UNIT-5

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Table of contents

UNIT-4: Self Study Topics

Self study topics:

- Moments
- Moments Generating Functions
- Characteristics Function

QUIZ-2 will contain at most 50% questions from these topics.

Assignment-2

– Will be shared soon.
– Will contain 2 questions.
– Last Date: 30-November-2025

2D Random Variable

- So far, one random variable on a sample space.

Example 1

Interest in the height and weight of every person in a certain educational institution.

- $\Longrightarrow$ **More than one** random variable on the **same** sample space.

2D Random Variable

Definition 2

Let X and Y be two random variables defined on the same sample space S, then then function (X, Y) that assigns a point in $\mathbb{R}^2 (= R \times R)$, is called a two-dimensional random variable.

- The value of (X, Y) at $\omega \in S$ is defined as:

$$\{X(\omega), Y(\omega)\}$$

- Notation $\{X(\omega), Y(\omega)\}$: the set of all events $\omega \in S$ such that $X(\omega) \leq a$ and $Y(\omega) \leq b$.
- $\{a < X \leq b, c < Y \leq d\} = \{a < X \leq b\} \cap \{c < Y \leq d\} = A \cap B$

  – Probability can be defined in same way.
  text

2D Random Variable

Remarks

- Two dimentional discrete variables takes at most a countable number of points in $\mathbb{R}^2$.
- Two random variables are said to be **jointly distributed** if they are defined on the *same probability space*.
- In joint distribution, a sample point is represented by a $2-$tuple (e.g. (x, y)).

Joint Probability Mass Function

Definition 3

Let (X, Y) is a two-dimensional discrete random variable, then the joint discrete function of X, Y, also called the joint probability mass function of X, Y, denoted by $p_{X,Y}$ is defined as:

$$p_{XY}(x_i, y_j) = P(X = x_i, Y = y_j) \text{ for a value } (x_i, y_j) \text{ of } (X, Y),$$

and

$$p_{XY}(x_i, y_j) = 0, \text{ otherwise}$$

Remark

The sum of the probablities of all possible values of (X, Y) denoted as $\sum \sum p_{XY}(x_i, y_j)$ is 1.

Marginal Probability Function of JPMF

Definition 4

Let (X, Y) be a discrete two-dimensional r.v. which takes up countable number of values (x_i, y_i). Then the probability distribution of X is determined as follows:

$$\begin{aligned} p_X(x_i) = P(X = x_i) &= P(X = x_i \cap Y = y_1) + \ldots + P(X = x_i \cap Y = y_m) \\ &= p_{i1} + p_{i2} + \ldots + p_{im} = \sum_{j=1}^{m} p_{ij} = \sum_{j=1}^{m} p(x_i, y_j) \end{aligned}$$

It is also known as the *marginal probability mass function* of X.
– Similarly, it can be proved for Y.

Conditional Probability Function

Definition 5

Let (X, Y) be a discrete two-dimensional random variable. Then the conditional probability mass function of X, given $Y = y$, denoted by $p_{X|Y}(x|y)$, is defined as:

$$p_{X|Y}(x|y) = \frac{P(X = x|Y = y)}{P(Y = y)} = \frac{P(x, y)}{P(y)}, \text{ provided } P(Y = y) \neq 0$$

Two-Dimensional Distribution Function

Definition 6

The distribution function of the two-dimensional r.v. (X, Y) is a real valued functon F defined for all real x and y by the relation:

$$F_{XY}(x, y) = P(X \leq x, Y \leq y)$$

Properties:

1. $0 \leq F(x, y) \leq 1, -\infty < x < \infty, -\infty < y < \infty$
2. $F(x, y)$ is monotone increasing in both the variables; that is if $x_1 \leq x_2$ and $y_1 \leq y_2$, then $F(x_1, y_1) \leq F(x_2, y_2)$.
3. If either x or y approaches $-\infty$, then $F(x, y)$ approaches 0, and if both x and y approach ∞, then $F(x, y)$ approaches 1.
4. $P(a < X \leq b \text{ and } c < Y \leq d) = F(b, d) - F(a, d) - F(b, c) + F(a, c)$.
5. If X and Y are continuous r.v. then $F(x, y)$ is continuous.

References I