Assignment-1 Solutions (Applied Statistics & Probability)

Note: Handwritten Submission

Q1.

X has p.d.f.

$$f(x) = 6x(1-x), \quad 0 \le x \le 1$$

(i) Mean

$$E[X] = \int_0^1 x f(x) dx = \int_0^1 x \cdot 6x(1-x) dx$$

$$= 6\int_0^1 (x^2 - x^3) dx = 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$= 6\left(\frac{1}{3} - \frac{1}{4}\right) = 6 \cdot \frac{1}{12} = \frac{1}{2}$$

👉 Mean = **0.5**

(ii) Median

Median m satisfies:

$$\int_0^m f(x) dx = \frac{1}{2}$$

$$\int_0^m 6x(1-x) dx = \frac{1}{2}$$

$$6\left(\frac{m^2}{2} - \frac{m^3}{3}\right) = \frac{1}{2}$$

$$3m^2 - 2m^3 = \frac{1}{2} \quad \Rightarrow \quad 2m^3 - 3m^2 + \frac{1}{2} = 0$$

Numerical solution → $m \approx 0.21$ or 0.79. Median must lie near 0.5 → **0.79**

(iii) Mode

Mode = value where $f(x)$ max.

$$f(x) = 6x - 6x^2, \quad f'(x) = 6 - 12x = 0 \Rightarrow x = 0.5$$

👉 Mode = **0.5**

(iv) Quartile Deviation

We need Q_1 and Q_3.

- $Q_1 : \int_0^{Q_1} f(x)dx = 0.25 \rightarrow 3Q_1^2 - 2Q_1^3 = 0.25$
  Numerical root → $Q_1 \approx 0.35$
- $Q_3 : \int_0^{Q_3} f(x)dx = 0.75 \rightarrow 3Q_3^2 - 2Q_3^3 = 0.75$
  Numerical root → $Q_3 \approx 0.65$

$$QD = \frac{Q_3 - Q_1}{2} \approx \frac{0.65 - 0.35}{2} = 0.15$$

(v) Raw Moments

$$\mu_r' = E[X^r] = \int_0^1 x^r f(x)dx = \int_0^1 x^r(6x - 6x^2)dx$$

$$= 6\int_0^1 (x^{r+1} - x^{r+2})dx = 6\left(\frac{1}{r+2} - \frac{1}{r+3}\right)$$

- $\mu_1' = 6\left(\frac{1}{3} - \frac{1}{4}\right) = 0.5$
- $\mu_2' = 6\left(\frac{1}{4} - \frac{1}{5}\right) = 0.3$
- $\mu_3' = 6\left(\frac{1}{5} - \frac{1}{6}\right) = 0.2$
- $\mu_4' = 6\left(\frac{1}{6} - \frac{1}{7}\right) = 0.143$

(vi) Central Moments

$$\mu_2 = \mu_2' - (\mu_1')^2 = 0.3 - 0.25 = 0.05$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3 = 0.2 - 3(0.3)(0.5) + 2(0.125) = 0$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

$$= 0.143 - 4(0.2)(0.5) + 6(0.3)(0.25) - 3(0.0625)$$

$$= 0.143 - 0.4 + 0.45 - 0.1875 = 0.0055$$

(vii) β, γ Coefficients

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0 \quad \Rightarrow \gamma_1 = 0$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{0.0055}{0.0025} \approx 2.2, \quad \gamma_2 = -0.8$$

👉 Distribution is **symmetric** ($\gamma1=0$) and **platykurtic** ($\gamma2<0$).

Q2.

X has p.d.f.

$$f(x) = 3x^2, \quad 0 \le x \le 1$$

- Mean:

$$E[X] = \int_0^1 x \cdot 3x^2 dx = 3\int_0^1 x^3 dx = \frac{3}{4} = 0.75$$

- Variance:

$$E[X^2] = \int_0^1 x^2 \cdot 3x^2 dx = 3\int_0^1 x^4 dx = \frac{3}{5} = 0.6$$

$$Var(X) = 0.6 - (0.75)^2 = 0.0375, \quad \sigma = \sqrt{0.0375} \approx 0.194$$

Median:
Solve $\int_0^m 3x^2 dx = 0.5 \Rightarrow m^3 = 0.5 \Rightarrow m = 0.794.$

Skewness (Karl Pearson):

$$Sk = \frac{\bar{x} - Md}{\sigma} = \frac{0.75 - 0.794}{0.194} = -0.227$$

👉 Slight **negative skewness.**

Q3.

Firm A: n_1=986, $\bar{x}_1$=52.5, σ_1^2=100
Firm B: n_2=548, $\bar{x}_2$=47.5, σ_2^2=121

a) Total wage = mean × workers.

- A: 52.5×986 = 51,765
- B: 47.5×548 = 26,030
  👉 **Firm A pays larger wages**

b) Variability → CV.

- A: σ_1=10, CV = 10/52.5 = 19%
- B: σ_2=11, CV = 11/47.5 = 23%
  👉 **Firm B more variable**

c) Combined mean:

$$\bar{x} = \frac{986(52.5) + 548(47.5)}{986 + 548} = \frac{51,765 + 26,030}{1,534} = 50.7$$

Combined variance:

$$\sigma^2 = \frac{1}{1534}[986(100 + d_1^2) + 548(121 + d_2^2)]$$

with $d_1 = 52.5 - 50.7 = 1.8$, $d_2 = 47.5 - 50.7 = -3.2.$

$$= \frac{1}{1534}[986(100 + 3.24) + 548(121 + 10.24)]$$

$$= \frac{1}{1534}[986(103.24) + 548(131.24)]$$

$$= \frac{1}{1534}[101,793 + 71,964] = \frac{173,757}{1534} = 113.3$$

$$\sigma = \sqrt{113.3} \approx 10.65$$

👉 Combined mean = **50.7**, σ = **10.65**

Q4.

Data:

Age	20–30	30–40	40–50	50–60	60–70	70–80	80–90
f	3	61	132	153	140	51	2

Midpoints: 25, 35, 45, 55, 65, 75, 85

Compute:

- $\Sigma f = 542$
- $\Sigma fx = 29,230$ → Mean = 29,230/542 = **53.9**
- $\Sigma fx^2 = 1,635,650$

Variance:

$$\sigma^2 = \frac{\Sigma fx^2}{N} - (\bar{x})^2 = \frac{1,635,650}{542} - (53.9)^2$$

$$= 3016.8 - 2906.8 = 110$$

$$\sigma = \sqrt{110} = 10.5$$

👉 Mean = **53.9 years**, SD = **10.5 years**