.t eigenstrains $\varepsilon^*$ yields

$$\varepsilon^* = P\overline{\varepsilon}_0 + Q\varepsilon_{in} \tag{14}$$

With

$$P = (A^{\Omega} - S^{\Omega})^{-1}, Q = (A^{\Omega} - S^{\Omega})^{-1}(S^{\Omega} - I) \tag{15}$$

Putting back the value of eigenstrains from eqn. (14) into eqn. (11) gives the total strain and elastic strains inside the inclusion by Mori-Tanaka method.

$$\varepsilon_{el}^{\Omega} = \overline{\varepsilon}_{el}^{\Omega} = A^{\Omega}(P\overline{\varepsilon}_0 + Q\varepsilon_{in}) \tag{16}$$

The total average strain in the composite material is given by weighted averages of the average strains in the matrix and average strains in the inclusion, with weights corresponds to matrix and inclusion individual volume fractions.

$$\overline{\varepsilon} = V_m\overline{\varepsilon}^M + V_f\overline{\varepsilon}^{\Omega} \tag{17}$$

$$\overline{\varepsilon} = V_m\overline{\varepsilon}_0 + V_f(\varepsilon_{in} + A^{\Omega}(P\overline{\varepsilon}_0 + Q\varepsilon_{in})) \tag{18}$$

Similar to eqn. (18) the total average stress into the composite is given as:

$$\overline{\sigma} = V_m\overline{\sigma}^M + V_f\overline{\sigma}^{\Omega} \tag{19}$$

Substituting equation for average strains inside the inclusion which is equal to the inelastic strains and addition of weighted average matrix strains and weighted inelastic strains into corresponding tangent stiffness of the inclusion and relating the average matrix stress to average matrix strain by linear relation, we can evaluate average stress in the composite by eqn. (19). The upper bounds on the stresses calculated by the Mori-Tanaka has also been obtained by Voigt method for the comparison of the response obtained from the homogenization technique. The average stresses by Voigt bound is given by the equation:

$$\bar{\sigma}_{voigt} = (V_f \frac{d\sigma}{d\varepsilon} \bar{\varepsilon}_{el}^{\Omega}) + V_m E^M \bar{\varepsilon}^0 \tag{20}$$

## 4. NUMERICAL RESULTS

In this section numerical results, stress vs strain curves for the homogenized material under the monotonic loading and unloading has been shown by both Mori-Tanaka and Voigt bounds, for different range of matrix properties. The energy dissipation curve for different volume fraction of the inelastic inclusions has been shown. The material properties considered are as follows:

**MATERIAL PROPERTIES**

| Inclusion (Al) Properties | | Glass-fibre/ Matrix | |
|---|---|---|---|
| Young's Modulus (E) | 69 GPa | Young's Modulus | 85 GPa |
| Shear Modulus | 26 GPa | Shear Modulus | 36 GPa |
| Poisson's ratio | 0.3 | Poisson's ratio | 0.2203 |
| Yield strength | 50 MPa | Poisson's ratio (Epoxy) | 0.2995 |
| Kinematic Hardening Mod. | 17.2 GPa | Young's Modulus(Epoxy) | 3.4 GPa |

### 4.1 Inelastic composite stress- strain response

At a single point, or RVE, the problem is considered to be strain-controlled. The strain has been incremented in steps as a function of sine in the longitudinal direction. The graphs has been plotted by incrementing strains to the peak value and then decrementing them again to zero. Figure 1. Shows the stress vs strain behavior for the different volume fractions of the aluminum (Al). At $V_f = 1$, the response of aluminum undergoing kinematic hardening coincides with the stresses calculated from the composite homogenization. This shows that the homogenization model gives good response for the composites. The homogenization method has been developed for two phases. Now to homogenize the glass-fiber/epoxy composite embedded with Al. inclusions, we can first homogenize the glass-fiber/epoxy composite, and then obtain its properties after homogenization and can consider them to be as one phase. This one phase is fed as a matrix to the RVE, which consist of Al. as the inclusions behaving inelastically after reaching yield stress. At $V_f = 0$, we can see that there is a straight line, which gives relation between stress and strain when just matrix is present. Its slope is higher than the slope at $V_f = 1$, because matrix is considered to be stiffer than the Al. As we increase the amount of fibers into the matrix, we can see that slope started decreasing and inelastic behavior of the inclusions starts getting visible. Considering complete aluminum ,decrement have been done into the strains, such that stresses go down to zero, but same amount of decrement is not enough to bring the stresses into the composites to zero and as volume fraction decreases , strain requirement is increasing to complete a zero stress mark. It shows that stress accumulation into the composites are more and due to this result strain needed while unloading back to zero stress increases. We can also observe that, the area keeps on increasing with increase in $V_f$ which leads to more energy dissipations when material behaves inelastically. In Figure 2. Matrix is considered to be epoxy whose young's modulus is 3.4 GPa, and thus the slope of stress-strain response at $V_f = 0$ is less. The figure shows homogenized response by Mori-Tanaka and how response gets converged to aluminum's at $V_f = 1$. Again there is increment in the area as volume fraction increases. For the softer material, the amount of strain decremented while unloading is sufficient to bring it down to zero stresses, which was not the case earlier. Interesting point to note is due to inelasticity of the material, the behavior turns out to be non-linear after the yield stress, but if the material is just linear and stiffer, it can break before realizing the complete half cycle of the strain. The variation in the matrix properties of inelastic composite, effects the amount of energy dissipation happening into the material, the advantage is more dissipation happens when the material becomes stiffer.

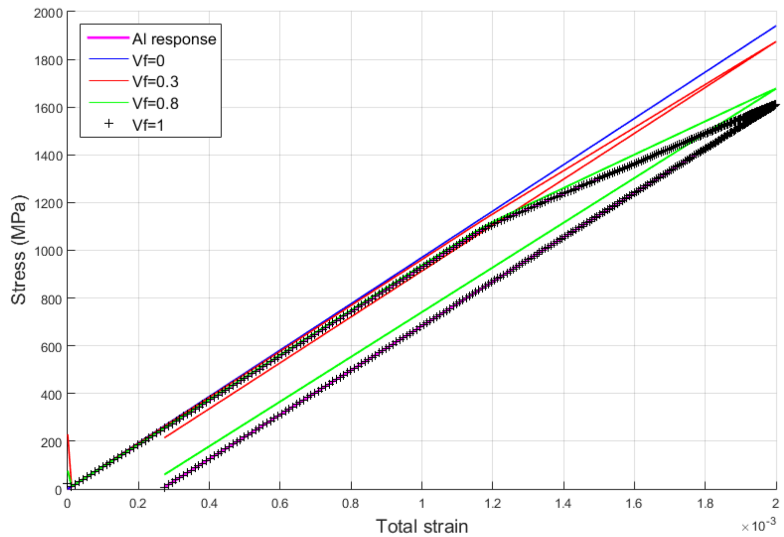


Figure 1. Mori-Tanaka homogenized stress – strain response in longitudinal direction for glass-fiber/epoxy as a matrix aluminum as an inelastic inclusion into the matrix

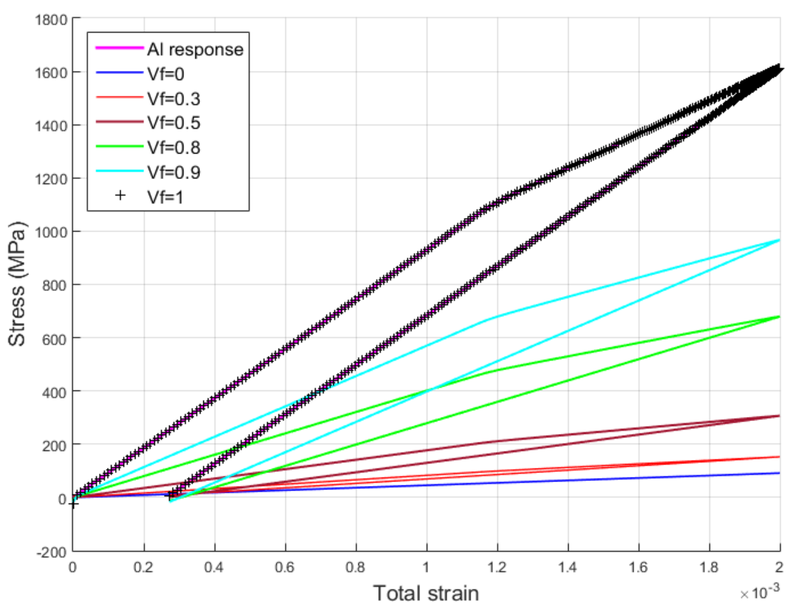


Figure 2. Mori-Tanaka homogenized stress – strain response in longitudinal direction for epoxy as a matrix and aluminum as an inelastic inclusion into the matrix

Lot of energy is required by the disturbances to float around in the stiffer material and to communicate with other inclusions, and hence as we can see the benefits of increasing matrix stiffness. Just looking at the epoxy as a matrix, in table 1. We can see energy dissipation is really less, and as volume fraction of inelastic material increases, there is an increase in dissipation, which should happen, but to get an accountful dissipation we need to put lot more fibers, than in comparison to the matrix which is stiffer than epoxy. The plot shows that at $V_f$=1, all the curves with whatever matrix material gives same energy dissipation should definitely be the case as just aluminum is left as a fiber. Energy dissipation at different volume fractions of inelastic fiber for varying matrix modulus. Energy dissipation change is quite significant for the volume fraction 0.1, which is an important advantage of increasing the stiffness of the matrix material. The increase in the energy dissipation from zero at volume fraction zero and be seen in the columns first and second. Significant change happen as we add aluminum into the metal. For both matrix stiffness at volume fraction zero and one, there is no change in the dissipation, which validate the calculation as such should happen for specific material.

Table 1. Energy dissipation at different volume fractions of inelastic fiber for varying matrix modulus.

| **Volume Fraction of Al.** | **Energy dissipation at Em = 5 GPa (Pascal)** | **Energy dissipation at Em = 85 Gpa (Pascal)** | **Percentage Increase (%)** |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0.1 | 1.33e4 | 6.02e4 | 352 |
| 0.3 | 4.29e4 | 1.26e5 | 193 |
| 0.5 | 8.57e4 | 1.91e5 | 122 |
| 0.8 | 2.00e5 | 2.86e5 | 43 |
| 1 | 3.494e5 | 3.498e5 | 0.1 |

## 5. CONCLUSIONS

In this paper we discussed an approach on how to homogenize a nonlinear composite using Mori-Tanaka mean field homogenization techniques. An example of glass-fiber composite is considered and inelastic inclusions have been added in to, thus making it a three phase composite. It is homogenized as two phase with matrix being considered as glass-fibers and epoxy and second phase is the non-linear material, when strains are being applied in the longitudinal directions. It has been showed in the figures and discussed in fourth section, how homogenization gave good results, as the two extreme volume fractions, the graph is giving properties of individual materials, and how these stress-strain behavior changes on changing again the properties of one of the phase of the composite from glass-fiber and epoxy as a matrix to the epoxy acting as a matrix alone. After homogenization energy dissipation in the homogenized composite has been found out and it can be seen that when there is no inelastic material in the linear composite, at volume fraction zero, the energy dissipation is zero. As the inelastic material's volume fraction increases, considerable increase in the amount of dissipation can be seen which proves the point of adding such a non-linearity to the composites and the effect is further enhanced by considering the stiffer matrix material.

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