Multichannel Random Access in OFDMA Wireless Networks

Young-June Choi, Suho Park, and Saewoong Bahk, *Member, IEEE*

***Abstract*—Orthogonal frequency-division multiple access (OFDMA) systems are considered promising candidates for implementing next-generation wireless communication systems. They provide multiple channels that can be accessed via random access schemes. However, traditional random access schemes could result in an excessive amount of access delay. To address this issue, we develop a fast retrial scheme that is based on slotted Aloha and exploits the structure of OFDMA. A salient feature of this scheme is that when collisions occur instead of retrials occuring randomly in time, they occur randomly in frequency, i.e., the scheme randomly selects the subchannels for retrial. To further achieve fast access, retrials are designed to follow the 1-persistent type, i.e., no exponential backoff. To achieve the maximum throughput, we limit the maximum number of allowed retrials according to the load condition. We also consider the issue of designing for an appropriate reuse factor for random access channels in order to overcome the intercell interference problem in OFDMA multicell environments. Our finding is that full sharing, i.e., a reuse factor of one, performs best for given random access channels. Through analysis and simulation, we confirm that our fast retrial algorithm has the advantage of high throughput and low access delay, and the full sharing policy for random access channels shows high throughput as well as low collision.**

***Index Terms*—Aloha, frequency reuse factor, orthogonal frequency-division multiple access (OFDMA), random access communication.**

I. Introduction

NEXT-GENERATION wireless communication technologies, such as B3G (beyond third-generation) and 4G (fourth-generation), are beginning to emerge. The IEEE 802.16 [1] for high-data rate fixed wireless services and the IEEE 802.20 [2] for mobile broadband wireless access service already have working groups in place for the development of B3G systems. Although code-division multiple-access (CDMA)-based systems have been developed in some research groups, it appears that orthogonal frequency-division multiplexing (OFDM) systems will play a dominant role in implementing B3G systems. IEEE 802.11a/g [3], [4] and IEEE 802.16 [1] devices are also OFDM based. The high bandwidth of OFDM comes from thousands of orthogonal subcarriers [12]. By grouping subcarriers, orthogonal frequency-division multiple access (OFDMA) systems can have many logical channels on the link layer and also exploit multiuser diversity.

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Y.-J. Choi and S. Bahk are with the Network Laboratory, Seoul National University, Seoul 151-742, Korea (e-mail: yjchoi@netlab.snu.ac.kr; sbahk@netlab.snu.ac.kr).

S. Park is with the Convergence Laboratory, KT, Seoul 137-792, Korea (e-mail: suhopark@kt.co.kr).

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Since OFDMA systems use frequency-division multiplexing, it needs to ensure an appropriate frequency reuse factor for multicell environments [9], [13]. If two neighboring cells use the same subcarrier channels, the transmission in a cell interferes with that in the other. To overcome the interference problem, the system can employ techniques like spectrum spreading [14] and multiple-receiver-based interference suppression [15]. The most widely accepted approach is to design the frequency reuse factor such that the two neighboring cells allocate subcarriers exclusively. In this paper, we design the reuse factor for random access channels in an OFDMA system. We believe that designing random access-based systems are important to be able to handle: 1) initial access; 2) bursty traffic; and 3) short packets. We briefly provide the reasons.

For a long time now, random access has been a popular approach for medium access control. Local area networks (LANs) and wireless local area networks (WLANs) use carrier-sensing multiple access with collision detection (CSMA/CD) and with collision avoidance (CSMA/CA) protocols, respectively. While the CSMA type of protocol is well applied for unlicensed band systems, it is not used in cellular networks because of channel efficiencies that use a licensed band. Therefore, 2G and 3G cellular systems use a slotted Aloha type of solution for initial uplink access. To lower the collision probability, reservation protocols such as packet reservation multiple access (PRMA) have been considered [23]–[25]. While reservation-based protocols are adequate for periodic voice traffic, they are not suitable for data traffic because of the bursty nature of data traffic. Therefore, in this paper, we will consider a random access scheme that is especially tailored for wireless data networks.

It is known that the random access schemes perform well for delivering short packets [8]. If many relatively long data packets are competing for random access, the collision probability goes up, resulting in low-link utilization. To solve this problem, a system like CDMA-high data rate (HDR) (or cdma2000 1x EV-DO) dedicates some uplink channels to each user by assigning a unique code [5]. This may also cause low channel utilization because data packets are not usually generated continuously. Therefore, next-generation systems are expected to support more uplink shared channels that are managed by uplink scheduling [27].

Uplink scheduling, when adopted, inevitably incurs more random access attempts because each mobile with pending data packets should send short channel request packets to the base station (BS). Accordingly, the effective use of random access channels is becoming important for uplink scheduling as well as initial access and short message transmission. In this paper, our contributions are threefold. First, we consider the uplink data communication in OFDMA environments and apply the random channel access for it. Second, we propose a fast retrial algorithm that is adaptive to loading condition and resolves contention. Our scheme basically uses the 1-persistent slotted Aloha and achieves fast access and high throughput for multichannel random access. Third, we find the most adequate reuse factor for random access channels in multicell OFDMA environments.

Some related works are as follows. While much research has been devoted to the slotted Aloha in single-channel environments, our work focuses on the multichannel slotted Aloha like in [16]–[18]. In [16], the multichannel slotted Aloha is analyzed for fixed bandwidth per channel or fixed total bandwidth, which is designed for multichannel satellite communication. In [17], it targets reducing the number of connections. In [18], reservation is considered.

The random access protocol has also been modified for various environments [19]–[22]. In [19], the retransmission probability is dynamically adjusted according to the transmission result in the previous slot. In [20], redundant transmissions are exploited to meet a user's deadline requirement. In [21], a random access protocol is analyzed when long-range dependent traffic is transmitted over random access channels. In [22], a cross-layer technique combined with signal processing is proposed. In CDMA systems, the random access scheme uses power control to overcome the near–far effect that basically uses a power ramping algorithm [6], [7]. Similarly, the IEEE 802.16 standard uses the power ramping for initial random access.

The fundamental difference between previous work and our paper is in the objective of using random access. In future cellular networks, we expect to require both fast access and high utilization under multiple random access channel environments that will support communications for channel requests, short messages, and other signaling messages. This provides us with the main motivation to design a *simple* and *efficient* mechanism for utilizing random access channels under normal loading conditions. Our major contribution in this paper has been to extend randomness from the *time domain* to the *frequency domain*, which allows us to improve throughput and the access delay in multiple random access channel environments.

We organize the remainder of this paper as follows. Section II illustrates an uplink access model and describes a system model. In Section III, we present our collision resolution algorithm for multichannel random access and analyze it. In Section IV, we design the frequency reuse factor for random access channels and conclude that the reuse factor of one is best. Section V compares simulation results with analysis results, and Section VI discusses further development of our work, followed by our conclusions in Section VII.

II. System Description

A. Uplink Access Model

In cellular networks, downlink communication is relatively straightforward because the BS coordinates events in a centralized way. When the BS has packets to transmit over the downlink, it notifies the corresponding user and transmits them according to a scheduling scheme or the channel allocation being used.

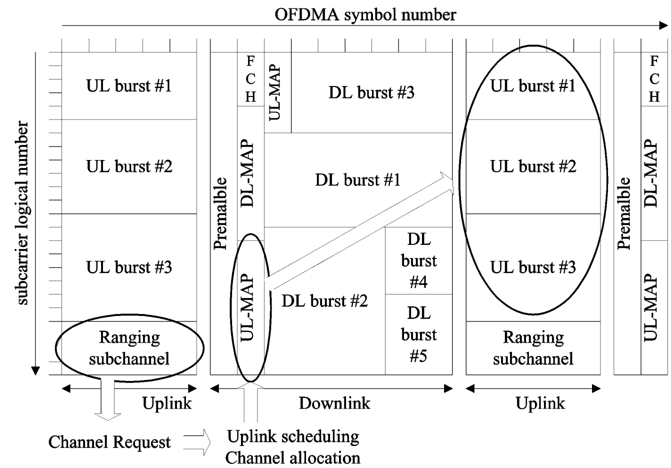

Fig. 1. Frame structure model in IEEE 802.16 TDD-OFDMA standard.

Scheduling is applicable to general data packet service over shared channels and channel allocation to voice or streaming service over dedicated channels. As new services become dominant in wireless networks, uplink transmission also needs to take advantage of shared channels. To request shared or dedicated channels, a user terminal sends control messages over contention-based common channels like in the IEEE 802.16 standard [1]. So, random access can be a good candidate for sending channel requests. Besides, as we know, it is useful for sending short message service, location update, initial access, etc. The difficulty with uplink transmission is that it cannot avoid contention unless a user specifically has a channel dedicated to it. The Aloha type that is widely used for random access has the drawback of having high-collision probability when the packet size is big, so it is good for short message service and channel request messages.

B. Ranging

OFDMA systems use a ranging technique to adjust the uplink timing offset of a symbol. The IEEE 802.16-OFDMA uses CDMA-based ranging and defines three types of ranging: *initial ranging, periodic ranging*, and *bandwidth request ranging* [1]. Initial ranging uses power ramping to search for the appropriate power level, which increases transmission power step by step. When a terminal enters the network, it synchronizes the uplink offset by the initial ranging and then adjusts it periodically (periodic ranging) by using ranging. The bandwidth request also exploits ranging that plays a role in random access.

So, the ranging follows random access procedures. Given a code set, a user terminal randomly selects a code and transmits a ranging request. The BS then broadcasts the ranging response that includes information about OFDM symbols and subchannels. When a collision occurs, the terminal waits for some time and then retries. In the IEEE 802.16 standard, an exponential backoff algorithm is used to resolve collisions.

C. Channel Structure

Fig. 1 shows the frame structure defined in the IEEE 802.16 time-division duplex (TDD)-OFDMA standard. The uplink interval consists of data subchannels and ranging (or random access) subchannels. When a terminal has packets to send, it sends a channel request message to the BS. If the BS receives the request successfully, it broadcasts the assignment result through the uplink map in the next frame. Then, the terminal transmits the packets over the allocated channel(s). The procedures are the same for the dedicated channel request. To reduce the number of random access trials, the IEEE 802.16 standard also defines a polling scheme.

To serve various random access needs, future cellular networks are expected to provide multiple channels for random access. The ranging is also a kind of random access. In CDMA-based ranging, the number of codes corresponds to that of random access channels. As the number of codes grows, the conventional exponential backoff algorithm may no longer be adequate for collision resolution. This motivates us to propose a fast retrial algorithm that becomes more powerful as the number of random access channels increases.

III. Collision Resolution

Existing systems such as second-generation (2G) and third-generation (3G) cellular systems have adopted the slotted Aloha for random access. Its appeal is in large part its simplicity of implementation. We will, therefore, also assume that the random access strategy being used is the slotted Aloha. Hence, we assume that a packet that arrives during time slot k is transmitted at time slot $k+1$, and the terminal learns whether or not access is successful immediately at the end of each slot. Our work does not consider the capture effect [28], [29].[1]

A. Fast Retrial Algorithm

Since the OFDMA system has multiple random access channels in the frequency domain,[2] we propose a *fast retrial algorithm* that exploits the nature of multiple channels for resolving contention. When a random access fails, which is mainly caused by collision, a user terminal retries the access by random channel selection instead of conventional random backoff time. If the utilization is not very high, this scheme can be highly efficient because the probability of experiencing consecutively collisions will be low. In a conventional random access scheme for a single channel, the collision is resolved by using random backoff in the time domain as shown in Fig. 2(a). In contrast, in the multichannel environment, we resolve the collision at time k by hopping in the frequency domain at time $k+1$, as shown in Fig. 2(b). Hence, we refer to this approach as a fast retrial scheme.

Our algorithm runs in the same manner as the 1-persistent scheme in the time domain, except that in the multichannel system, our approach spreads randomness over the frequency domain. We call this type of arrival a frequency-domain backoff or fast retrial. Through this approach, we can obtain high throughput and low access delay even under increased collision probability.

[1]A random access trial with a strong SINR is regarded as being successful even when collision occurs.

[2]Our mechanism can be directly applied to the code domain in the case of CDMA-based ranging.

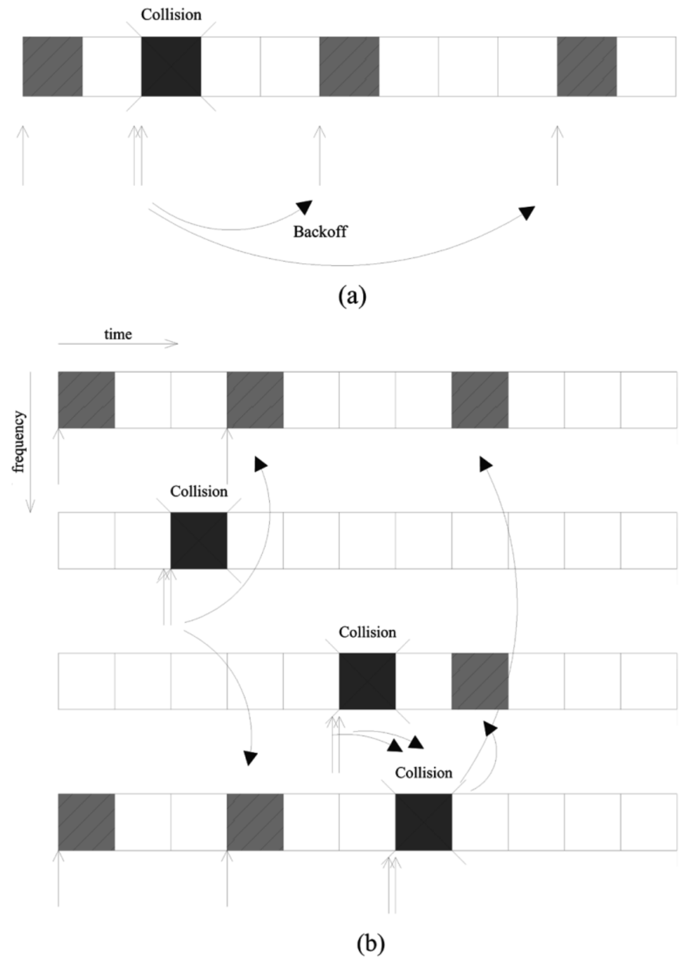

Fig. 2. Randomness in time and frequency domain: (a) Time-domain backoff in single channel. (b) Frequency-domain backoff in multiple channels.

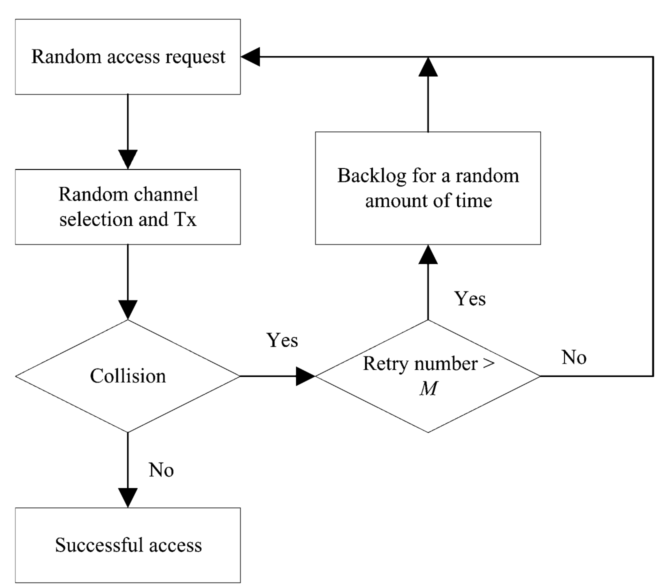

Fig. 3. Our proposed fast retrial algorithm.

The fast retrial algorithm is illustrated in Fig. 3. This algorithm limits the maximum number of retrials to M. That is, if the retrial number is larger than M, it uses random backoff in the time domain, i.e., waits for a random amount of time and then gives a retrial like in the slotted Aloha. We call this type of arrival a time-domain backoff. Our algorithm operates well under normal loading conditions, where the combined rate of new arrivals and retransmissions is not far above the full loading. A detailed numerical investigation is provided in Section V.

We now define the following notations. These notations will be used to analytically obtain the collision probability and the throughput:

- N number of random access channels;
- M maximum number of allowed fast retrials;
- λ combined rate of new and time-domain backlogged arrivals;
- λ_T total arrival rate;
- T throughput;
- p user collision probability (ratio of collisions to total trials);
- q slot collision probability (ratio of collision slots to total slots);
- γ ratio of fast retrials to the number of collisions in the previous slot.

Assume that the total of the new and time-domain backlogged arrivals in a slot follows a Poisson distribution with mean λ. Then, the total arrival rate at slot i is nothing but the sum of λ and fast retrials caused by collision at slot $i-1$. Although, in this case, fast retrials do not follow a Poisson distribution, to simplify analysis,[3] we assume that the aggregate arrivals in a time slot are Poisson with mean λ_T. So, the total arrival rate at slot i is written as

$$\lambda_T(i) = \lambda + p \cdot \gamma \cdot \lambda_T(i-1). \tag{1}$$

Since the maximum number of fast retrials is bounded by M, the added arrival rate is reduced by a factor γ, which is the ratio of fast retrials at slot i to the number of collisions at slot $i-1$. We obtain γ by

$$\gamma = \frac{\sum_{i=0}^{M-1} p^i}{\sum_{i=0}^{M} p^i}. \tag{2}$$

In steady state, we express λ_T by omitting the slot index i as

$$\lambda_T = \frac{\lambda}{1 - \frac{1-p^M}{1-p^{M+1}} \cdot p}. \tag{3}$$

Assuming that channels are independent, we obtain the total throughput of N channels by N times the throughput of a single channel [17]. Therefore, we have

$$T = N\frac{\lambda_T}{N}\exp\left(-\frac{\lambda_T}{N}\right) = \lambda_T \exp\left(-\frac{\lambda_T}{N}\right). \tag{4}$$

Since the user collision probability p is defined as the ratio of collisions to total trials in number, we have

$$p = \frac{E[\text{collisions}]}{E[\text{ successes}] + E[\text{collisions}]} \tag{5}$$

[3]Similar assumptions have been used in the past to analyze the throughput of the Aloha and slotted Aloha [10].

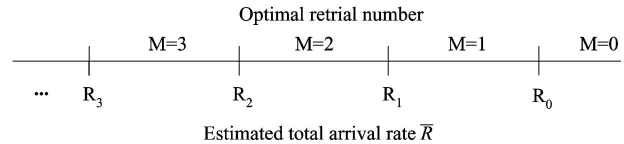

Fig. 4. Load adaptive M selection for estimated arrival rate $\bar{R}$.

where

$$\begin{aligned} E[\text{collisions}] &= \sum_{k=2}^{\infty} k\frac{\left(\frac{\lambda_T}{N}\right)^k}{k!}\exp\left(-\frac{\lambda_T}{N}\right) \\ &= \frac{\lambda_T}{N}\left(1-\exp\left(-\frac{\lambda_T}{N}\right)\right) \end{aligned} \tag{6}$$

$$E[\text{successes}] = \frac{\lambda_T}{N}\exp\left(-\frac{\lambda_T}{N}\right). \tag{7}$$

Therefore

$$p = 1 - \exp\left(-\frac{\lambda_T}{N}\right). \tag{8}$$

Meanwhile, in a single channel, slot collision occurs when two or more users arrive at the same slot. Thus, we can represent the slot collision probability q as

$$q = 1 - \left(1 + \frac{\lambda_T}{N}\right)\exp\left(-\frac{\lambda_T}{N}\right). \tag{9}$$

Unlike the conventional slotted Aloha, the throughput in (4) is a function of λ_T, which is a function of p. Therefore, we obtain p and T in a recursive manner.

From p, we can obtain the probability distribution of the access delay, i.e., the probability that m time slots are required to successfully access the channel. Therefore

$$Pr\{\text{access delay} = m+1\} = (1-p)p^m, \quad 0 \le m \le M. \tag{10}$$

Accordingly, the probability that the access delay is smaller than or equal to $M+1$ is given by

$$\begin{aligned} Pr\{\text{access delay} \le M+1\} &= (1-p)\sum_{m=0}^{M} p^m \\ &= 1 - p^{M+1}. \end{aligned} \tag{11}$$

B. Load Adaptivity

For a given N, there exist appropriate operating ranges for λ and M. It is intuitively obvious that if M goes high, the average offered load for a given time interval also increases due to the increased fast retrials. So, we obtain the following proposition.

Proposition 1: As M increases, the maximum throughput is obtained at a lower arrival rate.[4]

Hence, we divide the optimal operating range of total arrival rate according to M as shown in Fig. 4. Let R_i denote the threshold arrival rate for the ith operating region. Then, we obtain the following proposition.

[4]We omit the proof, but readers can refer to [30] for mathematical comprehension.

Proposition 2: $\max_i\{T_i(\lambda)\} = T_n(\lambda)$ if $R_n < \lambda \leq R_{n-1}$ for $n = 1, 2, 3, \ldots$, and $\max_i\{T_i(\lambda)\} = T_0(\lambda)$ if $\lambda > R_0$.

The proof is given in Appendix A. Comparing the throughput curve of $M = n$ with that of $M = n+1$, we can find R_n. That is, the throughput achieves its maximum at $M = n$ for $R_n < \bar{R} \leq R_{n+1}$, where $\bar{R}$ is the estimated total arrival rate. Therefore, it is necessary for the system to estimate $\bar{R}$ properly for good operation. We can think of two ways to estimate $\bar{R}$. First, the BS measures and estimates the average arrival rate for a given interval such as superframe interval. Second, a user measures p and estimates λ_T by (8). Besides, the BS can determine M more exactly if the user reports the estimated p to the BS.

Under normal loads, we use the slotted Aloha in the 1-persistent manner to resolve contention. For the heavy load case, we can additionally use some contention resolution algorithm like a p persistent [8] or a load-adaptive algorithm [19].

IV. Reuse Factor of Random Access Channel

In FDMA cellular networks, the uplink transmission of a user near the cell boundary interferes with that of some other users in neighboring cells. The intercell interference hinders the BS from decoding corresponding signals properly. The same problem occurs in OFDMA networks since OFDMA is a kind of FDMA scheme. As a way of overcoming the interference problem without sacrificing efficiency, we design the frequency reuse factor in this section. Differently from the previous work that has dealt with data channels, our paper considers the reuse factor for random access channels. We describe here our solution for three different types of reuse: *nonsharing*, *full sharing*, and *partial sharing*. Although the first two cases are special cases of the last, we present them separately for the purpose of illustration.

A. Nonsharing

The traditional way of frequency reuse is to allocate a frequency band to a cell and to reassign the same frequency band to nonneighboring cells. Hence, each cell has its own random access channels. Fig. 5(a) shows an example of a reuse factor of seven. Denoting the reuse factor by F, we can obtain the cell throughput for the nonsharing case, T_{ns}, by following the same procedures in Section III

$$T_{\text{ns}} = \lambda \exp\left(-\frac{\lambda F}{N}\right). \tag{12}$$

Assuming that the arrival rate is homogeneous for each cell, we obtain the user collision probability of the nonsharing case p_{ns} as

$$p_{\text{ns}} = 1 - \exp(-\lambda) \tag{13}$$

where λ is the arrival rate at each cell.

B. Full Sharing

In the full sharing case, each cell shares random access channels fully with neighboring cells. Therefore, some arrivals generated at a cell can influence the performances of neighboring cells. To simplify the analysis, we model the interference range

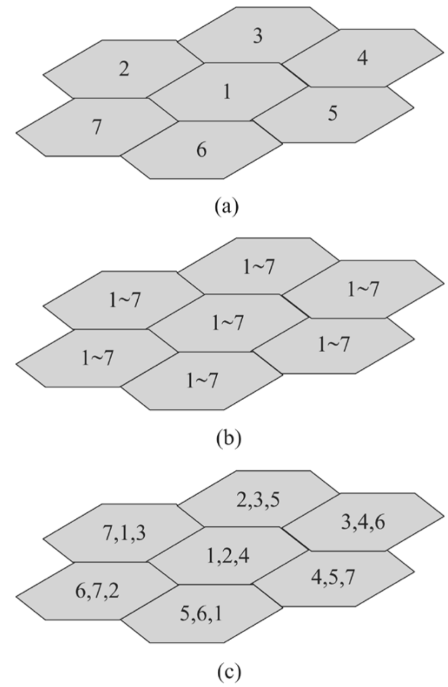

Fig. 5. Channel reuse policies. (a) Nonsharing. (b) Full sharing. (c) Partial sharing.

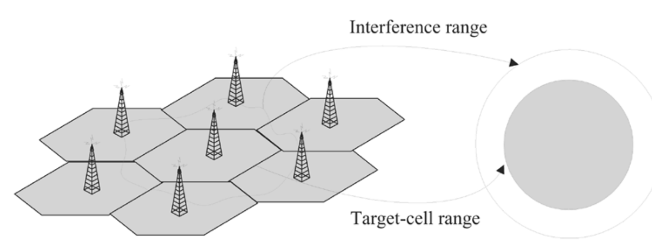

Fig. 6. Multicell model for full sharing and partial sharing.

as in Fig. 6.[5] Let B denote the number of neighboring cells and α $(0 \leq \alpha \leq 1)$ the average ratio of interference range to cell coverage per cell. α approaches zero if the interference range shrinks, and the arrival rate induced by arrivals at neighboring cells is $\alpha B \lambda$ when users are uniformly distributed. So, the combined total arrival rate λ_C is given by

$$\lambda_C = \lambda + \alpha B \lambda. \tag{14}$$

Then, we can express the throughput for the full sharing case T_{fs} as

$$\begin{aligned} T_{\text{fs}} &= \frac{\lambda_C}{N} \exp\left(-\frac{\lambda_C}{N}\right) \cdot N \frac{\lambda}{\lambda_C} \\ &= \lambda \exp\left(-\frac{1+\alpha B}{N}\lambda\right). \end{aligned} \tag{15}$$

Accordingly, we obtain the user collision probability for the full sharing case p_{fs} as

$$p_{\text{fs}} = 1 - \exp\left(-\frac{1+\alpha B}{N}\lambda\right). \tag{16}$$

Comparing the results of full sharing with those of nonsharing, we obtain the following lemma.

[5] A similar multicell model is found in [26].

Lemma 1: If N channels are given for random access, the full sharing policy has higher throughput and lower user collision probability than the nonsharing policy.

The proof is given in Appendix B.

C. Partial Sharing

Partial sharing allows a cell to share a part of its channels with neighboring cells. To design a partial sharing policy, we propose a new method that uses a *difference set* [11]. To form a difference set, we allocate a subset of random access channels to each cell such that the number of shared channels at each cell is the same.

Definition 1: Let $\Gamma = \{0, 1, \ldots, \nu - 1\}$, and $D(\neq \emptyset)$ a κ subset of Γ with $0 < \kappa < v$. Then, D is called a (ν, κ, ζ) *difference set* if it satisfies that the list of differences $d - d' \neq 0$ $(d, d' \in D,\ d \neq d')$ and contains each nonzero element of Γ precisely ζ times [11].

Lemma 2: Let D be a (ν, κ, ζ) difference set in Γ. Then, [11]

$$\zeta(\nu - 1) = \kappa(\kappa - 1). \tag{17}$$

Lemma 3: Let D be a (ν, κ, ζ) difference set in a group Γ. Then, $\mathbf{B} = \{D + g(\mathrm{mod} v) : g \in \Gamma\}$ is a symmetric set of (ν, κ, ζ) [11].

Lemma 4: There exist exactly ζ elements that are common between any two subsets X and Y such that $X, Y \in \mathbf{B}(X \neq Y)$.

The proof of Lemma 4 is straightforward; therefore, we omit it here.

Example 1: For a (7,3,1) difference set, we can select a subset (1,2,4) arbitrarily. Then, (2,3,5), (3,4,6), (4,5,0), (5,6,1), (6,0,2), and (0,1,3) are also the subsets that satisfy the difference set property in Lemma 3. From Lemma 4, there exists exactly one element that is common for any two subsets.

Fig. 5(c) shows an example of partial sharing by using the difference set (7,3,1). Each cell exploits three random access channels among seven and any two neighboring cells share one channel. When the network uses the difference set (N, N_d, v), we have

$$\lambda_C = \lambda + \alpha B \frac{v}{N_d} \lambda. \tag{18}$$

With some manipulations, we can express the throughput for partial sharing $T_{\rm ps}$ as

$$T_{\rm ps} = \lambda \exp\left(-\frac{1 + \frac{\alpha B v}{N_d}}{N_d} \lambda\right) \tag{19}$$

and the user collision probability for partial sharing $p_{\rm ps}$ as

$$p_{\rm ps} = 1 - \exp\left(-\frac{1 + \frac{\alpha B v}{N_d}}{N_d} \lambda\right). \tag{20}$$

Comparing the results of full sharing and partial sharing, we obtain the following lemma.

Lemma 5: When N channels are given for random access, the full sharing policy has higher throughput and lower user collision probability than the partial sharing policy.

The proof is given in Appendix C. From Lemmas 1 and 5, we obtain the following proposition.

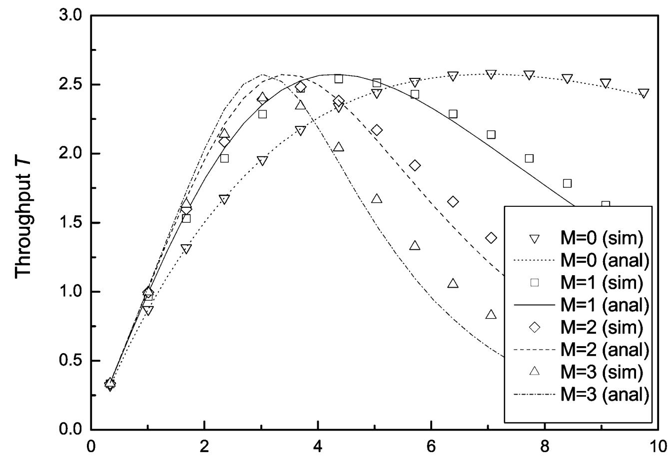

Fig. 7. Throughput versus new arrival rate $(N = 7)$.

TABLE I
OPTIMAL M ACCORDING TO OPERATING RANGE OF λ ($M_{\max} = 4, N = 7$)

Region	Optimal M
$\lambda > 5.26$	$M = 0$
$3.69 < \lambda \leq 5.26$	$M = 1$
$3.15 < \lambda \leq 3.69$	$M = 2$
$2.90 < \lambda \leq 3.15$	$M = 3$
$\lambda \leq 2.90$	$M = 4$

Proposition 3: For given random access channels in multicell OFDMA environments, the frequency reuse factor of one is best.

Additionally, comparing the performance among several partial sharing schemes, we have the following lemma.

Lemma 6: For several difference sets with the same ν, the performance increases with the increase of κ or ζ.

The proof is given in Appendix D.

V. NUMERICAL RESULTS

A. Fast Retrial Algorithm

We compare our analytical results with simulation results. A cell has seven channels $(N = 7)$, and this implies that λ of seven is the total arrival rate that achieves maximum throughput in the slotted Aloha.[6] In our simulations, we assume that users are distributed uniformly in each cell without mobility. Users in wireless communications experience different channel conditions according to path loss and fading in reality. However, as we focus on the access failure due to the collision only, we assume that the BS receives a random access perfectly if there is no collision. This means that the channel model deciding the channel condition is beyond the scope of this paper. In Figs. 7–13, we use the symbols to present simulation results and the lines to present analytic results.

Fig. 7 shows the throughput curve as a function of λ. The case of $M = 0$ corresponds to conventional slotted Aloha. When we increase M, more retrials contribute to the total arrival rate. So, the maximum throughput is obtained at a lower arrival rate. Table I shows the optimal M that depends on the operating re-

[6]Slotted Aloha obtains its maximum throughput at $\lambda = N$.

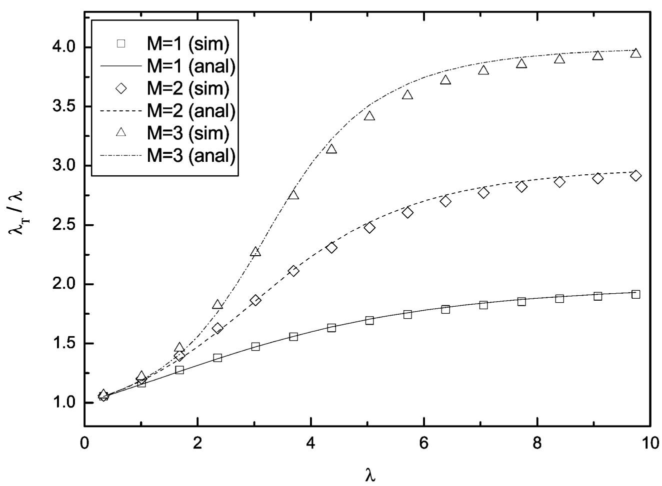

Fig. 8. λ_T/λ versus new arrival rate $(N = 7)$.

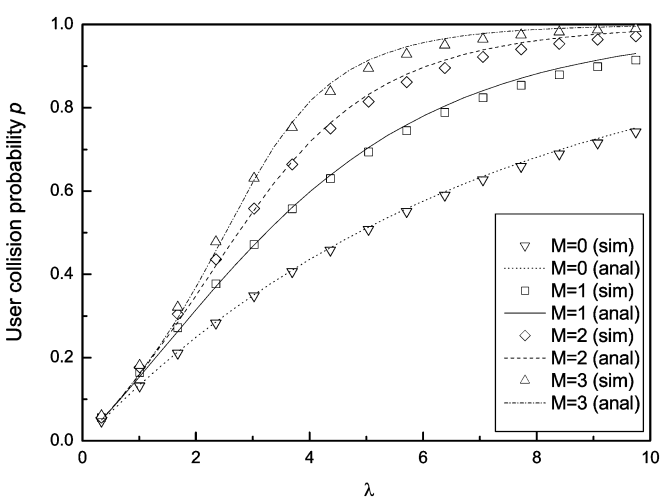

Fig. 9. User collision probability versus new arrival rate $(N = 7)$.

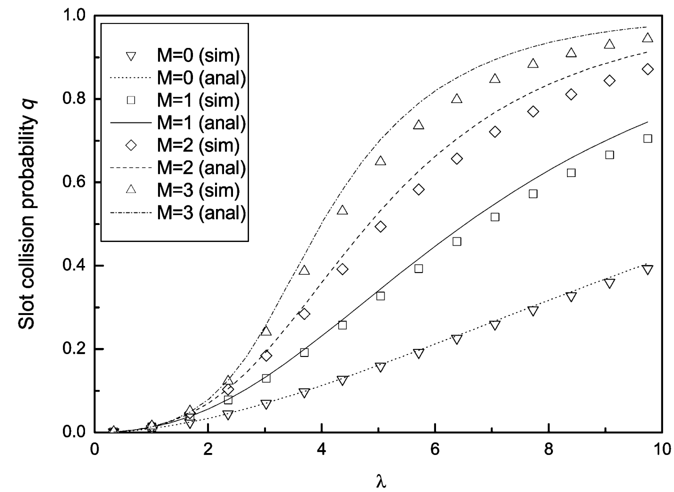

Fig. 10. Slot collision probability versus new arrival rate $(N = 7)$.

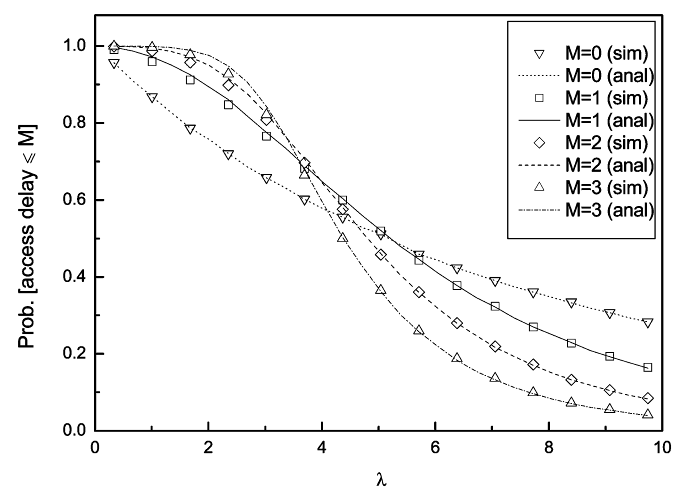

Fig. 11. Prob. [access delay $\leq M + 1$] versus new arrival rate $(N = 7)$.

gion of λ for the case of $M_{\max} = 4$. By using adaptive M, our scheme achieves better performance than the conventional Aloha scheme.

Fig. 8 shows the ratio of λ_T to λ as a function of λ. The higher the value of M, the higher is this ratio because of more retrials. As λ increases, the ratio converges at $M + 1$. This is because extremely high-collision probability of near one results in $\gamma \simeq M/(M+1)$ in (2), thus λ_T/λ approaches $M + 1$ in (3). Figs. 9 and 10 show the user collision probability and the slot collision probability, respectively. Although both collision probabilities increase with an increase in M, the throughput performance becomes better under the normal load.

In reality, it is recommended that random access channels are to operate below 20% of the slot collision probability. So, our interest regarding λ lies in the normal load which is up to a rate where the throughput curve starts falling down. We call this load the stable loading condition. Fig. 7 shows the throughput increase with respect to M at the light load. This implies that the fast retrial algorithm performs very well in spite of the increased collision probability shown in Fig. 9.

Fig. 11 plots the probability of access delay smaller than $M+1$ slots according to λ. In the conventional Aloha type, each access can be exponentially backlogged over future slots, so the access delay cannot be guaranteed. However, in the fast retrial algorithm, most access trials succeed within $M + 1$ slots under a stable loading.

Some of the figures show a slight gap between the analysis and simulation results, especially under the heavy load. This is because of the Poisson assumption on the total number of arrivals in a time slot. As mentioned before, this is not true because of retrials. Thus, the gap grows as M increases, since the number of retrials also increases. In the simulation, we emulated a queue per user to generate new arrivals.

B. Reuse Factor for Multicell Environments

We compare the throughput and the user collision probability for nonsharing, partial sharing (PS), and full sharing (FS) policies. For the nonsharing policy, we set $F = 7$ for a structure of seven hexagonal cells. Let $N = 7$, $B = 6$, and $\alpha = 1/6$ for full sharing. Then, the performance of the nonsharing policy is the same as that of single-channel slotted Aloha. For the partial sharing, we use the difference sets (7,5,4), (7,4,2), and (7,3,1). Figs. 12 and 13 show the comparison results for T and p, respectively. As shown in Proposition 3, the full sharing policy performs best. Also, as proven in Lemma 6, the performance is

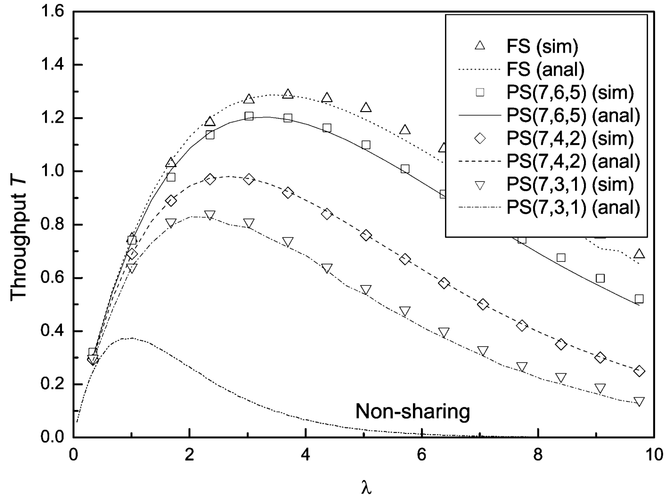

Fig. 12. Throughput comparison of channel sharing policies.

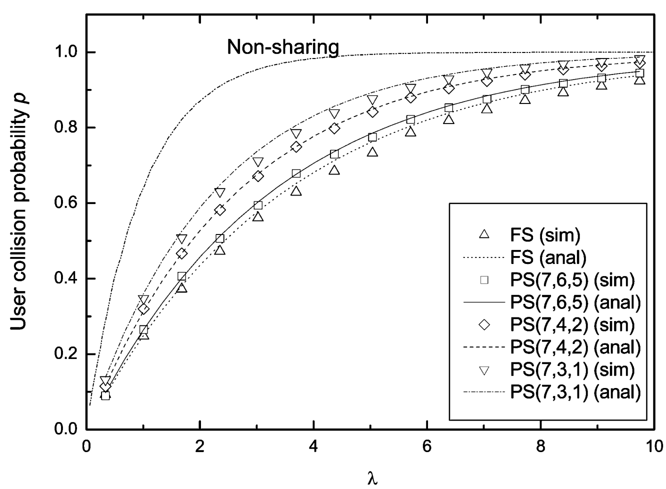

Fig. 13. Comparison of user collision probability for channel sharing policies.

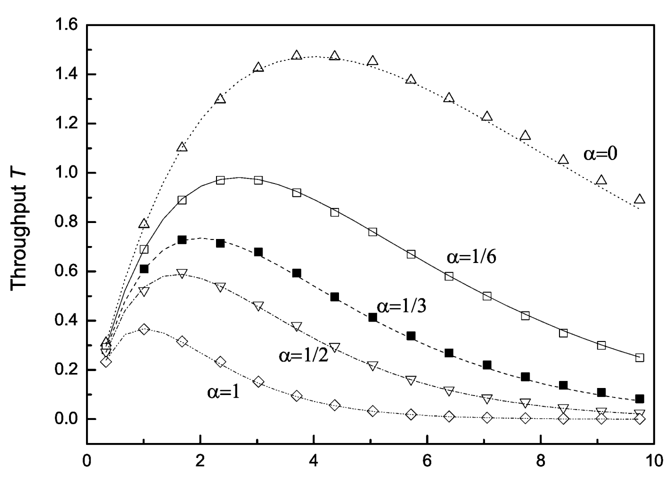

Fig. 14. Throughput comparison according to α in PS(7,4,2).

improved when κ or ζ is high for several difference sets with the same ν.

We measure the impact of intercell interference range α for the same environment. Figs. 14 and 15 show T and p, respectively, for the partial sharing with the (7,4,2) difference set when

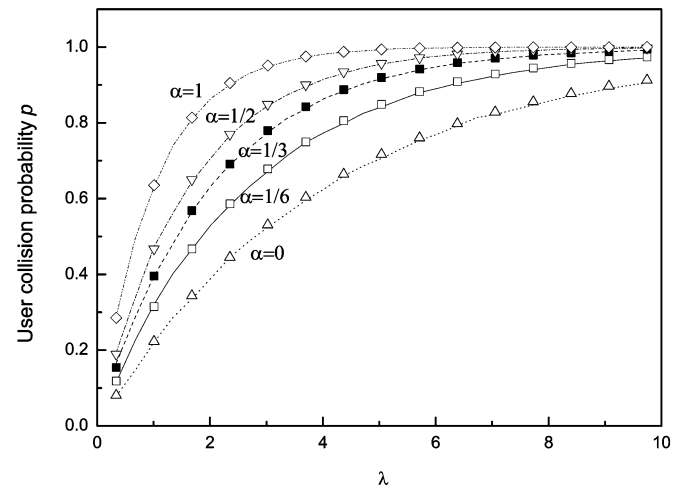

Fig. 15. Comparison of user collision probabilities according to α in PS(7,4,2).

α varies from zero to one. As expected, the performance increases with a decrease of α, and the case of $\alpha = 1$ yields the same performance as the nonsharing case.

VI. Further Discussions

A. Random Access With Power Control

Until now, we have not considered random access with power control. Typically, retrial in a CDMA type of a system increases the power level according to a power ramping algorithm in order to take advantage of the capture effect. Our fast retrial scheme can be combined with power ramping in two ways. The first is an aggressive approach that increases the power level per retrial, and the second is a conservative one that increases the power level only for random backlogged retrials, but not for fast retrials. The former makes use of the capture effect aggressively, while the latter concentrates on the collision resolution. The evaluations of these algorithms are left for future work.

B. Further Application of Partial Sharing Policy

Although partial sharing is worse than the full sharing regarding reusing random access channels, it can be efficient for reusing data channels. While fully sharing data channels may cause excessive intercell interference, partial sharing can meet a target number of overlapping channels among the cells. The (ν, κ, ζ) difference set satisfies the ratio of overlapping channels at ζ/κ between any two cells, so it is possible to ensure a predefined interference level. Hence, it provides a flexible reuse factor of middle levels while the conventional reuse factors just fall into one, three, four, seven, and so on.

The reuse factor one of data channels may bring low signal quality for cell boundary users due to the intercell interference. To overcome this problem, the Korean WiBro system ,based on the IEEE 802.16 OFDMA/TDD, introduces the concept of safety channels that are dedicated to mobile users under bad channel environments. We can simply consider a reuse factor of three or seven for users near the cell boundary and one for users elsewhere. This kind of a hybrid assignment can be made possible by flexibly using several difference sets. For instance, two difference sets can be used for creating less and more overlapping channels, each for bad and good channel conditions, respectively.

VII. Concluding Remarks

There has been a voluminous amount of research that has been devoted to the study of random access methods such as the slotted Aloha. For satellite communications, where data is transmitted over random access channels, it was important to lower the collision probability to achieve high throughput at full load for a single channel. In the next generation of cellular systems, fast access becomes especially important in the design of random access schemes. In particular, this is true for multichannel environments like OFDMA systems.

In this paper, we developed a fast access algorithm that handles retrials in a system with multiple random access channels. This algorithm retransmits 1-persistently within the allowed maximum number of retrials based on the estimated total arrival rate. The algorithm is derived to take advantage of the structural property of OFDMA systems where multiple channels are available. Thus, instead of retransmissions occurring in time, they occur over a different frequency.

Our algorithm operates very well under the stable load condition. For a heavy load, our scheme can work like a conventional competitive scheme. Numerical results demonstrate that our algorithm achieves high throughput and low access delay compared to conventional slotted Aloha. Moreover, the operating points can be easily controlled by varying the parameters of our algorithm.

We also considered the reuse factor for random access channels in multicell environments. Among the policies of nonsharing, full sharing, and partial sharing, we confirmed that the full sharing policy indeed demonstrates the best performance. This policy is also important from the point of view of initial access because user terminals are expected to use random access channels initially without having detailed channel information about the cell. Lastly, we briefly addressed the possible advantage of using the partial sharing policy for data channels. A detailed investigation is left for future work.

Appendix A
Proof of Proposition 2

Let us denote λ_T^m for integer $m \geq 0$ as

$$\lambda_T^m = \frac{\lambda}{1 - p\left(\frac{1-p^m}{1-p^{m+1}}\right)}. \tag{21}$$

Definition 2: For the given function $k(x) = x\exp(-x/N)$, we say two positive real numbers x and y are dual if $k(x) = k(y)$. We denote this by $\bar{x} = y$. An example is given in Fig. 16.

Lemma 7: $x, y (> 0)$ are dual iff $x = y$ or $(\ln x - \ln y)/(x-y) = 1/N$.

As it is obvious that $\lambda_T^m(\lambda) \neq \lambda_T^{m-1}(\lambda)$, we can find dual numbers, i.e., $\lambda_T^m(\lambda) = \overline{\lambda_T^{m-1}(\lambda)}$ that satisfy the latter in Lemma 7. We define $\{\alpha_n(x) | \alpha_n : [0,\infty) \longrightarrow \mathbb{R}\} \subset C^\infty[0,1)$ for $n = 0, 1, 2, \ldots$ that satisfies $\alpha_n(x) < \alpha_{n+1}(x)$ and $\alpha_n'(x) > 0$ for $x > 0$ with $\alpha_0(x) = x$, $\alpha_n(0) = 0$, and

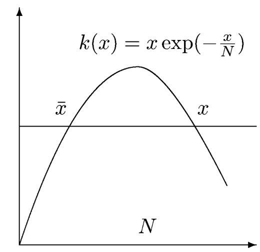

Fig. 16. Function $k(x)$.

$\lim_{x\to\infty} \alpha_n(x) = \infty$ for $n = 0, 1, 2, \ldots$. For a given N, we define $\beta_n(x) = \alpha_n(x)\exp(-\alpha_n(x)/N)$.

Lemma 8: There is a unique nonzero x_n satisfying $\alpha_n(x) = \overline{\alpha_{n+1}(x)}$.

Proof:

1) Existence: Let us consider u_n, $v_n (> 0)$ satisfying $\alpha_n(u_n) = N$ and $\alpha_{n+1}(v_n) = N$. This is feasible because for each n, $\alpha_n(x)$ has the image $[0, \infty)$. Then, $k(\alpha_n(u_n)) = k(N) > k(\alpha_{n+1}(u_n))$. This holds because $\alpha_{n+1}(u_n) = N$ implies $\alpha_n(u_n) = \alpha_{n+1}(u_n)$, which is contradictory. Accordingly, we have $k(\alpha_{n+1}(v_n)) = k(N) > k(\alpha_n(v_n))$ and the function $k(\alpha_{n+1}(x)) - k(\alpha_n(x))$ that is smooth in $(0,\infty)$ has different signs at u_n and v_n. Hence, we have at least one positive real value x_n satisfying $\alpha_n(x) = \overline{\alpha_{n+1}(x)}$.
2) Uniqueness: Because $k(x)$ increases in $(0, N]$ and decreases in $[N, \infty)$, the equation $k(\alpha_n(x)) = k(\alpha_{n+1}(x))$ implies that $\alpha_n(x) < N < \alpha_{n+1}(x)$. Indeed, if both are in $(0, N]$ or $[N, \infty)$, then $\alpha_n(x) < \alpha_{n+1}(x)$ implies strict inequality between $k(\alpha_n(x))$ and $k(\alpha_{n+1}(x))$. Hence, if there are two values $x_{n,1}$ and $x_{n,2}$ $(x_{n,1} < x_{n,2})$, each satisfying $k(\alpha_n(x)) = k(\alpha_{n+1}(x))$, the increase of $k(x)$ in $(0, N]$ implies that $k(\alpha_n(x_{n,1})) < k(\alpha_n(x_{n,2})) = k(\alpha_{n+1}(x_{n,2}))$. Because $k(x)$ decreases in $[N, \infty)$, we have $k(\alpha_{n+1}(x_{n,2})) < k(\alpha_{n+1}(x_{n,1}))$, which is contradictory. ■

Lemma 9: Let us denote the nonzero solution of $\beta_{n+1}(x) = \beta_n(x)$ as x_n that is unique. Then, the following holds.

1) The sequence $\{x_n\}$ is decreasing.
2) $\beta_n(x) > \beta_{n+1}(x)$ if $x > x_n$.

Proof:

1) $\beta_{n+1}(x_n) = \beta_n(x_n)$ implies that $\alpha_n(x_n) < N < \alpha_{n+1}(x_n)$. Hence, $\alpha_{n+1}(x_{n+1}) < N < \alpha_{n+2}(x_{n+1})$, which implies that $\alpha_{n+1}(x_{n+1}) < \alpha_{n+1}(x_n)$. Because $\alpha_{n+1}'(x) > 0$, we have $x_{n+1} < x_n$.
2) Because $\beta_{n+1}(x) = \beta_n(x)$ has a unique solution x_n, we can check the existence of a point $x' (> x_n)$ that satisfies $\beta_n(x') > \beta_{n+1}(x')$. For a fixed n, we choose $x' > x_n$ that satisfies $\alpha_n(x') > \alpha_{n+1}(x_n)$. This is feasible because $\lim_{x\to\infty}\alpha_n(x) = \infty$.

By the mean value theorem, we obtain

$$\frac{\ln(\alpha_{n+1}(x')) - \ln(\alpha_n(x'))}{\alpha_{n+1}(x') - \alpha_n(x')} = \frac{1}{c'} \tag{22}$$

where $\alpha_n(x') < c' < \alpha_{n+1}(x')$. We also have

$$\frac{\ln(\alpha_{n+1}(x_n)) - \ln(\alpha_n(x_n))}{\alpha_{n+1}(x_n) - \alpha_n(x_n)} = \frac{1}{N} \tag{23}$$

where $\alpha_n(x_n) < N < \alpha_{n+1}(x_n)$. The condition $N < c'$ leads to $\beta_n(x') > \beta_{n+1}(x')$ because the inequality $\beta_n(x) > \beta_{n+1}(x)$ is equivalent to

$$\frac{\ln(\alpha_{n+1}(x)) - \ln(\alpha_n(x))}{\alpha_{n+1}(x) - \alpha_n(x)} < \frac{1}{N}. \tag{24}$$

■

Applying Lemmas 8 and 9 to the sequence $\lambda_T^n(\lambda)$ and the throughput $T_n(\lambda) = \lambda_T^n(\lambda)\exp(-\lambda_T^n(\lambda)/N)$, we obtain the following proof.

Proof of Proposition 2: If $R_n < \lambda \le R_{n-1}$, we have $R_i < \lambda$ for $i = n, n+1, \ldots$ as R_i decreases with the increase of i. Hence, we have $T_i(\lambda) > T_{i+1}(\lambda)$ for $i = n, n+1, \ldots$. This implies that $T_n(\lambda) > T_i(\lambda)$ for $i = n+1, n+2, \ldots$. So, we have $\max_i\{T_i(\lambda)\} = \max\{T_0(\lambda), T_1(\lambda), \ldots, T_n(\lambda)\}$. The fact that $\lambda \le R_{n-1}$ implies that $\lambda \le R_i$ for $i = 0, 1, \ldots, n-1$. Hence, we obtain $T_i(\lambda) \le T_{i+1}(\lambda)$ for $i = 0, 1, \ldots, n-1$ by Lemma 9. This implies that $T_i(\lambda) \le T_n(\lambda)$ for $i = 0, 1, \ldots, n-1$. Therefore, we have $\max_i\{T_i(\lambda)\} = T_n(\lambda)$ if $R_n < \lambda \le R_{n-1}$. If $\lambda > R_0$, then $\lambda > R_i$ for $i = 0, 1, \ldots$ so that $T_i(\lambda) > T_{i+1}(\lambda)$ for $i = 0, 1, \ldots$. This implies that $T_0(\lambda) > T_i(\lambda)$ for $i = 1, 2, \ldots$. ■

APPENDIX B
PROOF OF LEMMA 1

To claim $T_{\text{fs}} \ge T_{\text{ns}}$, we calculate the ratio by

$$\frac{T_{\text{fs}}}{T_{\text{ns}}} = \exp\left(\frac{F - (1 + \alpha B)}{N}\lambda\right). \tag{25}$$

For the nonsharing policy, the reuse factor F should be $B+1$. With the condition of $0 \le \alpha \le 1$ and $F - 1 = B$, $F - (1+\alpha B)$ is larger than or equal to zero. This implies $T_{\text{fs}}/T_{\text{ns}}$ is always larger than or equal to one. Therefore, the condition $T_{\text{fs}} \ge T_{\text{ns}}$ is true. Through the same approach, we state that $p_{\text{fs}} \le p_{\text{ns}}$ is true.

APPENDIX C
PROOF OF LEMMA 5

To claim $T_{\text{fs}} \ge T_{\text{ps}}$, we calculate

$$\begin{aligned}\frac{T_{\text{fs}}}{T_{\text{ps}}} &= \exp\left(\frac{1 + \frac{\alpha B v}{N_d}}{N_d}\lambda - \frac{1 + \alpha B}{N}\lambda\right)\\ &= \exp(\Phi\lambda)\end{aligned} \tag{26}$$

where we define Φ for the following.

From the principle of difference set in Lemma 2, we have $v = N_d(N_d - 1)/(N - 1)$. Inserting this into (26), we obtain

$$\begin{aligned}\Phi &= \frac{N - 1 + \alpha B(N_d - 1)}{(N-1)N_d} - \frac{1 + \alpha B}{N}\\ &= \frac{(N - N_d)(N - 1 - \alpha B)}{N_d(N-1)N}.\end{aligned} \tag{27}$$

Since $N > N_d, \Phi \ge 0$ only if $N - 1 - \alpha B \ge 0$. This condition holds since $B + 1$ cells should have at least N shared channels for even distribution in multicell environments. Therefore, the condition $T_{\text{fs}} \ge T_{\text{ps}}$ is true. By following the same approach, we have $p_{\text{fs}} \le p_{\text{ps}}$.

APPENDIX D
PROOF OF LEMMA 6

By following similar manipulations in Appendix C, we obtain

$$T_{\text{ps}} = \lambda \exp\left(-\frac{1 + \frac{\alpha B(N_d - 1)}{(N-1)}}{N_d}\lambda\right) \triangleq f(N_d). \tag{28}$$

We obtain $f'(N_d) > 0$ easily from the condition $N - 1 - \alpha B \ge 0$.

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Young-June Choi received the B.S. and M.S. degrees from Seoul National University, Seoul, Korea, in 2000 and 2002, respectively. He is expected to receive the Ph.D. degree in the School of Electrical Engineering and Computer Science, Seoul National University, in 2006.

His research interests include fourth-generation wireless networks, wireless resource management, and cross-layer system design.

Suho Park received the B.S. degree from the School of Mathematics, Korea Advanced Institute of Science and Technology, Korea, in 1994, and the M.S. and Ph.D. degrees in from the School of Mathematics, Seoul National University, Seoul, Korea, in 1996 and 2003, respectively.

From 2003 to 2004, he worked as a Postdoctoral Faculty Member in the School of Electrical Engineering and Computer Science, Seoul National University. He is currently with Convergence Laboratory, Seoul. His research interests include geometric topology and network mobility.

Saewoong Bahk (M'94) received the B.S. and M.B. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 1984 and 1986, respectively, and the Ph.D. degree from the University of Pennsylvania, College Park, in 1991.

From 1991 to 1994, he was with the Department of Network Operations Systems, AT&T Bell Laboratories as an MTS, where he worked for AT&T network management. In 1994, he joined the School of Electrical Engineering, Seoul National University and currently serves as a Professor. His research interests include performance analysis of communication networks and network security.