STA237H1F TUTORIAL 5 PROBLEMS (Nov 1-2, 2023)

Beta Probability Distribution:

A random variable Y is said to have a **beta probability distribution** with shape parameters $\alpha > 0$ and $\beta > 0$ if and only if the density function of Y is

$$f_Y(y) = \begin{cases} \frac{1}{B(\alpha,\beta)} y^{\alpha-1}(1-y)^{\beta-1} & 0 \le y \le 1 \\ 0, & otherwise \end{cases}, \text{ where}$$

$$B(\alpha,\beta) = \int_0^1 y^{\alpha-1}(1-y)^{\beta-1}\,dy = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}.$$

It follows that the mean and variance of Y are $E(Y) = \frac{\alpha}{\alpha+\beta}$ and $V(Y) = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$.

Note that Beta probabilities and values can be computed in R:

- dbeta(y,α,β) # computes $f_Y(y;\ \alpha,\ \beta)$
- pbeta(y,α,β) # computes $P(Y \le y;\ \alpha,\ \beta)$
- qbeta(p,α,β) # compute pth quantile of Beta(α, β) distribution
- rbeta(n,α,β) # generates n realizations of Beta$(\alpha,\ \beta)$ distribution.

Problems:

1. The proportion of his daily study time that Sean devotes to probability can be modeled with a density function proportional to $x^3(1-x)$, for $0 < x < 1$.

a. Write down the distribution and its parameters.

b. On a randomly selected day what is the probability that Sean will spend more than half his study time on probability? Calculate this by hand and using R.

c. Compute the mean of the proportion of Sean's daily study time spent on probability.

d. Calculate the standard deviation of the proportion of Sean's daily study time spent on probability.

2. Contracts for two construction jobs are randomly assigned to one or more of three firms: A, B, and C. Let Y_1 denote the number of contracts assigned to firm A and Y_2 the number of contracts assigned to firm B. Recall that each firm can receive $0, 1,$ or 2 contracts.

a. Find the joint probability function for Y_1 and Y_2.

b. Find $F(1,0)$.

c. Find $F(1,1)$.