TRANSACTIONS ON EMERGING TELECOMMUNICATIONS TECHNOLOGIES
Trans. Emerging Tel. Tech. 2012; **00**:1–12
DOI: 10.1002/ett

RESEARCH ARTICLE

Code-Expanded Radio Access Protocol for M2M Communications

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ABSTRACT

The random access methods used for support of Machine-to-Machine (M2M), also referred to as Machine-Type Communications (MTC), in current cellular standards are derivatives of traditional framed slotted ALOHA and therefore do not support high user loads efficiently. We propose an approach that is motivated by the random access method employed in LTE, which significantly increases the amount of contention resources without increasing the system resources, such as contention sub-frames and preambles. This is accomplished by a logical, rather than physical, extension of the access method in which the available system resources are interpreted in a novel manner. Specifically, in the proposed scheme, users perform random access by transmitting orthogonal preambles in multiple random access sub-frames, in this way creating access codewords that are used for contention. We show that, for the same number of random access sub-frames and orthogonal preambles, the amount of available contention resources is drastically increased, enabling the massive support of MTC users that is beyond the reach of current systems.

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1. INTRODUCTION

In the past few years there has been an increase in the number of networked machines on current networks designed for human-centric communications. This has led to a shift in the conventional perception on human-centric communications towards device-centric communications, which are independent of human interaction [1, 2]. While the common requirements of traditional human-centric communications are high bit-rates and lower latency, in device-centric communications, one of the main requirements is the massive transmission of simultaneous low data rate messages.

Within 3GPP, there is an ongoing study on the adaptation of the 3GPP cellular networks to handle Machine-Type Communications (MTC) traffic [3, 4], through the inclusion of enhanced load control mechanisms in the Radio Access Network (RAN). This is paramount, because due to the expected large number of deployed MTC devices, the cellular networks are expected to withstand traffic bursts [5]. In such situations, radio and signalling network congestions may occur due to mass concurrent transmissions [6], which can lead to large delays, packet loss and, in the extreme case, service unavailability.

1.1. 3GPP Load Control Approach

In 3GPP there have been proposed several solutions for managing the Random Access Channel (RACH) load [3], which are here listed briefly:

- **Access Class Barring** - where the network controls the load by restricting the access of devices based on their class. In case of overload due to MTC traffic, the network can restrict the access of those same devices, while allowing other devices to continue normal network access [7, 8];
- **Orthogonal Resources** - where the network provides separated RACH resources for MTC devices and traditional devices. In LTE this can be accomplished by dividing the random access slots and the preamble sequences, i.e. in the time and preamble domain [9];
- **Dynamic Resources Allocation** - where the network allocates dynamically additional RACH resources based on the RACH load condition and overall traffic load [2]. The scheme proposed in this paper is of this type, although it is accomplished through a non-traditional approach, which will be exposed later in the paper;

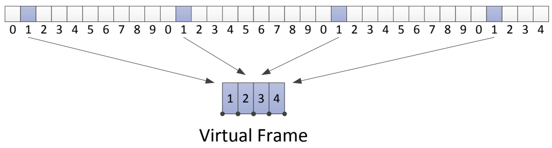

Figure 1. Example of PRACH opportunities organized in virtual frames.

- **Backoff** - where the network in case of overload enacts a backoff request making the connecting devices retry the random access at a later stage. This scheme is less effective upon massive batch arrivals, as expected from MTC traffic [5];
- **Slotted access** - where the network assigns specific random access slots to a MTC device or a group of MTC devices and where each MTC device is only allowed to access on its dedicated access slot;
- **Pull-based** - where an entity within the core network triggers the eNodeB to page the intended MTC terminals. Upon receiving the paging signal, the MTC devices initiate the random access. Here the eNodeB only proceeds with the paging when the traffic load is favourable.

Among these, the 3GPP decided to extend the Access Class Barring method to handle the MTC traffic. In this method, denoted as Extended Access Barring (EAB), the network is able to selectively control the access attempts from multiple MTC User Equipment (M-UE) configured for EAB [3]. This approach, according to 3GPP [3], allows to control the access network load without the need to introduce new access classes.

1.2. Proposed Scheme

In contrast with the current 3GPP direction, in this paper we propose an extension to the LTE random access method, which is in line with the dynamic resource allocation approach. Here the dynamic resource allocation is not accomplished through traditional means, such as the increase of the number of available preambles and/or random access sub-frames, but instead by introducing the concept of Code-Expanded random access scheme. Where, this scheme is an application of the Protocol Coding approach, first introduced in [10], [11], [12], [13]. The motivation behind this proposal is to enable existing systems to sustain a large and bursty random access loads, while preserving the physical layer unchanged and incurring minimal changes in the medium access control layer.

To illustrate what is meant by code-expanded random access, first consider that the random access in LTE occurs only in specific sub-frames, here denoted as contention sub-frames. Further, as depicted in the Fig. 1, we assume that a group of consecutive contention sub-frames are grouped together in a contention *virtual frame*. Now consider the diagrams in Fig. 2, where the selection

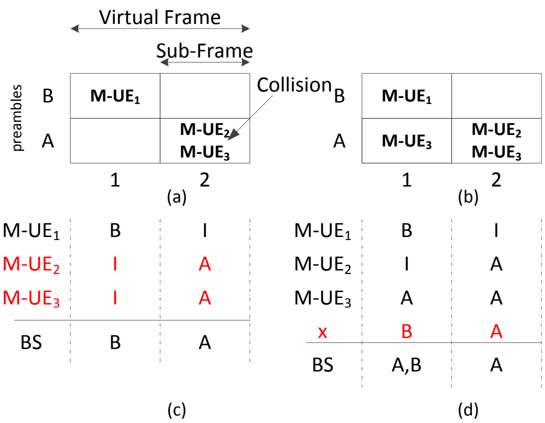

Figure 2. (a) Reference random access, (b) Code-expanded random access, (c) Reference random access codewords, with collisions in red, (d) Code-expanded codewords, with phantom codewords in red.

of preambles and sub-frames in the reference and in the proposed code-expanded random access schemes are depicted. In both schemes the sub-frames are grouped in contention virtual frames and the random access is performed always at the virtual frame level.

In the reference scheme, the M-UE performs the random access by selecting one of the available preambles, in this example denoted as A and B, and then selecting one of the virtual frame sub-frames to transmit the chosen preamble, as depicted in Fig. 2(a). The proposed code-expanded random access scheme is a generalization of the reference scheme, where the M-UE transmits one or none of the available preambles in each of the virtual frame sub-frames, as depicted in Fig. 2(b). For both schemes, when the M-UE does not transmit any preamble in the sub-frame, we assume that the M-UE actually transmits an *idle* preamble, henceforth denoted as I. Therefore, in both schemes each M-UE contends using an *access codeword* with length equal to the length of the virtual frame.

When multiple M-UEs transmit the same preamble in the same sub-frame the BS is still able to detect that preamble [14], i.e. the collisions are assumed to be non-destructive. Therefore, the Base Station (BS) discerns between M-UEs according to the observed preambles in each random access sub-frame, as shown in Fig. 2(c) and Fig. 2(d), i.e., observes a set of access codewords that are then used to discern between M-UEs.

In the reference scheme the codewords are composed of just one preamble sent in one of the sub-frames of the virtual frame, with the remaining sub-frames at idle. Taking as example the case depicted in Fig. 2(c), the BS perceives the preamble B in the first sub-frame and the preamble A in the second sub-frame. Therefore, the BS perceives the transmission of the codewords (B, I) and (I, A).

In the proposed scheme, each M-UE selects a codeword consisting of a randomly chosen preamble (including the

DOI: 10.1002/ett
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idle preamble) in every sub-frame of the virtual frame. In this way, the number of contention resources are expanded and the amount of collisions is reduced - a collision occurs when two or more M-UEs select the same codeword, as depicted in Fig. 2(c) where both M-UE$_2$ and M-UE$_3$ choose the codeword (I, A).

While reducing the amount of collisions, the code-expanded generalization introduces a shortcoming not present in the reference scheme. In the reference scheme, the codewords do not introduce ambiguities at the BS in regards to which codewords where transmitted, i.e., based on the observation of the preambles in the virtual frame, the BS can always discern which codewords were actually sent. In the proposed scheme, such ambiguities exist - based on the observation, the BS may conclude that there are more codewords present in the virtual frame than there were actually sent. Namely, multiple combinations of transmitted codewords can yield the same observation, introducing phantom codewords which were not sent by any of the transmitting M-UEs.

In Fig. 2(d) is depicted a combination of the three codewords used by M-UEs ((B, I), (I, A) and (A, A)), which misleads the BS to perceive the phantom codeword (B, A). The existence of phantom codewords affects adversely the performance, however, we show that this is significant only when the network is experiencing low user loads. Specifically, we show that the efficiency of the proposed approach (i.e., the fraction of M-UEs that successfully contended for access) for moderate and high loads and for the same number of used preambles and sub-frames in the virtual frame, is substantially higher than the efficiency of the reference scheme, despite the drawbacks caused by phantom codewords. We also show that by choosing the operating random access method according to the user load, it is possible to maintain an efficient random access over a large load region using the same number of preambles and sub-frames.

The work presented in this paper is an extended version of the material first presented in [15]. The extension pertains to the following: a comprehensive explanation of the reference random access mechanism; a generalization of the method used to compute the distribution that models the number of perceived codewords by the BS for any codebook size; and a extension of that method to the case where subsets of the codebook are used.

1.3. Related Work

The Code-Expanded random access scheme belongs to a broader class of novel advanced random access schemes. The recent advancements include the work found in [16] and [17], where the authors propose a random access protocol that takes advantage of the analogies that have been established between use of successive interference cancellation in slotted ALOHA based access schemes and iterative belief-propagation erasure-decoding, enabling application of theory and tools from codes-on-graphs for the design of random access schemes

There are several works found in the literature that deal with the machine-to-machine (M2M) challenges in different settings and following different approaches than the one here considered. In [18] the authors examine and propose schedule based solutions for dealing with diverse M2M time-controlled traffic profiles within a LTE context. This approach complements the one here presented, since we focus instead on the enhancement of the random access part for the serving of M2M traffic generated by a massive amount of M-UEs.

In [19], a new concept of M2M communicatios infrastructure is introduced, which is accomplished via airliners. This work presents a first study on possible coverage within Europe and North America using airliners. This approach can be used to complement the coverage of M2M communications currently provided mainly by cellular networks.

In [20] the authors discuss how sensor and electronic-health networks can be served by the 3GPP networks (GSM, UMTS, LTE), with the goal of offering quality-of-service to this kind of applications. The proposed code-expanded approach enables to address some of this issues, since it enables

Finally, in [21] the authors revisit the LTE random access procedure in regards to its application to the synchronization of the user equipment within the network. The authors propose a novel scheme that provides superior performance in comparison with the existing methods, by taking in account the frequency selectivity of the channel. It should be noted that in this paper we do not consider how the proposed code-expanded scheme can be used to enable synchronization within the network. Such extension is left for future work.

1.4. Article Structure

The remainder of this paper is organized as follows. In Section 2 we model and analyse both the reference and the proposed random access scheme. In Section 3 we discuss how the proposed code-expanded scheme can be used to enable a random access mechanism which adapts according to the load in the random access. Finally, Section 4 concludes the paper.

2. SYSTEM MODEL

In LTE the random access is performed through the Random Access Channel (RACH), which is mapped in the physical layer to the Physical Random Access Channel (PRACH) [22].

The random access procedure is depicted in Figure 3, and is composed of the following steps [23, 24, 25, 26]:

1. The UE selects one of the preamble sequences available in the cell, [25], and transmits the preamble in one of the sub-frames reserved for the PRACH transmissions;

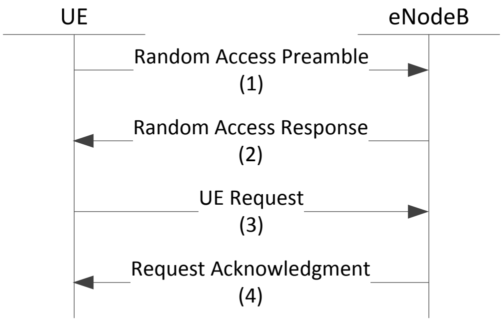

Figure 3. LTE Random Access Procedure

2. The eNodeB decodes the preamble and responds to the UE by sending a Random Access Response. This response includes the matching preamble information together with the information about the uplink resource that the UE should use for further information exchange as well as the timing advance to be used;
3. The UE proceeds with the information exchange on the resources indicated by the PRACH response;
4. Finally. the eNodeB acknowledges the information received from the UE.

The periodicity of the PRACH resources can scale according to the expected RACH load [24]. Therefore, the PRACH resources can occur from once in every sub-frame (1000 RACH sub-frames per second) to once in every two frames (50 RACH sub-frames per second), where a frame consists of 10 sequential sub-frames lasting 1 ms [27].

As previously mentioned, we assume that a group of consecutive sub-frames is organized in a virtual frame reserved for PRACH (i.e., random access), as depicted in the Fig. 1. We denote the number of sub-frames within a virtual frame by L and the number of available preambles by M.

2.1. Reference Random Access Scheme

The reference scheme considered here is inspired by the LTE random access [23, 24]. We model it as an access reservation scheme [28], as depicted in Figure 4. We focus on the contention phase, which consists of L sub-frames (contention slots), and these sub-frames constitute a single virtual frame. To ease the analysis, we assume that in the non-contention phase there are always available enough data slots to serve the incoming requests. We assume that there are N contending M-UEs; where each M-UE randomly chooses a preamble and a sub-frame of the virtual frame in which the chosen preamble is sent.

In the LTE random access scheme, the contention occurs at the sub-frame level, in contrast with the reference scheme presented here, where the contention occurs at the virtual frame level. However, the amount of contention opportunities is the same in both cases, therefore one could argue that for high user loads, the efficiency of the

Figure 4. Access reservation.

contention phase of the LTE scheme is worse than of the reference scheme, as the number of collisions in the former is higher, due to the M-UEs being able to contend more than once within the virtual frame.

The expected fraction of the M-UEs that succeed in the contention phase is denoted as the expected efficiency of the contention phase S_r, which is given by:

$$S_r = \frac{N_S}{N_P}, \tag{1}$$

where N_S is the expected number of codewords used by a single M-UE (singles) and N_P is the expected number of perceived codewords by the BS. *

The expected number of perceived codewords, N_P, in the reference random access scheme accounts for all the single and collision codewords is given by:

$$N_P = N_S + N_C \tag{2}$$

where N_C is the number of codewords used by multiple M-UEs (collisions), $N_P \leq A_r$, and A_r is the number of codewords available in the reference scheme. As outlined earlier, in a contention phase consisting of L sub-frames, each M-UE sends only one preamble, therefore the number of available codewords, A_r, is given by:

$$A_r = M \cdot L. \tag{3}$$

Assuming that the number of M-UEs contending per codeword is modeled by the random variable X, then the probability that in a given contention codeword there are k M-UEs contending is:

$$\Pr[X = k] = \binom{N}{k} \left(\frac{1}{A_r}\right)^k \left(1 - \frac{1}{A_r}\right)^{N-k}. \tag{4}$$

The expected number of codewords chosen by a single M-UE, N_S, is given by:

$$\begin{aligned} N_S &= \Pr[X = 1] \cdot A_r \\ &= \binom{N}{1} \left(\frac{1}{A_r}\right)^1 \left(1 - \frac{1}{A_r}\right)^{N-1} \cdot A_r \\ &= N \left(1 - \frac{1}{A_r}\right)^{N-1}, \end{aligned} \tag{5}$$

*We note that the strict definition of the expected efficiency is the expectation of the ratio between N_S and N_P, instead of the ratio of the expectations as here considered. This approximation holds with the increase of the number of contending M-UEs.

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DOI: 10.1002/ett
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and, similarly, the expected number of codewords chosen by multiple M-UEs, N_C, is:

$$\begin{aligned} N_C &= \Pr[X > 1] \cdot A_r \\ &= (1 - \Pr[X = 0] - \Pr[X = 1]) \cdot A_r \\ &= \left(1 - \left(1 - \frac{1}{A_r}\right)^N - \frac{N}{A_r}\left(1 - \frac{1}{A_r}\right)^{N-1}\right) \cdot A_r. \end{aligned} \tag{6}$$

Substituting (5) and (6) in (1), we get the estimate of the efficiency of the contention phase.

2.2. Code-Expanded Random Access

In the code-expanded random access, the outcome of the contention phase is the set of the codewords perceived by the BS, which the BS assumes to belong to the M-UEs that successfully contended for the access. However, the set of perceived codewords actually contains the codewords that are used just by a single M-UE (singles), codewords that are used by multiple M-UEs (collisions) and codewords that are used by none of the M-UEs (phantom codewords, see Fig. 2). The expected efficiency of the code-expanded random access S_e is:

$$S_e = \frac{N_S}{N_P}. \tag{7}$$

where the N_S is the expected number of single codewords, and N_P is the expected number of codewords that the BS perceives, i.e., N_P accounts for all single, collision and phantom codewords. As in the reference case, we note that the strict definition of the expected efficiency is the expectation of the ratio between N_S and N_P, instead of the ratio of the expectations. As shown in Fig. 8, this approximation holds with the increase of the number of contending M-UEs.

In the code-expanded approach, the M-UEs send in each sub-frame of the virtual frame either one of the M preambles or the idle preamble I, so the total number of available codewords is:

$$A_e = (M+1)^L - 1, \tag{8}$$

where the all-idle codeword is excluded. Furthermore, $N_P \leq A_e$.

The distribution of devices contending per virtual frame is modeled in the same way as in (4), with the difference that the number of available codewords is now higher.

The expected number of codewords chosen by a single M-UE, N_S, is given by:

$$\begin{aligned} N_S &= \Pr[X = 1] \cdot A_e \\ &= \binom{N}{1}\left(\frac{1}{A_e}\right)^1\left(1 - \frac{1}{A_e}\right)^{N-1} \cdot A_e \\ &= N\left(1 - \frac{1}{A_e}\right)^{N-1}. \end{aligned} \tag{9}$$

The method to obtain the expected value of N_P is discussed in the next subsection.

Codeword	L	
	1	2
1	I	A
2	I	B
3	A	I
4	A	A
5	A	B
6	B	I
7	B	A
8	B	B

Table I. Codebook, $L = 2$, $M = 2$.

2.3. Calculation of N_P

For the calculation of N_P we use a representation based on a Markov Chain (MC) that describes the evolution of the perceived number of codewords by the BS when the number of contending M-UEs increases sequentially from 1 to N. We note that it is assumed that the M-UEs select their codewords independently and uniformly at random from the set of available codewords.

The MC states are determined by the configuration that corresponds to the number of the observed preambles in the sub-frames of the virtual frame, which is created by the actual codewords selected by the M-UEs. The configuration is denoted by $(C_1, C_2, C_3, ..., C_L)$, where C_j is the number of observed preambles by the BS in the j-th subframe. We note here that C_j always includes the idle preamble, i.e., if the number of unique preambles in the j-th sub-frame is C_j, then the number of actually observed preambles is $C_j - 1$. Each state is characterized by a cardinality, which is the cardinality of the set of codewords perceived by the BS that is created by the given configuration.

We define the vector $\boldsymbol{\alpha}$ as a vector with s entries, where s is the number of states in the MC. Also, if the number L of subframes is given, then we indicate this with a superscript, i.e. $\boldsymbol{\alpha}^{(L)}$. For the i-th state configuration $(C_1^i, C_2^i, C_3^i, ..., C_L^i)$ the corresponding cardinality is the i'th entry in $\boldsymbol{\alpha}$, and it is given by

$$\alpha_i = \prod_{j=1}^{L} C_j^i - 1, \tag{10}$$

where, once again, we assumed that the all-idle codeword is not used by any of the M-UEs. Using this model, N_P can be assessed as the average cardinality of the set of the codewords perceived after $N-1$ transitions of the MC, averaged over the probability distribution of the MC states after $N-1$ transitions.

To ease the explanation, we focus on an example case where $L = 2$ and $M = 2$, and therefore $A_e = 8$. The full codebook is shown in Table I, while the MC representation including the state configurations, cardinalities and possible state transitions is shown in Table II. For example, the state configuration of the state 2 is $(1, 3)$, implying that in the first sub-frame of the

Starting Symbols
(I,I)

Selected Code		Transition State
IA	→	(1,2)
IB	→	(1,2)
AI	→	(2,1)
AA	→	(2,2)
AB	→	(2,2)
BI	→	(2,1)
BA	→	(2,2)
BB	→	(2,2)

Figure 5. Initial state vector selection procedure.

virtual frame there is an idle preamble I, and in the second sub-frame there is an idle preamble and both available preambles, A and B. The cardinality of state 2 is then $\alpha_2 = 2$.

Initially, when there is only one M-UE attempting random access, the system can be in the states 1, 3 and 4. The probability of the system being in any of those states is simply the ratio of the number of codewords from the codebook which provides the corresponding state configuration and the total number of available codewords A_e, as depicted in Figure 5.

For example, for state 1, where the state configuration is $(1, 2)$, from Table I and Figure 5 it can be seen that there are only two codewords that satisfy the state configuration, which are (I, A) and (I, B). Therefore, the probability of the BS perceiving this state upon the M-UE transmission of one of the available codewords is $\frac{2}{8}$. A similar reasoning is done for the remaining possible initial states, and the following initial state probability vector $\boldsymbol{\pi}^{(1)}$ is:

$$\boldsymbol{\pi}^{(1)} = \frac{1}{8}\begin{bmatrix}2 & 0 & 2 & 4 & 0 & 0 & 0 & 0\end{bmatrix} \tag{11}$$

where $\pi_i^{(1)}$ is the probability that the system is initially in the i-th state. Therefore, when one M-UE attempts random access, i.e. when $N = 1$, N_P is obtained as:

$$N_P = \sum_{i=1}^{A_e} \alpha_i \cdot \pi_i^{(1)} = 2 \tag{12}$$

where α_i is the cardinality of the i-th state, obtained from (10) and listed in Table II.

In the case where there are two M-UEs attempting random access, it is assumed that the selection of the transmitted codewords is sequential and independent. Therefore, the codeword selected by the first M-UE leads the system to be in one of the possible initial states, which are 1, 3 and 4. When the second M-UE selects the codeword, the system can transit to any of the states listed in Table II. The transition between each state and the corresponding transition probabilities are depicted in Figure 6.

State (i)	C_1^i	C_2^i	α_i	Transitions
1	1	2	1	1,2,4,5
2	1	3	2	2,5
3	2	1	1	3,4,6,7
4	2	2	3	4,5,7,8
5	2	3	5	5,8
6	3	1	2	6,7
7	3	2	5	7,8
8	3	3	8	8

Table II. Markov Chain Model, $L = 2$, $M = 2$.

The transition probabilities can be obtained following the same reasoning as the one used to obtain $\boldsymbol{\pi}^{(1)}$. Consider that the first M-UE selects a codeword that puts the system in state 1, i.e. the M-UE selected either (I, A) or (I, B). Now, whatever codeword the second M-UE selects, the system can only transit to the states $1, 2, 4, 5$, as depicted in Figure 7(a). For the system to transit from state 1 to state 5, it means that the second M-UE has to select a codeword that consists of either preamble A or B in the first sub-frame, and the remaining yet unused preamble in the second sub-frame. For the considered example, this is the codeword (A, B) or (B, B) if the initial codeword was (I, A), or the codeword (A, A) or (B, A) if the initial codeword was (I, B), thus making the configuration become $(2, 3)$. From the preceding discussion, we see that no matter which codeword caused the chain to be in state 1, the transition to state 5 can be caused by the second M-UE selecting (one of) two codewords from the set of all codewords. Therefore, this transition probability is equal to $\frac{2}{8}$ and it does not depend on which codeword the first M-UE selected to bring the system to state 1.

To further illustrate the described procedure, consider that the second M-UE puts the system in state 5. From Figure 7(b), it can be seen what are the possible state transitions based on both the state configuration of the system as well as on the selection of a codeword by a third M-UE.

Using similar reasoning, it can be shown that the transition probabilities do not depend on the choices of codewords that brought the system to a given state and, for every state transition, the transition probability is the ratio of the number of codewords that enable the transition and the total number of available codewords A_e. For the considered example, the transition matrix $\boldsymbol{P}$ is:

$$\boldsymbol{P} = \frac{1}{8}\begin{bmatrix}1 & 1 & 0 & 4 & 2 & 0 & 0 & 0\\ 0 & 2 & 0 & 0 & 6 & 0 & 0 & 0\\ 0 & 0 & 1 & 4 & 0 & 1 & 2 & 0\\ 0 & 0 & 0 & 3 & 2 & 0 & 2 & 1\\ 0 & 0 & 0 & 0 & 5 & 0 & 0 & 3\\ 0 & 0 & 0 & 0 & 0 & 2 & 6 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 5 & 3\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8\end{bmatrix}$$

where the (i, j)'th entry is the transition probability for going from state i to j.

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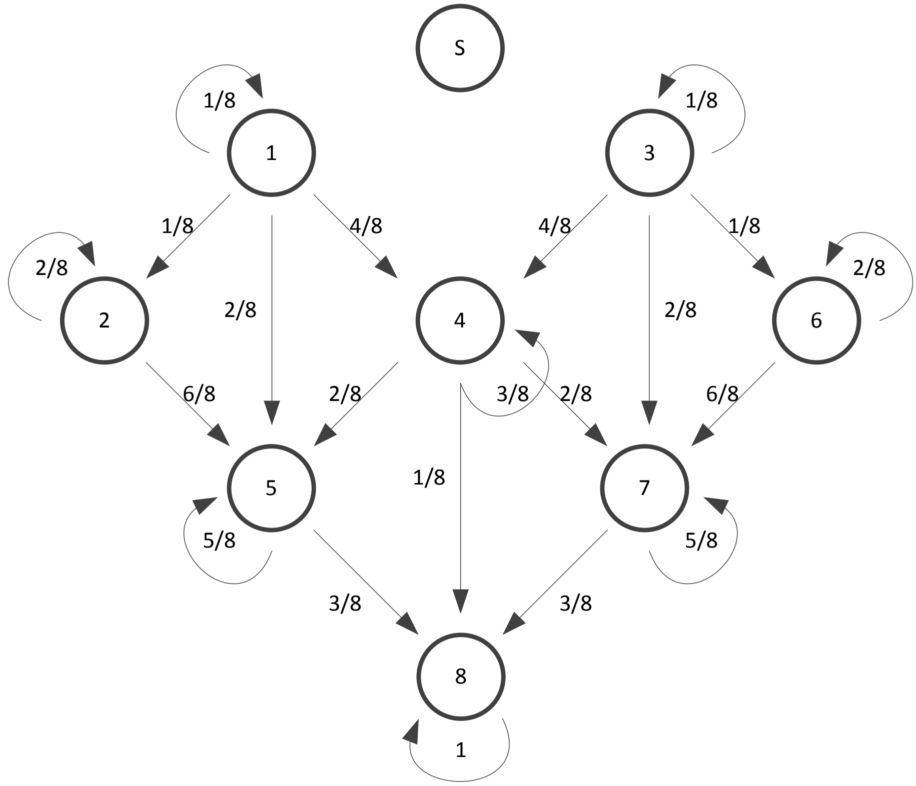

Figure 6. Markov Chain Transition Model, $L = 2$, $M = 2$; where S represents the state.

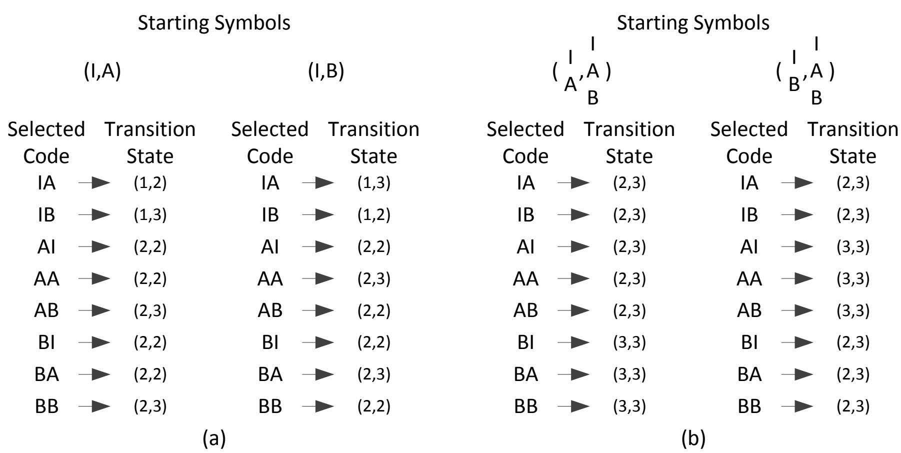

Figure 7. Transition matrix element deduction for configuration state (a) (1,2) and (b) (2,3).

The expected number of perceived codewords N_P can be obtained as follows:

$$N_P = \sum_{i=1}^{A_e} \alpha_i \cdot \pi_i^{(N)}, \tag{13}$$

where $\pi_i^{(N)}$ is the i-th element of the state vector $\boldsymbol{\pi}^{(N)}$, i.e., the probability of the BS perceiving a state configuration given by the i-th state after the N-th user has chosen his codeword. The α_i is the cardinality of the i-th state, given by (10). As for any MC, the state probability distribution $\boldsymbol{\pi}^{(N)}$ is:

$$\boldsymbol{\pi}^{(\boldsymbol{N})} = \boldsymbol{\pi}^{(\mathbf{1})} \cdot \boldsymbol{P}^{(N-1)}. \tag{14}$$

Fig. 8 depicts the comparison of the efficiencies for the reference scheme calculated using (1), code-expanded scheme calculated using (7) and (13), and code-expanded scheme that is obtained using Monte Carlo simulations,

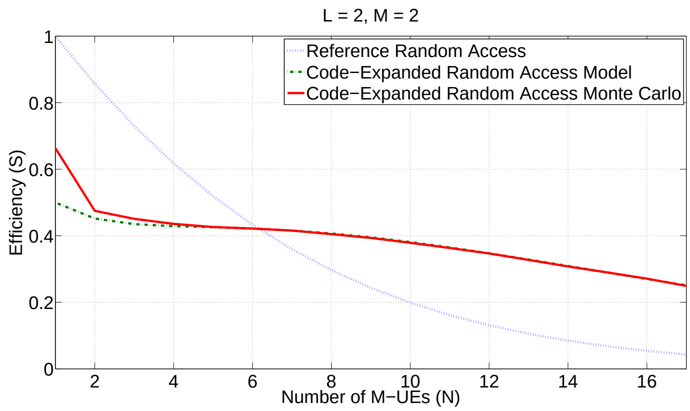

Figure 8. Reference random access and code-expanded random access comparison.

for the example case when $L = 2$ and $M = 2$. It is obvious that, as the number of M-UEs grows, the efficiency of the code-expanded method outperforms the reference one. Also, both curves corresponding to the code-expanded method converge, justifying the use of (13) when computing the expected efficiency using (7).

Finally, we note that the described methodology for modelling N_P can be generalized for arbitrary L and M, as shown in the next sub-section.

2.4. Derivation of N_P for the general case

We now generalize the N_P model to any arbitrary L and M. The generalized model is constructed in a three step approach:

1. Construction of the model for the case where all the possible codewords are considered;
2. Removal of the all idle codeword from the transition matrix;
3. Construction of the initial state vector $\boldsymbol{\pi}^{(1)}$.

The transition matrix is constructed iteratively, with the base case being $L = 1$. The matrix for the base case is:

$$\boldsymbol{P}_1 = \frac{1}{M+1}\begin{bmatrix} 1 & M & & & \\ & 2 & M-1 & & \\ & & \ddots & \ddots & \\ & & & M & 1 \\ & & & & M+1 \end{bmatrix}, \tag{15}$$

where the rows and columns are ordered according to how many distinct preambles are used. The first row corresponds to the state 0, i.e. when only the *Idle* symbol is present. If a user selects the codeword 0, then the system stays in this state. Otherwise, as there are M orthogonal symbols excluding the idle symbol, there are M possibilities for the system going to the next state. We note that for $L = 1$ the number of codewords and orthogonal symbols is the same. For the second row, the $(2, 2)$ matrix entry counts the number of ways the second user can select a codeword, given that the first user has selected one. The second user can either select the all-idle codeword or the same codeword as the first user. This accounts for the 2 in the second collumn of the second row of the matrix (15). The entry with $M - 1$ counts the number of other codewords that the second user can select. Continuing in this way, the matrix for M symbols and $L = 1$ is as given in (15).

Now we consider the case where the codeword is now of length $L = 2$. When the second sub-frame is added, we get that this sub-frame can contain $0, 1, 2, \ldots, M$, corresponding to how many distinct orthogonal symbols were transmitted in that sub-frame. For the case of $L = 2$, the transition matrix is then:

$$\boldsymbol{P}_2 = \frac{1}{M+1}\cdot \begin{bmatrix} \boldsymbol{P}_1 & M\boldsymbol{P}_1 & & & \\ & 2\boldsymbol{P}_1 & (M-1)\boldsymbol{P}_1 & & \\ & & \ddots & \ddots & \\ & & & M\boldsymbol{P}_1 & \boldsymbol{P}_1 \\ & & & & (M+1)\boldsymbol{P}_1 \end{bmatrix},$$

by a similar counting argument as the case of $L = 1$.

The cardinality state vector, $\boldsymbol{\alpha}^{(1)}$ for the case of $L = 1$ is of the form

$$\boldsymbol{\alpha}^{(1)} = (1, 2, 3, \ldots, M+1), \tag{16}$$

where each entry corresponds to the number of distinct symbols transmitted. For the case where $L = 2$, the cardinality state vector is of the form

$$\boldsymbol{\alpha}^{(2)} = (\boldsymbol{\alpha}^{(1)}, 2\boldsymbol{\alpha}^{(1)}, 3\boldsymbol{\alpha}^{(1)}, \ldots, (M+1)\boldsymbol{\alpha}^{(1)}). \tag{17}$$

We now explain the meaning of the entries in this vector. Firstly, this vector is composed of $M + 1$ blocks of the form $k\boldsymbol{\alpha}^{(1)}$, where $1 \le k \le M + 1$. The k'th block in $\boldsymbol{\alpha}^{(2)}$, $k\boldsymbol{\alpha}^{(1)}$, corresponds to the case where k preambles, including the idle preamble, are used in the second slot.

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DOI: 10.1002/ett
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The transition matrix and $\boldsymbol{\alpha}^{(L+1)}$ for the case of $L+1$ slots can then be constructed from the case of L slots, in the same manner shown. For the case of $L+1$ slots, the transition matrix is:

$$\boldsymbol{P}_{L+1} = \frac{1}{M+1} \cdot \begin{bmatrix} \boldsymbol{P}_L & M\boldsymbol{P}_L & & & \\ & 2\boldsymbol{P}_L & (M-1)\boldsymbol{P}_L & & \\ & & \ddots & \ddots & \\ & & & M\boldsymbol{P}_L & \boldsymbol{P}_L \\ & & & & (M+1)\boldsymbol{P}_L \end{bmatrix},$$

and the $\boldsymbol{\alpha}$ is

$$\boldsymbol{\alpha}^{(L+1)} = (\boldsymbol{\alpha}^{(L)}, 2\boldsymbol{\alpha}^{(L)}, 3\boldsymbol{\alpha}^{(L)}, \ldots, (M+1)\boldsymbol{\alpha}^{(L)}). \tag{18}$$

Because the way the transition matrix $\boldsymbol{P}_L$ is constructed, we can write it as:

$$\boldsymbol{P}_L = \frac{1}{(M+1)^L}\boldsymbol{Q}, \tag{19}$$

where $\boldsymbol{Q}$ is a $(M+1)^L \times (M+1)^L$ matrix.

The next step is to remove the all-idle codeword from the transition matrix and state cardinality vector. This is accomplished through the following three steps:

1. For the transition matrix $\boldsymbol{P}_L$, the first row and first column are deleted;
2. Then all entries on the diagonal of the resulting matrix are decreased by one, since the all-idle codeword is no longer a valid choice;
3. The transition matrix is normalized with $\frac{1}{(M+1)^L-1}$ instead of $\frac{1}{(M+1)^L}$;
4. The first entry of the state cardinality vector, $\boldsymbol{\alpha}$, is removed, and the other entries are decreased by one.

The next step is to determine the initial vector $\boldsymbol{\pi}^{(1)}$. From the case when the idle is a valid codeword, this vector is $(1, 0, \ldots, 0)$. The row that is removed from the matrix, contains in its entries the probability for going from the state configuration $(0, \ldots, 0)$ to another state when selecting a new codeword. Since the all-idle codeword is removed, the starting probabilities of the current case are contained in this row. Therefore, the $\boldsymbol{\pi}^{(1)}$ vector is set equal to the removed row of the transition matrix.

3. ADAPTIVE CODE-EXPANDED RANDOM ACCESS

In the previous subsection we considered using only the codebook consisting of all the available codewords (i.e., the full codebook). In this subsection we consider the case where a subset of the full codebook is selected based on the system input load. A subset is obtained by restricting the number of preambles that could be used in each of the sub-frames of the virtual frame (we note that these restrictions

State (i)	C_1^i	C_2^i	α_i	Transitions
1	1	2	1	1,2,5,6
2	1	3	2	2,3,7
3	1	4	3	3,4,8
4	1	5	4	4,9
5	2	1	1	5,6,10,11
6	2	2	3	6,7,11,12
7	2	3	5	7,8,12,13
8	2	4	7	8,9,13,14
9	2	5	9	9,14
10	3	1	2	10,11,15,16
11	3	2	5	11,12,16,17
12	3	3	8	12,13,17,18
13	3	4	11	13,14,18,19
14	3	5	14	14,19
15	4	1	3	15,16,20,21
16	4	2	7	16,17,21,22
17	4	3	11	17,18,22,23
18	4	4	15	18,19,23,24
19	4	5	19	19,24
20	5	1	4	20,21
21	5	2	9	21,22
22	5	3	14	22,23
23	5	4	19	23,24
24	5	5	24	24

Table III. Markov Chain Model, $L = 2$, $M = 4$.

are not necessarily the same for every sub-frame). We note that the strategy used here to define the codebook subsets may not be the optimal one. The efficiency in this case can be computed following the procedure described in the previous subsections.

As an illustrative example consider the case where $L = 2$ and $M = 4$, for which the MC model is given in Table III. The possible cardinality values of the states for this MC are then $\{1, 2, 3, 4, 5, 7, 8, 9, 11, 14, 15, 19, 24\}$. This sequence represents all the possible subsets in regards to the number of codes that can be used. For $L = 2$ and $M = 4$ the number of codewords for the reference scheme is $A_r = 8$, so the cardinalities of interest are the ones where $A_e = \{9, 11, 14, 15, 19, 24\}$. In Figure 9 is depicted the efficiency for the given example, both for the reference scheme and for the code-expanded scheme when cardinalities of interest are used. As it can be observed, there is a set of threshold values in regards to the number of M-UEs, where the system should increase the cardinality of the codebook in use, in order to maintain the efficiency. Overall, it could be concluded that an adaptive approach, where the codebook is selected based on the estimate of the user load is the preferred operating strategy.

In Figure 10 is depicted the efficiency of the adaptive code-expanded approach for $L = 4$ and $M = 4$. Comparing these results with the results given in Fig. 9, it can be observed that the region where the gain of the adaptive approach over the case when the full codebook is

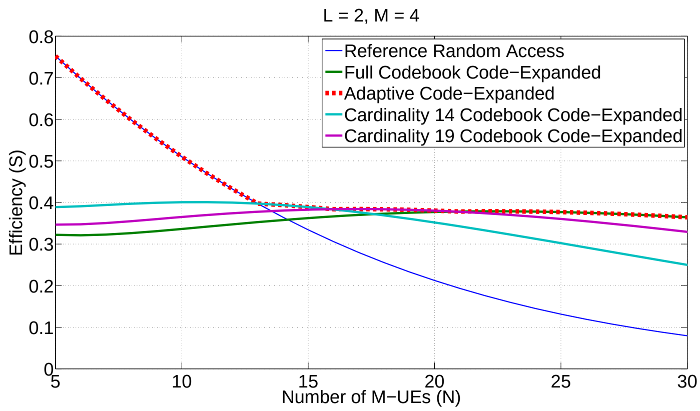

Figure 9. Adaptive Random Access example with $L = 2$ and $M = 4$.

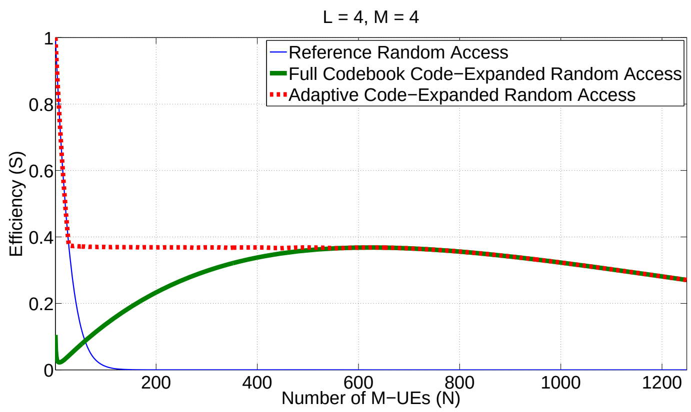

Figure 10. Adaptive Random Access example with $L = 4$ and $M = 4$.

used is more obvious. Also, the increased M provides for significant expansion of the supported user load region.

In next subsection we give the method to compute N_P for the case where a subset of the codebook is used.

3.1. Derivation of N_P for a subset of the codebook

The method exposed in sub-section 2.4, can be extended to compute the expected value of N_P when only a restricted subset of the codewords available in the codebook are used.

We define the vector $\boldsymbol{v}$ as:

$$\boldsymbol{v} = (v_1, v_2, \ldots, v_L), \tag{20}$$

where v_j is the maximum number of distinct orthogonal symbols that are allowed to be used in sub-frame j. For example, $\boldsymbol{v} = (1, 3, 2)$ means that only the idle symbol is allowed in sub-frame 1, two orthogonal symbols are allowed in sub-frame 2, and one orthogonal symbol is allowed in sub-frame 3.

The construction of the transition matrix $\boldsymbol{P}_L$ is then as follows. For the base case:

$$\boldsymbol{P}_1 = \frac{1}{v_1 + 1} \begin{bmatrix} 1 & v_1 & & & \\ & 2 & v_1 - 1 & & \\ & & \ddots & \ddots & \\ & & & v_1 & 1 \\ & & & & v_1 + 1 \end{bmatrix}, \tag{21}$$

and the matrix for the case when the $(j+1)$'th sub-frame is added is

$$\boldsymbol{P}_{j+1} = \frac{1}{v_j + 1} \cdot \begin{bmatrix} \boldsymbol{P}_j & v_j \boldsymbol{P}_j & & & \\ & 2\boldsymbol{P}_j & (v_j - 1)\boldsymbol{P}_j & & \\ & & \ddots & \ddots & \\ & & & v_j \boldsymbol{P}_j & \boldsymbol{P}_j \\ & & & & (v_j + 1)\boldsymbol{P}_j. \end{bmatrix}.$$

The $\boldsymbol{\alpha}$ for the base case is:

$$\boldsymbol{\alpha}^{(1)} = (1, 2, 3, \ldots, v_1), \tag{22}$$

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DOI: 10.1002/ett
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and the cardinality state vector $\boldsymbol{\alpha}$ for the $(j+1)^{th}$ step is obtained from the j'th step as:

$$\boldsymbol{\alpha}^{(j+1)} = (\boldsymbol{\alpha}^{(j)}, 2\boldsymbol{\alpha}^{(j)}, \ldots, v_j\boldsymbol{\alpha}^{(j)}). \qquad (23)$$

The initial vector $\boldsymbol{\pi}^{(1)}$ is obtained as in the non-restricted case, i.e. it is obtained from the first row that is removed from the transition matrix which includes the all idle codeword. Also, the computation of the expected N_P proceeds as in the non-restricted case.

4. CONCLUSION

In this paper we proposed a code-expanded random access scheme inspired by the LTE random access. The proposed scheme increases the amount of available contention resources, without resorting to the increase of system resources, such as contention sub-frames and preambles. This increase is accomplished by expanding the contention space to the code domain, through the creation of random access codewords.

It was shown that for high user loads the proposed scheme is significantly more efficient than the reference scheme. Also, it was shown that by selecting the appropriate number of random access codewords it is possible to maintain the random access scheme efficiency over a large load region. This suggests the usage of an adaptive random access scheme, i.e., a combination of the reference and the code-expanded random access, which allows to maintain the efficiency both for low and high user load regions.

Finally, we note that for optimal operation of the proposed adaptive random access scheme, it is necessary to obtain the estimates of the user loads. The design of the adaptive random access scheme that implements the estimation of the user load is a topic of ongoing research. Considerations on the energy usage due to the transmission of codewords needs to be investigated in the future.

ACKNOWLEDGMENT

The research presented in this paper was supported by the Danish Council for Independent Research (Det Frie Forskningsråd) within the Sapere Aude Research Leader program, Grant No. 11-105159 "Dependable Wireless Bits for Machine-to-Machine (M2M) Communications".

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Trans. Emerging Tel. Tech. 2012; **00**:1–12 © 2012 John Wiley & Sons, Ltd.
DOI: 10.1002/ett
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