HERMES-INTERNAL 07-001

Simultaneous Extraction of the BSAs and the BCA Associated with DVCS with the Extended Maximumn Likelihood Method*

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Abstract

Both the Deeply Virtual Compton Scattering (DVCS) process and the Beithe-Heitler (BH) process contribute to exclusive electroproduction of a real photon off an unpolarized nucleon, $lN \to lN\gamma$. Here not only the DVCS process but also its interference with the BH process depends on the beam helicity. The BH-DVCS interference depends also on the beam charge. These dependences give access to Generalized Parton Distributions (GPDs) and are of physics interest.

In this report, we present the first data analysis to simultaneously extract the Beam-Spin Asymmetry (BSA) induced by the beam-helicity dependence of the pure DVCS cross section, the BSA induced by the beam-helicity dependence of the BH-DVCS interference, and the Beam-Charge Asymmetry induced by the beam-charge dependence of the BH-DVCS interference. The analysis is based on the HERMES 2000/2005 data accumulated with $e^{\pm}$ beams and hydrogen targets. The method of Extended Maximum Likelihood, which takes into account the fact that the total number of observed events has a Poisson variation, is used in the analysis. This method can provide satifactory estimates for constant terms of the asymmetries, for which the standard Maximum Likelihood method may not.

*For updated analysis of the asymmetry extraction method, please refer to *The Maximum Likelihood Estimation Method* given by X.-G. Lu in the HERMES DVCS Week Sep. 2007. It also describes some relevant issues which are not included in this note: **grouping of data sample**, **weight and the corresponding estimation error**, etc. Some important conclusions: **(1) EML and SML are consistent to a large extent; the discrepancy observed in this note is due to inappropriate grouping of data sample for SML; (2) with weights, UML can also be used (WUML described in the lecture)**.

Contents

1 Introduction

1.1 Kinematics and amplitudes

In exclusive electroproduction of a real photon off an unpolarized nucleon

$$l(k) + N(p) \to l(k') + N(p') + \gamma(q'), \tag{1}$$

where k, p, k', p' and q' denote the four-momenta of the corresponding particles, the four-fold cross section is given by [1]

$$\frac{d\sigma}{dx_B dy d|t| d\phi} = \frac{\alpha_{em}^3 x_B y}{8\pi Q^2 \sqrt{1+\epsilon^2}} \left| \frac{\tau}{e^3} \right|^2, \quad \epsilon \equiv 2x_B \frac{M_N}{Q}. \tag{2}$$

with the fine-structure constant α_{em}, the positron charge e, and the nucleon mass M_N. This cross section depends on the Bjorken variable $x_B = -q^2/(2p \cdot q)$, with $q = k - k'$, the lepton energy fraction $y = (p \cdot q)/(p \cdot k)$, the squared 4-momentum transfer $t = (p' - p)^2$, the photon virtuality $Q^2 = -q^2$, and the azimuthal angle ϕ between the lepton scattering plane and photon production plane (see Fig. 1).

The amplitude τ is given by the coherent sum of the DVCS amplitude τ_{DVCS} and the BH amplitude τ_{BH}. Thus

$$|\tau|^2 = |\tau_{BH}|^2 + |\tau_{DVCS}|^2 + \underbrace{\tau_{DVCS}\tau_{BH}^* + \tau_{DVCS}^*\tau_{BH}}_{\mathcal{I}}. \tag{3}$$

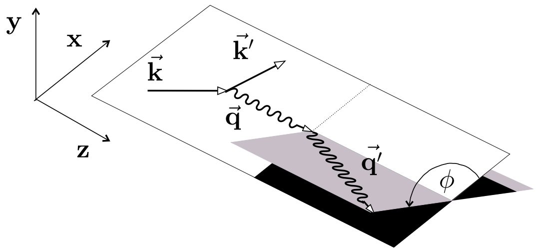

Figure 1: Kinematics of real photon production in the target rest frame. The z-direction is chosen along the three-momentum of the virtual photon $\vec{q}$.

The contributions on the right hand side of the above equation can be Fourier expanded in ϕ to twist-three approximation [1]:

$$|\tau_{BH}|^2 = \frac{e^6}{x_B^2 y^2 (1+\epsilon^2)^2 t P_1(\phi) P_2(\phi)} (c_0^{BH} + \sum_{n=1}^{2} c_n^{BH} \cos n\phi), \tag{4}$$

$$|\tau_{DVCS}|^2 = \frac{e^6}{y^2 Q^2} (c_0^{DVCS} + \sum_{n=1}^{2} c_n^{DVCS} \cos n\phi + \lambda s_1^{DVCS} \sin\phi), \tag{5}$$

$$\mathcal{I} = -\frac{\eta e^6}{x_B y^3 P_1(\phi) P_2(\phi) t} (c_0^{\mathcal{I}} + \sum_{n=1}^{3} c_n^{\mathcal{I}} \cos n\phi + \lambda \sum_{n=1}^{2} s_n^{\mathcal{I}} \sin n\phi), \tag{6}$$

Here η and λ denote the beam charge and the beam helicity respectively, and $P_{1,2}(\phi)$ are defined in the lepton BH propagators:

$$Q^2 P_1 \equiv (k - q')^2, \tag{7}$$

$$Q^2 P_2 \equiv \left[k - (p' - p)\right]^2. \tag{8}$$

1.2 BSAs with a certain beam charge

By defining the cross section below which depends on the beam charge and is indepedent of the beam helicity[1]

$$\sigma_{UU}^{\eta}(\phi) = \frac{\alpha_{em}^3 x_B}{8\pi Q^2 \sqrt{1+\epsilon^2}} \Big[\frac{c_0^{BH} + \sum_{n=1}^{2} c_n^{BH} \cos n\phi}{x_B^2 y (1+\epsilon^2)^2 t P_1(\phi) P_2(\phi)} + \frac{c_0^{DVCS} + \sum_{n=1}^{2} c_n^{DVCS} \cos n\phi}{y Q^2} - \eta \frac{c_0^{\mathcal{I}} + \sum_{n=1}^{3} c_n^{\mathcal{I}} \cos n\phi}{x_B y^2 P_1(\phi) P_2(\phi) t} \Big], \tag{9}$$

[1]The subscript UU stands for Unpolarized beams and Unpolarized targets.

the cross section[2] $\sigma_{LU}(\phi;\lambda,\eta)$ in Eq. (2) can be expressed as

$$\begin{aligned}\sigma_{LU}(\phi;\lambda,\eta) =&\sigma^{\eta}_{UU}(\phi)\Big\{1+\lambda\big[K_1\frac{s_1^{DVCS}\sin\phi}{\sigma^{\eta}_{UU}(\phi)}-\eta K_2\frac{s_1^{I}\sin\phi+s_2^{I}\sin 2\phi}{\sigma^{\eta}_{UU}(\phi)}\ \big]\Big\}\\ =&\sigma^{\eta}_{UU}(\phi)\big[1+\lambda A^{\eta}_{LU}(\phi)\big],\end{aligned} \tag{10}$$

here $K_{1,2}$ are kinematic factors that are independent of ϕ, and $A^{\eta}_{LU}(\phi)$ is the BSA with certan beam charge η defined as

$$A^{\eta}_{LU}(\phi)=\frac{\frac{s_1^{DVCS}\sin\phi}{Q^2}-\eta\frac{s_1^{\mathcal{I}}\sin\phi+s_2^{\mathcal{I}}\sin 2\phi}{x_B y P_1(\phi)P_2(\phi)t}}{\frac{c_0^{BH}+\sum_{n=1}^{2}c_n^{BH}\cos n\phi}{x_B^2(1+\epsilon^2)^2 tP_1(\phi)P_2(\phi)}+\frac{c_0^{DVCS}+\sum_{n=1}^{2}c_n^{DVCS}\cos n\phi}{Q^2}-\eta\frac{c_0^{\mathcal{I}}+\sum_{n=1}^{3}c_n^{\mathcal{I}}\cos n\phi}{x_B y P_1(\phi)P_2(\phi)t}}. \tag{11}$$

It can be seen that $A^{\eta}_{LU}(\phi)$ is a mixture of the beam-charge dependent interference and beam-charge independent DVCS effects both in its numerator and denominator. Such beam-charge dependent BSAs have been measured by HERMES with positron beams [2] and by CLAS with electron beams [3].

As the BH part prevails in the denominator of $A^{\eta}_{LU}(\phi)$ with c_0^{BH} dominating over $c_{1,2}^{BH}$, and as the interference part prevails in the numerator with $s_1^{\mathcal{I}}$ dominating over $s_2^{\mathcal{I}}$, $A^{\eta}_{LU}(\phi)$ can be approximated as

$$A^{\eta}_{LU}(\phi)\approx-\eta\frac{x_B}{y}\frac{s_1^{\mathcal{I}}}{c_0^{BH}}\sin\phi. \tag{12}$$

1.3 BSAs induced by pure DVCS and interference contributions

Similar to the previous section, by defining the cross section below which is independent of the beam charge and helicity

$$\begin{aligned}\sigma^{0}_{UU}(\phi)=\frac{\alpha^3_{em}x_B}{8\pi yQ^2\sqrt{1+\epsilon^2}}\Big[&\frac{c_0^{BH}+\sum_{n=1}^{2}c_n^{BH}\cos n\phi}{x_B^2(1+\epsilon^2)^2 tP_1(\phi)P_2(\phi)}\\ &+\frac{c_0^{DVCS}+\sum_{n=1}^{2}c_n^{DVCS}\cos n\phi}{Q^2}\Big],\end{aligned} \tag{13}$$

$\sigma_{LU}(\phi;\lambda,\eta)$ can be expressed as

$$\begin{aligned}\sigma_{LU}(\phi;\lambda,\eta) =&\sigma^{0}_{UU}(\phi)\Big\{1+\lambda K_1\frac{s_1^{DVCS}\sin\phi}{\sigma^{0}_{UU}(\phi)}-\eta K_2\frac{c_0^{I}+\sum_{n=1}^{3}c_n^{I}\cos n\phi}{\sigma^{0}_{UU}(\phi)}\\ &-\eta\lambda K_2\frac{s_1^{I}\sin\phi+s_2^{I}\sin 2\phi}{\sigma^{0}_{UU}(\phi)}\ \big]\Big\}\\ =&\sigma^{0}_{UU}(\phi)\big[1+\lambda A^{DVCS}_{LU}(\phi)+\eta A_C(\phi)+\eta\lambda A^{\mathcal{I}}_{LU}(\phi)\big],\end{aligned} \tag{14}$$

[2]The subscript LU stands for Longitudinally polarized beams and Unpolarized targets.

in which the Beam-Charge Asymmetry BCA $A_C(\phi)$ and BSAs $A_{LU}^{DVCS}(\phi)$ and $A_{LU}^{\mathcal{I}}(\phi)$ are defined as

$$
\begin{aligned}
A_C(\phi) &= -\frac{\frac{x_B}{y}\sum_{n=0}^{3} c_n^I \cos n\phi}{\frac{\sum_{n=0}^{2} c_n^{BH}\cos n\phi}{(1+\epsilon^2)^2} + \frac{x_B^2 t P_1(\phi)P_2(\phi)}{Q^2}\sum_{n=0}^{2} c_n^{DVCS}\cos n\phi}, && (15)\\
A_{LU}^{DVCS}(\phi) &= \frac{\frac{x_B^2 t P_1(\phi)P_2(\phi)}{Q^2} s_1^{DVCS}\sin\phi}{\frac{\sum_{n=0}^{2} c_n^{BH}\cos n\phi}{(1+\epsilon^2)^2} + \frac{x_B^2 t P_1(\phi)P_2(\phi)}{Q^2}\sum_{n=0}^{2} c_n^{DVCS}\cos n\phi}, && (16)\\
A_{LU}^{\mathcal{I}}(\phi) &= -\frac{\frac{x_B}{y}\sum_{n=1}^{2} s_n^I \sin n\phi}{\frac{\sum_{n=0}^{2} c_n^{BH}\cos n\phi}{(1+\epsilon^2)^2} + \frac{x_B^2 t P_1(\phi)P_2(\phi)}{Q^2}\sum_{n=0}^{2} c_n^{DVCS}\cos n\phi}. && (17)
\end{aligned}
$$

It can be seen that $A_{LU}^{DVCS}(\phi)$ and $A_{LU}^{\mathcal{I}}(\phi)$ are independent of the beam charge, and that $A_{LU}^{DVCS}(\phi)$ give information about the beam-helicity dependence of the pure DVCS cross section, while $A_{LU}^{\mathcal{I}}(\phi)$ gives information about the beam-helicity dependence of the BH-DVCS interference. Hence measurements of $A_{LU}^{DVCS}(\phi)$ and $A_{LU}^{\mathcal{I}}(\phi)$ will be of physics interest[3].

As the BH part prevails in the denominators of the three asymmetries with c_0^{BH} dominating over $c_{1,2}^{BH}$, and as in numerators $c_1^{\mathcal{I}}$ dominates over $c_{0,2,3}^I$ and $s_1^{\mathcal{I}}$ over $s_2^{\mathcal{I}}$, the three asymmetries can be approximated as [4]

$$
\begin{aligned}
A_C(\phi) &\approx -\frac{x_B}{y}\frac{c_1^{\mathcal{I}}}{c_0^{BH}}\cos\phi, && (18)\\
A_{LU}^{DVCS}(\phi) &\approx \frac{x_B^2 t P_1(\phi)P_2(\phi)}{Q^2}\frac{s_1^{DVCS}}{c_0^{BH}}\sin\phi, && (19)\\
A_{LU}^{\mathcal{I}}(\phi) &\approx -\frac{x_B}{y}\frac{s_1^I}{c_0^{BH}}\sin\phi. && (20)
\end{aligned}
$$

Comparing Eqs. (12) and (20), one has

$$
A_{LU}^{\eta}(\phi) \approx \eta A_{LU}^{\mathcal{I}}(\phi). \qquad (21)
$$

In the following, we will present results on the beam-charge dependent BSAs $A_{LU}^{\pm}$ and examine the relation in Eq. (12). We will then present the first results on the beam-charge independent BSAs A_{LU}^{DVCS} and $A_{LU}^{\mathcal{I}}$, and the BCA A_C. The results on the BCA are compared to the HERMES published results [4].

[3]Note that interest in studying $A_{LU}^{\mathcal{I}}(\phi)$ though not $A_{LU}^{DVCS}(\phi)$ has been pointed out previously [6].

[4]The parameterizations of $A_C(\phi)$, $A_{LU}^{DVCS}(\phi)$, $A_{LU}^{\mathcal{I}}(\phi)$ and $A_{LU}^{\eta}(\phi)$ in this analysis are not confined by such approximations (also see Eq.(12)); higher order harmonics are taken into account in the asymmetry extraction.

2 Extraction Methods

Suppose that N sets of independently measured quantities $\mathbf{x}_1, \ldots, \mathbf{x}_i, \ldots, \mathbf{x}_N$ with $\mathbf{x}_i = \{x_{1i}, \ldots, x_{ni}\}$ come from a probability density function (p.d.f.) $p(\mathbf{x}; \boldsymbol{\theta})$, where $\boldsymbol{\theta} = \{\theta_1, \ldots, \theta_m\}$ is the set of m unknown parameters. The p.d.f. $p(\mathbf{x}; \boldsymbol{\theta})$ is normalized over the entire range of $\mathbf{x}$:

$$\int p(\mathbf{x}; \boldsymbol{\theta}) d\mathbf{x} = 1. \tag{22}$$

The parameter set $\boldsymbol{\theta}$ can be estimated by maximizing the likelihood function $\mathscr{L}(\boldsymbol{\theta})$, which is the joint p.d.f. for $\mathbf{x}_1, \ldots, \mathbf{x}_N$. The estimators $\hat{\boldsymbol{\theta}}$ are the solution of the following m equations [5]:

$$\frac{\partial \ln \mathscr{L}}{\partial \theta_j} = 0, \;\; j = 1, \ldots, m. \tag{23}$$

The covariance matrix $V_{ij} = \mathrm{cov}[\hat{\theta}_i, \hat{\theta}_j]$ is given by

$$(\hat{V}^{-1})_{ij} = - \left. \frac{\partial^2 \ln \mathscr{L}}{\partial \theta_i \partial \theta_j} \right|_{\hat{\boldsymbol{\theta}}}. \tag{24}$$

For the standard Maximum Likelihood (SML) method,

$$\mathscr{L}(\boldsymbol{\theta}) = \prod_i^N p(\mathbf{x}_i; \boldsymbol{\theta}). \tag{25}$$

In nuclear and particle physics experiments, the observed number of events often has a Poisson fluctuation about its expected value $\mathbb{N}(\boldsymbol{\theta})$, *which may depend on the parameters*. Taking this into account, the standard maximum likelihood function is extended to include the Poisson p.d.f. $\frac{\mathbb{N}^N e^{-\mathbb{N}}}{N!}$, that is

$$\mathscr{L}(\boldsymbol{\theta}) = \frac{[\mathbb{N}(\boldsymbol{\theta})]^N e^{-\mathbb{N}(\boldsymbol{\theta})}}{N!} \prod_i^N p(\mathbf{x}_i; \boldsymbol{\theta}), \tag{26}$$

the Extended Maximum Likelihood (EML) function [7].

In both the ML methods, $\mathbb{N}$ can be interpreted as the normalization of the extended p.d.f. $\mathscr{P}(\mathbf{x}; \boldsymbol{\theta}) \equiv p(\mathbf{x}; \boldsymbol{\theta}) \mathbb{N}(\boldsymbol{\theta})$:

$$\mathbb{N}(\boldsymbol{\theta}) = \int \mathscr{P}(\mathbf{x}; \boldsymbol{\theta}) d\mathbf{x}. \tag{27}$$

The negative log-likelihood functions to be minimized read

$$- \ln \mathscr{L}_{SML}(\boldsymbol{\theta}) = - \sum_i^N \ln \mathscr{P}(\mathbf{x}_i; \boldsymbol{\theta}) + N \ln \mathbb{N}(\boldsymbol{\theta}), \tag{28}$$

$$- \ln \mathscr{L}_{EML}(\boldsymbol{\theta}) = - \sum_i^N \ln \mathscr{P}(\mathbf{x}_i; \boldsymbol{\theta}) + \mathbb{N}(\boldsymbol{\theta}). \tag{29}$$

In the case that the normalization $\mathbb{N}$ is independent of $\boldsymbol{\theta}$, the estimators $\hat{\boldsymbol{\theta}}$ will not be affected if the last term in Eq. (28) or (29) is dropped, as can be seen from Eqs. (23) and (24). In this report, the method that ignores the normalization term in the negative log-likelihood function is referred to as the unnormalized ML (UML) method.

The likelihood and negative log-likelihood functions of the three ML methods are summarized in Table 1. Because the SML method only takes into account the shape of the distribution, it can not properly estimate parameters that describe the integral of the distribution, e.g. when $\mathscr{P}(\mathbf{x};\boldsymbol{\theta}) = \theta_1$, $\mathscr{L}_{SML}$ is independent of $\boldsymbol{\theta}$ by definition, and thus the SML method is not applicable. The UML method fails to give the correct results if the normalization depends on the parameters, because the estimators then depend on the normalization.

	SML	EML	UML
$\mathscr{L}(\boldsymbol{\theta})$	$\prod_i^N \frac{\mathscr{P}(\mathbf{x}_i;\boldsymbol{\theta})}{\mathbb{N}(\boldsymbol{\theta})}$	$\frac{\mathbb{N}(\boldsymbol{\theta})^N e^{-\mathbb{N}(\boldsymbol{\theta})}}{N!} \prod_i^N \frac{\mathscr{P}(\mathbf{x}_i;\boldsymbol{\theta})}{\mathbb{N}(\boldsymbol{\theta})}$	$-$
$-\ln \mathscr{L}(\boldsymbol{\theta})$	$-\sum_i^N \ln \mathscr{P}(\mathbf{x}_i;\boldsymbol{\theta}) + N \ln \mathbb{N}(\boldsymbol{\theta})$	$-\sum_i^N \ln \mathscr{P}(\mathbf{x}_i;\boldsymbol{\theta}) + \mathbb{N}(\boldsymbol{\theta})$	$-\sum_i^N \ln \mathscr{P}(\mathbf{x}_i;\boldsymbol{\theta})$

Table 1: Likelihood and negative log-likelihood functions of the methods of SML, EML and UML.

3 Data Analysis

The analysis is performed with the data accumulated in the year 2000 with positron beams and in the year 2005 with electron beams. Unpolarized H data in the 00d0 production and both unpolarized and transversely polarized H data in the 05b1 production are used. The net target polarization of the polarized H data is negligible ($\langle g1Target.rPol \rangle = 0.6$ %) and hence the whole data set is regarded as target being unpolarized.

3.1 Data quality and event selection

The data quality and event selection criteria are given elsewhere [8].

3.2 BSAs with a certain beam charge

As the discussions in this section involve only with one beam charge, the superscript η is omitted to avoid complexity.

3.2.1 Event Number Distribution and Normalization

During the infinitesimal time interval $[t,\ t+dt]$, the expected number of the exclusive events observed in the phase space region $[\mathbf{x},\ \mathbf{x}+d\mathbf{x}]$ is proportional to the luminosity L, the detection efficiency ϵ and the cross section given in Eq. (10), i.e.,

$$L(t)\epsilon(\mathbf{x},t)\sigma_{UU}(\mathbf{x})[1+P(t)A_{LU}(\mathbf{x};\boldsymbol{\theta})]dtd\mathbf{x}, \tag{30}$$

where $\mathbf{x}$ denotes the set of quantities determining the kinematics of the events, and $P(t)$ the beam polarization. Thus the extended p.d.f. of the total observed exclusive events in $\mathbf{x}$ and P reads

$$\mathcal{N}(\mathbf{x},P;\boldsymbol{\theta}) = \mathcal{L}(P)\epsilon(\mathbf{x},P)\sigma_{UU}(\mathbf{x})[1+PA_{LU}(\mathbf{x};\boldsymbol{\theta})], \tag{31}$$

where

$$\mathcal{L}(P')dP = \sum_{P'<P(t)<P'+dP} L(t)dt. \tag{32}$$

The normalization

$$\mathbb{N}(\boldsymbol{\theta}) = \iint \mathcal{N}(\mathbf{x},P;\boldsymbol{\theta})d\mathbf{x}dP \tag{33}$$

can be approximated as[5]

$$\mathbb{N}(\boldsymbol{\theta}) = \frac{\mathbb{L}}{\overrightarrow{\mathbb{L}}}\sum_i^{\overrightarrow{N}} \frac{\langle\epsilon\rangle(\mathbf{x}_i)\left[1+\langle P\rangle(\mathbf{x}_i)A_{LU}(\mathbf{x}_i;\boldsymbol{\theta})\right]}{\langle\overrightarrow{\epsilon}\rangle(\mathbf{x}_i)\left[1-\langle\overrightarrow{P}\rangle(\mathbf{x}_i)/\langle\overleftarrow{P}\rangle(\mathbf{x}_i)\right]} + \frac{\mathbb{L}}{\overleftarrow{\mathbb{L}}}\sum_i^{\overleftarrow{N}} \frac{\langle\epsilon\rangle(\mathbf{x}_i)\left[1+\langle P\rangle(\mathbf{x}_i)A_{LU}(\mathbf{x}_i;\boldsymbol{\theta})\right]}{\langle\overleftarrow{\epsilon}\rangle(\mathbf{x}_i)\left[1-\langle\overleftarrow{P}\rangle(\mathbf{x}_i)/\langle\overrightarrow{P}\rangle(\mathbf{x}_i)\right]}, \tag{38}$$

[5] Integrate Eq. (31) over P for different helicity states, we have

$$\int_{P>0} \mathcal{N}(\mathbf{x},P;\boldsymbol{\theta})dP = \overrightarrow{\mathbb{L}}\langle\overrightarrow{\epsilon}\rangle(\mathbf{x})\sigma_{UU}(\mathbf{x})[1+\langle\overrightarrow{P}\rangle(\mathbf{x})A_{LU}(\mathbf{x};\boldsymbol{\theta})] \equiv \overrightarrow{\mathcal{N}}(\mathbf{x};\boldsymbol{\theta}), \tag{34}$$

$$\int_{P<0} \mathcal{N}(\mathbf{x},P;\boldsymbol{\theta})dP = \overleftarrow{\mathbb{L}}\langle\overleftarrow{\epsilon}\rangle(\mathbf{x})\sigma_{UU}(\mathbf{x})[1+\langle\overleftarrow{P}\rangle(\mathbf{x})A_{LU}(\mathbf{x};\boldsymbol{\theta})] \equiv \overleftarrow{\mathcal{N}}(\mathbf{x};\boldsymbol{\theta}). \tag{35}$$

$\sigma_{UU}(\mathbf{x})$ and $A_{LU}(\mathbf{x};\boldsymbol{\theta})$ can be solved from the above two equations as unknown variables, that is

$$\sigma_{UU}(\mathbf{x}) = \frac{\frac{\overleftarrow{\mathcal{N}}(\mathbf{x};\boldsymbol{\theta})}{\overleftarrow{\mathbb{L}}\langle\overleftarrow{\epsilon}\rangle(\mathbf{x})}\langle\overrightarrow{P}\rangle(\mathbf{x}) - \frac{\overrightarrow{\mathcal{N}}(\mathbf{x};\boldsymbol{\theta})}{\overrightarrow{\mathbb{L}}\langle\overrightarrow{\epsilon}\rangle(\mathbf{x})}\langle\overleftarrow{P}\rangle(\mathbf{x})}{\langle\overrightarrow{P}\rangle(\mathbf{x}) - \langle\overleftarrow{P}\rangle(\mathbf{x})}, \tag{36}$$

$$A_{LU}(\mathbf{x};\boldsymbol{\theta}) = \frac{\frac{\overrightarrow{\mathcal{N}}(\mathbf{x};\boldsymbol{\theta})}{\overrightarrow{\mathbb{L}}\langle\overrightarrow{\epsilon}\rangle(\mathbf{x})} - \frac{\overleftarrow{\mathcal{N}}(\mathbf{x};\boldsymbol{\theta})}{\overleftarrow{\mathbb{L}}\langle\overleftarrow{\epsilon}\rangle(\mathbf{x})}}{\frac{\overleftarrow{\mathcal{N}}(\mathbf{x};\boldsymbol{\theta})}{\overleftarrow{\mathbb{L}}\langle\overleftarrow{\epsilon}\rangle(\mathbf{x})}\langle\overrightarrow{P}\rangle(\mathbf{x}) - \frac{\overrightarrow{\mathcal{N}}(\mathbf{x};\boldsymbol{\theta})}{\overrightarrow{\mathbb{L}}\langle\overrightarrow{\epsilon}\rangle(\mathbf{x})}\langle\overleftarrow{P}\rangle(\mathbf{x})}. \tag{37}$$

Substitute Eq. (36) into Eq. (31) and integrate over the entire range of P, we thus obtain the estimation of $\mathbb{N}(\boldsymbol{\theta})$ in Eq. (38). Eq. (37) can be used for the estimation for $A_{LU}(\mathbf{x})$ using the Least Square (LS) method.

where N is the observed number of the exclusive events, $\mathbb{L}$ the integrated luminosity, $\langle\epsilon\rangle(\mathbf{x})$ the averaged detection efficiency and $\langle P\rangle(\mathbf{x})$ the averaged polarization :

$$\mathbb{L} = \int \mathcal{L}(P)dP = \int L(t)dt, \tag{39}$$

$$\langle\epsilon\rangle(\mathbf{x}) = \frac{\int \epsilon(\mathbf{x}, P)\mathcal{L}(P)dP}{\int \mathcal{L}(P)dP} = \frac{\int \epsilon(\mathbf{x}, t)L(t)dt}{\int L(t)dt}, \tag{40}$$

$$\langle P\rangle(\mathbf{x}) = \frac{\int P\epsilon(\mathbf{x}, P)\mathcal{L}(P)dP}{\int \epsilon(\mathbf{x}, P)\mathcal{L}(P)dP} = \frac{\int P(t)\epsilon(\mathbf{x}, t)L(t)dt}{\int \epsilon(\mathbf{x}, t)L(t)dt}. \tag{41}$$

The right/left arrow indicates that quantities are averaged or integrated over the positive/negative polarization, e.g. $\overrightarrow{\mathbb{L}} \equiv \int_{P>0} \mathcal{L}(P)dP$. The standard p.d.f. $p(\mathbf{x}, P; \boldsymbol{\theta})$ reads

$$p(\mathbf{x}, P; \boldsymbol{\theta}) = \frac{\mathcal{N}(\mathbf{x}, P; \boldsymbol{\theta})}{\mathbb{N}(\boldsymbol{\theta})}. \tag{42}$$

The negative log-likelihood function $-\ln \mathscr{L}(\boldsymbol{\theta})$ for the SML, EML and UML methods are given as

$$-\ln \mathscr{L}_{SML}(\boldsymbol{\theta}) = -\sum_{i}^{N} \ln[1 + P_i A_{LU}(\mathbf{x}_i; \boldsymbol{\theta})] + N \ln \mathbb{N}(\boldsymbol{\theta}), \tag{43}$$

$$-\ln \mathscr{L}_{EML}(\boldsymbol{\theta}) = -\sum_{i}^{N} \ln[1 + P_i A_{LU}(\mathbf{x}_i; \boldsymbol{\theta})] + \mathbb{N}(\boldsymbol{\theta}), \tag{44}$$

$$-\ln \mathscr{L}_{UML}(\boldsymbol{\theta}) = -\sum_{i}^{N} \ln[1 + P_i A_{LU}(\mathbf{x}_i; \boldsymbol{\theta})]. \tag{45}$$

In the case that the detection efficiency is set to unity, Eq. (38) reduces to a simpler form:

$$\mathbb{N}(\boldsymbol{\theta}) = \frac{\mathbb{L}}{\overrightarrow{\mathbb{L}}} \frac{\sum_i^{\overrightarrow{N}} [1 + \langle P\rangle A_{LU}(\mathbf{x}_i; \boldsymbol{\theta})]}{\left[1 - \frac{\langle\overrightarrow{P}\rangle}{\langle\overleftarrow{P}\rangle}\right]} + \frac{\mathbb{L}}{\overleftarrow{\mathbb{L}}} \frac{\sum_i^{\overleftarrow{N}} [1 + \langle P\rangle A_{LU}(\mathbf{x}_i; \boldsymbol{\theta})]}{\left[1 - \frac{\langle\overleftarrow{P}\rangle}{\langle\overrightarrow{P}\rangle}\right]} \tag{46}$$

As can be seen from Eq. (46), if the net polarization $\langle P\rangle$ were zero or the $\boldsymbol{\theta}$-dependence of $\mathbb{N}(\boldsymbol{\theta})$ vanished after $A_{LU}(\mathbf{x}; \boldsymbol{\theta})$ is integrated over the entire range of $\mathbf{x}$, $\mathbb{N}$ would be independent of $\boldsymbol{\theta}$ and therefore the UML method could be applied [9].

3.2.2 Results without Detection Efficiency Correction

Not considering the detection inefficiency, we performed the aysmmetry extraction, in which the integrated luminosities in Eqs. (43)-(45) were substituted by the numbers of DIS events.

	00d0 unpol.	05b1 un.+pol.
	$s_1 \pm (stat.)$	$s_1 \pm (stat.)$
LS	-0.18 ± 0.03	0.25 ± 0.06
SML	-0.18 ± 0.03	0.25 ± 0.06
EML	-0.18 ± 0.03	0.26 ± 0.06
UML	-0.19 ± 0.03	0.24 ± 0.06

Table 2: Fitting results of $A_{LU}(\phi; \boldsymbol{\theta}) = s_1 \sin\phi$ by the methods of LS, SML, EML and UML (without beam-helicity balancing).

For $A_{LU}(\phi; \boldsymbol{\theta})$ parameterized by $s_1 \sin\phi$, the fitting results by the methods of Least Square (LS), SML, EML, and UML are summarized in Table 2. An overall consistency can be seen among the results by different methods.

For $A_{LU}(\phi; \boldsymbol{\theta})$ parameterized by $c_0 + s_1 \sin\phi + c_1 \cos\phi$ and $c_0 + s_1 \sin\phi + c_1 \cos\phi + s_2 \sin 2\phi + c_2 \cos 2\phi$, in which c_0 denotes a constant term in parameterization of $A_{LU}(\phi; \boldsymbol{\theta})$, results by the methods of LS, SML, EML and UML are given in Table 3 and Table 4.

It can be seen that the estimates for c_0 by the SML method and the UML method are unreasonable, which can be understood according to Section 2 as follows:

- *for the SML method:* The c_0 term describes the integral of the distribution and can not be properly estimated by minimizing $-\ln \mathscr{L}_{SML}$.
- *for the UML method:* The net polarizations are -14.7% and 12.5% for the 00d0 unpol. and 05b1 un.+pol. data respectively; neither does the c_0 term vanish in $\int_{-\pi}^{\pi} A_{LU}(\phi; \boldsymbol{\theta}) d\phi$. Therefore the normalization depends on $\boldsymbol{\theta}$ and can not be ignored.

The c_0 and c_1 in $A_{LU}(\phi)$ are not at all expected by theory, as they are even in ϕ while the BSA is odd in Eq. (11). Their deviating from 0 by 2σ observed in Table 3 and Table 4 can be statistical fluctuations. Nevertheless a non-vanishing c_0 may be clue to improper estimates of the relative luminosities between two beam helicity states, while the c_1 remains a mystery.

	00d0 unpol. e^+			05b1 un.+pol. e^-		
	$c_0 \pm (stat.)$	$s_1 \pm (stat.)$	$c_1 \pm (stat.)$	$c_0 \pm (stat.)$	$s_1 \pm (stat.)$	$c_1 \pm (stat.)$
LS	-0.03 ± 0.02	-0.18 ± 0.03	-0.05 ± 0.03	-0.08 ± 0.04	0.24 ± 0.06	-0.13 ± 0.06
SML	$-0.05 \pm \mathit{0.28}$	-0.18 ± 0.03	-0.04 ± 0.03	$-0.08 \pm \mathit{0.67}$	0.25 ± 0.06	-0.13 ± 0.06
EML	-0.03 ± 0.02	-0.18 ± 0.03	-0.04 ± 0.03	-0.08 ± 0.04	0.25 ± 0.05	-0.13 ± 0.05
UML	$\mathit{-0.52} \pm 0.02$	-0.17 ± 0.03	-0.03 ± 0.03	$\mathit{1.10} \pm 0.04$	0.23 ± 0.05	-0.11 ± 0.05

Table 3: Fitting results of $A_{LU}(\phi; \boldsymbol{\theta}) = c_0 + s_1 \sin\phi + c_1 \cos\phi$ by the methods of LS, SML, EML and UML (without beam-helicity balancing).

	00d0 unpol. e^+				
	$c_0 \pm (stat.)$	$s_1 \pm (stat.)$	$c_1 \pm (stat.)$	$s_2 \pm (stat.)$	$c_2 \pm (stat.)$
LS	-0.03 ± 0.02	-0.18 ± 0.03	-0.05 ± 0.03	0.01 ± 0.03	0.03 ± 0.03
SML	-0.03 ± 0.28	-0.18 ± 0.03	-0.04 ± 0.03	-0.00 ± 0.03	0.04 ± 0.03
EML	-0.03 ± 0.02	-0.18 ± 0.03	-0.04 ± 0.03	-0.00 ± 0.03	0.03 ± 0.03
UML	-0.52 ± 0.02	-0.17 ± 0.03	-0.04 ± 0.03	-0.00 ± 0.03	0.03 ± 0.03
	05b1 un.+pol. e^-				
	$c_0 \pm (stat.)$	$s_1 \pm (stat.)$	$c_1 \pm (stat.)$	$s_2 \pm (stat.)$	$c_2 \pm (stat.)$
LS	-0.07 ± 0.04	0.25 ± 0.06	-0.13 ± 0.06	0.07 ± 0.06	-0.07 ± 0.06
SML	-0.08 ± 0.66	0.25 ± 0.06	-0.13 ± 0.06	0.05 ± 0.06	-0.04 ± 0.06
EML	-0.08 ± 0.04	0.25 ± 0.05	-0.13 ± 0.05	0.06 ± 0.05	-0.02 ± 0.05
UML	1.10 ± 0.04	0.24 ± 0.05	-0.12 ± 0.05	0.06 ± 0.05	-0.00 ± 0.05

Table 4: Fitting results of $A_{LU}(\phi;\boldsymbol{\theta}) = c_0 + s_1 \sin\phi + c_1 \cos\phi + s_2 \sin 2\phi + c_2 \cos 2\phi$ by the methods of LS, SML, EML and UML (without beam-helicity balancing).

The results with the three parameterizations by the LS and the EML methods are shown in Fig. 2. And shown in Fig. 3 is the kinematic dependence of the $\sin\phi$ moments of $A^{\pm}_{LU}(\phi;\boldsymbol{\theta})$ obtained by the EML method.

3.2.3 Results with Detection Efficiency Correction

Detailed results with detection efficiency correction will be described in future reports for systematic studies. Preliminary results show that the asymmetries are not sensitive to the H0 efficiency correction.

3.3 Simultaneous extraction of BCA and BSAs

Considering the beam-charge effect, during the infinitesimal time interval $[t,\, t+dt]$, the expected number of the exclusive events observed in the phase space region $[\mathbf{x},\, \mathbf{x}+d\mathbf{x}]$ is proportional to the luminosity L, the detection efficiency ϵ and the cross section given in Eq. (14), i.e.

$$L(t)\epsilon(\mathbf{x},t)\sigma^0_{UU}(\mathbf{x})[1+\eta(t)A_C(\mathbf{x};\boldsymbol{\theta}) + P(t)A^{DVCS}_{LU}(\mathbf{x};\boldsymbol{\theta}) + \eta(t)P(t)A^{\mathcal{I}}_{LU}(\mathbf{x};\boldsymbol{\theta})], \quad (47)$$

where $\eta(t)$ is the beam charge depending on the data taking periods.

Thus the extended p.d.f. of the total observed exclusive events in $\mathbf{x}$, P and η reads

$$\begin{aligned}&\mathcal{N}(\mathbf{x},P,\eta;\boldsymbol{\theta})\\ &= \mathcal{L}(P,\eta)\epsilon(\mathbf{x},P,\eta)\sigma^0_{UU}(\mathbf{x})[1+\eta A_C(\mathbf{x};\boldsymbol{\theta}) + PA^{DVCS}_{LU}(\mathbf{x};\boldsymbol{\theta}) + \eta P A^{\mathcal{I}}_{LU}(\mathbf{x};\boldsymbol{\theta})], \quad (48)\end{aligned}$$

where

$$\mathcal{L}(P',\eta')dP = \sum_{P'<P(t)<P'+dP,\eta(t)=\eta'} L(t)dt. \quad (49)$$

Similarly to the deduction of Eq. (38), the normalization

$$\mathbb{N}(\boldsymbol{\theta}) = \iint_{\eta=1} \mathcal{N}(\mathbf{x}, P, \eta; \boldsymbol{\theta}) d\mathbf{x} dP + \iint_{\eta=-1} \mathcal{N}(\mathbf{x}, P, \eta; \boldsymbol{\theta}) d\mathbf{x} dP \tag{50}$$

is approximated as

$$\mathbb{N}(\boldsymbol{\theta}) = \sum_{i}^{N} K(\mathbf{x}_i; P_i, \eta_i) \times [M_1(\mathbf{x}_i) + M_2(\mathbf{x}_i) A_C(\mathbf{x}_i; \boldsymbol{\theta}) + M_3(\mathbf{x}_i) A_{LU}^{DVCS}(\mathbf{x}_i; \boldsymbol{\theta}) + M_4(\mathbf{x}_i) A_{LU}^{\mathcal{I}}(\mathbf{x}_i; \boldsymbol{\theta})], \tag{51}$$

with

$$K(\mathbf{x}; P, \eta) = \begin{cases} \frac{1}{2} \frac{1}{\overrightarrow{\mathcal{L}}^{+} \langle \overrightarrow{\epsilon}^{+} \rangle(\mathbf{x})} \frac{1}{1 - \langle \overrightarrow{P}^{+} \rangle(\mathbf{x}) / \langle \overleftarrow{P}^{+} \rangle(\mathbf{x})} & (P > 0, \eta = 1) \\ \frac{1}{2} \frac{1}{\overleftarrow{\mathcal{L}}^{+} \langle \overleftarrow{\epsilon}^{+} \rangle(\mathbf{x})} \frac{1}{1 - \langle \overleftarrow{P}^{+} \rangle(\mathbf{x}) / \langle \overrightarrow{P}^{+} \rangle(\mathbf{x})} & (P < 0, \eta = 1) \\ \frac{1}{2} \frac{1}{\overrightarrow{\mathcal{L}}^{-} \langle \overrightarrow{\epsilon}^{-} \rangle(\mathbf{x})} \frac{1}{1 - \langle \overrightarrow{P}^{-} \rangle(\mathbf{x}) / \langle \overleftarrow{P}^{-} \rangle(\mathbf{x})} & (P > 0, \eta = -1) \\ \frac{1}{2} \frac{1}{\overleftarrow{\mathcal{L}}^{-} \langle \overleftarrow{\epsilon}^{-} \rangle(\mathbf{x})} \frac{1}{1 - \langle \overleftarrow{P}^{-} \rangle(\mathbf{x}) / \langle \overrightarrow{P}^{-} \rangle(\mathbf{x})} & (P < 0, \eta = -1) \end{cases}, \tag{52}$$

and

$$M_1(\mathbf{x}) = \mathbb{L}^{+} \langle \epsilon^{+} \rangle(\mathbf{x}) + \mathbb{L}^{-} \langle \epsilon^{-} \rangle(\mathbf{x}) \tag{53}$$
$$M_2(\mathbf{x}) = \mathbb{L}^{+} \langle \epsilon^{+} \rangle(\mathbf{x}) - \mathbb{L}^{-} \langle \epsilon^{-} \rangle(\mathbf{x}), \tag{54}$$
$$M_3(\mathbf{x}) = \mathbb{L}^{+} \langle \epsilon^{+} \rangle(\mathbf{x}) \langle P^{+} \rangle(\mathbf{x}) + \mathbb{L}^{-} \langle \epsilon^{-} \rangle(\mathbf{x}) \langle P^{-} \rangle(\mathbf{x}), \tag{55}$$
$$M_4(\mathbf{x}) = \mathbb{L}^{+} \langle \epsilon^{+} \rangle(\mathbf{x}) \langle P^{+} \rangle(\mathbf{x}) - \mathbb{L}^{-} \langle \epsilon^{-} \rangle(\mathbf{x}) \langle P^{-} \rangle(\mathbf{x}), \tag{56}$$

where the superscripts $\pm$ in $\mathbb{L}$, ϵ and P indicate that the quantities are integrated or averaged over the $e^{\pm}$ beam data sample.

The standard p.d.f. $p(\mathbf{x}, P, \eta; \boldsymbol{\theta})$ reads

$$p(\mathbf{x}, P, \eta; \boldsymbol{\theta}) = \frac{\mathcal{N}(\mathbf{x}, P, \eta; \boldsymbol{\theta})}{\mathbb{N}(\boldsymbol{\theta})}. \tag{57}$$

The negative log-likelihood function $-\ln \mathscr{L}(\boldsymbol{\theta})$ of the SML, EML and UML methods are given as

$$-\ln \mathscr{L}_{SML}(\boldsymbol{\theta}) = -\sum_{i}^{N} \ln[1 + \eta_i A_C(\mathbf{x}_i; \boldsymbol{\theta}) + P_i A_{LU}^{DVCS}(\mathbf{x}_i; \boldsymbol{\theta}) + \eta_i P_i A_{LU}^{\mathcal{I}}(\mathbf{x}_i; \boldsymbol{\theta})] + N \ln \mathbb{N}(\boldsymbol{\theta}) \tag{58}$$

$$-\ln \mathscr{L}_{EML}(\boldsymbol{\theta}) = -\sum_{i}^{N} \ln[1 + \eta_i A_C(\mathbf{x}_i; \boldsymbol{\theta}) + P_i A_{LU}^{DVCS}(\mathbf{x}_i; \boldsymbol{\theta}) + \eta_i P_i A_{LU}^{\mathcal{I}}(\mathbf{x}_i; \boldsymbol{\theta})] + \mathbb{N}(\boldsymbol{\theta}) \tag{59}$$

$$-\ln \mathscr{L}_{UML}(\boldsymbol{\theta}) = -\sum_{i}^{N} \ln[1 + \eta_i A_C(\mathbf{x}_i; \boldsymbol{\theta}) + P_i A_{LU}^{DVCS}(\mathbf{x}_i; \boldsymbol{\theta}) + \eta_i P_i A_{LU}^{\mathcal{I}}(\mathbf{x}_i; \boldsymbol{\theta})]. \tag{60}$$

In the case that the detection efficiency is set to unity in Eqs. (52)-(56), Eq. (51) reduces to

$$\mathbb{N}(\boldsymbol{\theta}) = \sum_{i}^{N} K(P_i, \eta_i)[M_1 + M_2 A_C(\mathbf{x}_i; \boldsymbol{\theta}) + M_3 A_{LU}^{DVCS}(\mathbf{x}_i; \boldsymbol{\theta}) + M_4 A_{LU}^{\mathcal{I}}(\mathbf{x}_i; \boldsymbol{\theta})], \tag{61}$$

with

$$K(P, \eta) = \begin{cases} \frac{1}{2}\frac{1}{\overrightarrow{\mathcal{L}}^{+}}\frac{1}{1-\langle\overrightarrow{P}^{+}\rangle/\langle\overleftarrow{P}^{+}\rangle} & (P > 0, \eta = 1) \\ \frac{1}{2}\frac{1}{\overleftarrow{\mathcal{L}}^{+}}\frac{1}{1-\langle\overleftarrow{P}^{+}\rangle/\langle\overrightarrow{P}^{+}\rangle} & (P < 0, \eta = 1) \\ \frac{1}{2}\frac{1}{\overrightarrow{\mathcal{L}}^{-}}\frac{1}{1-\langle\overrightarrow{P}^{-}\rangle/\langle\overleftarrow{P}^{-}\rangle} & (P > 0, \eta = -1) \\ \frac{1}{2}\frac{1}{\overleftarrow{\mathcal{L}}^{-}}\frac{1}{1-\langle\overleftarrow{P}^{-}\rangle/\langle\overrightarrow{P}^{-}\rangle} & (P < 0, \eta = -1) \end{cases}, \tag{62}$$

and

$$M_1 = \mathbb{L}^{+} + \mathbb{L}^{-}, \tag{63}$$

$$M_2 = \mathbb{L}^{+} - \mathbb{L}^{-}, \tag{64}$$

$$M_3 = \mathbb{L}^{+}\langle P^{+}\rangle + \mathbb{L}^{-}\langle P^{-}\rangle, \tag{65}$$

$$M_4 = \mathbb{L}^{+}\langle P^{+}\rangle - \mathbb{L}^{-}\langle P^{-}\rangle. \tag{66}$$

3.3.1 Results without Detection Efficiency Correction

$A_C(\phi; \boldsymbol{\theta})$, $A_{LU}^{DVCS}(\phi; \boldsymbol{\theta})$ and $A_{LU}^{\mathcal{I}}(\phi; \boldsymbol{\theta})$ can be calculated in each ϕ bin [6], and thus $\boldsymbol{\theta}$ can be extracted with the LS method. The results are shown in the left panels of Figs. 4 and 5.

Fitting results by the EML method are shown in the right panels of Figs. 4 and 5. The results are insensitive to the fitting functions, as can be seen in the kinematic dependence shown in Fig. 6. Within statistical uncertainties, $A_{LU}^{DVCS, \sin\phi} \simeq 0$ and $A_{LU}^{\eta, \sin\phi} \approx \eta A_{LU}^{\mathcal{I}, \sin\phi}$ are observed (see Fig. 7).

Again, the non-vanishing c_0 and c_1 are not expected by theory in Eqs. (16) and (17). They should be related to the non-vanishing c_0 and c_1 in $A_{LU}^{\pm}(\mathbf{x}; \boldsymbol{\theta})$ observed in the previous section.

[6] The asymmetries can also be obtained from Eq. (48):

$$A_C = \frac{-\overleftarrow{P}^{+}(\overrightarrow{P}^{-} - \overleftarrow{P}^{-})\overrightarrow{n}^{+} + \overrightarrow{P}^{+}(\overrightarrow{P}^{-} - \overleftarrow{P}^{-})\overleftarrow{n}^{+} + \overleftarrow{P}^{-}(\overrightarrow{P}^{+} - \overleftarrow{P}^{+})\overrightarrow{n}^{-} - \overrightarrow{P}^{-}(\overrightarrow{P}^{+} - \overleftarrow{P}^{+})\overleftarrow{n}^{-}}{-\overleftarrow{P}^{+}(\overrightarrow{P}^{-} - \overleftarrow{P}^{-})\overrightarrow{n}^{+} + \overrightarrow{P}^{+}(\overrightarrow{P}^{-} - \overleftarrow{P}^{-})\overleftarrow{n}^{+} - \overleftarrow{P}^{-}(\overrightarrow{P}^{+} - \overleftarrow{P}^{+})\overrightarrow{n}^{-} + \overrightarrow{P}^{-}(\overrightarrow{P}^{+} - \overleftarrow{P}^{+})\overleftarrow{n}^{-}}, \tag{67}$$

$$A_{LU}^{DVCS} = \frac{(\overrightarrow{P}^{-} - \overleftarrow{P}^{-})\overrightarrow{n}^{+} - (\overrightarrow{P}^{-} - \overleftarrow{P}^{-})\overleftarrow{n}^{+} + (\overrightarrow{P}^{+} - \overleftarrow{P}^{+})\overrightarrow{n}^{-} - (\overrightarrow{P}^{+} - \overleftarrow{P}^{+})\overleftarrow{n}^{-}}{-\overleftarrow{P}^{+}(\overrightarrow{P}^{-} - \overleftarrow{P}^{-})\overrightarrow{n}^{+} + \overrightarrow{P}^{+}(\overrightarrow{P}^{-} - \overleftarrow{P}^{-})\overleftarrow{n}^{+} - \overleftarrow{P}^{-}(\overrightarrow{P}^{+} - \overleftarrow{P}^{+})\overrightarrow{n}^{-} + \overrightarrow{P}^{-}(\overrightarrow{P}^{+} - \overleftarrow{P}^{+})\overleftarrow{n}^{-}}, \tag{68}$$

$$A_{LU}^{\mathcal{I}} = \frac{(\overrightarrow{P}^{-} - \overleftarrow{P}^{-})\overrightarrow{n}^{+} - (\overrightarrow{P}^{-} - \overleftarrow{P}^{-})\overleftarrow{n}^{+} - (\overrightarrow{P}^{+} - \overleftarrow{P}^{+})\overrightarrow{n}^{-} + (\overrightarrow{P}^{+} - \overleftarrow{P}^{+})\overleftarrow{n}^{-}}{-\overleftarrow{P}^{+}(\overrightarrow{P}^{-} - \overleftarrow{P}^{-})\overrightarrow{n}^{+} + \overrightarrow{P}^{+}(\overrightarrow{P}^{-} - \overleftarrow{P}^{-})\overleftarrow{n}^{+} - \overleftarrow{P}^{-}(\overrightarrow{P}^{+} - \overleftarrow{P}^{+})\overrightarrow{n}^{-} + \overrightarrow{P}^{-}(\overrightarrow{P}^{+} - \overleftarrow{P}^{+})\overleftarrow{n}^{-}}. \tag{69}$$

with $P = \langle P\rangle(\mathbf{x})$, $n = \frac{\int \mathcal{N}(\mathbf{x}, P, \eta; \boldsymbol{\theta}) dP}{\mathbb{L}\langle\epsilon\rangle(\mathbf{x})}$.

4 Summary

In this report, we introduce the BSAs A_{LU}^{DVCS} and $A_{LU}^{\mathcal{I}}$ induced by pure DVCS and interference contributions. We discuss the extraction methods to simultaneously extract these BSAs and the BCA and present the first analysis results of these asymmetries. Comparing to the other Maximum Likelihood methods, we found that only the Extended Maximum Likelihood method gives correct estimates for the asymmetries when constant terms are allowed. Comparison among the methods of LS, SML, EML and UML are summarized as follows:

- results are consistent if the parameterization includes a constant term;
- when a constant term is allowed, the methods of SML and UML fail, while the EML method and LS method provide correct estimates;
- generally, the EML method is better than the LS method: the latter may involve with worse resolution in ϕ due to binning; when the statistics is low, the estimated mean values of the parameters might be biased and the uncertainties might be underestimated [10]. On the other hand, the EML method is free of these problems.

The analysis was based on a sample of about 7700 positron and 5800 electron exclusive events from the 2000 and 2005 data with the proton target. Systematic studies are not performed yet, hence the uncertainties given are statistical only and the results are very preliminary. Nevertheless, we found that

- The results of $A_C^{\cos\phi}$ are consistent with the DC38 ones (Fig. 8), which were based on a sample of about 9500 positron and 700 electron exclusive events. The statistical uncertainties of the current ones are smaller by about a factor of two.
- Within statistical uncertainties, $A_{LU}^{e^{\pm},\sin\phi} \approx \pm A_{LU}^{\mathcal{I},\sin\phi}$ and $A_{LU}^{DVCS,\sin\phi} \simeq 0$ are observed, which are in agreement with theoretical expectation.

A problem reveled in this analysis is the 2σ effect of the non-physical even harmonic coefficients c_0 and c_1 in $A_{LU}^{\pm}(\mathbf{x};\boldsymbol{\theta})$, $A_{LU}^{DVCS}(\mathbf{x};\boldsymbol{\theta})$ and $A_{LU}^{\mathcal{I}}(\mathbf{x};\boldsymbol{\theta})$. They need to be investigated in future studies.

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Appendix

Normalization for BSA Extraction

Let $\mathbf{x}$ denote the set of kinematic observables of detected particles in exclusive events, $P(t)$ the beam polarization ($-1 < P(t) < 1$), $\epsilon(\mathbf{x}, t)$ the detection efficiency, $dL(t)/dt$ the luminosity. From Eq. (10), we can write down the event number distribution:

$$d\mathcal{N}(\mathbf{x}, t) = \epsilon(\mathbf{x}, t) d\sigma_{UU}(\mathbf{x})[1 + P(t)A_{LU}(\mathbf{x})]dL(t). \tag{70}$$

And the DIS event number distribution reads

$$dN_{DIS}(\mathbf{x}, t) = \epsilon(\mathbf{x}, t) d\sigma_{DIS} dL(t). \tag{71}$$

In the following deduction, we define the integrated luminosity $\mathcal{L}$, the averaged detection efficiency $\epsilon(\mathbf{x})$ and the averaged polarization $\langle P\rangle(\mathbf{x})$:

$$\mathcal{L} = \int_t dL(t), \tag{72}$$

$$\epsilon(\mathbf{x}) = \frac{\int_t \epsilon(\mathbf{x},t)dL(t)}{\int_t dL(t)}, \tag{73}$$

$$\langle P\rangle(\mathbf{x}) = \frac{\int_t P(t)\epsilon(\mathbf{x},t)dL(t)}{\int_t \epsilon(\mathbf{x},t)dL(t)}. \tag{74}$$

and the notation of right/left arrows to indicate that quantities are averaged or integrated over the time when the beam polarization is positive/negative, e.g. $\overrightarrow{\mathcal{L}} \equiv \int_{\overrightarrow{t}} dL(t)$ with $\overrightarrow{t} \equiv t\big|_{P(t)>0}$.

From Eq. (70),

$$d\overrightarrow{\mathcal{N}}(\mathbf{x}) = d\sigma_{UU}(\mathbf{x})\int_{\overrightarrow{t}} \epsilon(\mathbf{x},t)[1+P(t)A_{LU}(\mathbf{x})]dL(t) \tag{75}$$

$$= d\sigma_{UU}(\mathbf{x})\int_{\overrightarrow{t}} \epsilon(\mathbf{x},t)dL(t) \times [1 + A_{LU}(\mathbf{x})\frac{\int_{\overrightarrow{t}} P(t)\epsilon(\mathbf{x},t)dL(t)}{\int_{\overrightarrow{t}} \epsilon(\mathbf{x},t)dL(t)}], \tag{76}$$

in which

$$\int_{\overrightarrow{t}} \epsilon(\mathbf{x},t)dL(t) \tag{77}$$

$$= \overrightarrow{\mathcal{L}}\frac{\int_{\overrightarrow{t}} \epsilon(\mathbf{x},t)dL(t)}{\int_{\overrightarrow{t}} dL(t)} \tag{78}$$

$$= \overrightarrow{\mathcal{L}}\frac{\int_{\overrightarrow{t}} \epsilon(\mathbf{x},t)d\sigma_{DIS}(\mathbf{x})dL(t)}{\int_{\overrightarrow{t}} d\sigma_{DIS}(\mathbf{x})dL(t)} \tag{79}$$

$$\equiv \overrightarrow{\mathcal{L}}\,\overrightarrow{\epsilon}(\mathbf{x}), \tag{80}$$

$$\frac{\int_{\overrightarrow{t}} P(t)\epsilon(\mathbf{x},t)dL(t)}{\int_{\overrightarrow{t}} \epsilon(\mathbf{x},t)dL(t)} \tag{81}$$

$$= \frac{\int_{\overrightarrow{t}} P(t)\epsilon(\mathbf{x},t)d\sigma_{DIS}(\mathbf{x})dL(t)}{\int_{\overrightarrow{t}} \epsilon(\mathbf{x},t)d\sigma_{DIS}(\mathbf{x})dL(t)} \tag{82}$$

$$= \frac{\int_{\overrightarrow{t}} P(t)d\mathcal{N}_{DIS}(\mathbf{x},t)}{\int_{\overrightarrow{t}} dN_{DIS}(\mathbf{x},t)} \tag{83}$$

$$\equiv \langle\overrightarrow{P}\rangle(\mathbf{x}). \tag{84}$$

Thus we have

$$d\overrightarrow{\mathcal{N}}(\mathbf{x}) = d\sigma_{UU}(\mathbf{x})\overrightarrow{\mathcal{L}}\,\overrightarrow{\epsilon}(\mathbf{x})[1+\langle\overrightarrow{P}\rangle(\mathbf{x})A_{LU}(\mathbf{x})], \tag{85}$$

and similarly

$$d\overleftarrow{\mathcal{N}}(\mathbf{x}) = d\sigma_{UU}(\mathbf{x})\overleftarrow{L}\,\overleftarrow{\epsilon}(\mathbf{x})[1+\langle\overleftarrow{P}\rangle(\mathbf{x})A_{LU}(\mathbf{x})]. \tag{86}$$

From the above two equations, we can solve $d\sigma_{UU}(\mathbf{x})$:

$$d\sigma_{UU}(\mathbf{x}) = \frac{1}{\frac{1}{\langle\overrightarrow{P}\rangle(\mathbf{x})} - \frac{1}{\langle\overleftarrow{P}\rangle(\mathbf{x})}} \times [\frac{d\overrightarrow{\mathcal{N}}(\mathbf{x})}{\overrightarrow{\mathcal{L}}\,\overrightarrow{\epsilon}(\mathbf{x})\langle\overrightarrow{P}\rangle(\mathbf{x})} - \frac{d\overleftarrow{\mathcal{N}}(\mathbf{x})}{\overleftarrow{L}\,\overleftarrow{\epsilon}(\mathbf{x})\langle\overleftarrow{P}\rangle(\mathbf{x})}] \tag{87}$$

$$\equiv \overrightarrow{K}(\mathbf{x})d\overrightarrow{\mathcal{N}}(\mathbf{x}) + \overleftarrow{K}(\mathbf{x})d\overleftarrow{\mathcal{N}}(\mathbf{x}). \tag{88}$$

The normalization can then be calculated:

$$d\mathscr{N}(\mathbf{x}) = \int_t d\mathscr{N}(\mathbf{x}, t) \tag{89}$$
$$= d\sigma_{UU}(\mathbf{x})\left[\int_t \epsilon(\mathbf{x}, t)dL(t) + A_{LU}(\mathbf{x})\int_t \epsilon(\mathbf{x}, t)P(t)dL(t)\right] \tag{90}$$
$$= d\sigma_{UU}(\mathbf{x})[M_1(\mathbf{x}) + M_2(\mathbf{x})A_{LU}(\mathbf{x})] \tag{91}$$
$$= [\overrightarrow{K}(\mathbf{x})d\overrightarrow{\mathscr{N}}(\mathbf{x}) + \overleftarrow{K}(\mathbf{x})d\overleftarrow{\mathscr{N}}(\mathbf{x})] \times [M_1(\mathbf{x}) + M_2(\mathbf{x})A_{LU}(\mathbf{x})], \tag{92}$$

$$\mathscr{N} = \int d\mathscr{N}(\mathbf{x}) = \sum_i^N K'(\mathbf{x}_i; P_i)[M_1(\mathbf{x}_i) + M_2(\mathbf{x}_i)A_{LU}(\mathbf{x}_i)], \tag{93}$$

with

$$K'(\mathbf{x}; P) = \begin{cases} \frac{1}{\overrightarrow{\mathcal{L}}\overrightarrow{\epsilon}(\mathbf{x})}\frac{1}{1-\frac{\langle\overrightarrow{P}\rangle(\mathbf{x})}{\langle\overleftarrow{P}\rangle(\mathbf{x})}} & (P > 0) \\ \frac{1}{\overleftarrow{L}\overleftarrow{\epsilon}(\mathbf{x})}\frac{1}{1-\frac{\langle\overleftarrow{P}\rangle(\mathbf{x})}{\langle\overrightarrow{P}\rangle(\mathbf{x})}} & (P < 0) \end{cases}, \tag{94}$$

and

$$M_1(\mathbf{x}) = \mathcal{L}\epsilon(\mathbf{x}), \tag{95}$$
$$M_2(\mathbf{x}) = \mathcal{L}\epsilon(\mathbf{x})\langle P\rangle(\mathbf{x}). \tag{96}$$

The EML minimization function reads

$$-\ln\mathscr{L}_{EML}(\boldsymbol{\theta}) = -\sum_i^N \ln[1 + P_i A_{LU}(\mathbf{x}_i; \boldsymbol{\theta})] + \mathscr{N} \tag{97}$$
$$= \sum_i^N \Big\{ -\ln[1 + P_i A_{LU}(\mathbf{x}_i; \boldsymbol{\theta})] + K'(\mathbf{x}_i; P_i)[M_1(\mathbf{x}_i) + M_2(\mathbf{x}_i)A_{LU}(\mathbf{x}_i)]\Big\}. \tag{98}$$

Normalization for Simultaneous Extraction

From Eq. (14), we have the event number distribution

$$d\mathscr{N}(\mathbf{x}, t) \tag{99}$$
$$= \epsilon(\mathbf{x}, t)d\sigma^0_{UU}(\mathbf{x}) \tag{100}$$
$$\times [1 + \eta(t)A_C(\mathbf{x}) + P(t)A^{DVCS}_{LU}(\mathbf{x}) + \eta(t)P(t)A^{\mathcal{I}}_{LU}(\mathbf{x})]dL(t), \tag{101}$$

in which

$$\eta(t) = \begin{cases} \eta^+ (=1) \\ \eta^- (=-1) \end{cases} . \tag{102}$$

$+/-$ denotes that the quantity is integrated or averaged over the time when the beam charge is positive/negative. And $t^+ \equiv t|_{\eta(t)=1}$.

$$d\overrightarrow{\mathcal{N}}^+(\mathbf{x}) = \int_{\overrightarrow{t}^+} d\mathcal{N}(\mathbf{x}, t) \tag{103}$$

$$= d\sigma_{UU}^0(\mathbf{x}) \int_{\overrightarrow{t}^+} \epsilon(\mathbf{x}, t) dL(t) \times \left[1 + A_C(\mathbf{x}) \frac{\int_{\overrightarrow{t}^+} \epsilon(\mathbf{x}, t)\eta(t) dL(t)}{\int_{\overrightarrow{t}^+} \epsilon(\mathbf{x}, t) dL(t)}\right. \tag{104}$$

$$\left. + A_{LU}^{DVCS}(\mathbf{x}) \frac{\int_{\overrightarrow{t}^+} \epsilon(\mathbf{x}, t) P(t) dL(t)}{\int_{\overrightarrow{t}^+} \epsilon(\mathbf{x}, t) dL(t)} + A_{LU}^{\mathcal{I}}(\mathbf{x}) \frac{\int_{\overrightarrow{t}^+} \epsilon(\mathbf{x}, t)\eta(t) P(t) dL(t)}{\int_{\overrightarrow{t}^+} \epsilon(\mathbf{x}, t) dL(t)}\right] \tag{105}$$

$$= d\sigma_{UU}^0(\mathbf{x}) \overrightarrow{\mathcal{L}}^+ \overrightarrow{\epsilon}^+(\mathbf{x}) \tag{106}$$

$$\times \left[1 + \eta^+ A_C(\mathbf{x}) + \langle \overrightarrow{P} \rangle^+(\mathbf{x}) A_{LU}^{DVCS}(\mathbf{x}) + \eta^+ \langle \overrightarrow{P} \rangle^+(\mathbf{x}) A_{LU}^{\mathcal{I}}(\mathbf{x})\right]. \tag{107}$$

And similarly,

$$d\overleftarrow{\mathcal{N}}^+(\mathbf{x}) = d\sigma_{UU}^0(\mathbf{x}) \overleftarrow{L}^+ \overleftarrow{\epsilon}^+(\mathbf{x}) \tag{108}$$

$$\times \left[1 + \eta^+ A_C(\mathbf{x}) + \langle \overleftarrow{P} \rangle^+(\mathbf{x}) A_{LU}^{DVCS}(\mathbf{x}) + \eta^+ \langle \overleftarrow{P} \rangle^+(\mathbf{x}) A_{LU}^{\mathcal{I}}(\mathbf{x})\right], \tag{109}$$

$$d\overrightarrow{\mathcal{N}}^-(\mathbf{x}) = d\sigma_{UU}^0(\mathbf{x}) \overrightarrow{\mathcal{L}}^- \overrightarrow{\epsilon}^-(\mathbf{x}) \tag{110}$$

$$\times \left[1 + \eta^- A_C(\mathbf{x}) + \langle \overrightarrow{P} \rangle^-(\mathbf{x}) A_{LU}^{DVCS}(\mathbf{x}) + \eta^- \langle \overrightarrow{P} \rangle^-(\mathbf{x}) A_{LU}^{\mathcal{I}}(\mathbf{x})\right], \tag{111}$$

$$d\overleftarrow{\mathcal{N}}^-(\mathbf{x}) = d\sigma_{UU}^0(\mathbf{x}) \overleftarrow{L}^- \overleftarrow{\epsilon}^-(\mathbf{x}) \tag{112}$$

$$\times \left[1 + \eta^- A_C(\mathbf{x}) + \langle \overleftarrow{P} \rangle^-(\mathbf{x}) A_{LU}^{DVCS}(\mathbf{x}) + \eta^- \langle \overleftarrow{P} \rangle^-(\mathbf{x}) A_{LU}^{\mathcal{I}}(\mathbf{x})\right]. \tag{113}$$

$d\sigma^0_{UU}(\mathbf{x})$ can be solved from the above 4 equations:[7]

$$
\begin{aligned}
d\sigma^0_{UU}(\mathbf{x}) =& \overrightarrow{K}^{+}(\mathbf{x})\frac{d\overrightarrow{\mathcal{N}}^{+}(\mathbf{x})}{\overrightarrow{\mathcal{L}}^{+}\overrightarrow{\epsilon}^{+}(\mathbf{x})} + \overleftarrow{K}^{+}(\mathbf{x})\frac{d\overleftarrow{\mathcal{N}}^{+}(\mathbf{x})}{\overleftarrow{L}^{+}\overleftarrow{\epsilon}^{+}(\mathbf{x})} && (117)\\
&+ \overrightarrow{K}^{-}(\mathbf{x})\frac{d\overrightarrow{\mathcal{N}}^{-}(\mathbf{x})}{\overrightarrow{\mathcal{L}}^{-}\overrightarrow{\epsilon}^{-}(\mathbf{x})} + \overleftarrow{K}^{-}(\mathbf{x})\frac{d\overleftarrow{\mathcal{N}}^{-}(\mathbf{x})}{\overleftarrow{L}^{-}\overleftarrow{\epsilon}^{-}(\mathbf{x})}, && (118)
\end{aligned}
$$

with

$$
\begin{aligned}
\overrightarrow{K}^{+}(\mathbf{x}) &= \frac{1}{1-\frac{\eta^+}{\eta^-}}\frac{1}{1-\frac{\langle\overrightarrow{P}\rangle^{+}(\mathbf{x})}{\langle\overleftarrow{P}\rangle^{+}(\mathbf{x})}} = \frac{1}{2}\frac{1}{1-\frac{\langle\overrightarrow{P}\rangle^{+}(\mathbf{x})}{\langle\overleftarrow{P}\rangle^{+}(\mathbf{x})}}, && (119)\\
\overleftarrow{K}^{+}(\mathbf{x}) &= \frac{1}{1-\frac{\eta^+}{\eta^-}}\frac{1}{1-\frac{\langle\overleftarrow{P}\rangle^{+}(\mathbf{x})}{\langle\overrightarrow{P}\rangle^{+}(\mathbf{x})}} = \frac{1}{2}\frac{1}{1-\frac{\langle\overleftarrow{P}\rangle^{+}(\mathbf{x})}{\langle\overrightarrow{P}\rangle^{+}(\mathbf{x})}}, && (120)\\
\overrightarrow{K}^{-}(\mathbf{x}) &= \frac{1}{1-\frac{\eta^-}{\eta^+}}\frac{1}{1-\frac{\langle\overrightarrow{P}\rangle^{-}(\mathbf{x})}{\langle\overleftarrow{P}\rangle^{-}(\mathbf{x})}} = \frac{1}{2}\frac{1}{1-\frac{\langle\overrightarrow{P}\rangle^{-}(\mathbf{x})}{\langle\overleftarrow{P}\rangle^{-}(\mathbf{x})}}, && (121)\\
\overleftarrow{K}^{-}(\mathbf{x}) &= \frac{1}{1-\frac{\eta^-}{\eta^+}}\frac{1}{1-\frac{\langle\overleftarrow{P}\rangle^{-}(\mathbf{x})}{\langle\overrightarrow{P}\rangle^{-}(\mathbf{x})}} = \frac{1}{2}\frac{1}{1-\frac{\langle\overleftarrow{P}\rangle^{-}(\mathbf{x})}{\langle\overrightarrow{P}\rangle^{-}(\mathbf{x})}}. && (122)
\end{aligned}
$$

By defining

$$
K'(\mathbf{x}) \equiv \frac{K(\mathbf{x})}{L\epsilon(\mathbf{x})}, \tag{123}
$$

Eq. (118) reads

$$
d\sigma^0_{UU}(\mathbf{x}) = \overrightarrow{K'}^{+}(\mathbf{x})d\overrightarrow{\mathcal{N}}^{+}(\mathbf{x}) + (\leftarrow^{+}) + (\rightarrow^{-}) + (\leftarrow^{-}). \tag{124}
$$

[7]Also, the asymmetries can also be solved as the following. For better illustration, P is the absolute value of the mean polarization, and $n = \frac{d\mathcal{N}(\mathbf{x})}{L\epsilon(\mathbf{x})}$.

$$
\begin{aligned}
A_C =& \frac{(\overrightarrow{P}^{-}+\overleftarrow{P}^{-})\overleftarrow{P}^{+}\overrightarrow{n}^{+} + (\overrightarrow{P}^{-}+\overleftarrow{P}^{-})\overrightarrow{P}^{+}\overleftarrow{n}^{+} - (\overrightarrow{P}^{+}+\overleftarrow{P}^{+})\overleftarrow{P}^{-}\overrightarrow{n}^{-} - (\overrightarrow{P}^{+}+\overleftarrow{P}^{+})\overrightarrow{P}^{-}\overleftarrow{n}^{-}}{(\overrightarrow{P}^{-}+\overleftarrow{P}^{-})\overleftarrow{P}^{+}\overrightarrow{n}^{+} + (\overrightarrow{P}^{-}+\overleftarrow{P}^{-})\overrightarrow{P}^{+}\overleftarrow{n}^{+} + (\overrightarrow{P}^{+}+\overleftarrow{P}^{+})\overleftarrow{P}^{-}\overrightarrow{n}^{-} + (\overrightarrow{P}^{+}+\overleftarrow{P}^{+})\overrightarrow{P}^{-}\overleftarrow{n}^{-}}, && (114)\\
A_{LU}^{DVCS} =& \frac{(\overrightarrow{P}^{-}+\overleftarrow{P}^{-})\overrightarrow{n}^{+} - (\overrightarrow{P}^{-}+\overleftarrow{P}^{-})\overleftarrow{n}^{+} + (\overrightarrow{P}^{+}+\overleftarrow{P}^{+})\overrightarrow{n}^{-} - (\overrightarrow{P}^{+}+\overleftarrow{P}^{+})\overleftarrow{n}^{-}}{(\overrightarrow{P}^{-}+\overleftarrow{P}^{-})\overleftarrow{P}^{+}\overrightarrow{n}^{+} + (\overrightarrow{P}^{-}+\overleftarrow{P}^{-})\overrightarrow{P}^{+}\overleftarrow{n}^{+} + (\overrightarrow{P}^{+}+\overleftarrow{P}^{+})\overleftarrow{P}^{-}\overrightarrow{n}^{-} + (\overrightarrow{P}^{+}+\overleftarrow{P}^{+})\overrightarrow{P}^{-}\overleftarrow{n}^{-}}, && (115)\\
A_{LU}^{\mathcal{I}} =& \frac{(\overrightarrow{P}^{-}+\overleftarrow{P}^{-})\overrightarrow{n}^{+} - (\overrightarrow{P}^{-}+\overleftarrow{P}^{-})\overleftarrow{n}^{+} - (\overrightarrow{P}^{+}+\overleftarrow{P}^{+})\overrightarrow{n}^{-} + (\overrightarrow{P}^{+}+\overleftarrow{P}^{+})\overleftarrow{n}^{-}}{(\overrightarrow{P}^{-}+\overleftarrow{P}^{-})\overleftarrow{P}^{+}\overrightarrow{n}^{+} + (\overrightarrow{P}^{-}+\overleftarrow{P}^{-})\overrightarrow{P}^{+}\overleftarrow{n}^{+} + (\overrightarrow{P}^{+}+\overleftarrow{P}^{+})\overleftarrow{P}^{-}\overrightarrow{n}^{-} + (\overrightarrow{P}^{+}+\overleftarrow{P}^{+})\overrightarrow{P}^{-}\overleftarrow{n}^{-}}. && (116)
\end{aligned}
$$

The normalization can then be calculated:

$$
\begin{aligned}
d\mathscr{N}(\mathbf{x}) &= \int_t d\mathscr{N}(\mathbf{x},t) && (125)\\
&= d\sigma^0_{UU}(\mathbf{x})\Big[\int_t \epsilon(\mathbf{x},t)dL(t) + A_C(\mathbf{x})\int_t \epsilon(\mathbf{x},t)\eta(t)dL(t) && (126)\\
&+ A_{LU}^{DVCS}(\mathbf{x})\int_t \epsilon(\mathbf{x},t)P(t)dL(t) + A_{LU}^{\mathcal{I}}(\mathbf{x})\int_t \epsilon(\mathbf{x},t)\eta(t)P(t)dL(t)\Big] && (127)\\
&= d\sigma^0_{UU}(\mathbf{x})[M_1(\mathbf{x}) + M_2(\mathbf{x})A_C(\mathbf{x}) + M_3(\mathbf{x})A_{LU}^{DVCS}(\mathbf{x}) + M_4(\mathbf{x})A_{LU}^{\mathcal{I}}(\mathbf{x})] && (128)\\
&= [\overrightarrow{K'}^{+}\overrightarrow{d\mathscr{N}}^{+}(\mathbf{x}) + (\longleftarrow^{+}) + (\longrightarrow^{-}) + (\longleftarrow^{-})] && (129)\\
&\times [M_1(\mathbf{x}) + M_2(\mathbf{x})A_C(\mathbf{x}) + M_3(\mathbf{x})A_{LU}^{DVCS}(\mathbf{x}) + M_4(\mathbf{x})A_{LU}^{\mathcal{I}}(\mathbf{x})]. && (130)
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{N} &= \int d\mathscr{N}(\mathbf{x}) && (131)\\
&= \sum_i^N K'(\mathbf{x}_i; P_i, \eta_i)[M_1(\mathbf{x}_i) + M_2(\mathbf{x}_i)A_C(\mathbf{x}_i) && (132)\\
&+ M_3(\mathbf{x}_i)A_{LU}^{DVCS}(\mathbf{x}_i) + M_4(\mathbf{x}_i)A_{LU}^{\mathcal{I}}(\mathbf{x}_i)], && (133)
\end{aligned}
$$

in which

$$
K'(\mathbf{x}; P, \eta) = \begin{cases}
\frac{1}{2}\frac{1}{\overrightarrow{\mathcal{L}}^{+}\overrightarrow{\epsilon}^{+}(\mathbf{x})}\frac{1}{1-\frac{\langle\overrightarrow{P}\rangle^{+}(\mathbf{x})}{\langle\overleftarrow{P}\rangle^{+}(\mathbf{x})}} & (P > 0, \eta = 1)\\
\frac{1}{2}\frac{1}{\overleftarrow{L}^{+}\overleftarrow{\epsilon}^{+}(\mathbf{x})}\frac{1}{1-\frac{\langle\overleftarrow{P}\rangle^{+}(\mathbf{x})}{\langle\overrightarrow{P}\rangle^{+}(\mathbf{x})}} & (P < 0, \eta = 1)\\
\frac{1}{2}\frac{1}{\overrightarrow{\mathcal{L}}^{-}\overrightarrow{\epsilon}^{-}(\mathbf{x})}\frac{1}{1-\frac{\langle\overrightarrow{P}\rangle^{-}(\mathbf{x})}{\langle\overleftarrow{P}\rangle^{-}(\mathbf{x})}} & (P > 0, \eta = -1)\\
\frac{1}{2}\frac{1}{\overleftarrow{\mathcal{L}}^{-}\overleftarrow{\epsilon}^{-}(\mathbf{x})}\frac{1}{1-\frac{\langle\overleftarrow{P}\rangle^{-}(\mathbf{x})}{\langle\overrightarrow{P}\rangle^{-}(\mathbf{x})}} & (P < 0, \eta = -1)
\end{cases}, \qquad (134)
$$

and

$$
\begin{aligned}
M_1(\mathbf{x}) &= L\epsilon(\mathbf{x}) && (135)\\
M_2(\mathbf{x}) &= L^{+}\epsilon^{+}(\mathbf{x}) - L^{-}\epsilon^{-}(\mathbf{x}), && (136)\\
M_3(\mathbf{x}) &= L\epsilon(\mathbf{x})\langle P\rangle(\mathbf{x}), && (137)\\
M_4(\mathbf{x}) &= L^{+}\epsilon^{+}(\mathbf{x})\langle P\rangle^{+}(\mathbf{x}) - L^{-}\epsilon^{-}(\mathbf{x})\langle P\rangle^{-}(\mathbf{x}). && (138)
\end{aligned}
$$

The EML minimization function reads

$$-\ln \mathscr{L}_{EML}(\boldsymbol{\theta}) \tag{139}$$

$$= -\sum_{i}^{N} \ln[1 + \eta_i A_C(\mathbf{x}_i) + P_i A_{LU}^{DVCS}(\mathbf{x}_i) + \eta_i P_i A_{LU}^{\mathcal{I}}(\mathbf{x}_i)] + \mathscr{N} \tag{140}$$

$$= \sum_{i}^{N} \Bigg\{ - \ln[1 + \eta_i A_C(\mathbf{x}_i) + P_i A_{LU}^{DVCS}(\mathbf{x}_i) + \eta_i P_i A_{LU}^{\mathcal{I}}(\mathbf{x}_i)] \tag{141}$$

$$+ K'(\mathbf{x}_i; P_i, \eta_i)[M_1(\mathbf{x}_i) + M_2(\mathbf{x}_i) A_C(\mathbf{x}_i) + M_3(\mathbf{x}_i) A_{LU}^{DVCS}(\mathbf{x}_i) + M_4(\mathbf{x}_i) A_{LU}^{\mathcal{I}}(\mathbf{x}_i)] \Bigg\}. \tag{142}$$

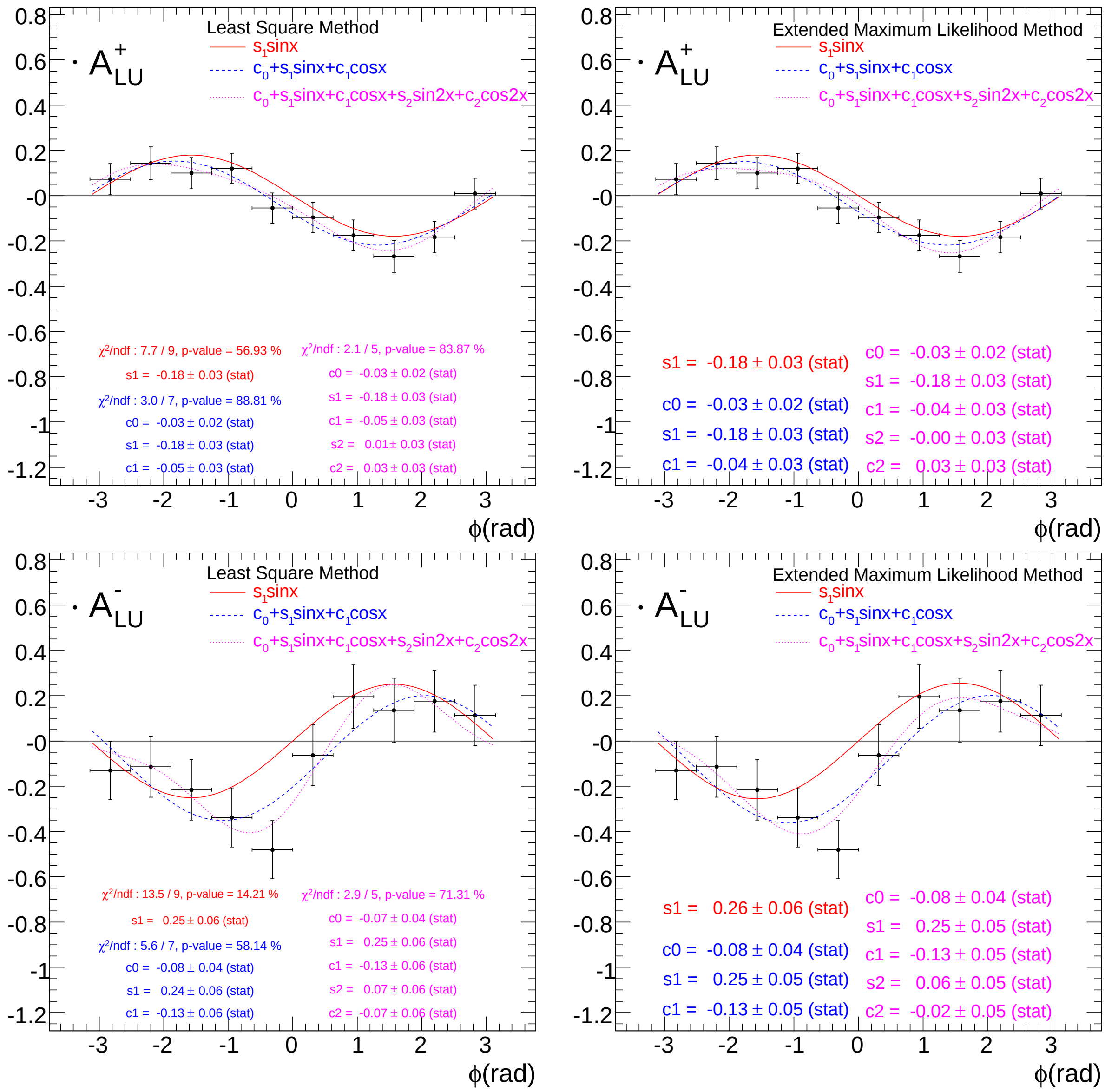

Figure 2: Results of $A_{LU}(\phi;\boldsymbol{\theta})$ obtained by the LS method and the EML method with different parameterizations.

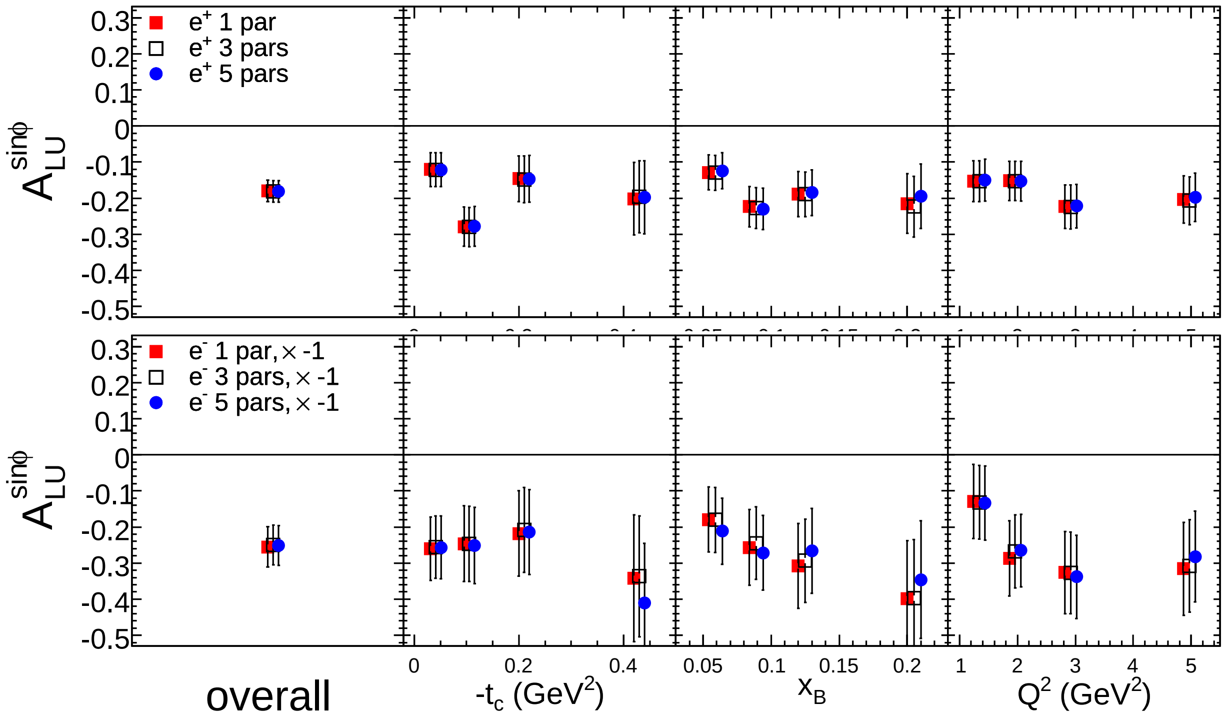

Figure 3: Kinematic dependence of the $\sin\phi$ moments of $A^{\pm}_{LU}(\phi;\boldsymbol{\theta})$. Different fit functions, which correspond to the parameterizations in Fig. 2, are denoted by the number of parameters. $A^{-,\,\sin\phi}_{LU}$ is scaled by -1.

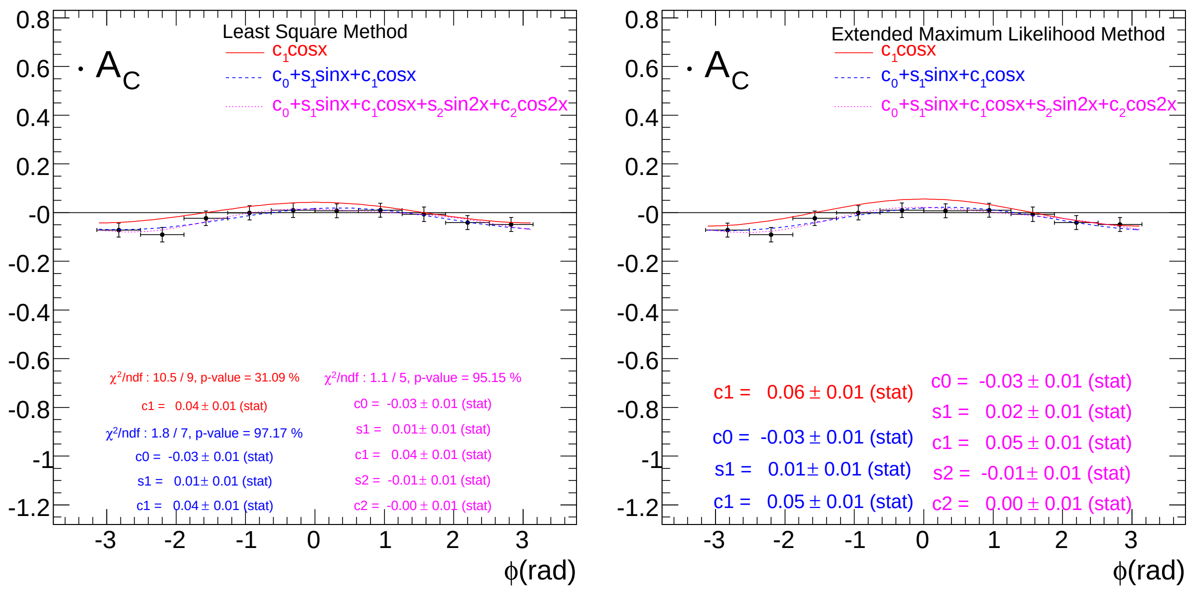

Figure 4: Results of $A_C(\phi;\boldsymbol{\theta})$ obtained by the LS method and the EML method with different parameterizations.

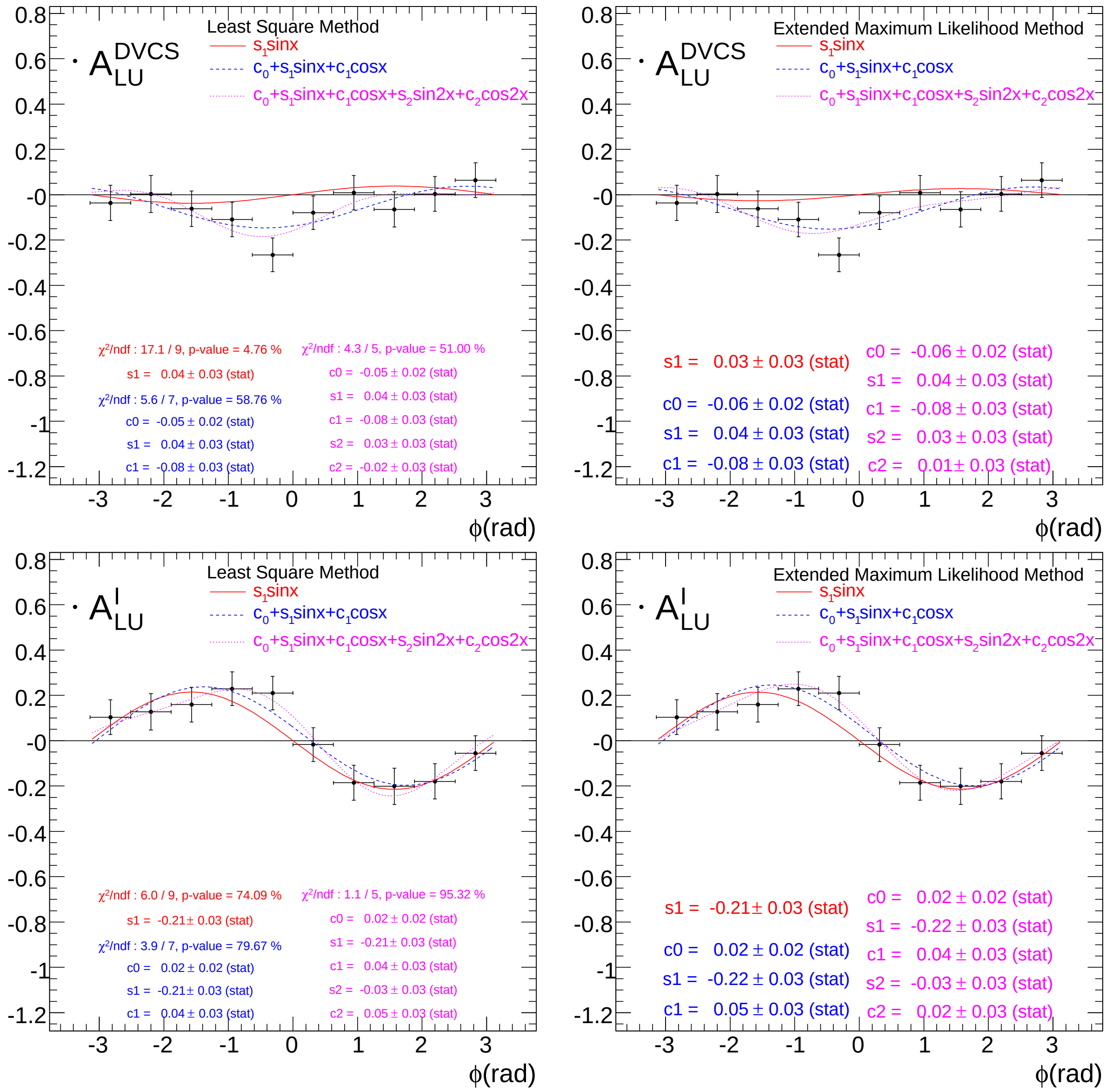

Figure 5: Results of $A_{LU}^{DVCS}(\phi;\boldsymbol{\theta})$ and $A_{LU}^{\mathcal{I}}(\phi;\boldsymbol{\theta})$ obtained by the LS method and the EML method with different parameterizations.

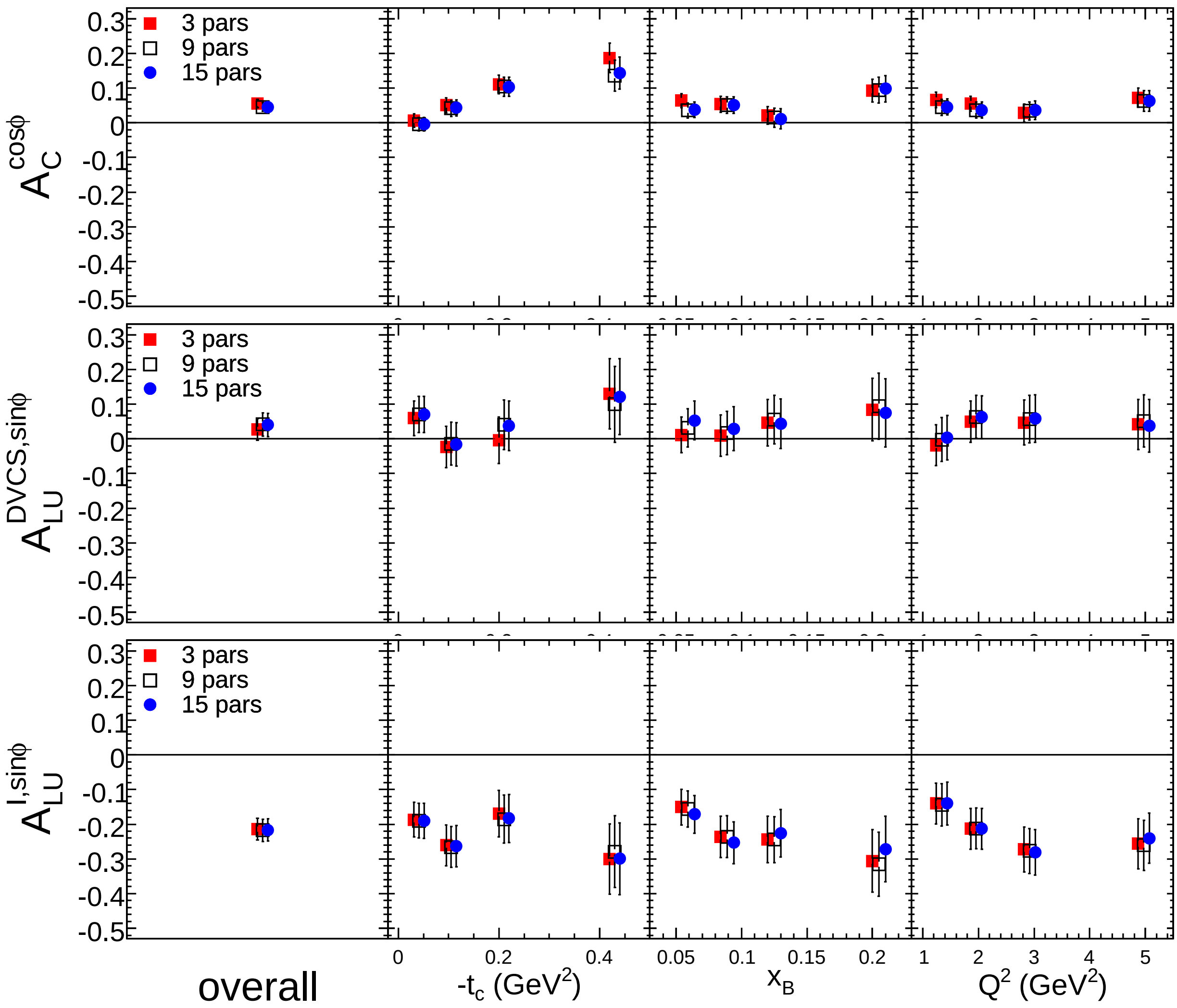

Figure 6: Kinematic dependence of the $\cos\phi$ moment of $A_C(\phi;\boldsymbol{\theta})$ and the $\sin\phi$ moments of $A_{LU}^{DVCS}(\phi;\boldsymbol{\theta})$ and $A_{LU}^{\mathcal{I}}(\phi;\boldsymbol{\theta})$.

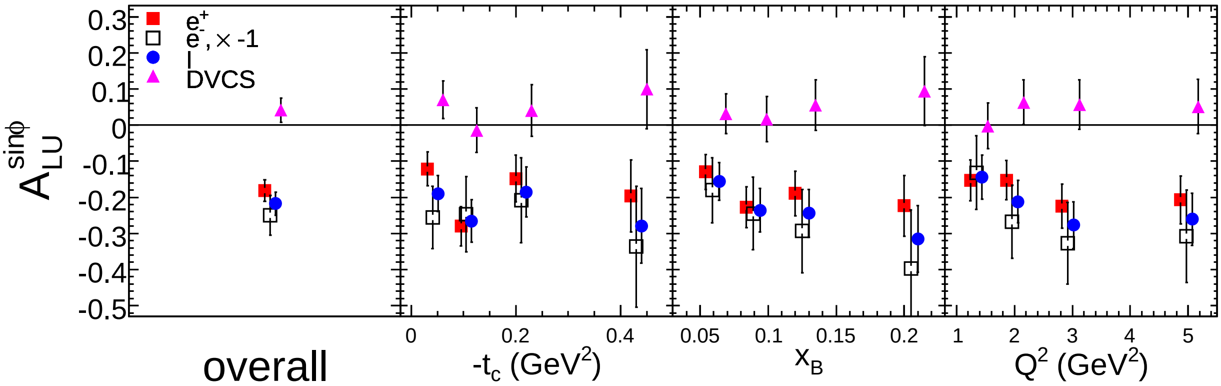

Figure 7: Comparison among the kinematic dependence of the $\sin\phi$ moments of $A_{LU}^{+}(\phi;\boldsymbol{\theta})$, $A_{LU}^{-}(\phi;\boldsymbol{\theta})$ (scaled by -1), $A_{LU}^{\mathcal{I}}(\phi;\boldsymbol{\theta})$ and $A_{LU}^{DVCS}(\phi;\boldsymbol{\theta})$.

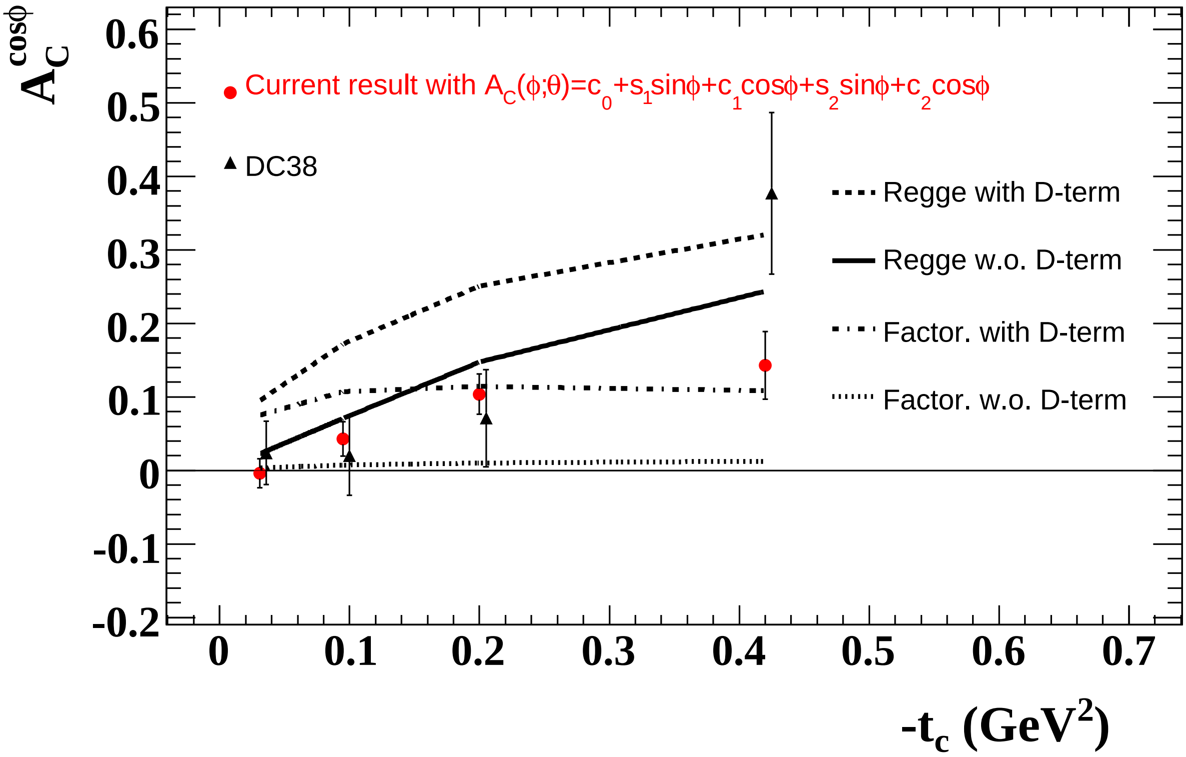

Figure 8: Current results of $A_C^{\cos\phi}$, compared to the published results DC38. Overall result of DC38: $(-0.011 \pm 0.019) + (0.060 \pm 0.027)\cos\phi + (0.016 \pm 0.026)\cos 2\phi + (0.034 \pm 0.027)\cos 3\phi$. Curves by different theoretical calculations [12] are shown.