2016 International Conference on Computing, Analytics and Security Trends (CAST)
College of Engineering Pune, India. Dec 19-21, 2016

Resampling schemes for Rao-Blackwellization Particle Filters

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Abstract— **Particle filter is a general numerical algorithm to estimate the posterior probability density function. While the execution of particle filter is very simple, but its principle drawback is that, it is very computational expensive by increasing rapidly with the increasing in the state measurement. One solution for this issue is to minimize the states showing up directly in the flow. The primary commitment in this paper is to determine the subtle elements for the Rao-Blackwellization (marginalize) Particle Filter (RBPF) for a general nonlinear and non-gaussian state-space model. In this article a comparison is made between commonly encountered resampling algorithms for particle filters and Rao-Blackwellization (marginalize) Particle Filter (RBPF). This facilitates a similarity of the algorithms with respect to their resampling quality. Using wide-ranging Monte Carlo simulations the theoretical outcomes are verified.**

Keywords- State estimation, Particle filter, Monte Carlo simulations , RBPF, Marginalization.

I. INTRODUCTION

Particle filtering (PF) is a strategy for sequential or statistical signal processing. Since from two decades with the distribution on PF [1], it has turned out to be extremely prominent due to its capacity to process perceptions spoke to by nonlinear state-space models where the observations of the model can be either non-Gaussian or Gaussian. In applications, the observation and evolution models may be cumbersome or impossible to linearization. For this type of problems we may try to use Monte Carlo methods to simulate the distributions by random samples. Such methods are called as particle filters.

In the Bayesian environment, each the data and the unidentified are model as random variables and the cause is to locate the posterior probability density function for the unidentified variable, conditioned on the look at of the information random variable. Efficiently, the parametric model which describe the spatio-temporal habits of the bio-magnetic subject is non-linear by means of the position of the point resources but linear with differentiate to their amplitude. On this case a rather easy assessment of variance show that a particle filter with set of rules compute the posterior probability density characteristic related to the bring role is alternatively greater correct than the particle clear out computing the possibility density function related to the complete current density[13].

Based on the foundation of this effect, we exercise a computational procedure, Rao- Blackwellization [3], the place a particle filter out approximates the possibility density operate related to the deliver function at the same time as suggest and variance of the supply amplitude are optimally determined through the use of set of Kalman filters.

There are a number of PF methods, and close to them all are mounted on three operations: 1) particle propagation, 2) weight computation, and three) resampling. Particle propagation and weight computation amount to the generation of particles and venture of weights [14]. Whereas resampling replaces one set of debris and their weights with a further set. Particle propagation and weight computation are computationally the most in depth steps. But, they may be software programmable and can additionally be without problems implemented in parallel if parallel hardware is on hand.

The resampling step is universal and in most cases state-dimension-free however it isn't always obviously suitable for parallel processing. The resampling is major for PF; without this step, PF will produce a degenerate set of particles, i.e., a fixed wherein multiple particles dominate the complete set of the particles with their weights [11]. Which means estimates might be inaccurate and may have unacceptably huge variances. With resampling, such deteriorations are avoided, that is why it's far quite crucial to PF. consequently, resampling has been broadly researched, and, as a end result, pretty quite a few resampling schemes had been proposed.

The arrangement of the paper is as per the following. In section 2 we in brief introduce the Bayesian filtering technique alongside standard particle filter. In area 3 the Rao-Blackwell procedure is specified. In segment 4 diverse resampling plans have been talked about and in segment 5 connected to simulated data. Eventually part 6 includes our conclusion.

978-1-5090-1338-8/16/$31.00 ©2016 IEEE

II. PARTICLE FILTER

Particle filtering is an ordinary Monte Carlo (inspecting) framework for performing deduction in state-space units where the condition of methodology develops in time and comprehension about the state is got by means of loud estimations set aside a few minutes step. In a typical discrete-time state-space mannequin, the condition of a framework advances with regards to:

$$x_{k+1}=f(x_k,w_k) \quad (2.1)$$
$$z_k=h(x_k,e_k) \quad (2.2)$$

Here, z_k is measurement variable at time *k*, x_k is the state variable at time k, w_k is the state noise, e_k is the measurement noise and *f, h* are two nonlinear functions.

The monitoring scenario is to recursively estimate Xk from a fixed of measurements Z1:k = z1,z2,…,zk as much as time k. From a Bayesian perspective, the tracking hassle is to recursively assemble the posterior likelihood density perform of the nation p(Xk/Z1:k), given the sequentially offered measurements. it's assumed that the initial risk density perform or the earlier of the kingdom vector p(X0/Z0)=p(X0)available.

The iterative Bayesian procedure includes steps to reap the posterior possibility density carry out p(Xk/Zk): prediction and replace(update). Assuming that the danger density carry out p(Xk−1/Z1:k-1) at time k − 1 is available, the prediction step makes use of the nation mannequin in Equation (2.1) to collect the prior probability density carry out of the nation at time k adequate following the Chapman-Kolmogorov equation

$$p(Xk/Z1{:}k\text{-}1)=\int p(Xk/Xk\text{-}1)\, p(Xk\text{-}1/Z1{:}k\text{-}1)\, dXk\text{-}1 \quad (2.3)$$

The p(Xk/Xk−1) represents the probabilistic evolution mannequin of the kingdom and is given via the state mannequin in Equation (2.1).

At time step k, the replace stage involves utilizing the state-of-the-art to be had size Zk to obtain the posterior possibility density function p(Xk/Z1:k) via the Bayes theorem

$$p(Xk/Z1{:}k)=\frac{p(Zk/Xk)P(Xk/Z1{:}k\text{-}1)}{p(Zk/Z1{:}k\text{-}1)} \quad (2.4)$$

Where $p(Zk|Xk)$ is the likelihood function defined with the aid of the measurement model in Equation (2.2) and

$$p(Zk/Z1{:}k\text{-}1)=\int p(Zk/Xk)\, p(Xk/Z1{:}k\text{-}1)\, dXk \quad (2.5)$$

The prediction and replace Equations (2.3) and (2.4) kind the groundwork for the Bayesian tracking algorithm. Options to these equations are to be had provided that the procedure and dimension items satisfy some stipulations, as summarized subsequent.

• When the state and measurement items of a method are linear and the modeling error and noise approaches are Gaussian, then the Kalman filtering (KF) can furnish most appropriate options to this predicament.

• When the state-space of the procedure is a finite, discrete-valued sequence, then grid-centered search methods can be utilized to estimate the unknown procedure state.

• When the posterior probability density function of a nonlinear dynamic procedure has a ample statistic, Bene and Daum filters can also be utilized to estimate the probability density of the method state.

PF is a sequential MC machine this is used to estimate the dynamic nation parameters of nonlinear and/or non-Gaussian techniques[15]. The estimation is executed by way of manner of approximating the posterior possibility density function of the unknown nation parameters at at any time whilst step given measurements up to that factor step. notably, for a dynamic technique defined in Equations (2.1) and (2.2), the PF approximates the joint posterior probability density carry out of Xk at time k using a set of N random samples or particles, $X^{(\ell)}_k$ and their corresponding weights, $W^{(\ell)}_k$, $\ell = 1, \ldots, N$, as:

$$p(Xk/Zk) \approx \sum_{l=1}^{N} W^{(l)}k\ \delta(Xk - X^{(l)}k) \quad (2.6)$$

where $\delta(\bullet)$ is the Dirac delta function.

There are unique PF algorithms, relying on the option of importance density used to compute the weights. Some of the almost constantly used algorithms is the sequential importance resampling (SIR) PF that consists of the subsequent common 3 steps [7].

1. Particle creation (propagation): The debris $X^{(\ell)}_k$ are drawn from an importance density characteristic

$q(Xk|X^{(\ell)}k-1,Z1{:}k)$,where $Z1{:}k = \{z1, \ldots, zk\}$.

2. Weight calculation: The weights of the particles are calculated as

$$Wk \approx W^{(l)}k\frac{p(Zk/X^{(l)}k)P(X^{(l)}k/X^{(l)}k-1)}{q(X^{(l)}k/X^{(l)}k-1,Z1{:}k)} \quad (2.7)$$

And then normalized to one so that

$$\sum_{l=1}^{N} W^{(l)}k\ = 1,$$

Note that the substance density is chosen such that

$$q(X^{(l)}k/X^{(l)}k-1, Z1{:}k) = P(X^{(l)}k/X^{(l)}k-1).$$

Therefore equation (2.7) becomes

$$Wk \propto W^{(l)}k\ p\big(Zk/X^{(l)}k\big).$$

3. Resampling: The particles be resample to stay away from particle wickedness, which takes place while most particle weights are nearer to 0, main to a terrible illustration of the subsequent probability density perform. Resampling avoids degeneracy thru disposing of debris with low significance weights and replicating debris with excessive significance weights[15].

III. RAO-BLACKWELLIZED (MARGINALIZED) PARTICLE FILTER

The particle filters struggles while the state measurement grows, The Rao-Blackwellized (marginalized) particle filter is relevant to a conditionally linear-Gaussian structure. To boom the efficiency of particle filtering, making use of a system recognized as Rao- Blackwellisation. Almost, this samples one of the most variables, and marginalizes out the relaxation precisely, the usage of the Kalman filter, HMM clear out, junction tree algorithm, or every other finite dimensional optimal filter. We display that Rao-Blackwellised particle filters (RBPFs) result in extra correct estimates than regular PFs. The knowledge of this approach is that it can fairly decrease the size of the distance over which we need to sample.

The fundamental proposal of the marginalized PF (MPF) is to detachment the state vector as Xk = $[(X^l k)^t; (X^n k)^t]t$, the place $X^l k$ represent the state variable panel with conditionally linear dynamics and $X^n k$ represent the state variable panel with non-linear dynamics.

Allow us to bear in mind the next marginalization mannequin:

$$\begin{cases} X^n k = f(X^n k-1) + V^n k \\ X^l k = A(X^n k-1)X^l k-1 + V^l k \\ Yk = h(X^n k) + C(X^n k)X^l k + Ek \end{cases} \quad (3.1)$$

Where f and h are arbitrarily non linear functions, whereas $A(X^n k-1)$ is conditionally linear on $X^l k-1$ and $C(X^n k)$is on $X^l k$. The system and measurement noise are assumed to be Gaussian.

The subsequent pdf of the state Xk can be observed as $p(X^l k; \boldsymbol{X}^{n,k}/\boldsymbol{Y^k})$, where $\boldsymbol{X}^{n,k} = \{x^n 0, x^n 1, \dots x^n k\}$ and the marginal of which is$p(X^l k; \boldsymbol{x}^n \boldsymbol{k}/\boldsymbol{Y^k})$. By marginalizing out the conditionally linear states $X^l k$ using Bayes' theorem, we have

$$p\left(X^l k; \boldsymbol{X}^{n,k}/\boldsymbol{Y^k}\right) = p\left(X^l k/\boldsymbol{X}^{n,k}; \boldsymbol{Y^k}\right)p\left(\boldsymbol{X}^{n,k}/\boldsymbol{Y^k}\right) \quad (3.2)$$

The distribution $p\left(X^l k/\boldsymbol{X}^{n,k}; \boldsymbol{Y^k}\right)$ is analytically tractable on account that it is conditioned on the non-linear states $\boldsymbol{X}^{n,k}$ and, hence, can be determined optimally making use of the Kalman Filter [3]. The distribution $p\left(\boldsymbol{X}^{n,k}/\boldsymbol{Y^k}\right)$ depends best on the nonlinear states and can also be estimated utilizing the PF.

Considering the pdf $p\left(X^l k/\boldsymbol{X}^{n,k}; \boldsymbol{Y^k}\right)$ is conditioned on the nonlinear states $\boldsymbol{X}^{n,k}$, we have $A(X^n k-1)$ and $C(X^n k)$ as constant regular matrices, accordingly permitting for a statistically greatest approximation of the linear state. Kalman Filter algorithm to compute the most excellent estimate of $X^l k$ given the non-linear states is outlined under.

Given the preliminary conditions $X^l 0/0$ and P0/0, compute the prediction equations

$$\begin{aligned} x^l_{k|k-1} &= A_k(x^n_{k-1})x^l_{k-1} \\ P_{k|k-1} &= A_k(x^n_{k-1})P_{k|k-1}A_k(x^n_{k-1})^T + Q^l_k \end{aligned} \quad (3.3)$$

And the update equations are

$$\begin{aligned} S_k &= C_k(x^n_k)P_{k|k-1}C_k(x^n_k)^T + R_k \\ K_k &= P_{k|k-1}C_k(x^n_k)^T S_k^{-1} \\ x^l_{k|k} &= x^l_{k|k-1} + K_k(y_k - h_k(x^n_k) - C_k(x^n_k)x^l_{k|k-1}) \\ P_{k|k} &= P_{k|k-1} - K_k C_k(x^n_k)P_{k|k-1} \end{aligned} \quad (3.4)$$

The best possible estimate of $X^l k$ is then given by $X^l k/k$. The conditional probability $p\left(\boldsymbol{X}^{n,k}/\boldsymbol{Y^k}\right)$can be expressed as

$$\begin{aligned} &p(X^{n,k}|Y^k) \propto \\ &p(y_k|X^{n,k}, Y^{k-1})p(x_k|X^{n,k-1}, Y^{k-1})p(X^{n,k-1}|Y^{k-1}) \end{aligned} \quad (3.5)$$

The weights of the particles are calculated as

$$\widetilde{w}_k^{(i)} = w^i_{k-1}p(y_k|X^{n,k}, Y^{k-1}) \quad (3.6)$$

Where,

$$p(y_k|X^{n,k}, Y^{k-1}) = N(h_k(x^n_k) + C_k(x^n_k)x^l_{k|k-1}, S_k)$$

The most favorable state estimation at time k is then given by

$$\hat{x}_k = \left[x^l_{k|k}, \sum_{i=1}^N w_k^{(i)} x_k^{n,(i)}\right]^T \quad (3.7)$$

A resampling step is also presented after normalizing the weights to avert degeneracy of the PF.

IV. RESAMPLING SCHEMES

Preferably, the weights of particles will have to all be equivalent. On the opposite severe, it's majority undesirable chance that every one of the particles contain weights equal to 0 or 1 or a some of particles have the majority of the mass and whatever remains of the molecule weights are unimportant. That is by and large called decadence and is decisively what in the end happens when PF is acknowledged by method for making utilization of least difficult the previously stated two stages. At that point, as the handling of the perceptions continues the fluctuation of the weights increments and achieves a variable at which the arbitrary compute is an exceptionally poor estimate of the sifting circulation.

We first collect the techniques deliberating their utilization as consecutive and parallel. The consecutive strategies are similarly arranged in view of whether or not the resampling is from a solitary dissemination or from two or greater conveyances acquired from amassing of the debris (compound testing). On the point when actualizing resampling, some picks need to be made. They comprise

selecting the circulate for resampling, indicating the trying out process, figuring out the resample degree, and deciding on the recurrence of resampling.

Conventional examining techniques have as of now been portrayed somewhere else; we introduce them here for culmination and in light of the fact that they are utilized as a part of the compound, uncommon, and parallel strategies depicted later. We accept that the weights W^m_t are standardized prior to resampling, i.e. $\sum_{m=1}^{M} w_t^m = 1$.

A) Multinomial Resampling

The middle considered multinomial resampling is to make self-ruling N unpredictable numbers, u^n_t from the uniform apportionment on (0, 1] and use them to pick particles from χ_t [14]. In the nth decision, the atom x^m_t is picked when the going with condition is satisfied:

$$Q_t^{(m-1)} < u_t^{(n)} \leq Q_t^{(m)} \quad (4.1)$$

Where

$$Q_t^{(m-1)} = \sum_{k=1}^{m} w_t^{(k)} \quad (4.2)$$

Along those lines, the probability of choosing x^m_t is similar to that of u^n_t being within the meantime limited through the mixture general of the standardized weights as appeared in (4.1). Multinomial resampling is additionally alluded to as trustworthy arbitrary resampling. Because the inspecting of each particle is abnormal, the higher and poorer breaking points of the quantity of instances a given particle is resampled are null (not tested) and N_t (inspect N_t times), one at a time. This yields the most intense fluctuation of the resampled particles.

B) Stratified Resampling

Stratified resampling partitions the entire populace of particles into sub parts called strata. It pre-partitions the $(0, 1]$ period into N displace sub-intervals $\left(0, \frac{1}{N}\right] U..U(1 - 1/N, 1]$. The random numbers $(u_t^n)_{n=1}^N = 1$, are attracted autonomously each of these subintervals, i.e.,

$$u_t^{(n)} \sim U\left(\frac{n-1}{N}, \frac{n}{N}\right), n = 1,2, \ldots . N \quad (4.3)$$

And after that the bouncing technique taking into account the aggregate whole of standardized weights as appeared in (4.1) is utilized.

C) Systematic Resampling

Efficient resampling likewise abuses the thought of strata yet differently. Presently, u_t^1 is drained from the regular distribution on (0,1/N], and whatever is left of the 'u_t' information are acquired conclusively, i.e.,

$$u_t^{(n)} \sim U\left(0, \frac{1}{N}\right),$$

$$u_t^{(n)} = u_t^{(1)} + \frac{n-1}{N}, \; n = 2,3, \ldots . . N \quad (4.4)$$

Note that the systematic resampling is computationally further helpful than stratified resampling system by virtue of the tinier numeral of irregular statistics that are made.

D) Residual Resampling

Residual resampling includes two phases. The first phase is a deterministic reproduction of each and every particle and its weight is more noteworthy than 1/N. The second phase is irregular examining utilizing the staying of weights (implied as residuals). Here where N_t^m speaks to the quantity of times the molecule x_t^m is imitated thusly. In this resampling, the m th molecule is resampled $N_t^m + R_t^m$ times, where N_t^m and R_t^m are the quantities of reproduction from first and second phase, independently, and where $N_t^m = |N\, w_t^m|$. The aggregate number of particles in the main phase is $N_t = \sum_{m=1}^{M} N_t^m$ and in second phase $R_t = N - N_t$. The rest is acquired from

$$\widetilde{w}_t^{(m)} = w_t^{(m)} - \frac{N_t^{(m)}}{N} \quad (4.5)$$

In second phase, particles are pulled in by remaining weights and by utilizing either the multinomial resampling or another unpredictable investigating methodology, where possibility for selecting x_t^m is comparing to the waiting weight of that atom. The principle phase addresses a deterministic reproduction; consequently the assortment of the amount of times a particle is resampled is just credited to second phase. Along these lines, the higher and lower breaking points of the capacity of period that the mth molecule is resampled are $|N\, w_t^m|$and $|N\, w_t^m| + R_t$ individually, if the second phase is actualized utilizing multinomial resampling.

E) Residual systematic resampling (RSR)

Residual systematic resampling aggregates the disconnected commitments of every molecule in the seeking grouping until it is sufficiently vast to produce a specimen (which is proportionate to the amassing thought utilized as a part of deliberate resampling). At that point, no extra system is required for the residuals. In this manner, one can have one cycle circle, and the intricacy is of request O(N) . In the event that it is an optional to remain the molecule size M steady at each point in time and rather, the dimension is permitted to shift, we encompass straightforward methods for managing the particles in parallel and in one circle.

F) Metropolis resampling

Metropolis and autonomous Metropolis–Hastings testing require just proportions between weights that don't should

be standardized and in this way strings can prepare in parallel, for instance. The arrangement displayed in address GP-GPU usage where the measure of parallel computational assets is copious, and it focuses on that despite the fact that Metropolis testing is extra computationally serious than customary resampling calculations, it is as quick on a GP-GPU in light of the fact that there are no reliant operations on the weights. There, testing from U(1,….M) gives back a worth arbitrarily chose from the set {1,…., M} .

V. IMPLEMENTATION

We reveal the overall performance of our proposed system the usage of three dimensional state-space version.

$$x_{t+1}^n = \arctan x_t^n + (1\ 0\ 0)x_t^n + w_t^n$$

$$x_{t+1}^1 = \begin{pmatrix} 1 & 0.3 & 0 \\ 0 & 0.92 & -0.3 \\ 0 & 0.3 & 0.92 \end{pmatrix} x_t^1 + w_t^1$$

$$y_t = \begin{pmatrix} 0.1(x_t^n)^2 sgn(x_t^n) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 1 & -1 & 1 \end{pmatrix} x_t^1 + e_t$$

Where $$w_t = \begin{pmatrix} w_t^n \\ w_t^l \end{pmatrix} \sim N(0, 0.01 I_{4*4})$$

$$e_t \sim N(0, 0.1 I_{2*2})$$

$$x_0^n \sim N(0,1)$$

$$x_0^l \sim N(0_{3*1}, 0_{3*3})$$

A) *Estimation performance*

We apply both standard particle filter and RBPF algorithms to the model specified, and also used different types of resampling algorithms. The corresponding estimation outcomes are shown in Fig. 1. Table 1, 2 and 3 shows the root mean-squared error (**RMSE**), peak signal to noise ratio (**PSNR**) and **Absolute Error** for the given model respectively.

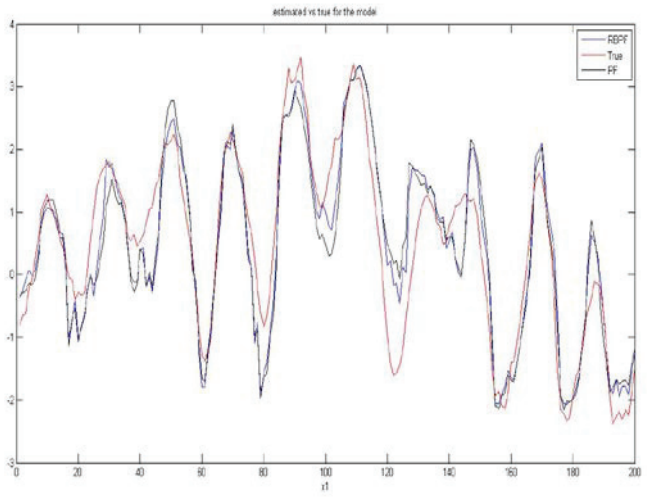

Fig1. Estimation result of the model for the PF and RBPF

Table 1: Comparison of the **RMSE** in PF and RBPF for the given model

	PF	RBPF
Multinomial	0.2076	0.2040
Systematic	0.2069	0.2017
Residual	0.2116	0.2045
RSR	0.2094	0.2021
Stratified	0.2152	0.2038
Metropolis	0.2096	0.2025

Table 2: Comparison of the **PSNR** in PF and RBPF for the given model

	PF	RBPF
Multinomial	57.6761	58.0793
Systematic	57.6620	58.2429
Residual	57.7234	58.1861
RSR	57.6728	58.2816
Stratified	57.8283	58.1907
Metropolis	57.3150	58.3248

Table 3: Comparison of the **Absolute Error** in PF and RBPF for the given model

	PF	RBPF
Multinomial	1.6415	1.6866
Systematic	1.8312	1.6140
Residual	1.7604	1.5567
RSR	1.7800	1.5661
Stratified	1.7532	1.5978
Metropolis	1.8264	1.5697

VI. CONCLUSION

In this paper six resampling algorithms were used. Multinomial, systematic, residual, RSR, stratified, metropolis, are found to provide comparable results between particle filter and RBPF. From these resampling schemes systematic resampling is often preferred because it is the simplest method to implement and also it is having low RMSE when compared to the remaining resampling schemes with the PF and RBPF. The design was implemented for picking between the different resampling schemes keeping in mind the end goal to accomplish a low execution and computational overhead while as yet safeguarding satisfactory execution precision.

The minimization strategies include successfully connected to standard nonlinear and non-Gaussian state-space models, with straight sub-systems. These have been completed within few levels, in which every step infers a specific alternate of the standard particle filter. The preliminary step become to associate Kalman filter which marginalize out with every particle. Those Kalman filters have been utilized to estimate the direct states. The next step

was to utilize the expectancy of the non1inear state as an extra estimation. This was applied to acquire higher gauges of the directly state variab1es. The complete subtle elements for the underestimated particle channel were determined for a general nonlinear and non-Gaussian state-space model. A few vital extraordinary cases were additionally depicted.

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