# Narrowband Interference Suppression Using Filter-Bank Analysis/Synthesis Techniques

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#### **Abstract**

In this paper, we consider the application of multirate digital filter banks to the suppression of narrowband interference in Direct Sequence Spread Spectrum systems. Analysis/synthesis methods have received a substantial amount of attention in the multirate and speech processing community where a rich theory exists on the design of these banks. In particular, conditions exists for perfect reconstruction of the received signal and for the elimination of time and frequency aliasing. In this paper, we apply these principles to the interference suppression problem. As will be seen, the traditional windowed and nonwindowed transform domain processing techniques are reduced cases of these more general structures. In addition to providing a general framework from which to study and design transform domain interference suppression systems, these banks are a particular time-frequency distribution making them applicable to a larger class of jamming scenarios. Here, we exploit the bank's perfect reconstruction property to mitigate on/off type jammers. Also, an understanding of the perfect reconstruction concept enables deeper insight into the relative performance of the two traditional techniques. Namely, windowed processing, although providing improved spectral containment, exhibits poor approximation to the theoretical conditions for perfect reconstruction. Whereas, nonwindowed processing satisfies the reconstruction condition but suffers heavily from spectral leakage. In contrast, quite practical filter banks provide near perfect reconstruction while, due to superior filtering, effectively mitigate the spectral leakage problem with near complete removal of the interference. In support, we provide in the paper bit error performance results which demonstrate the superiority of the filter banks for a number of scenarios. Finally, the mathematical-equivalent, computationally-efficient processing structure known as polyphase filter networks are put forth and shown to possess only a slight complexity increase from windowed transform processing. Thus, a real-time implementation of the superior performing suppression technique is seen to be quite feasible at today's technology.

#### I. Introduction

The performance of a Direct Sequence Spread Spectrum communication system against narrowband jamming can be improved above and beyond the system's spectrum spreading gain by employing interference suppression techniques. Among the many techniques which have been proposed in recent years includes the general method of transform domain processing [1]-[3][7]. This technique is premised upon the jammer's high spectral concentration about a narrow frequency range while the spread-spectrum signal and the ambient noise are wideband having a lower spectral density level within the jammer's bandwidth. In principle, transforming the received signal into the frequency domain enables straightforward identification and removal of the jammer's spectra by what has customarily been referred to as excision. In practice, however, the transformation is performed over a finite time interval resulting in the well known spectral leakage problem associated with the traditional spectral estimation method[4]. This has lead to the wide-spread adoption of windowed transform processing where it is believed that a window possessing better spectral characteristics can contain the leakage problem[5]. Unfortunately, windowing distorts the signal and, as a result, has not been shown to be the uniformly superior approach for all jamming scenarios[6]-[9].

In this paper, we propose a general framework for transform domain interference suppression based on multirate digital filter banks[10]. These signal processing structures consist of a bank of analysis digital bandpass filters, a spectral modification function, and a bank of synthesis digital bandpass filters. Analysis/synthesis methods have received a substantial amount of attention in the multirate and speech processing community where they have been used for spectrogram estimation[11] and subband coding of speech signals[12]. In this paper, we apply the filter bank principles to the interference suppression problem. As will be seen, both windowed and nonwindowed FFT processing are special cases of these more general structures. In addition to providing a general framework from which to study and design transform domain interference suppression systems, these banks are a particular time-frequency signal representation [13] making them applicable to a much broader class of jamming scenarios. Here, we exploit the perfect reconstruction capability associated with these banks to mitigate on/off-type jamming sources. Specifically, this property enables signal distortion to be avoided when the interference is not present and excision not performed. On the other hand, it will be seen that typical window functions exhibit a poor approximation to the theoretical conditions for perfect reconstruction causing this technique to experience the unnecessary signal distortion. When jamming is present or time-invariant, the filter bank approach has the particular advantage of concentrating the jammer's power to a minimum number of frequency bins. That is, due to superior filtering, spectral leakage is all but eliminated with quite practical filter designs in the bank. The significance of this capability has implication to both the jammer and the desired signal. Namely, frequency bins containing jammer power are more effectively identified and removed. This contrasts the FFT processing approach where a considerable amount of jammer power remains on sidelobes which do not exceed the excision threshold. With regard to the desired signal, the ability to completely remove the jammer in a minimum number of frequency bins minimizes signal distortion. With FFT processing at high J/S, many more bins corresponding to sidelobes are removed which increases the signal distortion. Provided in the paper are bit error rate performance results which quantify these conclusions and clearly demonstrate for a variety of jamming scenarios the superiority of the filter bank method to nonwindowed and windowed FFT interference suppression.

Finally, a direct mechanization of the analysis/synthesis filter bank structure would be a questionable real-time implementation with current technology. But, a computationally efficient structure known as polyphase filter networks exist which are mathematically equivalent to the direct filter bank structure[10]. The polyphase network equivalent is a time-variant digital structure which takes advantage of the multirate processing, performed within the filter bank, to reduce complexity and multiplier count. A complexity comparison is presented which indicates the efficient filter bank mechanization is only slightly more complex than windowed FFT processing. Thus, a real-time implementation is feasible which can

achieve the superior performance promised by the filter bank method to interference suppression.

#### II. Filter Bank Interference Suppression

Filter banks are multirate signal processing structures which spectrally decompose a digital signal into a bank of frequency channels. The term multirate arises from the fact that multiple sampling rates, due to decimation and interpolation operations, occur throughout the processing. The component signals are modified, depending upon the application, and then recombined producing an output signal with some desired property. It is clear that the banks are directly extensible to the transform domain interference suppression problem where the desired property is the accurate identification and complete removal of narrowband interference. In this paper, we consider uniform banks where the center frequencies of the M channels are equally spaced over the sampling bandwidth [10] (to the extent possible, nomenclature consistency will be maintained with [10]). Additionally, the channel bandwidths are equal having passbands which may or may not be designed to overlap. In an ideal case, the channels will be seen to be contiguous with zonal passbands.

Figure 1 functionally illustrates the receiver processing of a direct sequence spread spectrum communication system using filters banks to suppress interference. The received RF signal is downconverted, low-pass filtered and sampled at some multiple of the Nyquist rate. Carrier and timing synchronization are assumed. The complex valued baseband signal, x(n), is mathematically given by

$$x(n) = s(n) + j(n) + b(n)$$
(1)

where:

$$s(n) = \sqrt{S} \sum_{k=-\infty}^{\infty} c_k \sum_{j=-\infty}^{\infty} d_j \int_{-\infty}^{\infty} p_C(\tau - kT_C) p_d(\tau - jT_d) a(nT_S - \tau) d\tau \tag{2}$$

S is the signal power

 $c_k$  ,  $d_j$  are the chip and data sequences, respectively  $T_s$  ,  $T_c$  ,  $T_d$  are the sampling, chip, and data times, respectively

pc (t) is the chip pulse shape

pd (t) is data pulse shape

a(t) is the impulse response of the antialiasing filter

j(n) are samples of the narrowband interference having power J and b(n) are zero mean complex Gaussian random variables with

variance  $\sigma_n^2 = \frac{N_0}{T_s}$  where  $N_0$  is the single-sided power spectral density of the channel noise. Following downconversion, the digital baseband signal is then subjected to the filter bank for identification and removal of the interfering signal. Finally, the resulting signal,  $\hat{x}(n)$ , out of the bank is processed by the data recovery function where the PN code is removed and data decisions declared.

Within the filter bank structure, we identify three fundamental elements. First is the parallel analysis filters which generate the spectral decomposition. Each channel component within x(n) is translated to zero frequency via a bank of complex exponentials. The translated signals are low pass filtered by h(n). This analysis filter is typically designed for good passband and stopband characteristics. The uniform filter bank property dictates the low pass filtering be the same for each channel. Noting the bandwidth of each channel signal is much less than the sampling bandwidth allows us to reduce the sampling rate of the filter outputs. Since the decimation factor, M, is equal to the number of channels, the filter bank is said to be critically sampled or maximally decimated. Once the spectral decomposition is accomplished, the modification segment operates on the individual spectral components. In subband coding, the components are encoded for transmission. For narrowband interference suppression in wideband systems, the spectral modification becomes excision. We note that the general framework enables alternative modification functions, e.g., nonparametric median filtering techniques[14]. Using the modified signal components, the signal is reconstructed or synthesized by interpolating the individual components and modulating them to the appropriate spectral location. The interpolation is achieved with a common lowpass filter, f(n), termed the synthesis filter. The principal objective of this filter is the removal of aliasing and distortion introduced by the analysis filter such that an identity system is approximated.

Referring to Figure 1, if the analysis lowpass filter and decimator are replaced by a digital integrate and dump (sample sum), then the analysis segment is equivalent to a Discrete Fourier Transform (DFT). Similarly, the synthesis segment can be reduced to an inverse DFT. If the analysis filter is replaced by a time-variant scalar, then an equivalence is made to windowed transforms. Thus, the prevalent digital techniques for transform domain interference suppression are seen as special cases of the more general filter bank. Using the filter bank though can provide, as will be seen, much greater flexibility in terms of controllability of spectral leakage and in the mitigation of more diverse jamming scenarios. This in turn provides greater freedom in the design and optimization of the spectral modification function.

Of key theoretical interest with filter banks is the condition under which the output signal of the bank is a perfect replica of the input when no spectral modification is performed, i.e., conditions for

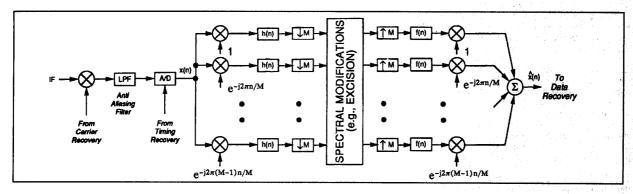


Figure 1. Interference Suppression using Analysis/Synthesis Filter Banks: Direct Mechanization

perfection reconstruction. This capability is of particular interest to the interference suppression problem since the interference may not always be present, as might be the case with an on/off or popup jammer. Clearly, an effective suppression technique should mitigate degraded performance attributable to jamming but should also not cause artifactual signal distortion when the jamming source is not radiating. This situation is not always addressed in the development of suppression techniques. Referring back to Figure 1 and ignoring spectral modification, it can be shown that the output signal is expressible as [10]

$$\hat{x}(n) = \sum_{s = -\infty}^{\infty} x(n - s \ M) \sum_{m = -\infty}^{\infty} f(n - m \ M) h(m \ M - n + s \ M)$$
(3)

From (3), it is clear that perfect reconstruction is satisfied when

$$\sum_{m=-\infty}^{\infty} f(n-m \ M)h(m \ M-n+s \ M) = \delta(s)$$
(4)

where  $\delta(s)$  is the Kronecker delta function. For nonwindowed FFT processing, the relation in (4) is satisfied as expected. On the other hand, for windowed processing the right-hand side of (4) becomes  $w(n)\delta(s)$  where w(n) is the periodic extension of the window function. Inserting into (3) yields

$$\hat{x}(n) = w(n)x(n) \tag{5}$$

Equation (5) demonstrates the distortion introduced when windowing. In the frequency domain, sufficient conditions for perfect reconstruction are satisfied by

$$H(e^{j\omega}) = \begin{cases} 1, 0 \le |\omega| \le \frac{\pi}{M} \\ 0, otherwise \end{cases}$$
 (6a)

and

$$F(e^{j\omega}) = \begin{cases} 1, 0 \le |\omega| \le \frac{\pi}{M} \\ 0, otherwise \end{cases}$$
 (6b)

In the following section, we will approximate these responses to evaluate and compare the performance capability of multirate filter banks as they apply to the interference suppression problem.

## **III. Performance Results**

To evaluate and quantify the performance of the filter bank interference suppression technique, a system simulation was developed on a commercially available signal processing simulation package. The simulation was performed at baseband with a normalized symbol rate of 1 Hz. A minimum shift keyed (MSK) DSPN waveform was selected for bandwidth containment. The spreading code used was a 1023 maximum length code and the number of chips per bit, N, was set to 8. The sampling rate within the system was 48 Hz. The number of channels, M, used in the filter bank was 16.

Consequently, the center frequencies of the filter bank are  $0, \pm 3, \pm 6$ , etc. The analysis and synthesis filters were designed to closely approximate the conditions in (6) and were 80 taps in length. The actual simulation used the computationally efficient implementation to be described in section IV. In addition to uncorrelated Gaussian noise, two jamming sources were considered -1) a complex tone and 2) a narrow bandpass Gaussian source generated by Butterworth lowpass filtering a random noise source and then frequency translating to a prescribed spectral location.

The first set of results demonstrate the reconstruction capability of the filter bank when a jamming source is not radiating and excision is not performed. Figure 2 illustrates the results. The theoretical curve

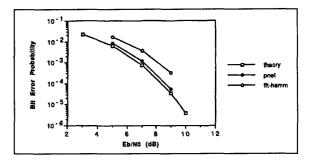


Figure 2. Bit Error Probability versus Eb/N0, No Jammer Power, No Excision

represents ideal performance. In addition, this curve corresponds to nonwindowed, or equivalently, rectangular windowed FFT processing as well as the ideal filter bank using the conditions in (6). The filter bank with actual filter designs is denoted 'pnet' which refers to the efficient polyphase implementation in section IV. The remaining curve corresponds to FFT processing with a Hamming window. As can be seen from the curves, the filter bank with realizable filters closely approximates theoretically performance with a loss of only about 0.2 dB while windowed processing incurs nearly a 1.3-dB loss.

Figure 3 illustrates performance of the suppression techniques when J/S is varied. In this simulation, the jamming source was a complex tone at 1.5 Hz (i.e., halfway between two frequency bins) and the Eb/NO was set at 9 dB. Also indicated in the figure are the performance levels when excision was turned on, but the jammer was not radiating. This represents signal distortion from both excision and inaccurate reconstruction and is indicated by a right-hand hash mark and a letter denoting the technique. A comparison with this level indicates the degree to which the jammer has been removed. Excision was fixed for all techniques with the 0, ±3 and 6 Hz frequency bins excised when using the two FFT processing techniques and only the 0 and 3 Hz bins excised when using the filter bank. A number of key conclusions can be drawn from these results. First, the filter bank is extremely effective at completely removing the jammer over a wide range of J/S with a minimum number of frequency bins excised (recall the processing gain was only a nominal 9 dB). Thus, not only is the suppression complete but the incurred distortion from excision minimized. Further, this is essentially the only signal distortion since the reconstruction property is approximately satisfied. On the other hand, for mild levels of jamming both FFT processing techniques are ineffective. Additionally, the signal distortion with these two techniques is greater than the filter bank technique since twice as many bins were excised yet the jammer was still not effectively

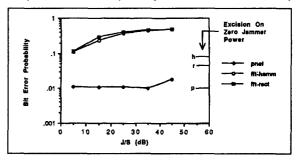


Figure 3. Bit Error Probability versus J/S, Tone Jammer Frequency = 1.5 Hz

removed. In this regard, the filter bank framework enables a clearer understanding of the relative performance between the two FFT techniques. Namely, windowing provides better spectral localization with respect to nonwindowed processing but fails at satisfying the reconstruction property, thus incurring greater signal distortion. Whereas for nonwindowed processing, signal distortion is solely due to excision since reconstruction is perfect, but, now spectral leakage becomes the dominate degradation. It is interesting to note, for this particular jamming scenario, the composite degradation is comparable

The spectral leakage problem is even better illustrated in the final two performance plots. Figure 4 illustrates performance curves when the J/S is fixed at 25 dB and the frequency of the tone was varied from zero to 3 Hz, i.e., from the center frequency of one bin to the center frequency of an adjacent bin. As expected, the FFT processing techniques suffer heavily from spectral leakage when the tone is shifted off the bin center frequencies. On the other hand, the superior filtering performed within the bank provides complete removal of the jammer independent of the tone's frequency. In Figure 5, we consider a bandpass jammer where the J/S has again been fixed at 25 dB, but the bandpass bandwidth has been allowed to vary. The center frequency of the jammer is 1.5 Hz. These curves indicate again the ability of the filter bank to contain the jammer's power such that effective excision is accomplished. Due to practical filters and the excision of only two bins, performance begins to degrade as the jammer's bandwidth expands about more than two bins. With FFT processing neither technique is effective against the jammer although performance is improved with windowing due to the better spectral containment.

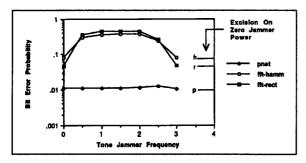


Figure 4. Bit Error Probability versus Tone Jammer Frequency, J/S = 25 dB

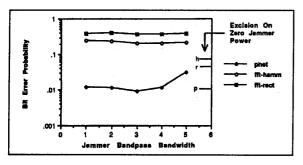


Figure 5. Bit Error Probability versus Jammer Bandpass Bandwidth, J/S = 25 dB, Jammer Center Frequency = 1.5 Hz

## IV. Polyphase Implementation

A direct realization of the filter bank in Figure 1 is a computationally intense structure due to the redundant processing performed in each channel. In addition, for good filter characteristics, the order of the lowpass filter can become quite large. Making comparisons to the traditional windowed and nonwindowed transform techniques, leads one to question the practical feasibility of this realization. Fortunately, a mathematical equivalence can be made between the processing of Figure 1 and a structure with considerably reduced complexity. Consider the kth channel in Figure 1. Using the commutative property of linear time-variant digital networks [10], the output of the decimator is given by

$$X_k(m) = \sum_{n=-\infty}^{\infty} h(m \ M-n) \exp\left(j\frac{2\pi \ k(m \ M-n)}{M}\right) x(n)$$
 (7)

Simplifying (7) yields

$$X_{k}(m) = \sum_{\varrho=0}^{M-1} \exp\left(-j\frac{2 \pi k \varrho}{M}\right) \sum_{r=-\infty}^{\infty} \overline{p_{\varrho}}(r) x_{\varrho}(m-r)$$
 (8)

where: 
$$\overline{p_{\varrho}}(m) = h(m \ M - \varrho)$$
 (9)

 $\overline{P_{\theta}}$  (m) is termed a polyphase filter and is seen to be a decimated sequence from the lowpass filter, h(n). A parallel set of these filters is referred to as a polyphase network. Inspection of (8) reveals that the channel component is the DFT of the decimated input convolved with the corresponding polyphase filter where the index of the DFT operates across the network for each decimated time instance. Thus, the analysis segment can be equivalently generated by the processing in Figure 6.

In a similar manner, an equivalence can be made with regard to the synthesis segment. Namely for the pth component, the synthesis polyphase filter is given by

$$q_{\varrho}(m) = f(m \ M + \varrho) \tag{10}$$

Again applying the commutative property for rate expansion [10] and simplifying, the output of the filter bank becomes

$$\hat{x}(n(r,\varrho)) = \sum_{m=-\infty}^{\infty} q_{\varrho}(r-m) \sum_{k=0}^{M-1} \hat{X}_{k}(m) \exp\left(j\frac{2\pi k \varrho}{M}\right)$$
 (11)

where: 
$$n(r, \varrho) = M r + \varrho$$
 (12)

$$X_k$$
 is a spectral modification of (8)

The index notation of (12) implies that at each decimated sample time r, M samples are produced at the output via the commutator. From (11) and (12), we see that the synthesis segment also has equivalent processing.

With equivalence made, complexity reduction from Figure 1 to the polyphase implementation manifests itself as follows. First, all the filtering performed by the polyphase filters are operating at the decimated rate, greatly influencing throughput performance in the signal processing hardware. Secondly, the filters themselves are decimated sequences of the original lowpass filter. For an M.L. length lowpass filter, the M polyphase filters have length L. Therefore, we effectively require a single filter achieving an M-fold reduction in memory and arithmetic processing.

To see if the polyphase implementation is a viable technical solution to the interference suppression problem, even with its advantages over the direct filter bank realization, we must make a quantified comparison with windowed transform processing. Figure 6 exhibits windowed FFT processing when the analysis polyphase filters become a constant multiplier which varies across the bank and the

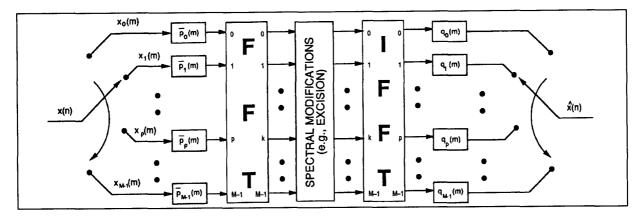


Figure 6. Polyphase Equivalent Filter Bank

synthesis filters become unity gain constants. Thus, for a complex input and ignoring the spectral modification function, the number of real multiplies per second is given by

$$K_{w-fft} = (2 M + 8 M \log M) R_S/M$$
 (13)

where: R<sub>S</sub> is the sampling rate

With the polyphase implementation, the number of multiplies per second is given by

$$K_{pnet} = (4 M L + 8 M log M) R_S/M$$
 (14)

From (14), the complexity increase arises from the more general filtering of the polyphase filter which, as shown in Section III, enables superior performance. Taking the ratio of (14) to (13), the relative performance of the two techniques can be quantified. Specifically,

$$\Gamma(L,M) = \frac{2L+4 \log M}{1+4 \log M} \tag{15}$$

The following table evaluates (15) for nominal values of the number of channels, M, and the length of the polyphase filters, L.

Table I

M	3	5	7
128	1.17	1.31	1.45
256	1.15	1.27	1.39
512	1.14	1.24	1.35
1024	1.12	1.22	1.32

Table I indicates, as expected, the fundamental tradeoff between complexity and performance. That is, better performance is achieved with better yet more complex polyphase filters which, with FIR filters, implies longer filters. Yet, from Section III, very simple designs achieved excellent performance (the polyphase filters used in the actual simulations had length 5).

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