Sets and Set Builder Notation Handout

1. Unpack the meaning of each of the following sets and provide a description of the elements that each set contains.
   (a) $A = \{x \in \mathbb{N} : 2 < x < 8\}$
   (b) $B = \{t \in \mathbb{R} : -2 < t \leq 3\}$
   (c) $C = \{\begin{bmatrix} x \\ 0 \end{bmatrix} \in \mathbb{R}^2 : x \in \mathbb{Z}\}$
   (d) $D = \{\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 : x \geq 0, y \geq 0\}$
2. Express the following sets in set builder notation
   (a) "The set of all real numbers greater than $-\sqrt{2}$."
   (b) "The set of all integers greater than 10 and less than 1000."
   (c) "The set of all points in the plane with x-coordinate greater than 10."
   (d) "The set of all points in $\mathbb{R}^3$ such that the x-coordinate is equal to 0, and the y-coordinate is less than the z-coordinate."
3. Unpack the meaning of each of the following sets and provide a description of the elements that each set contains. For these problems assume $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.
   (a) $E = \{x \in \mathbb{N} : x = 3k \text{ for some } k \in \mathbb{N}\}$
   (b) $F = \{t \in \mathbb{R} : t = 1 - \frac{1}{n} \text{ for some } n \in \mathbb{Z}\}$
   (c) $G = \{\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 : \begin{bmatrix} x \\ y \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ for some } t \in \mathbb{R}\}$
   (d) $H_1 = \{z \in \mathbb{R} : z \leq y \text{ for all } y \in \mathbb{R}\}$
   (e) $H_2 = \{z \in \mathbb{R} : z \leq y \text{ for some } y \in \mathbb{R}\}$
   (f) $J_1 = \{w \in \mathbb{N} : w \leq s \text{ for all } s \in \mathbb{N}\}$
   (g) $J_2 = \{w \in \mathbb{N} : w \leq s \text{ for some } s \in \mathbb{N}\}$
4. Express the following sets in set builder notation
   (a) "The set of even integers."
   (b) "The set of integers that are perfect squares."
   (c) "The x-axis in the plane."
   (d) "The line y=x in the plane."
5. Convert to set notation each of the sentences.
   (a) "The number 4 is an element of the set of integers."
   (b) "0.5 is not an element of the set of integers. "
   (c) "The set of natural numbers is a subset of the set of integers."
   (d) "There exists some number in the set of integers that is less than 2 and greater than or equal to 1."
6. Determine if these pairs of sets are equal.
   (a) $\{1, 2, 2, 3, 3, 3\}$, $\{1, 2, 3\}$
   (b) $\{1\}, \{\{1\}\}$
   (c) $\emptyset, \{\emptyset\}$

Solutions

1. (a) A is the set of natural numbers greater than 2 and less than 8, or $A = \{3,4,5,6,7\}$.
   (b) B is the set of real numbers greater than -2 and less than or equal to 3.
   (c) C is the set of vectors in the plane with integer x-coordinates and y-coordinate equal to zero.
   (d) D is the set of vectors in the plane with non-negative x and y coordinates (the first quadrant including the origin and the positive x and y axes.).

2. (a) $\{x \in \mathbb{R} : x > -\sqrt{2}\}$
   (b) $\{n \in \mathbb{Z} : 10 < n < 1000\}$
   (c) $\{\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 : x > 10\}$
   (d) $\{\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : x = 0, y < z\}$

3. (a) E is the set of natural numbers divisible by 3.
   (b) F is the set of real numbers of the form $1 - \frac{1}{n}$, where n is an integer. OR: A real number t is a member of F if there exists an integer n such that $t = 1 - \frac{1}{n}$.
   (c) G is the set of vectors in the plane on the line $x = y$. OR: G is the set of vectors in the plane whose x-coordinate is equal to its y-coordinate.
   (d) $H_1 = \emptyset$, the empty set, since there is no real number that is smaller than every other real number. In other words, $\mathbb{R}$ does not have a minimum.
   (e) $H_2 = \mathbb{R}$, the set of real numbers. This is because any real number z is less than or equal to itself (this is just one example, it's also less than or equal to any real number that is greater than z, we only need to find one example to satisfy the 'for some' condition).
   (f) $J_1 = \{0\}$, because the only natural number that is less than or equal to every other natural number is the minimum of $\mathbb{N}$, which is zero.
   (g) $J_2 = \mathbb{N}$, the set of natural numbers. This is because every natural number w is less than or equal to itself.

4. (a) $\{n \in \mathbb{Z} : n = 2k \text{ for some } k \in \mathbb{Z}\}$
   (b) $\{m \in \mathbb{Z} : m = t^2 \text{ for some} t \in \mathbb{Z}\}$
   (c) $\{\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 : y = 0\}$
   (d) $\{\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 : x = y\}$ OR $\{\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 : \begin{bmatrix} x \\ y \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ for some } t \in \mathbb{R}\}$

5. (a) $4 \in \mathbb{Z}$
   (b) $0.5 \notin \mathbb{Z}$
   (c) $\mathbb{N} \subseteq \mathbb{Z}$
   (d) $\{n \in \mathbb{Z} : 1 \leq n < 2\} \neq \emptyset$

6. (a) The sets are equal because each element of the first set is a member of the second and vice versa.
   (b) The sets are not equal.
   (c) The sets are not equal.