The utility of the smart jammer at time slot $k$ denoted by $u_J^{(k)}$ is given by

$$u_J^{(k)} = -\max\left(\frac{Ph_1^{(k)}}{\sigma + y^{(k)}h_3^{(k)}}, \min\left(\frac{Ph_2^{(k)}}{\sigma + y^{(k)}h_4^{(k)}}, \frac{x^{(k)}h_5^{(k)}}{\sigma}\right)\right) + x^{(k)}C_U - y^{(k)}C_J, \tag{1}$$

and the time index $k$ is omitted in the superscript.

**Theorem 1.** *The UAV applying Algorithm 1 in the dynamic UAV relay game can achieve the optimal policy* $(0, 0)$*, if*

$$\begin{aligned} &\max\left(\frac{Ph_1h_3}{\sigma^2}, \frac{Ph_2h_4}{\sigma^2}, \frac{Ph_2h_4(h_3\sqrt{h_2h_4} - h_4\sqrt{h_1h_3})^2}{\sigma^2(\sqrt{h_1h_3}(1-h_4) + \sqrt{h_2h_4}(h_3-1))^2}\right) \\ &< C_J \leq \frac{Ph_1h_3(h_3\sqrt{h_2h_4} - h_4\sqrt{h_1h_3})^2}{\sigma^2(\sqrt{h_1h_3}(1-h_4) + \sqrt{h_2h_4}(h_3-1))^2} \qquad (2) \\ &C_U \geq \frac{h_5}{\sigma}, \qquad (3) \end{aligned}$$

*and the corresponding BER with QPSK is given by*

$$P_e = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{Ph_1}{\sigma}}\right). \tag{4}$$

*Proof:* By (1), if (3) holds, we have

$$\frac{\partial u(x, 0)}{x} = \frac{h_5}{\sigma} - C_U \leq 0, \tag{5}$$

thus the utility $u$ is decreases with respect to (w.r.t.) $x$, and we have

$$u(0, 0) = \frac{Ph_1}{\sigma} \geq \max\left(\frac{Ph_1}{\sigma}, \min(\frac{Ph_2}{\sigma}, \frac{xh_5}{\sigma})\right) - xC_U = u(x, 0). \tag{6}$$

By (1), if (2) holds, we have

$$\frac{\partial u_J(0, y)}{y} = -\max\left(\frac{-Ph_1h_3}{(\sigma + yh_3)^2}, \frac{-Ph_2h_4}{(\sigma + yh_4)^2}\right) - C_J \tag{7}$$

$$\frac{\partial^2 u_J(0, y)}{\partial y^2} = -\max\left(\frac{2Ph_1h_3^2}{(\sigma + yh_3)^3}, \frac{2Ph_2h_4^2}{(\sigma + yh_4)^3}\right) \leq 0, \tag{8}$$

indicating that $u_J(0, y)$ is concave with respect to (w.r.t.) $y$. Thus, $\forall y \in [0, P_J^M]$, we have

$$u_J(0, 0) = -\frac{Ph_1}{\sigma} \geq -\frac{Ph_1}{\sigma + yh_3} = u_J(0, y). \tag{9}$$

Thus, the optimal policy is $(0, 0)$. According the QPSK modulation, we have

$$P_e = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{Ph_1}{\sigma}}\right). \tag{10}$$

■