Hints (proofs)

Sec 1.3 #8 Show that if W_1 and W_2 are subspaces of a vector space with $W_1 \cap W_2 = \{0\}$, then for each vector $x \in W_1 + W_2$ there are unique vectors $x_1 \in W_1$ and $x_2 \in W_2$ such that $x = x_1 + x_2$

Pf Suppose $x_1 + x_2 = y_1 + y_2$ where $x_1, y_1 \in W_1$ and $x_2, y_2 \in W_1$

Then $\underbrace{x_1 - y_1}_{\text{in } W_1} = \underbrace{y_2 - x_2}_{\text{in } W_2}$

Use the information $W_1 \cap W_2 = \{0\}$ to finish the proof

Sec 1.4 #8 Let W_1 and W_2 be subspaces of a vector space satisfying $W_1 \cap W_2 = \{0\}$. Show that if $S_1 \subset W_1$ and $S_2 \subset W_2$ are linearly independent, then $S_1 \cup S_2$ is linearly independent

Pf. Say $S_1 \cup S_2 = \{x_1, \dots x_n, y_1, \dots y_m\}$ $x_i \in S_1$ $y_j \in S_2$ $1 \le i \le n$, $1 \le j \le m$.

Let $a_1 x_1 + a_2 x_2 + \cdots + a_n x_n + b_1 y_1 + \cdots + b_m y_m = 0$ where $a_i, b_j \in \mathbb{R}$

Then $\underbrace{a_1 x_1 + a_2 x_2 + \cdots + a_n x_n}_{\text{in } W_1} = \underbrace{-b_1 y_1 - b_2 y_2 - \cdots - b_m y_m}_{\text{in } W_2}$ $1 \le i \le n$, $1 \le j \le m$.

Hints

Sec 1.1 #7.

a) Vector addition: $f +' g = fg$ Scalar multiplication $c \cdot f = c + f$

$$c \cdot (f +' g) \overset{?}{=} c \cdot f +' c \cdot g$$

For example. $c = 2$, $f = x$, $g = x^2$

$c \cdot (f +' g) = 2 \cdot (x +' x^2) = 2 + x^3$

$c \cdot f +' c \cdot g = 2 \cdot x +' 2 \cdot x^2 = (2+x)(2+x^2) = 4 + 2x^2 + 2x + x^3$

$$2 + x^3 \neq x^3 + 2x^2 + 2x + 4$$

there are many different examples

Sec 1.5 # 3 (d) $v = 2x^3 + x + 1$. $S = \{x^3+1,\ x^2+1,\ x+1\}$ in $P_3(\mathbb{R})$

Is v in Span(S) ?

If $v \in \text{span}(S)$, then there exists $(t_1, t_2, t_3) \in \mathbb{R}^3$ such that $\underbrace{2x^3 + x + 1 = t_1(x^3+1) + t_2(x^2+1) + t_3(x+1)}$

use $\{1, x, x^2, x^3\}$ is linearly independent.