Delay Versus Throughput Comparisons for Stabilized Slotted ALOHA

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Abstract—**Methods to stabilize the slotted ALOHA channel are simulated to compare delay versus throughput performance. It is shown that the best performance is provided by methods that use deferred first transmission and estimate the number of blocked terminals. Three such methods, previously thought to be independent, are shown to be the same scheme with different control parameters. It is shown that these three methods do not differ in delay versus throughput performance at the 95% confidence level. The optimal parameters to provide the smallest delay at a given throughput are explored, and it is shown that near optimal parameters provide a smaller delay than previously published.**

I. Introduction

The infinite population slotted ALOHA channel with Poisson arrivals and fixed retransmission probabilities was shown to be inherently unstable by Fayolle, Gelenbe, and Labetoulle [1]. Fayolle *et al.* [1], Mikhailov [2], Hajek and Van Loon [3], Clare [4], Rivest [5], and Thomopoulos [6] have published computationally feasible retransmission control schemes to stabilize the channel for all input loads $\lambda < 1/e$. The methods used in [1], [3], [5], and [6] for stabilization of the ALOHA channel have been extended to stabilize a variety of CSMA channels in [6]–[9]. In Section I of this paper we show that if the estimated input load $\hat{\lambda} = 1/e$, the methods of Rivest [5] and Thomopoulos [6] become members of a class proved stable by Clare [4]. Section II compares the delay versus throughput performance of these various ALOHA channel stabilization schemes, and, in Section III, we explore the control parameters to produce the best delay versus throughput performance.

Fayolle *et al.* [1], Clare [4] and Thomopoulos [6] prove stability for immediate first transmission (IFT) in which transmission of newly arrived messages is attempted at the beginning of the next slot. If the transmission fails, the terminal is said to have been involved in a collision and becomes blocked. Blocked terminals attempt retransmission in the following slots with some specified probability. Rivest [5] proves stability for deferred first transmission (DFT) in which a terminal with a newly arrived message immediately goes into the blocked mode. Hajek and Van Loon [3] consider both cases. We simulate both IFT and DFT for all the stabilization methods considered. For additional discussion of immediate and deferred first transmission see [3] and [11].

Fayolle *et al.* [1] assume that all terminals know the number of blocked terminals n. They show that if each blocked terminal attempts retransmission in any given slot with probability $p_r = (1 - \lambda)/(n - \lambda)$, then the probability of a successful transmission is maximized and the channel is stable for all input loads $\lambda < 1/e$. Because of the requirement for global knowledge of the number of blocked terminals, as well as knowledge of the exact input load λ, this method cannot be realized. It does, however, provide a lower bound to the delay for a given throughput. This is useful for comparing the performance of realizable methods to the best possible performance.

Mikhailov [2], Clare [4], Rivest [5], and Thomopoulos [6] all use idle ($z_t = 0$), success ($z_t = 1$), collision ($z_t \geq 2$) feedback to compute an estimate of the number of blocked terminals $\hat{n}$ from slot t to slot $t+1$ using the equation

$$\hat{n}_{t+1} = \max\left(\hat{n}_{\min}, \hat{n}_t + u_0 I[z_t = 0] + u_1 I[z_t = 1] + u_c I[z_t \geq 2]\right) \quad (1)$$

where $\hat{n}_{\min}$ is the minimum allowable value for $\hat{n}$, I is the indicator function, and u_0, u_1, and u_c are the control parameters. For example, if $u_0 = 0$, $u_1 = -1$, and $u_c = +1$ then $\hat{n}$ will remain the same after an idle slot, decrease by one (but not be less than $\hat{n}_{\min}$) after a success, and increase by one after a collision.

Clare [4] provides a method to prove that a system with the given set of parameters (u_0, u_1, u_c) which satisfy the equation

$$u_0 + u_1 + u_c(e-2) = 0 \text{ with } u_c > 0 \text{ and } u_0 \leq 0 \quad (2)$$

is stable for all $\lambda < 1/e$. This differs from Hajek and Van Loon [3], Rivest [5], and Thomopoulos [6] who prove the existence of stable schemes, but do not prove that any scheme with specific parameters is stable. According to Kelly [10], Mikhailov [2] has developed a scheme that appears to be similar to Clare's [4]. This author has not seen [2]. As a specific example, Clare [4] proves stable a scheme which updates the estimate in the number of blocked terminals with

$$(u_0, u_1, u_c) = (2 - e, 0, 1) \approx (-0.72, 0, 1) \quad (3)$$

with $\hat{n}_{\min} = 1$, and $p_r = (1 - 1/e)(\hat{n} - 1/e)$.

Rivest [5] develops a scheme called pseudo-Bayesian approximation that uses an estimate of the input load $\hat{\lambda}$ to update the estimated number of blocked terminals with

$$(u_0, u_1, u_c) = (\hat{\lambda} - 1, \hat{\lambda} - 1, \hat{\lambda} + 1/(e-2)) \quad (4)$$

with $\hat{n}_{\min} = 1$, and $p_r = 1/\hat{n}$. The estimate $\hat{\lambda}$ is computed by

$$\hat{\lambda}_{t+1} = 0.995\,\hat{\lambda}_t + 0.005\, I[z_t = 1]. \quad (5)$$

If we let $\hat{\lambda} = 1/e$ then pseudo-Bayesian approximation becomes

$$(u_0, u_1, u_c) = (1/e - 1, 1/e - 1, (1/e + 1)/(e-2)) \approx (-0.63, -0.63, 1.76) \quad (6)$$

which is a member of the class of schemes that Clare [4] proved stable.

Thomopoulos [6] develops an approximate asymptotic minimum mean square error predictor that also uses an estimate of the input load $\hat{\lambda}$ to estimate the number of blocked terminals. The estimated number of blocked terminals is updated with

$$(u_0, u_1, u_c) = (0, \hat{\lambda} - 1, \hat{\lambda}(e-1)/(e-2)) \quad (7)$$

with $\hat{n}_{\min} = (1 - \hat{\lambda})/(1 - (3\hat{\lambda}/2))$, and $p_r = (1 - 1/e)/\hat{n}$. The estimate $\hat{\lambda}$ is computed by

$$\hat{\lambda}_{t+1} = \frac{1}{t+1}\left[\hat{n}(t+1 \mid t, \hat{\lambda}_t) + \sum_{k=0}^{t} I[z_k = 1] + \hat{\lambda}_0\right] \quad (8)$$

where $0 < \hat{\lambda}_0 < 1/e$. Using $\hat{\lambda} = 1/e$, the Thomopoulos [6] scheme

Paper approved by the Editor for Multiple Access Strategies of the IEEE Communications Society. Manuscript received July 13, 1988; revised March 3, 1990.

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IEEE Log Number 9039077.

0090-6778/90/1100-1932$01.00

TABLE I
COMPARISON OF THE MEAN NUMBER OF BLOCKED TERMINALS VERSUS THROUGHPUT λ FOR VARIOUS CONTROL SCHEMES USING IMMEDIATE FIRST TRANSMISSION (IFT)

(u_0, u_1, u_c)	$\lambda = 0.20$	$\lambda = 0.30$	$\lambda = 0.32$	$\lambda = 0.34$	$\lambda = 0.35$	$\lambda = 0.36$
ideal	0.37 < 0.37 < 0.38	1.97 < 1.99 < 2.00	3.23 < 3.27 < 3.32	6.39 < 6.50 < 8.54	10.29 < 10.59 < 10.88	23.43 < 24.96 < 25.94
(-0.72, 0.00, 1.00)	0.43 < 0.43 < 0.44	2.31 < 2.33 < 2.35	3.75 < 3.80 < 3.84	7.39 < 7.53 < 7.67	12.46 < 12.73 < 12.99	27.74 < 29.44 < 31.14
aammse	0.46 < 0.46 < 0.46	2.42 < 2.45 < 2.48	3.96 < 4.23 < 4.50	7.72 < 7.86 < 7.99	12.50 < 12.92 < 13.33	not stable
(0.00,-0.63, 0.88)	0.43 < 0.43 <0.43	2.31 < 2.33 < 2.35	3.81 < 3.86 < 3.90	7.46 < 7.59 < 7.71	12.38 < 12.68 < 12.99	26.69 < 27.97 < 29.24
pseudo-Bayesian	0.43 < 0.43 < 0.43	2.30 < 2.32 < 2.34	3.83 < 3.88 < 3.92	7.44 < 7.61 < 7.78	12.35 < 12.63 < 12.90	28.16 < 29.74 < 30.77
(-0.63,-0.63, 1.76)	0.44 < 0.45 < 0.45	2.33 < 2.35 < 2.37	3.81 < 3.86 < 3.90	7.42 < 7.55 < 7.67	12.30 < 12.66 < 13.02	27.12 < 28.01 < 28.91
sa	0.44 < 0.44 < 0.44	2.43 < 2.45 < 2.46	4.10 < 4.15 < 4.21	8.81 < 9.00 < 9.19	16.53 < 17.15 < 17.76	not stable

TABLE II
COMPARISON OF THE MEAN NUMBER OF BLOCKED TERMINALS VERSUS THROUGHPUT λ FOR VARIOUS CONTROL SCHEMES USING DEFERRED FIRST TRANSMISSION (DFT)

(u_0, u_1, u_c)	$\lambda = 0.20$	$\lambda = 0.30$	$\lambda = 0.32$	$\lambda = 0.34$	$\lambda = 0.35$	$\lambda = 0.36$
ideal	0.24 < 0.24 < 0.24	0.97 < 0.98 < 0.99	1.50 < 1.52 < 1.55	2.84 < 2.94 < 3.03	4.63 < 4.81 < 5.00	10.17 < 10.83 < 11.49
(-0.72, 0.00, 1.00)	0.42 < 0.42 < 0.42	2.13 < 2.15 < 2.17	3.48 < 3.52 < 3.55	6.74 < 6.85 < 6.97	11.11 < 11.39 < 11.68	22.56 < 23.69 < 24.82
aammse	0.33 < 0.69 < 1.04	2.45 < 2.47 < 2.50	2.36 < 5.56 < 8.76	6.78 < 9.43 < 12.07	10.91 < 13.05 < 15.20	23.70 < 27.86 < 32.02
(0.00,-0.63, 0.88)	0.42 < 0.42 < 0.42	2.18 < 2.20 < 2.22	3.56 < 3.61 < 3.65	6.96 < 7.05 < 7.14	11.17 < 11.44 < 11.71	23.31 < 24.18 < 25.05
pseudo-Bayesain	0.42 < 0.42 < 0.42	2.17 < 2.19 < 2.21	3.58 < 3.62 < 3.67	6.81 < 6.93 < 7.04	11.34 < 11.66 < 11.98	24.32 < 24.33 < 25.23
(-0.63,-0.63, 1.76)	0.42 < 0.42 < 0.42	2.16 < 2.18 < 2.19	3.51 < 3.55 < 3.60	6.78 < 6.90 < 7.02	10.97 < 11.29 < 11.61	22.62 < 23.60 < 24.57
sa	0.55 < 0.55 < 0.55	3.00 < 3.02 < 3.04	4.91 < 4.98 < 5.05	10.11 < 10.35 < 10.58	20.99 < 21.76 < 22.52	not stable
(-0.20,-0.59, 1.10)	0.40 < 0.40 < 0.40	2.04 < 2.06 < 2.07	3.33 < 3.36 < 3.39	6.50 < 6.62 < 6.73	10.49 < 10.70 < 10.92	22.17 < 23.16 < 24.16
(-0.30,-0.60, 1.25)	0.40 < 0.40 < 0.40	2.04 < 2.05 < 2.07	3.30 < 3.34 < 3.38	6.48 < 6.56 < 6.64	10.57 < 10.82 < 11.07	22.19 < 22.92 < 23.65

becomes

$$(u_0, u_1, u_c) = (0, 1/e - 1, (1/e)(e - 1)/(e - 2)) \approx (0, -0.63, 0.88) \tag{9}$$

which is another member of the class of schemes that Clare [4] proved stable.

Hajek and Van Loon [3] develop a scheme called stochastic approximation that uses idle, success, collision feedback to directly estimate the probability that a blocked terminal will attempt retransmission in a given slot as follows:

$$p_{r_{t+1}} = \min\left(p_{\max}, p_{r_t} \times \left(a_0 I[z_t = 0] + a_1 I[z_t = 1] + a_c I[z_t \geq 2]\right)\right) \tag{10}$$

with

$$(a_0, a_1, a_c) = \left(\left(e^{\frac{1-2e^{-1}}{1-e^{-1}}}\right)^{\gamma}, 1, \left(e^{\frac{-e^{-1}}{1-e^{-1}}}\right)^{\gamma}\right) \tag{11}$$

and $p_{\max} = (e - 1)/(2e - 1) \approx 0.38$. As in [3] and [11], we let $\gamma = 0.3$ which gives $(a_0, a_1, a_c) \approx (1.13, 1, 0.84)$ for both IFT and DFT. Hence, the retransmission probability p_r will be multiplied by 1.13 (but not allowed to grow larger than $p_{\max}$) after an idle slot, it will not change after a success, and it will be multiplied by 0.84 after a collision. Merakos and Kazakos [11] show that this scheme, with these parameters, is not stable for λ larger than about 0.356. Using the techniques from [12], it can be shown that a sufficient condition for stability using stochastic approximation is

$$a_0 + a_1 + a_c(e - 2) = e \text{ with } a_0, a_1 \geq 1 \text{ and } 0 < a_c < 1. \tag{12}$$

This result, which is striking similar to (2), has not been previously noted.

II. DELAY VERSUS THROUGHPUT SIMULATIONS

Our simulations use thirty trials, of one million slots each, for a total of 30 million slots at each input load. The delay versus throughput results are displayed in Tables I and II, with the scheme by Fayolle *et al.* labeled "ideal," the approximate asymptotic minimum mean square error scheme labeled "aammse," stochastic approximation labeled "sa," and the other schemes labeled with their (u_0, u_1, u_c) parameters. The table shows the 95% lower confidence limit, the mean, and the 95% upper confidence limit of the number of blocked terminals for various input loads λ. The delay can be obtained directly from the number of blocked terminals using Little's formula. Table I contains the IFT results and Table II the DFT results. These data show that (1) the methods that use an estimate of the number of blocked terminals provide better performance than stochastic approximation, (2) when using a method that estimates the number of blocked terminals, DFT provides uniformly better delay versus throughput performance than IFT, (3) the different methods proposed to date for estimating the number of blocked terminals do not differ in delay versus throughput performance at the 95% confidence level, and (4) the effort required to compute an estimate of input load in pseudo-Bayesian and approximate asymptotic minimum mean square estimation is not rewarded with improved delay versus throughput performance.

III. OPTIMUM REALIZABLE DELAY VERSUS THROUGHPUT PERFORMANCE

Using (2), we explored the u_0, u_c parameter space looking for the smallest delay using DFT at input load $\lambda = 0.32$. The results of this exploration using DFT are shown in Figs. 1 and 2. Surfaces similar to the surfaces shown in Figs. 1 and 2 were also obtained for IFT. Fig. 1 is a plot of the mean number of blocked terminals as a function of u_0 and u_c for $-3 \leq u_0 \leq 0$, $0.2 \leq u_c \leq 6$ using DFT with an input load $\lambda = 0.32$. It shows that there is a minimum point

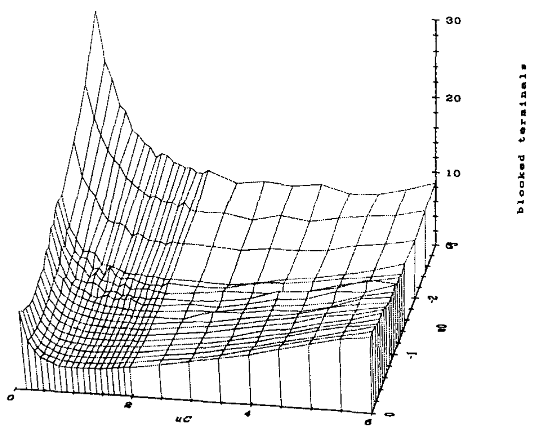

Fig. 1. Number of blocked terminals versus μ_0 and μ_c using DFT with an input load of $\lambda = 0.32$.

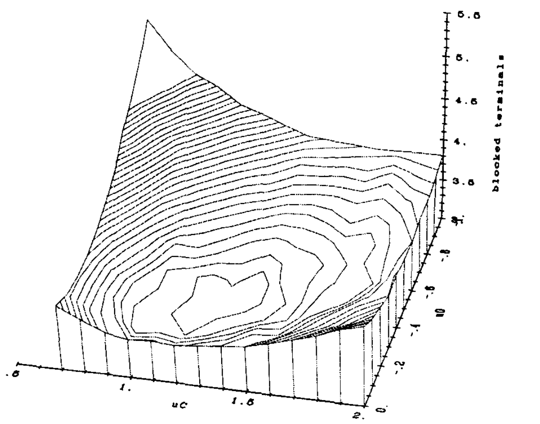

Fig. 2. Contour plot of the number of blocked terminals versus μ_0 and μ_c using DFT with an input load of $\lambda = 0.32$.

in the delay surface near $u_c \approx 1$. Fig. 2 is a contour plot of this same function over the region $-1 \leq u_0 \leq 0$, $0.7 \leq u_c \leq 2$. It shows a minimum near $u_0 = -0.3$, $u_c = 1.25$. Figs. 1 and 2 both show the surface is nearly flat around the minimum point. Hence, delays close to the minimum will be obtained for any choice of u_0, $0 \leq u_0 \leq 0.8$, and u_c, $0.7 \leq u_c \leq 2.0$. Simulations for $(u_0, u_1, u_c) = (-0.3, -0.60, 1.25)$ and $(u_0, u_1, u_c) = (-0.2, -0.59, 1.10)$ are included in Table II and they show that the mean delay is smaller than the mean delay using the previously proposed schemes for all input loads $\lambda \leq 0.34$ at the 95% confidence level.

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