The utility of the smart jammer at time slot k denoted by $u_J^{(k)}$ is given by

$$u_J^{(k)} = -\max\left(\frac{Ph_1^{(k)}}{\sigma + y^{(k)}h_3^{(k)}}, \min\left(\frac{Ph_2^{(k)}}{\sigma + y^{(k)}h_4^{(k)}}, \frac{x^{(k)}h_5^{(k)}}{\sigma}\right)\right) + x^{(k)}C_U - y^{(k)}C_J, \tag{1}$$

and the time index k is omitted in the superscript.

Theorem 1. *The UAV applying Algorithm 1 in the dynamic UAV relay game can achieve the optimal policy $(0,0)$, if*

$$\max\left(\frac{Ph_1h_3}{\sigma^2}, \frac{Ph_2h_4}{\sigma^2}, \frac{Ph_2h_4(h_3\sqrt{h_2h_4} - h_4\sqrt{h_1h_3})^2}{\sigma^2(\sqrt{h_1h_3}(1-h_4) + \sqrt{h_2h_4}(h_3-1))^2}\right)$$
$$< C_J \le \frac{Ph_1h_3(h_3\sqrt{h_2h_4} - h_4\sqrt{h_1h_3})^2}{\sigma^2(\sqrt{h_1h_3}(1-h_4) + \sqrt{h_2h_4}(h_3-1))^2} \tag{2}$$

$$C_U \ge \frac{h_5}{\sigma}, \tag{3}$$

and the corresponding BER with QPSK is given by

$$P_e = \frac{1}{2}\mathrm{erfc}\left(\frac{Ph_1}{\sigma}\right). \tag{4}$$

Proof: By (1), if (3) holds, we have

$$\frac{\partial u(x,0)}{x} = \frac{h_5}{\sigma} - C_U \le 0, \tag{5}$$

thus the utility u is decreases with respect to (w.r.t.) x, and we have

$$u(0,0) = \frac{Ph_1}{\sigma} \ge \max\left(\frac{Ph_1}{\sigma}, \min(\frac{Ph_2}{\sigma}, \frac{xh_5}{\sigma})\right) - xC_U = u(x,0). \tag{6}$$

By (1), if (2) holds, we have

$$\frac{\partial u_J(0,y)}{y} = -\max\left(\frac{-Ph_1h_3}{(\sigma+yh_3)^2}, \frac{-Ph_2h_4}{(\sigma+yh_4)^2}\right) - C_J \tag{7}$$

$$\frac{\partial^2 u_J(0,y)}{\partial y^2} = -\max\left(\frac{2Ph_1h_3^2}{(\sigma+yh_3)^3}, \frac{2Ph_2h_4^2}{(\sigma+yh_4)^3}\right) \le 0, \tag{8}$$

indicating that $u_J(0,y)$ is concave with respect to (w.r.t.) y. Thus, $\forall y \in [0, P_J^M]$, we have

$$u_J(0,0) = -\frac{Ph_1}{\sigma} \ge -\frac{Ph_1}{\sigma+yh_3} = u_J(0,y). \tag{9}$$

Thus, the optimal policy is $(0,0)$. According the QPSK modulation, we have

$$P_e = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{Ph_1}{\sigma}}\right). \tag{10}$$

■