UNIT-5 Part-3 Expectation of two Random Variables

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Example 1

In the random placement of three balls in three cells, describe the possible outcomes of the experiment. Let X_i denote the number of balls in cell $i; i = 1, 2, 3$; and N, the number of cells occupied. Obtain the joint distribution of:

1. (X_1, N), and
2. (X_1, X_2)

Example 1

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Hint:

– Determine what kind of distribution it is: i.e., whether "discrete" or "continuous".
– Determine possible outcomes.
– Create table that defined the probability for each value of first r.v. and second r.v.
– Follow the definition of joint distribution and compute the final value.

Independent Random Variable

Definition 2 (Independent Random Variable)

Two discrete random variables X and Y are defined to be independent provided their joint pmf is the product of their marginal pmf's:

$$p_{X,Y}(x,y) = p_X(x)p_Y(y) \qquad \text{for all } x \text{ and } y$$

- Assume that the event has been observed.
- Probability that a specific value of X will occur?

$$\begin{aligned} P(X = x|Y = y) &= \frac{P(X = x \cap Y = y)}{p_Y(y)} \\ &= \frac{p_{X,Y}(x,y)}{p_Y(y)} \\ &= p_X(x) \end{aligned}$$

Independence to r Random Variables

Definition 3

Let $X_1, X_2, \ldots, X_r$ be r discrete random variables with pmf's $p_{X_1}, p_{X_2}, \ldots, p_{X_r}$, respectively. These random variables are said to be **mutually independent** if their compound pmf p is given by:

$$p_{X_1, X_2, \ldots, X_r}(x_1, x_2, \ldots, x_r) = p_{X_1}(x_1) p_{X_2}(x_2) \ldots p_{X_r}(x_r)$$

Covariance

– Let X, Y are r.v., and $Y = \phi(X)$ (a function of X).

–

$$E[Y] = E[\phi(X)] = \begin{cases} \sum_i \phi(x_i) p_X(x_i), & \text{discrete } X \\ \int_{-\infty}^{\infty} \phi(x) f_X(x) dx, & \text{Continuous } X \end{cases}$$

– Special case: $\phi(X) = X^k$, then

– $E[X^k] :=$ kth moment of the random variable X.

– kth central moment: $\mu_k = E[(X - E[X])^k]$

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Definition 4 (Variance)

The variance of a random variable X is

$$Var[X] = \mu_2 = \sigma_X^2 = \begin{cases} \sum_i (x_i - E[X])^2 p(x_i) & \text{discrete } X \\ \int_{-\infty}^{\infty} (x - E[X])^2 f(x) dx & \text{continuous } X \end{cases}$$

Covariance

Definition 5

If X and Y are two random variables, then covariance between them is defined as

$$\begin{aligned} Cov(X, Y) &= E[\{X - E(X)\}\{Y - E(Y)\}] \\ &= E[XY] - E[X]E[Y] \end{aligned}$$

Covariance

- Covariance tells the direction in which r.v. moves together relative to their own averages.
- If covariance > 0 they move in the same direction.
- If covariance < 0 they move in the opposite directioin.
- If covariance ≈ 0: no consistent linear tendency of X and Y to move together or against each other.

NOTE: $Cov(X, Y)$ is always zero when X and Y are independent random variables.

Questions

Answer the following:

1. What will be the covaraince when X and Y are independent?
2. What is $Cov(aX, bY)$?
3. What is $Cov(a + X, b + Y)$?
4. What is $Cov(aX + b, cY + d)$?

Questions

Q1: Ten coins are thrown simultaneously. Find the probability of getting at least 6 coins.

Q2: A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson's distribution with mean 1.5. Calculate the proportion of days on which

1. neither car is used, and
2. the proportion of days on which some demand is refused.

Questions

Q3. The diameter of an electric cable, say X, is assumed to be a continuous random variable with pdf $p(x) = 6x(1-x), 0 \le x \le 1$.

1. Check that the above is a pdf.
2. Determine a number b such that $P(X < b) = P(X > b)$.

Q4. A random variable has the following probability distribution:

x	0	1	2	3	4	5	6	7
P(x)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

1. find k
2. Evaluate $P(X < 6), P(X \ge 6)$ and $P(0 < X < 5)$
3. if $P(X \le c) > 1/2$ find the minimum value of c
4. Find the distribution function of X.

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