3. Obtain the partial derivatives of

(a) $f(x,y) = \sin x + \cos y$

(b) $f(x,y) = x^2\sqrt{1-y^2}$

(c) $f(x,y) = e^{-(x^2+y^2)}$

(d) $f(x,y,z) = xyz + xy + z$

(e) $f(x,y,z) = e^x \ln y + z^4$

(f) $f(x,y) = e^{\sin x} + e^{\cos(x+y)}$

(a) $f(x,y) = \sin(x) + \cos(y)$

$$\frac{\partial}{\partial x} f(x,y) = \cos(x)$$

$$\frac{\partial}{\partial y} f(x,y) = -\sin(y)$$

(b) $f(x,y) = x^2\sqrt{1-y^2}$

$$\frac{\partial}{\partial x} f(x,y) = x\sqrt{1-y^2}$$

$$\frac{\partial}{\partial y} f(x,y) = -\frac{x^2 y}{2\sqrt{1-y^2}}$$

(c) $f(x,y) = e^{-(x^2+y^2)}$

$$\frac{\partial}{\partial x} f(x,y) = -2x\, e^{-(x^2+y^2)}$$

$$\frac{\partial}{\partial y} f(x,y) = -2y\, e^{-(x^2+y^2)}$$

(d) $f(x,y,z) = xyz + xy + z$

$$\frac{\partial}{\partial x} f(x,y,z) = yz + y$$

$$\frac{\partial}{\partial y} f(x,y,z) = xz + x$$

$$\frac{\partial}{\partial z} f(x,y,z) = xy + 1$$

(e) $f(x,y,z) = e^x \ln(y) + z^4$

$$\frac{\partial}{\partial x} f(x,y,z) = e^x \ln(y)$$

$$\frac{\partial}{\partial y} f(x,y,z) = \frac{e^x}{y}$$

$$\frac{\partial}{\partial z} f(x,y,z) = 4z^3$$

(f) $f(x,y) = e^{\sin(x)} + e^{\cos(x+y)}$

$$\frac{\partial}{\partial x} f(x,y) = \cos(x)\, e^{\sin(x)} - \sin(x+y) \cdot e^{\cos(x+y)}$$

$$\frac{\partial}{\partial y} f(x,y) = -\sin(x+y)\, e^{\cos(x+y)}$$

4. Determine the slope of the tangent in the x- and y-directions to the surface $z = x^2 + y^2$ at the point $\mathrm{P} = (0,1)$.

$$z(x=0, y=1) = 1$$

$$\frac{\partial}{\partial x} z = 2x \qquad \frac{\partial}{\partial y} z = 2y$$

For the point $P = (0;1)$:

Slope in x-direction:	Slope in y-direction:
$\frac{\partial}{\partial x} z = 0$	$\frac{\partial}{\partial y} z = 2$

5. Determine the partial derivatives f_{xx}, f_{xy}, f_{yx} and f_{yy} of the function

$$z = R^2 - x^2 - y^2$$

$$\frac{\partial^2 f}{\partial x^2} = -2$$

$$\frac{\partial^2 f}{\partial x \partial y} = -2x - 2y$$

$$\frac{\partial^2 f}{\partial y \partial x} = -2x - 2y$$

$$\frac{\partial^2 f}{\partial y^2} = -2$$

6. Show that the function $z = e^{(x/y)^2}$ satisfies the relation $x f_x + y f_y = 0$.

$$\frac{\partial z}{\partial x} = \frac{2}{y^2} x e^{\left(\frac{x}{y}\right)^2}$$

$$\frac{\partial z}{\partial y} = -\frac{2x^2}{y^3} e^{\left(\frac{x}{y}\right)^2}$$

$$x f_x + y f_y = x\left(\frac{2}{y^2} x e^{\left(\frac{x}{y}\right)^2}\right) + \cancel{y}\left(-\frac{2x^2}{y^{\cancel{3}}_{2}} e^{\left(\frac{x}{y}\right)^2}\right)$$

$$= \frac{2}{\cancel{y^2}} \cancel{x^2} \cancel{e^{\left(\frac{x}{y}\right)^2}} - \frac{2\cancel{x^2}}{\cancel{y^2}} \cancel{e^{\left(\frac{x}{y}\right)^2}}$$

$$= 0$$