Rejection of Narrow-Band Interferences in PN Spread Spectrum Systems Using an Eigenanalysis Approach

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Abstract

A new adaptive technique is suggested for rejecting narrow-band interferences in spread spectrum communications. When data is coded using a pseudo-noise code, the received signal consists of a wide-band signal with an almost white spectrum, and correlated narrow-band interference. The conventional approach to the interference suppression has been to exploit this correlation property to minimize the mean square error between predicted values of the signal and actual observations. The optimal solution is given by the Wiener filter. A different approach is suggested by the eigenanalysis of the data across the filter taps. While the energy of the spread spectrum signal is distributed across all the eigenvalues of the data correlation matrix, the energy of the interference is concentrated in a few large eigenvalues. The corresponding eigenvectors span the same signal subspace as the interference. The proposed method derives an error prediction filter with the additional constraint of orthogonality to these eigenvectors. The eigenanalysis based interference cancellation is sub-optimal for known correlation matrix, but is superior to the Wiener filter when the correlation matrix is estimated from a limited amount of data.

1 Introduction

In this paper we consider the rejection of narrow-band interferences in direct sequence (DS) spread spectrum systems. Spread spectrum communication systems are inherently capable of mitigating the effects of other users or intentional interferences. Cross-correlation of the received signal with a replica of the spreading code results in the collapse of the desired signal to the original data bandwidth and, simultaneously, spreads the interference over the DS bandwidth. The effect of this operation is a net processing gain advantage of the desired signal over the interference equal to the ratio of the DS bandwidth to the bandwidth of the signal before spreading.

The immunity of a spread spectrum communication system to interferences can be improved beyond the level inherent in the spreading operating, by applying a suppression filter prior to the despreading operation. For example, the additional signal processing may be required when, due to limited bandwidth, the processing gain alone does not reduce the interference to acceptable levels.

The cross-correlation operation is performed by a correlation receiver which is the match filter for detecting a known signal in white gaussian noise. In the presence of narrow-band interference the optimal receiver needs to be rederived. The maximum likelihood receiver for detecting a known signal in narrow-band interference is non-linear and difficult to implement [1]. When the statistics of the interference are assumed gaussian, the resulting optimal filter is linear but requires calculation of the noise and interference correlations separate from the signal [2]. A number of authors considered the application of notch filters to suppress narrow-band interferences [2]. While sub-optimal, this approach has the advantage of easy implementation using tapped delay line filters. The received signal consists of contributions of the DS signal, narrow-band interference, and thermal noise. A linear predictor will have little effect on the white spectra of the DS signal and the thermal noise, while providing an estimate of the narrow-band interference. This estimate is subsequently subtracted from the received data. It follows that the optimal linear suppression filter is a prediction-error filter which whitens the interference. When the statistics of the desired signal, interference, and noise, and in particular the correlations, are known, the filter coefficients are given by the solution to the Wiener equation.

Often in practice it is necessary to estimate the correlations which leads to adaptive forms of the Wiener Hopf equations. One approach is to recursively invert the sample correlation matrix [3], while another, that avoids matrix inversion, is to use the LMS algorithm [4]. Either way while the algorithms solutions provide asymptotically unbiased solutions to the Wiener equation, they are affected by measurement noise and are not necessarily locally optimal. This suggests that techniques which are less susceptible to measurement noise may be advantageous even if not based on the Wiener equation. In this paper we suggest an eigenanalysis based technique for suppression of narrow-band interferences in spread spectrum systems.

Eigenanalysis techniques gained popularity in spectral estimation [5] and direction finding [6] problems due to their ability to discriminate between closely spaced sources in frequency and angle, respectively. The eigenstructure of the received data correlation matrix can be also exploited for developing adaptive filters [7]. Indeed we will analyze the correlation matrix and show that it consists of a limited number of large eigenvalues contributed mainly be the narrow-band interference, and a large number of small and almost equal eigenvalues contributed by the DS data and thermal noise. The eigenanalysis interference canceler will be designed with a weight vector orthogonal to the eigenvectors corresponding to the large eigenvalues.

Section 2 of this paper presents the problem statement and the conventional Wiener filter for interference suppression. In Section 3 we develop the eigenanalysis based filter. Performance figures of merit are defined and evaluated in Section 4, and conclusions are summarized in Section 5.

2 Problem Statement

Consider the DS receiver with an interference suppression filter shown in Figure 1. After carrier demodulation the received signal is given by,

$$x(t) = s(t) + j(t) + n(t) \tag{1}$$

where $s(t)$ is a bi-polar spread spectrum waveform, $j(t)$ is a narrow-band interference, and $n(t)$ is the thermal noise. The desired signal $s(t)$ consists of the binary data sequence $d(t)$ modulated by the binary spreading sequence $c(t)$. The data sequence consists of *bits* of duration T_b and the spreading sequence consists of *chips* of duration $T_c = T_b/L$, where L is the spreading sequence length. After chip match filtering and sampling at the chip rate the received signal may be written $x_k = s_k + j_k + n_k$. The desired signal samples are $s_k = \sqrt{S} d_i c_k$, where $\sqrt{S}$ is the signal power at the input to the receiver and i is the index of the data sequence which changes each LT_c. The data sequence d_i, as well as the spreading sequence c_k, are random and white with unity power. For a tone interference, $j_k = \sqrt{2J}\cos(\Omega k T_c + \theta)$, where J is the interference average power, Ω is the interference frequency with respect to baseband, and θ is a random phase uniformly distributed in the interval $(0, 2\pi)$. The noise samples n_k are assumed independent, gaussian, zero mean and with σ_n^2 variance.

Consider an M-th order prediction error filter where the output is obtained as a linear combination of the delayed inputs and the first tap weight is set to unity. The output of the interference suppression filter is then given by

$$\begin{aligned} y_k &= \mathbf{w}^T \mathbf{x}_k \\ &= x_k + \sum_{m=1}^{M-1} w_m x_{k-m} \end{aligned} \tag{2}$$

where $\mathbf{w}^T = [w_o, w_1, \cdots, w_{M-1}]$ and $\mathbf{x}_k^T = [x_k, x_{k-1}, \cdots, x_{k-M+1}]$. The Wiener filter is obtained by solving for the weights that minimize the output power $\mathrm{E}\left[y_k^2\right] = \mathbf{w}^T \mathbf{R}_x \mathbf{w}$ subject to the constraint $\mathbf{w}^T \mathbf{u}_o = 1$ where $\mathbf{u}_o^T = [1, 0, \cdots, 0]$, and $\mathbf{R}_x = \mathrm{E}\left[\mathbf{x}_k \mathbf{x}_k^T\right]$. The solution is given by

$$\mathbf{w}_o = \alpha \mathbf{R}_x^{-1} \mathbf{u}_o \tag{3}$$

where $\alpha = \left(\mathbf{u}_o^T \mathbf{R}_x^{-1} \mathbf{u}_o\right)^{-1}$.

3 The Eigenanalysis Based Filter

The eigenvalue decomposition of the received data correlation matrix reveals some interesting properties. Consider $\mathbf{R}_j$ the correlation matrix of the interference contributions present in the filter structure. For a tone interference, $\mathbf{R}_j = \mathrm{E}\left[\mathbf{j}_k \mathbf{j}_k^T\right]$, where the expectation operation is over the random phase θ. Dropping the time subscript for brevity, $\mathbf{j}^T = [\cos\theta,\ \cos(\Omega T_c + \theta), \cdots, \cos(\Omega(M-1)T_c + \theta)]$. The interference vector can be expressed using the complex vectors $\mathbf{j} = \frac{1}{2}\left(e^{j\theta}\mathbf{f} + e^{-j\theta}\mathbf{f}^*\right)$, $\mathbf{f}^T = \left[1,\ e^{j\Omega T_c}, \cdots, e^{\Omega(M-1)T_c}\right]$, and the asterisk denotes complex conjugate. Then we have $\mathbf{R}_j = \frac{1}{4}\left(\mathbf{f}\,\mathbf{f}^H + \mathbf{f}^*\mathbf{f}^T\right)$, where the superscript H denotes complex conjugate and transposed. Since $\mathbf{f}$ and $\mathbf{f}^*$ are linearly independent, it follows that the interference correlation matrix, which is the sum of two rank one matrices, has rank two, i.e., it has only two non-zero eigenvalues. The eigenvalues of the data matrix $\mathbf{R}_x$ are determined by the combination of spread spectrum data, interference and thermal noise. The data and thermal noise are assumed white, and their effect is to add a constant level $\left(S + \sigma_n^2\right)$ to all the eigenvalues of $\mathbf{R}_j$. Consequently, the energy contributed by a tone interference is concentrated in the two largest eigenvalues of the correlation matrix $\mathbf{R}_x$. As the bandwidth of the interference is increased, the number of eigenvalues where most of the interference energy is contained is predicted by the Landau-Pollak theorem and is equal to $N = 2BT + 1$, where B is the interference bandwidth and T is its duration across the filter structure [8]. In the extreme wide-band case the bandwidth is equal to half the sampling rate $B_m = 1 / 2T_c$, and since $T = (M-1)T_c$, $N = M$, i.e., the interference energy is spread across all the system eigenvalues. When the interference occupies only part of the bandwidth, $B = \Delta / 2T_c$, where Δ is the fractional bandwidth with respect to B_m, and the number of interference-contributed eigenvalues is $N = \Delta(M-1) + 1 \leq M$. We refer to the subspace spanned by the eigenvectors associated with these eigenvalues as the *interference subspace.* The subspace spanned by the rest of the eigenvectors represents the white noise contributed by the DS signal and the thermal noise, hence it is named the *noise subspace.* Due to the orthogonality between eigenvectors, the interference and noise subspaces are orthogonal to each other. This suggests that if the filter weight vector is constrained to the noise subspace it will null the interference signal. Furthermore, the above analysis guarantees that the filter will utilize the smallest number of degrees of freedom required to reject the interference. We refer to the eigenanalysis based interference cancellation filter as the *eigencanceler.* While this technique is asymptotically sub-optimal, namely, the steady-state solution is sub-optimal with respect to the Wiener filter, it will be shown to have advantages in non-stationary environments.

The eigencanceler is obtained from a modification of the prediction error filter. Let $\mathbf{Q}_j$ and $\mathbf{Q}_n$ represent the matrices whose columns are the interference and noise eigenvectors, respectively. Then the eigencanceler is defined as the solution to the optimization problem

$$\min_{\mathbf{w}} \ \mathbf{w}^T\mathbf{w} \text{ subject to } \mathbf{w}^T\mathbf{Q}_j = 0 \text{ and } \mathbf{w}^T\mathbf{u}_o = 1. \tag{4}$$

In the appendix it is shown that the solution is given by:

$$\mathbf{w}_e = \beta\left(\mathbf{I} - \mathbf{Q}_j\mathbf{Q}_j^T\right)\mathbf{u}_o \tag{5}$$

where in order to satisfy the linear constraint, $\beta = \left(\mathbf{u}_o^T\left(\mathbf{I} - \mathbf{Q}_j\mathbf{Q}_j^T\right)\mathbf{u}_o\right)^{-1}$. To compute the eigencanceler's weight vector it is necessary to know or estimate the dimension of the interference subspace.

It is noteworthy to compare the Wiener filter weight vector in eq. (3) with the eigencanceler weight vector in eq. (5). To that end we rewrite eq. (3) to show explicit dependency on eigendecomposition. Using the relation: $\mathbf{R}_x^{-1} = \mathbf{Q}_j \mathbf{\Gamma}_j \mathbf{Q}_j^T + \mathbf{Q}_n \mathbf{\Gamma}_n \mathbf{Q}^T$, where $\mathbf{\Gamma}_j$ and $\mathbf{\Gamma}_n$ are diagonal matrices of the reciprocals of the interference and noise eigenvalues, respectively, we get:

$$\mathbf{w}_o = \alpha \left(\mathbf{Q}_j \mathbf{\Gamma}_j^{-1} \mathbf{Q}_j^T + \mathbf{Q}_n \mathbf{\Gamma}_n^{-1} \mathbf{Q}^T \right) \mathbf{u}_o. \tag{6}$$

From inspection of eqs. (6) and (5) it can be seen that the Wiener filter weight vector is a superposition of vectors in the noise subspace $\mathbf{Q}_n$, as well as vectors in the interference subspace $\mathbf{Q}_j$, and, furthermore, it depends on the reciprocals of the input correlation matrix eigenvalues. In contrast, the eigencanceler's weight vector (eq. (5)) is constructed using eigenvectors, solely. When the correlation matrix is estimated from a block of data, the eigenvalues and the eigenvectors will be perturbed from their theoretical values. If $\widehat{\mathbf{e}}_i$ is an eigenvector of the estimated correlation matrix $\widehat{\mathbf{R}}_x$, and $\widehat{\mathbf{e}}_i = \mathbf{e}_i + \Delta \mathbf{e}_i$, where $\mathbf{e}_i$ is the true eigenvector, then it has been shown that the error variance is given by [9],

$$\mathrm{var}\left[\Delta \mathbf{e}_i\right] = \frac{\lambda_i}{K} \sum_{j \neq i}^{M} \frac{\lambda_j}{(\lambda_j - \lambda_i)^2} \mathbf{e}_j \mathbf{e}_i^T \tag{7}$$

where K is the number of independent sample input vectors used in the estimation of the correlation matrix. From this relation it can be seen that this variance will be particularly large for the noise eigenvectors associated with the small and closely valued noise eigenvalues. Hence, the Wiener filter that makes use of those eigenvectors will be subject to higher noise fluctuations than the eigencanceler that is based on the dominant eigenvalues only. The Wiener filter is also affected by perturbations in the eigenvalues while the eigencanceler is not. In reference [10] we show that for the Wiener filter the normalized weight error when the correlation matrix is estimated using a large K (number of samples) is given by the expression

$$\mathrm{E}\left[\gamma_o^2\right] = \frac{1}{K} \tag{8}$$

where $\mathrm{E}\left[\gamma_o^2\right] = \mathrm{E}\left[\|\Delta \mathbf{w}_o\|^2 / \|\mathbf{w}_o\|^2 \right]$ and $\Delta \mathbf{w}_o = \widehat{\mathbf{w}}_o - \mathbf{w}_o$. For the eigencanceler, it is shown that:

$$\mathrm{E}\left[\gamma_e^2\right] \simeq \frac{1}{K} \frac{\lambda_{\min}(\mathbf{R}_x)}{\lambda_{\max}(\mathbf{R}_x)} \tag{9}$$

where $\mathrm{E}\left[\gamma_e^2\right] = \mathrm{E}\left[\|\Delta \mathbf{w}_e\|^2 / \|\mathbf{w}_e\|^2 \right]$, $\Delta \mathbf{w}_e = \widehat{\mathbf{w}}_e - \mathbf{w}_e$, and $\widehat{\mathbf{w}}_e$ is the eigencanceler's vector obtained from the eigendecomposition of the estimated correlation matrix $\widehat{\mathbf{R}}_x$. Comparison of the error variances in expressions (8) and (9) respectively, reveals a much lower variance for the eigencanceler. This robustness explains the superior performance exhibited by the eigencanceler.

4 Performance Evaluation

This section discusses the performance of the proposed eigenanalysis based interference suppression filter and evaluates it with respect to the performance of the Wiener filter.

The eigencanceler's operation is based on the eigendecomposition of the correlation matrix of the received signal samples at the filter taps. Figure 2 shows the eigenvalues corresponding to the theoretical correlation matrix associated with the sum of a DS signal, a narrow-band interference with fractional bandwidth $\Delta = 0.2$ and SIR = -10 dB, and thermal noise with SNR = 10 dB. The number of filter taps is $M = 15$. The plot substantiates the number of dominant eigenvalues predicted by the Landau-Pollak theorem, ($N = 0.2\,(15 - 1) + 1 \cong 4$). The large fluctuations of small eigenvalues is illustrated in Figure 3 which plots the normalized variance $(\mathrm{var}(\lambda_i)/\lambda_i)$ of the eigenvalues based on 100 independent runs, with 225 data and interference samples used to estimate each sample correlation matrix.

The interference suppression filter's performance can be measured in terms of SNIR (signal-to-noise-and-interference ratio) improvement and probability of bit error. The SNIR improvement is defined as the ratio of the SNIR after interference cancellation and spread spectrum demodulation to the SNIR at the in input of the receiver. At the input we have

$$\mathrm{SNIR}_i = \frac{\mathrm{E}^2\left[x_k/d_i\right]}{\mathrm{E}\left[\left(x_k - \mathrm{E}^2\left[x_k/d_i\right]\right)^2/d_i\right]} = \frac{S}{J + \sigma_n^2} \tag{10}$$

where the expected values are conditioned on the input data bit. The output of the interference suppression filter is cross-correlated with the L chips spreading sequence. The correlator output is given by $z_k = \sum_{l=0}^{L-1} c_k y_{k-l}$. The SNIR at the output of the correlator has been calculated in [3] and is given by:

$$\begin{aligned} \mathrm{SNIR}_c &= \frac{\mathrm{E}^2\left[z_k/d_i\right]}{\mathrm{E}\left[\left(z_k - \mathrm{E}^2\left[z_k/d_i\right]\right)^2/d_i\right]} \\ &= \frac{SL}{\mathbf{w}^T \mathbf{R}_j \mathbf{w} + (\sigma_n^2 + 1)\,\mathbf{w}^T \mathbf{w} - 1}. \end{aligned} \tag{11}$$

The numerator of (11) includes the processing gain equal to L, the length of the spreading sequence. In the denominator we find the interference power $\mathbf{w}^T \mathbf{R}_j \mathbf{w}$, the thermal noise power $\sigma_n^2 \mathbf{w}^T \mathbf{w}$, as well as the interchip interference power $(\mathbf{w}^T \mathbf{w} - |w_o|^2 = \mathbf{w}^T \mathbf{w} - 1)$, due to dispersion of the DS sequence in the interference suppression filter. The overall improvement factor (combination filter-correlator) is defined $\eta = \mathrm{SNIR}_c/\mathrm{SNIR}_i$. The probability of error can be determined analytically by assuming that the noise components in the denominator of (11) are gaussian. Then

$$P_e = Q\left(\sqrt{\mathrm{SNIR}_c}\right) \tag{12}$$

where $Q(x) \equiv \frac{1}{2}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$.

The improvement factor and the probability of error were used with simulation data to evaluate the performance of the Wiener filter and the eigencanceler. The simulations model consisted of a desired signal, a narrow-band interference with a fractional bandwidth $\Delta = 0.2$, and white noise. Each figure is the result of averaging 100 independent runs. Equations (3) and (5), correlation matrices estimated from blocks of data of specified size, were used to compute the weight vectors for a $M = 15$-tap Wiener filter and eigencanceler, respectively. Based on Figure 2, the eigencanceler was designed using the eigenvectors belonging to the four largest eigenvalues. Figure 4 presents the improvement factor η as a function of the SIR at the input for a fixed noise level of SNR = 10 dB, and for weight vectors estimated from blocks of $K = 225$ samples and $K = 450$ samples. At low SNIR, and for $K = 200$ the advantage of the eigencanceler reaches 16 dB. This advantage decreases to 10 dB as the data block size is doubled and estimates of the correlation matrix become more accurate. Figure 5 shows the probability of bit error as a function of SNR, at fixed SIR = -10 dB, and for $K = 225$ and $K = 450$ size blocks. Curve 1 is for ideal BPSK without interference. Curves 2 and 4 show the eigencanceler's performance for a tone interference and narrow-band interference, respectively. Curves 3 and 5 show that for $K = 225$ the Wiener filter hardly suppresses the interference. The advantage of the eigencanceler is evident.

5 Conclusions

In this paper we introduced an eigenanalysis based tapped delay line filter for suppression of narrow-band interferences in spread spectrum systems. The weight vector for this filter is constructed from the eigenvectors of the largest eigenvalues of the data across the filter structure. This filter was shown to be superior to the Wiener filter when the correlation matrix is estimated from data blocks whose size is only several times larger than the matrix dimension. This effect is explained by the reduced sensitivity to measurement noise of the eigenanalysis based method.

Appendix

In this appendix we provide the solution to the optimization problem in eq. (4). Using the method of Lagrange multipliers we define the objective function:

$$J = \mathbf{w}^T\mathbf{w} - \left[\mathbf{w}^T\mathbf{u}_o - 1\right]\lambda_1 - \mathbf{w}^T\mathbf{Q}_j\lambda_2 \qquad (13)$$

where λ_1 is a scalar and λ_2, is a vector. The minimum of J is found from the gradient with respect to $\mathbf{w}^T$:

$$\nabla_{w^T} J = 2\mathbf{w} - \mathbf{u}_o\lambda_1 - \mathbf{Q}_j\lambda_2 = 0. \qquad (14)$$

It follows that the weight vector $\mathbf{w}$ is given by:

$$\mathbf{w} = \frac{1}{2}\left[\mathbf{u}_o\lambda_1 + \mathbf{Q}_j\lambda_2\right]. \qquad (15)$$

The two Lagrange multipliers are determined from the two constraints:

$$\begin{aligned} \mathbf{w}^T\mathbf{u}_o &= \tfrac{1}{2}\left(\lambda_1 + \lambda_2^T\mathbf{Q}_j^T\mathbf{u}_o\right) &= 1 \\ \mathbf{w}^T\mathbf{Q}_j &= \tfrac{1}{2}\left(\lambda_1\mathbf{u}_o^T\mathbf{Q}_j + \lambda_2^T\mathbf{Q}_j^T\mathbf{Q}_j\right) &= 0. \end{aligned} \qquad (16)$$

From which we find

$$\begin{aligned} \lambda_1 &= \frac{2}{1 - \mathbf{u}_o^T\mathbf{Q}_j\mathbf{Q}_j^T\mathbf{u}_o} \\ \lambda_2 &= \frac{-2\mathbf{Q}_j^T\mathbf{u}_o}{1 - \mathbf{u}_o^T\mathbf{Q}_j\mathbf{Q}_j^T\mathbf{u}_o}. \end{aligned} \qquad (17)$$

When λ_1 and λ_2 are substituted back in eq. (15), expression (5) results.

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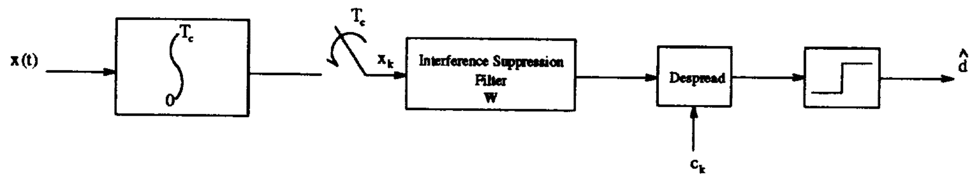

Figure 1: DS receiver with interference suppression filter.

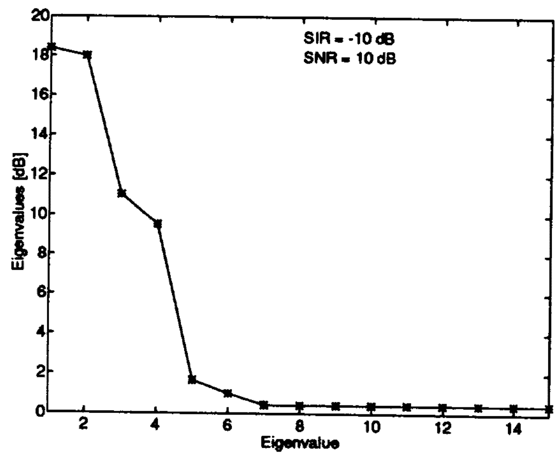

Figure 2: Eigenvalues of the theoretical correlation matrix.

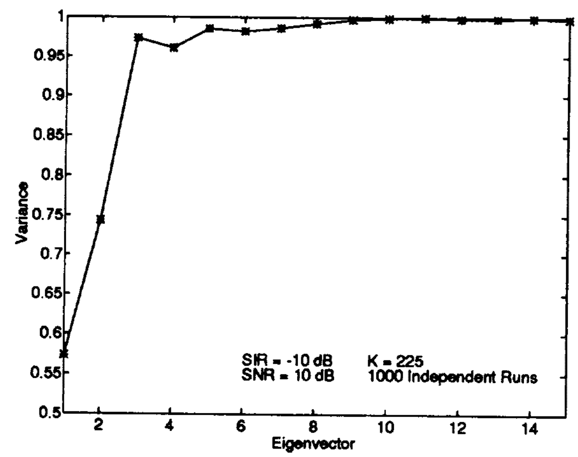

Figure 3: Variance of norm of the eigenvectors of sample correlation matrix.

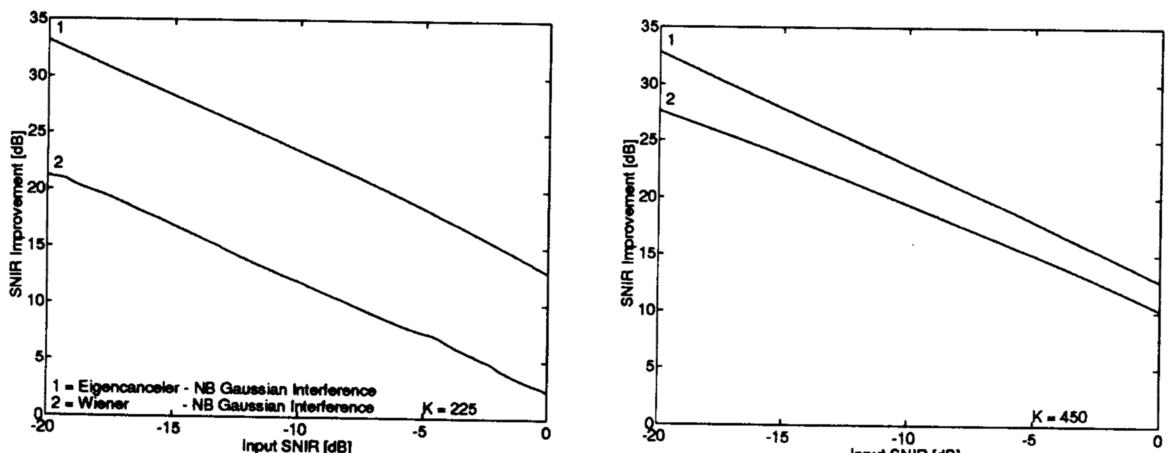

Figure 4: SNIR improvement vs. input SNIR, SNR = 10 dB.

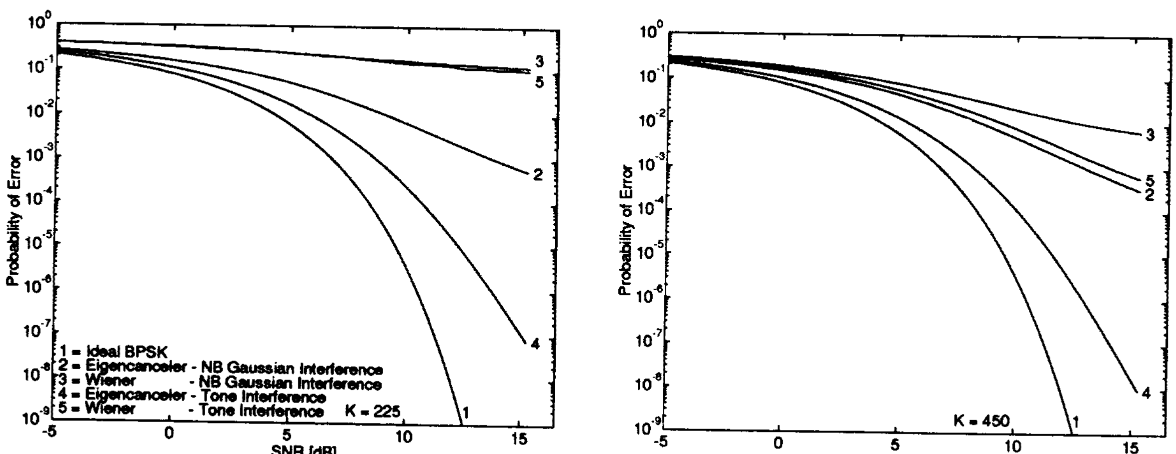

Figure 5: Probability of error vs. SNR, SIR = -10 dB.