**Problem Set #1**

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## Problem 1

**Cited article** Matthias Doepke and Fabian Kindermann (Sept. 2019). "Bargaining over Babies: Theory, Evidence, and Policy Implications". In: *American Economic Review* 109.9, pp. 3264–3306. DOI: 10.1257/aer.20160328. URL: http://www.aeaweb.org/articles?id=10.1257/aer.20160328

**A Theoretical Model: Fertility Choice** This paper set up a bargaining model for fertility to explain the negotiation and decision of a couple in a household on whether to give birth to a or another baby. The model conceptualizes the decision-making of the man and the woman in a household as rational choices as a result of comparing the utilities under different outcomes.

Model settings:

- $f$, $m$: the woman and the man in a household.
- $w_f$, $w_m$: the market wage earned by $f$ and $m$.
- $\phi$: the cost of the child in terms of consumption.

Each partner $g \in \{f, m\}$ has their utility:

$$u_g(c_g, b) = c_g + bv_g$$

- $c_g \geq 0$: consumption of $g$.
- $b \in \{0, 1\}$: indicator of whether they have a child .
- $v_g$: the additional utility that partner $g$ receives from having a child.

The outside options (opportunity costs) are:

$$\bar{u_f}(0) = w_f \text{ and } \bar{u_m}(0) = w_m$$

The fertility choice of this household is characterized by the utility maximization problem[1]:

$$\begin{aligned} \max_{b, c_f, c_m} \quad & (c_f + bv_f - w_f)^{0.5}(c_m + bv_m - w_m)^{0.5} \\ \text{s.t.} \quad & c_f + c_m = (1 + \alpha)(w_f + w_m - \phi b) \end{aligned}$$

, where

- $\alpha$: the factor by which the effective income of this household increases through increasing return to scale.

[1] The model has been adapted to conserve writing.

**Exogenous and Endogenous Variables** This model simulates the decision making of fertility choice by taking the wages, cost of raising a child, utility brought by having a child, and the efficiency scale factor as inputs and generating the choices as outputs. Explicitly, we have:

- Exogenous variables: market wages $w_f$ and $w_m$, cost of raising a child $\phi$, and additional utility of having a child $v_g$.
- Endogenous variables: fertility decision $b$ and other consumptions $c_f, c_m$.

**Classify the Model** This is a **static**, **non-linear**, and **deterministic** model.

**Missing variables** This model assumes the cost of raising a child is shared by the couple jointly. However, it is possible in reality that one of the partner in a household takes more burden than the other due to different levels of commitment to their responsibility in a family. Therefore, another variable that is missing in the current model can be:

- $\pi_f$, $\pi_m$: the share of cost of raising a child taken by $f$ and $m$, respectively.

A further developed model in Doepke and Kindermann (2019) characterizes this problem by generalizing the current model into a "lack of commitment" setting through a two-stage decision-making process. However, even in the current model, which is under a "full commitment" setting, the cost can be actually shared unevenly due to cultural reasons, occupations of the agents, contributions of other family members, etc, resulting in differentiated utility levels. To introduce this factor into the current model, $\pi_f$ and $\pi_m$ can be included as another term in the utility functions.

**Problem 2**

**A Theoretical Model: Marriage Choice** The choice of marriage can be characterized by a utility maximization model. We set up the following model: let $i \in \{a, b\}$ denote one of the two agents in a marriage relationship and $j \in \mathcal{J}$ denote a good for consumption *(can be spiritual or emotional)*. $I_i$ is the economic income of $i$. $\alpha$ is the return to scale in a marriage relationship. $p$ is the price vector. An agent has the utility function:

$$U_i = f(x_i) + \pi_i \pi_{-i} g(x_i, x_{-i})$$

, where $f(x_i) = [\sum_{j \in \mathcal{J}} a_i^{\frac{1}{s}} x_{ij}^{\frac{s-1}{s}}]^{\frac{s}{1-s}}$ is the utility gained by self consumption, and $g(x_i, x_{-i}) = f(x_i)^m f(x_{-i})^n$ is the additional utility gained from marriage, which depends on the choices of both agents. $\pi_i \in \{0, 1\}$ indicates agent $i$'s decision of whether to get marriage.

Each agent maximizes their utility by choosing whether to get married and their self consumption:

$$\begin{aligned} \max_{\pi_i, x_i} \quad & U_i = f(x_i) + \pi_i \pi_{-i} g(x_i, x_{-i}) \\ \text{s.t.} \quad & p x_i + \pi_i \pi_{-i} p x_{-i} \leq (1 + \pi_i \pi_{-i} \alpha)(I_i + \pi_i \pi_{-i} I_{-i}) \end{aligned}$$

Given the other agent's choice outcome, one agent's choice of whether to get married can be characterized by the optimization problem above:

$$\pi_i^*(\pi_{-i}, x_{-i}, I_i, I_{-i}, \alpha, p) \in \{0, 1\}$$

**Data Generating Process** This model simulates one's decision of whether to get marriage by taking his/her potential partner's decision, income levels of both agents, scale effects of marriage (can be benefits or costs), and the market prices as inputs. Given all the empirically estimated parameters, the optimization problem would give an optimal solution.

**Key factors and Unimportant factors** The key factors that influence the outcome can be:

- Economic income of both agents captured by $I_i$ and $I_{-i}$.
- Benefits or costs of marriage captured by $\alpha$: reduced life expenditure through cohabitation, partner's social position, appearance, personality, and the matching of two agents' education, interests, family backgrounds, physical conditions, etc.

There can be other factors that may influence one's decision of whether to get married such as parents' expectation, cultural stereotypes, peer pressure, etc. To capture such factors, another term can be included in the utility function. However, these factors become less important nowadays as the value of individualism gets more prevailing and acceptable. Nevertheless, whether to take such factors into accounts depends on specific cultural backgrounds and requires further assessments.

**Preliminary Test** To test the efficiency of model specification, we can collect empirical data through surveys. With the data set, we can first use a random subset of the sample to estimate the parameters in the model. The difficulty here is to estimate the return to scale $\alpha$, which may be determined by many factors such as those listed in the previous part. After the parameters are estimated, we can use this model to predict the decision of individuals

in the other subset of the sample, and evaluate the efficiency of the model by comparing the predicted values with the actual data. Re-sampling techniques such as cross-validation can be used to conduct the preliminary test. We can improve the model specification based on the result of the preliminary test.

## References

Doepke, Matthias and Fabian Kindermann (Sept. 2019). "Bargaining over Babies: Theory, Evidence, and Policy Implications". In: *American Economic Review* 109.9, pp. 3264–3306. DOI: 10.1257/aer.20160328. URL: http://www.aeaweb.org/articles?id=10.1257/aer.20160328.