SUBSPACE PROJECTION TECHNIQUES FOR ANTI-FM JAMMING GPS RECEIVERS

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ABSTRACT

This paper applies subspace projection techniques as a pre-correlation signal processing method for the FM interference suppressions in GPS receivers. The FM jammers are instantaneous narrowband and have clear time-frequency (t-f) signatures that are distinct from the GPS C/A spread spectrum code. In the proposed technique, the instantaneous frequency (IF) of the jammer is estimated and used to construct a rotated signal space in which the jammer occupies one dimension. The anti-jamming system is implemented by projecting the received sequence onto the jammer-free subspace. This paper focuses on the characteristics of the GPS C/A code and derives the signal to interference and noise ratio (SINR) of the GPS receivers implementing the subspace projection techniques.

1. INTRODUCTION

The Global Positioning System (GPS) is a satellite-based, worldwide, all-weather navigation and timing system [1]. The ever-increasing reliance on GPS for navigation and guidance has created a growing awareness of the need for adequate protection against both unintentional and intentional interference. Jamming is a procedure that attempts to block reception of the desired signal by the intended receiver. In general terms, it is high power signal that occupies the same frequency as the desired signal, making reception by the intended receiver difficult or impossible. Designers of military as well as commercial communication systems have, through the years, developed numerous anti-jamming techniques to counter these threats. As these techniques become effective for interference removal and mitigation, jammers themselves have become smarter and more sophisticated, and generate signals, which are difficult to combat.

The GPS system employs BPSK-modulated direct sequence spread spectrum (DSSS) signals. The DSSS systems are implicitly able to provide a certain degree of protection against intentional or non-intentional jammers. However, in many cases, the jammer may be much stronger than the GPS signal, and the spreading gain might be insufficient to decode the useful data reliably [2]. There are several methods that have been proposed for interference suppression in DSSS communications [3, 4, 5]. The recent development of the bilinear time-frequency distributions (TFDs) for improved signal power localization in the time-frequency plane has motivated several new effective approaches, based on instantaneous frequency (IF) estimation, for non-stationary interference excisions [6]. One of the important IF-based interference rejection techniques uses the jammer IF to construct a time-varying excision notch filter that effectively removes the interference [7]. However, this notch filtering excision technique causes significant distortions to the desired signal, leading to undesired receiver performance.

Recently, subspace projection techniques, which are also based on IF estimation, have been devised for non-stationary FM interference excision in DSSS communications [8]. The techniques assume clear jammer time-frequency signatures and rely on the distinct differences in the localization properties between the jammer and the spread spectrum signals. The jammer instantaneous frequency, whether provided by the time-frequency distributions or any other IF estimator, is used to form an interference subspace. Projection can then be performed to excise the jammer from the incoming signal prior to correlation with the receiver PN sequence. The result is improved receiver SINR and reduced BERs.

In this paper, we apply the subspace projection techniques as a pre-correlation signal processing method to the FM interference suppression in GPS receivers. The GPS receiver and signal structure impose new constraints on the problem since the spreading code from each satellite is known and periodic within one navigation data symbol. This structure and the signal model are reviewed in Section 2. In Section 3, we depict the received GPS signal properties in time-frequency domain. The SINR of the GPS receiver implementing the subspace projection techniques

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is derived in Section 4, which shows improved performance in strong interference environments.

2. SIGNAL MODEL

GPS employs BPSK-modulated DSSS signals. The navigation data is transmitted at a symbol rate of 50 bps. It is spread by a coarse acquisition (C/A) code and a precision (P) code. The C/A code is a Gold sequence with a chip rate of 1.023 MHz and a period of 1023 chips, i.e. its period is 1ms, and there are 20 periods within one data symbol. The P code is a pseudorandom code at the rate of 10.23 MHz and with a period of 1 week. These two spreading codes are multiplexed in quadrature phases. Figure 1 shows the signal structure. The carrier L1 is modulated by both C/A code and P code, whereas the carrier L2 is only modulated by P code. In this paper, we will mainly address the problem of anti-jamming for the C/A code, for which the peak power spectral density exceeds that of the P code by about 13 dB [1]. The transmitted GPS signal is also very weak with Jammer-to-Signal Ratio (JSR) often larger than 40 dB and Signal-to-Noise Ratio (SNR) in the range -14 to -20 dB [2, 9]. Due to the high JSR, the FM jammer often has a clear signature in the time-frequency domain as shown in Section 3. As the P code is very weak compared to the C/A code, noise and jammer, we can ignore its presence in our analysis.

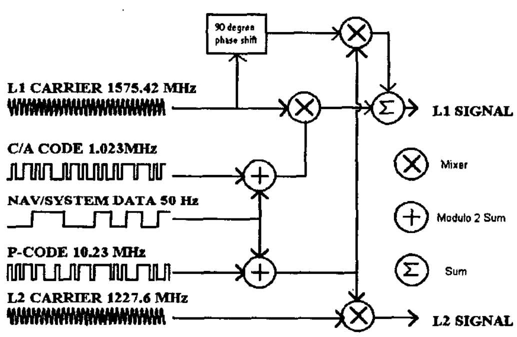

Figure 1: The GPS signal structure.

The BPSK-modulated DSSS signal may be expressed as

$$s(t) = \sum_i I_i \, b_i(t - iT_b) \qquad I_i \in \{-1, 1\} \forall i \tag{1}$$

where I_i represents the binary information sequence and T_b is the bit interval, which is 20ms in the case of GPS system. The i^{th} binary information bit, $b_i(t)$ is further decomposed as a superposition of L spreading codes, $p(n)$, pulse shaped by a unit-energy function, $q(t)$, of duration of τ_c, which is 1/1023 ms in the case of C/A code. Accordingly,

$$b_i(t) = \sum_{n=1}^{L} p(n) \; q(t - n\tau_c) \tag{2}$$

The signal for one data bit at the receiver, after demodulation, and sampling at chip rate, becomes

$$x(n) = p(n) + w(n) + j(n) \qquad 1 \le n \le L \tag{3}$$

where $p(n)$ is the chip sequence, $w(n)$ is the white noise, and $j(n)$ is the interfering signal. The above equation can be written in the vector form

$$\mathbf{x} = \mathbf{p} + \mathbf{w} + \mathbf{j} \tag{4}$$

where

$$\begin{aligned} \mathbf{x} &= [\; x(1) \quad x(2) \quad x(3) \quad \cdots \quad x(L) \;]^T, \\ \mathbf{p} &= [\; p(1) \quad p(2) \quad p(3) \quad \cdots \quad p(L) \;]^T \\ \mathbf{w} &= [\; w(1) \quad w(2) \quad w(3) \quad \cdots \quad w(L) \;]^T, \\ \mathbf{j} &= [\; j(1) \quad j(2) \quad j(3) \quad \cdots \quad j(L) \;]^T \end{aligned}$$

All vectors are of dimension $L \times 1$, and 'T' denotes vector or matrix transposition. It should be noted that the P vector is real, whereas all other vectors in the above equation have complex enteries.

3. PERIODIC SIGNAL PLUS JAMMER IN THE TIME-FREQUENCY DOMAIN

For GPS C/A code, the PN sequence is periodic. The PN code of length 1023 repeats itself 20 times within one symbol of the 50 bps navigation data. Consequently, it is no longer of a continuous spectrum in the frequency domain, but rather of spectral lines. The case is the same for periodic jammers. Figure 2 and Figure 3 show the effect of periodicity of the signal and the jammer on their respective power distribution over time and frequency, using Wigner-Ville distribution. In both figures, a PN sequence of length 32 samples that repeats 8 times is used. A non-periodic chirp jammer of a 50dB JSR (jammer-to-signal ratio) is added in Figure 2. A periodic chirp jammer of 50 dB JSR with the same period as the C/A code is included in Figure 3. We note that the chosen value of 50dB JSR has a practical significance. The spread spectrum systems in a typical GPS C/A code receiver can tolerate a narrowband interference of approximately 40 dB JSR without interference mitigation processing. However, field tests show that jammer strength often exceeds that number due to the weakness of the signal. SNR in both figures are -20dB, which is also close to its practical value [2, 9]. Due to high JSR, the jammer is dominant in both figures. From Figure 3, it is clear that the periodicity of the jammer brings more difficulty to IF estimation than the non-periodic jammers. This problem can be solved by applying a short data window when using Wigner-Ville distribution. Note that the window length should be less than the jammer period. Figure 4 shows the result of applying a window of length 31 to the same data used in Fig. 3. It is evident from the Fig. 4 that the horizontal discrete harmonic lines have disappeared.

4. GPS ANTI-JAMMING USING PROJECTION TECHNIQUES

The concept of subspace projection for instantaneously narrowband jammer suppression is to remove the jammer com-

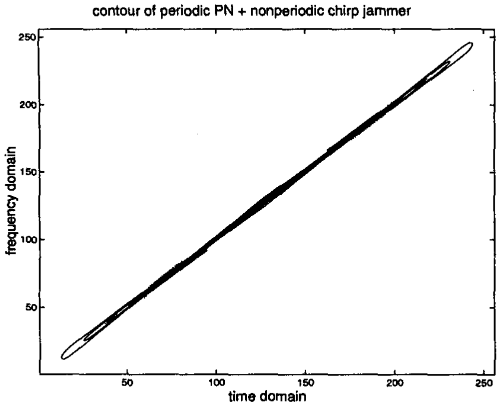

Figure 2: Periodic signal corrupted by a non-periodic jammer in time-frequency domain

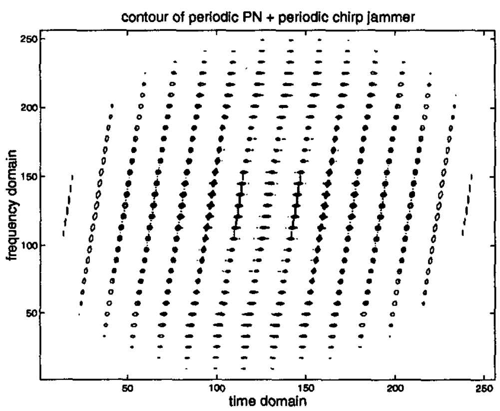

Figure 3: Periodic signal corrupted by a periodic jammer in time-frequency domain

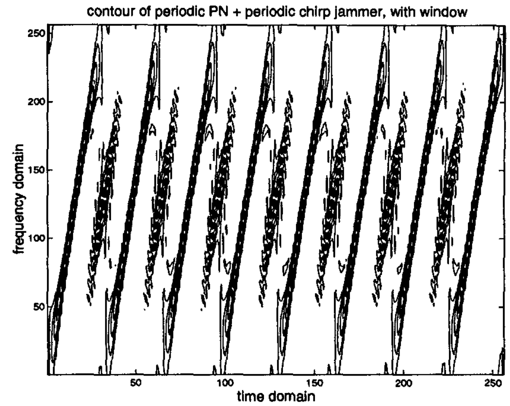

Figure 4: Periodic signal corrupted by a periodic jammer in time-frequency domain (with window)

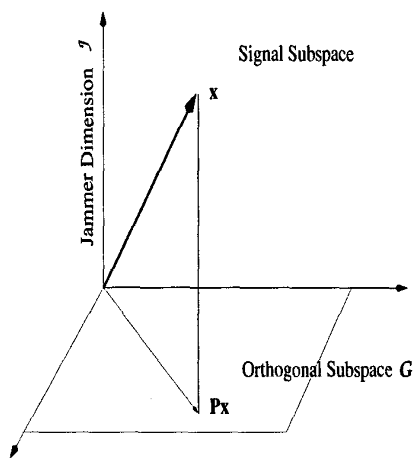

Figure 5: Jammer excision by subspace projection

ponents from the received data by projecting it onto the subspace that is orthogonal to the jammer subspace, as illustrated in Fig. 5.

Once the instantaneous frequency (IF) of the non-stationary jammer is estimated from the time-frequency domain, or by using any other IF estimator [10, 11, 12, 13], the interference signal vector $\mathbf{j}$ in (4) can be constructed, up to ambiguity in phase and possibly in amplitude. In the proposed interference excision approach, the data vector is partitioned into Q blocks, each of length P, i.e. L=PQ. For the GPS C/A code, Q=20, P=1023, and all Q blocks are identical, i.e., the signal PN sequence is periodic. Block-processing provides the flexibility to discard the portions of the data bit, over whih there are significant errors in the IF estimates. The orthogonal projection method makes use of the fact that, in each block, the jammer has a one-dimensional subspace $\mathcal{J}$ in the P-dimensional space $\mathcal{V}$, which is spanned by the received data vector. The interference can be removed from each block by projecting the received data on the corresponding orthogonal subspace $\mathcal{G}$ of the interference subspace $\mathcal{J}$. The subspace $\mathcal{J}$ is estimated using the IF information. The projection matrix for the k^{th} block is given by

$$\mathbf{V}_k = \mathbf{I} - \mathbf{u}_k \ \mathbf{u}_k^H \qquad (5)$$

The vector $\mathbf{u}_k$ is the unit norm basis vector in the direction of the interference vector of the k^{th} block, and 'H' denotes vector or matrix Hermitian. Since the FM jammer signals are uniquely characterized by their IFs, the i^{th} FM jammer in the k^{th} block can be expressed as

$$u_k(i) = \frac{1}{\sqrt{P}} exp[j\phi_k(i)] \qquad (6)$$

The result of the projection over the k^{th} data block is

$$\bar{\mathbf{x}}_k = \mathbf{V}_k \ \mathbf{x}_k \qquad (7)$$

where $\mathbf{x}_k$ is the input data vector. Using the three different components that make up the input vector in (4), the output of the projection filter $\mathbf{V}_k$ can be written as

$$\bar{\mathbf{x}}_k = \mathbf{V}_k \; [\mathbf{p}_k + \mathbf{w}_k + \mathbf{j}_k] \tag{8}$$

The noise is assumed to be complex white Gaussian with zero-mean,

$$E[w(n)] = 0, \;\; E[w(n)^* w(n+l)] = \sigma^2 \delta(l), \forall l \tag{9}$$

Since we assume total interference excision through the projection operation, then

$$\mathbf{V}_k \mathbf{j}_k = \mathbf{0}, \qquad \bar{\mathbf{x}}_k = \mathbf{V}_k \; \mathbf{p}_k + \mathbf{V}_k \; \mathbf{w}_k \tag{10}$$

The decision variable y_r is the real part of y that is obtained by correlating the filter output $\bar{\mathbf{x}}_k$ with the corresponding k^{th} block of the receiver PN sequence and summing the results over the K blocks. That is,

$$y = \sum_{k=0}^{K-1} \bar{\mathbf{x}}_k^H \; \mathbf{p}_k \tag{11}$$

Since the PN code is periodic, we can strip off the subscript k in p_k. The above variable can be written in terms of the constituent signals as

$$y = \sum_{k=0}^{Q-1} \mathbf{p}^T \mathbf{V}_k \mathbf{p} + \sum_{k=0}^{Q-1} \mathbf{w}^H \mathbf{V}_k \mathbf{p} \triangleq y_1 + y_2 \tag{12}$$

where y_1 and y_2 are the contributions of the PN and noise sequences to the decision variable, respectively. In [8], y_1 is considered as a random variable. However, in GPS system, due to the fact that each satellite is assigned a fixed Gold code [1], and that the Gold code is the same for every navigation data symbol, y_1 can no longer be treated as a random variable, but rather a deterministic value. This is a key difference between the GPS system and other spread spectrum systems. The value of y_1 is given by

$$\begin{aligned} y_1 &= \sum_{k=0}^{Q-1} \mathbf{p}^T \mathbf{V}_k \mathbf{p} \\ &= \sum_{k=0}^{Q-1} \mathbf{p}^T (\mathbf{I} - \mathbf{u}_k \mathbf{u}_k^H) \mathbf{p} \\ &= \sum_{k=0}^{Q-1} (\mathbf{p}^T \mathbf{p} - \mathbf{p}^T \mathbf{u}_k \mathbf{u}_k^H \mathbf{p}) \\ &= QP - \sum_{k=0}^{Q-1} (\mathbf{p}^T \mathbf{u}_k \mathbf{u}_k^H \mathbf{p}) \end{aligned} \tag{13}$$

Define

$$\beta_k = \frac{\mathbf{p}^T \mathbf{u}_k}{\sqrt{P}} \tag{14}$$

as the correlation coefficient between the PN sequence vector $\mathbf{p}$ and the jammer vector $\mathbf{u}$. β_k reflects the the compenent of the signal that is in the jammer subspace, and represents the degree of resemblance between the signal sequence and the jammer sequence. Since the signal is a PN sequence, and the jammer is a non-stationary FM signal, the correlation coefficient is typically very small. With the above definition, y_1 can be expressed as

$$y_1 = P(Q - \sum_{k=0}^{Q-1} |\beta_k|^2) \tag{15}$$

From (15), it is clear that y_1 is a real value, which is the result of the fact that the projection matrix V is Hermitian. With the assumptions in (9), y_2 is complex white Gaussian with zero-mean. Therefore,

$$\begin{aligned} \sigma_{y_2}^2 &= E\left[|y_2|^2\right] \\ &= E\left[(\sum_{k=0}^{Q-1} \mathbf{w}^H \mathbf{V}_k \mathbf{p})^H (\sum_{l=0}^{Q-1} \mathbf{w}^H \mathbf{V}_l \mathbf{p}) \right] \\ &= \sum_{k=0}^{Q-1} \sum_{l=0}^{Q-1} \mathbf{p}^T \mathbf{V}_k E\left[\mathbf{w}_k \mathbf{w}_l^H\right] \mathbf{V}_l \mathbf{p} \\ &= \sum_{k=0}^{Q-1} \mathbf{p}^T \mathbf{V}_k E\left[\mathbf{w}_k \mathbf{w}_k^H\right] \mathbf{V}_k \mathbf{p} \\ &= \sigma^2 \sum_{k=0}^{Q-1} \mathbf{p}^T \mathbf{V}_k \mathbf{V}_k \mathbf{p} \\ &= \sigma^2 \sum_{k=0}^{Q-1} \mathbf{p}^T \mathbf{V}_k \mathbf{p} = \sigma^2 y_1 \end{aligned} \tag{16}$$

the above equations make use of the noise assumptions in (9) and the properties of the projection matrix. The decision variable y_r is the real part of y. Consequently, y_r is given by

$$y_r = y_1 + Re\{y_2\} \tag{17}$$

where $Re\{y_2\}$ denotes the real part of y_2. $Re\{y_2\}$ is real white Gaussian with zero-mean and variance $\frac{1}{2}\sigma_{y_2}^2$. Therefore, the SINR is

$$\begin{aligned} SINR &= \frac{y_1^2}{var\{Re\{y_2\}\}} \\ &= \frac{y_1^2}{\frac{1}{2}\sigma_{y_2}^2} = \frac{2y_1}{\sigma^2} \\ &= \frac{2P(Q - \sum_{k=0}^{Q-1} |\beta_k|^2)}{\sigma^2} \end{aligned} \tag{18}$$

In the absence of jammers, no excision is necessary, and the SINR(SNR) of the receiver output will become $2PQ/\sigma^2$, which represents the upper bound for the anti-jamming performance. Clearly, $\frac{2P\sum_{k=0}^{Q-1} |\beta_k|^2}{\sigma^2}$ is the reduction in the receiver performance caused by the proposed jammer suppression techniques. It reflects the energy of the power of the signal component that is in the jammer subspace. If the jammer and spread spectrum signals are orthogonal, i.e., their correlation coefficient $|\beta| = 0$, then interference suppression is achieved with no loss in performance. However, as stated above, in the general case, β_k is often very small, so the projection technique can excise FM jammers

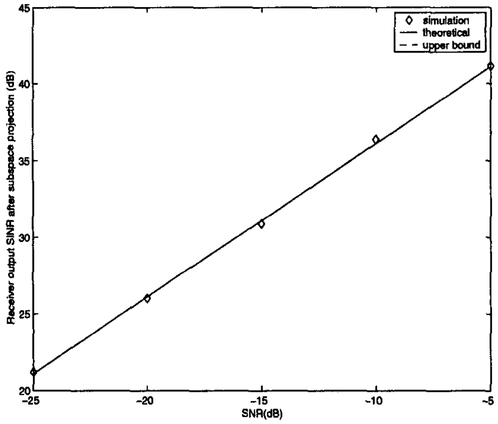

Figure 6: Receiver SINR vs SNR.

effectively with only very insignificant signal loss. The lower bound of SINR is zero and corresponds to $|\beta| = 1$. This case requires the jammer to assume the C/A code, i.e., identical and synchronous with actual one. Figure 6 depicts the theoretical SINR in (18), its upper bound, and estimated values using computer simulation. The SNR assumes five different values [-25, -20, -15, -10,-5] dB. In this figure, the signal is the Gold code of satellite SV#1, and the jammer is a periodic chirp FM signal with frequency 0–0.5 and has the same period as the C/A code. For this case, the correlation coefficiant β is very small, $|\beta| = 0.0387$. JSR used in the computer simulation is set to 50dB. Due to the large computation involved, we have used 1000 realizations for each SNR value. Figure 6 demonstrates that the theoretical value of SINRs is almost the same as the upper bound and both are very close to the simulation result. In the simulation as well as in the derivation of equation (18), we have assumed exact knowledge of the jammer IF. Inaccuracies in the IF estimation will have an effect on the receiver performance [8].

5. CONCLUSIONS

GPS receivers are vulnerable to strong interferences. In this paper, subspace projection techniques are adapted for the anti-FM jamming GPS receiver. These techniques are based on IF estimation of the jammer signal, which can be easily achieved, providing that the C/A code and the jammer have distinct time-frequency signatures. The IF information is used to construct the FM interference subspace which, because of signal nonstationarities, is otherwise difficult to obtain. Due to the characteristic of the GPS spread spectrum signal structure and the fact that the C/A codes are fixed for the different satellites and known to all, the analysis of the receiver SINR becomes different from common spread spectrum systems. The theoretical and simulation results suggest that the subspace projection techniques can effectively excise FM jammers for GPS receivers with insignificant loss in the spreading gain.

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