DIFFERNTIAL EQUATIONS

First Order Linear Differential Equation

A first order linear differntial equation has the following form:

$$\frac{dy}{dx} + P\left(x\right)y = Q\left(x\right)$$

Integrating Factor

$$\frac{dy}{dx} + P\left(x\right)y = Q\left(x\right)$$

multiyplying the expression by the integrating factor $\mu(x)$

$$\mu(x)\frac{dy}{dx} + \mu(x)P\left(x\right)y = \mu(x)Q\left(x\right)$$

Setting: $\mu'(x) := \mu(x)P\left(x\right)$ and finding the integrating factor:

$$\mu'(x) = \mu(x)P\left(x\right)$$

$$\Leftrightarrow \frac{\mu'(x)}{\mu(x)} = P\left(x\right)$$

$$\Leftrightarrow \int \frac{\mu'(x)}{\mu(x)}dx = \int P\left(x\right)dx$$

$$\Leftrightarrow \ln\left(\left|\mu(x)\right|\right) = \int P\left(x\right)dx$$

$$\Rightarrow \mu(x) = e^{\int P(x)dx}$$