CSPC15 MIDTERMS

Quantifiers

- A predicate becomes a proposition when we assign it with fixed values. However, another way to make a predicate into a proposition is to quantify it. That is, the predicate is true (or false) for all possible values in the universe of discourse or for some value(s) in the universe of discourse.

Universal Quantifiers

- The universal quantification of a predicate P(x) is the proposition "P(x) is true for all values of x in the universe of discourse" We use the notation

$\forall xP(x)$ - which can be read "for all x"

- If the universe of discourse is finite, say {n1, n2, . . . , nk}, then the universal quantifier is simply the conjunction of all elements:

$\forall\ xP(x)$ () $P(n_1)$ ^ $P(n2)$ ^ · · · ^ $P(n_k)$

Example

"Man is mortal" can be transformed into the propositional form $\forall xP(x)$ where $P(x)$ is the predicate which denotes x is mortal and the universe of discourse is all men.

Existential Quantifiers

- Existential quantifier states that the statements within its scope are true
for some values of the specific variable. It is denoted by the symbol $\exists$

$\exists xP(x)$ is read as for some values of x, $P(x)$ is true.

Example

"Some people are dishonest" can be transformed into the propositional form $\exists xP(x)$ where $P(x)$ is the predicate which denotes x is dishonest and the universe of discourse is some people.

- The existential quantification of a predicate P(x) is the proposition "There exists an x in the universe of discourse such that P(x) is true."We use the notation

$\exists xP(x)$ which can be read "there exists an x"

- Again, if the universe of discourse is finite, {n1, n2, . . . , nk}, then the existential quantifier is simply the disjunction of all elements:

$\exists\ xP(x)$ () P(n1) _ P(n2) _ · · · _ P(nk)

Quantifiers: Truth Values

In general, when are quantified statements true/false?

Statement	True When	False When
$\forall xP(x)$	P(x) is true for every x.	There is an x for which P(x) is false.
$\exists xP(x)$	There is an x for which P(x) is true.	P(x) is false for every x.

Mixing Quantifiers

- Existential and universal quantifiers can be used together to quantify a predicate statement; for example,

$\forall x \exists yP(x, y)$

is perfectly valid. However, you must be careful—it must be read left to right.
For example, $\forall x \exists y\ P(x, y)$ is not equivalent to $\exists x \forall y\ P(x, y)$. Thus, ordering is important.

Example:

- $\forall x\ \exists y$ Loves(x, y) : everybody loves somebody
- $\exists x \forall y$ Loves(x, y) : There is someone loved by everyone

Mixing Quantifiers: Truth Values

Statement	True When	False When
$\forall x \forall y\ P(x, y)$	P(x, y) is true for every pair x, y.	There is at least one pair, x, y for which P(x, y) is false.
$\forall x \exists y\ P(x, y)$	For every x, there is a y for which P(x, y) is true.	There is an x for which P(x, y) is false for every y.
$\exists x \forall y\ P(x, y)$	There is an x for which P(x, y) is true for every y.	For every x, there is a y for which P(x, y) is false.
$\exists x \exists y\ P(x, y)$	There is at least one pair x, y for which P(x, y) is true.	P(x, y) is false for every pair x, y.

Binding Variables

- When a quantifier is used on a variable x, we say that x is bound. If no quantifier is used on a variable in a predicate statement, it is called free.

Example:

- In the expression $\exists\ x \forall\ yP(x, y)$ both x and y are <u>bound</u>.
- In the expression $\forall xP(x, y)$, x is bound, but y is <u>free</u>.

- A statement is called a well-formed formula, when all variables are properly quantified.

- The set of all variables bound by a common quantifier is the scope of that quantifier.

Example:

In the expression $\exists x,y \forall zP(x, y, z, c)$ the scope of the existential quantifier is $\{x, y\}$, the scope of the universal quantifier is just z and c has no scope since it is free.

Sets

Set

- an unordered collection of different elements.
- listed using brackets.
- if the order of the elements is changed or any element of a set is repeated, it doesn't make any changes in the set.

Examples:
- set of all positive integers
- set of all the planets in the solar system
- set of all values

Representation of a Set
- sets can be represented in two ways.

Roster or Tabular Form
- The set is represented by listing all the elements comprising it.
- The elements are enclosed in curly braces and are separated by commas.

Example:
*set of vowels can be written as
A = {a,e,i,o,u}

*set of odd numbers less than 10 can be written as
B = {1,3,5,7,9}

Set Builder Notation
- The set is defined by specifying a property that elements of the set have in common. The set is described as A = {x: P(x)}

Example:
*set A = {a,e,i,o,u} can be written as
A = {x: x is a vowel in the English Alphabet}

*set B = {1,3,5,7,9} can be written as
B = {x: 1<x<10 and (x%2) ≠0}

NOTE
- If an element x is a member of any set S, it is denoted by x Є S and if an element Y is not a member of any set S, it is denoted by y Є S.

Example:
If S={1,1.2,1.7,2}
1 Є S but 1.5 Є S
* 2 Є S?

Some Important Sets
1. **N** – set of all natural numbers
{1,2,3,4…}
2. **Z** – set of all integers
{…-3,-2,-1,0,1,2,3,…}
3. **Z+** – set of all positive integers
{1,2,3,4…}
4. **Q** – set of all rational numbers (notation: p/q where p and q are integers provided that q ≠0)
{natural nos., integers, fractions, decimals}
5. **R** – set of all real numbers
{rational nos, irrational nos (square root, pi, etc.)}
6. **W** – set of all whole numbers
{0,1,2,3,4,…}

Cardinality of a Set
- denoted by |S|
- is the number of elements of the set. The number is also referred to as cardinal number.
- if a set has an infinite no. of elements, its cardinality is ∞.

Example:
*|{1,2,3,4,5}| = 4
* |{1,2,3,4,5…}| = ∞

NOTE:
- If there are two sets X and Y
*|X| = |Y| - It denotes two sets X and Y having same cardinality. It occurs when the no. of elements in X is exactly equal to the no. of elements in Y.
- In this case there exists a BIJECTIVE FUNCTION 'f' from X to Y.

*|X| ≤ |Y| - It denotes that set X's cardinality is less than or equal to set Y's cardinality.
- It occurs when no. of elements in X is less than or equal to the no. of elements in Y.
- in this case there exists an INJECTIVE FUNCTION 'f' from X to Y.

*|X| < |Y| - It denotes that set X's cardinality is less than to set Y's cardinality.
- It occurs when no. of elements in X is less than to the no. of elements in Y.
- in this case there exists an INJECTIVE FUNCTION 'f' from X to Y but not BIJECTIVE

*If |X| ≤ |Y| and |X|≥ |Y| then |X|=|Y|
The sets X and Y are commonly equivalent sets.

Types of Sets
Finite Set
- Contains a definite number of elements

Example:
* S = {x|x Є N and 60 > x >50}

Infinite Set
- Contains infinite no. of elements.

Example:
* S = {x|x Є N and x >10}

Subset
- Set X is a subset of Y can be written as X ⊆ Y if every element of X is an element of set Y

Example:
1. Let X= {1,2,3,4,5,6} and Y = {1,2} Here set Y is a subset of set X as all the elements of set Y is in set X. Hence we can write Y ⊆ X
* X ⊆ Y?
2. Let X= {1,2,3} and Y = {1,2,3} Here set Y is a subset (not a proper subset) of set X as all the elements of set Y is in set X. Hence we can write Y ⊆ X
* X ⊆ Y ?

Proper Subset
- defined as "subset of but not equal to"
- set X is a proper subset of set Y, written as X ⊂ Y if every element of X is an element of Set Y and |X|=|Y|

Example:
*X = {1,2,3,4,5,6} and Y= {1,2} Here Y ⊂ X since all elements in Y are contained in X too and X has at least one element and is more than set Y.
*|Y| ⊂ |X| = ?

Universal Set
- It is a collection of all elements in particular context or application. All the sets in that context or application are essentially subsets of this universal set.

- represented by U.
Example:
We may define U as the set of all animals on earth. In this case all mammals are a subset of U, set of all fishes are also a subset of U, etc.

Empty Set or Null Set
- contains no elements.
- denoted by ∅
- as the no. of elements in an empty set is finite, empty set is a finite set.
- the cardinality of empty set is zero (0)

Example:
S = {x|x Є N and 7 < x <8}
cardinality: 0
tabular form: ________?

Singleton Set or Unit Set
- contains only ONE element.
- denoted by {S}

Example: S={x|x Є N, 7 < x < 9}
S = {8}

Equal Set
- if two sets contain the same element.

Example:
If A = {1,2,6} and B = {6,2,1}, they are equal as every element of set A is also an element in set B and vice versa

Equivalent Set
- If the cardinalities of two sets are the same

Example:
If A = {1,2,6} and B = {16,17,22} they are equivalent as the cardinality of Set A is equal to the cardinality in set B.
|A|=|B| □ 3 = 3

Overlapping Set
- If two sets have at least one common element

Example:
Let A = {1,2,6} and B = {6,12,42} there is a common element '6', hence these sets are overlapping.

Disjoint Set
- Two sets, A and B are called disjoint sets if they do not have one element in common

Example:
Let A = {1,2,6} and B = {7,9,14} there is no element in common, hence these are disjoint.

Set Operators

Union (∪)
- combination of the elements of the given sets

Intersection (∩)
- same elements within the given sets

Complement (A', A^C)
- value in the universal set that are not in the given set

Difference of Relative Complement (A-B , A/B)
- elements that are left when you subtract set A in set B

Symmetric Difference (Δ , ⊖)
- elements that are in set A and set B but not both

Good luck, CS 1-2!