

The utility of the UAV at time slot k denoted by $u^{(k)}$ is given by

$$u^{(k)} = -\frac{1}{2}\text{erfc} \left(\max \left(\sqrt{\frac{Ph_1^{(k)}}{\sigma + y^{(k)}h_3^{(k)}}}, \min \left(\sqrt{\frac{Ph_2^{(k)}}{\sigma + y^{(k)}h_4^{(k)}}}, \sqrt{\frac{x^{(k)}h_5^{(k)}}{\sigma}} \right) \right) \right) - x^{(k)}C_U, \quad (1)$$

and the utility of the smart jammer at time slot k denoted by $u_J^{(k)}$ is given by

$$u_J^{(k)} = \frac{1}{2}\text{erfc} \left(\max \left(\sqrt{\frac{Ph_1^{(k)}}{\sigma + y^{(k)}h_3^{(k)}}}, \min \left(\sqrt{\frac{Ph_2^{(k)}}{\sigma + y^{(k)}h_4^{(k)}}}, \sqrt{\frac{x^{(k)}h_5^{(k)}}{\sigma}} \right) \right) \right) + x^{(k)}C_U - y^{(k)}C_J, \quad (2)$$

and the time index k is omitted in the superscript.

Theorem 1. *The UAV applying Algorithm 1 in the dynamic UAV relay game can achieve the optimal policy $(0, 0)$, if*

$$C_J \geq \frac{3\exp(\frac{-3}{2})h_3}{2\sqrt{\pi}Ph_1} \quad (3)$$

$$C_U \geq \frac{2h_5}{\sqrt{\pi}\sigma}, \quad (4)$$

and the corresponding BER with QPSK is given by

$$P_e = \frac{1}{2}\text{erfc} \left(\sqrt{\frac{Ph_1}{\sigma}} \right). \quad (5)$$

Proof: By (1), if (4) holds, we have

$$\frac{\partial u(x, 0)}{x} = \frac{2h_5\exp(\frac{-xh_5}{\sigma})}{\sqrt{\pi}\sigma} - C_U, \quad (6)$$

$$\frac{\partial^2 u(x, 0)}{\partial x^2} = -\frac{2h_5^2\exp(\frac{-xh_5}{\sigma})}{\sqrt{\pi}\sigma^2} \leq 0, \quad (7)$$

indicating that $u(x, 0)$ is concave with respect to (w.r.t.) x . Thus, $\forall x \in [0, P_U^M]$, we have

$$u(0, 0) = -\frac{1}{2}\text{erfc} \left(\sqrt{\frac{Ph_1}{\sigma}} \right) \geq -\frac{1}{2}\text{erfc} \left(\max \left(\sqrt{\frac{Ph_1}{\sigma}}, \min \left(\sqrt{\frac{Ph_2}{\sigma}}, \sqrt{\frac{xh_5}{\sigma}} \right) \right) \right) - xC_U = u(x, 0). \quad (8)$$

By (2), if (3) holds, we have

$$\frac{\partial u_J(0, y)}{y} = \frac{\sqrt{Ph_1}h_3\exp(\frac{-Ph_1}{\sigma+yh_3})}{\sqrt{\pi}(\sigma+yh_3)^{\frac{3}{2}}} - C_J \quad (9)$$

$$\frac{\partial^2 u_J(0, y)}{\partial y^2} = \frac{\sqrt{Ph_1}h_3^2 \left(\frac{Ph_1}{\sigma+yh_3} - \frac{3}{2} \right) \exp(\frac{-Ph_1}{\sigma+yh_3})}{\sqrt{\pi}(\sigma+yh_3)^{\frac{5}{2}}} \leq 0, \quad (10)$$

indicating that $u_J(0, y)$ is concave with respect to (w.r.t.) y . Thus, $\forall y \in [0, P_J^M]$, we have

$$u_J(0, 0) = \frac{1}{2}\text{erfc} \left(\sqrt{\frac{Ph_1}{\sigma}} \right) \geq \frac{1}{2}\text{erfc} \left(\sqrt{\frac{Ph_1}{\sigma+yh_3}} \right) - yC_J = u_J(0, y). \quad (11)$$

Thus, the optimal policy is $(0, 0)$. According the QPSK modulation, we have

$$P_e = \frac{1}{2}\text{erfc} \left(\sqrt{\frac{Ph_1}{\sigma}} \right). \quad (12)$$

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