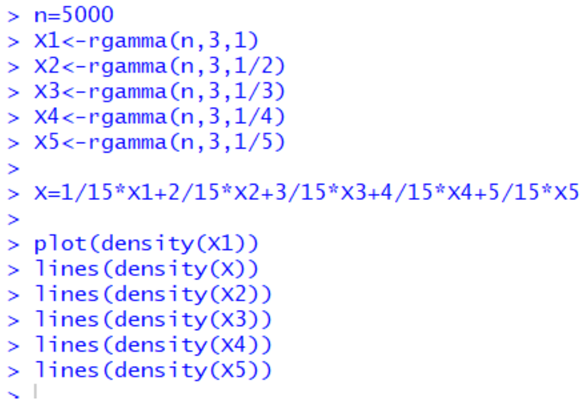
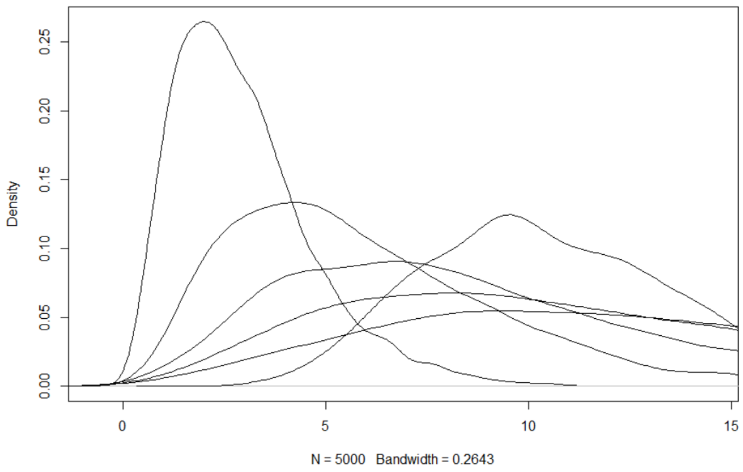
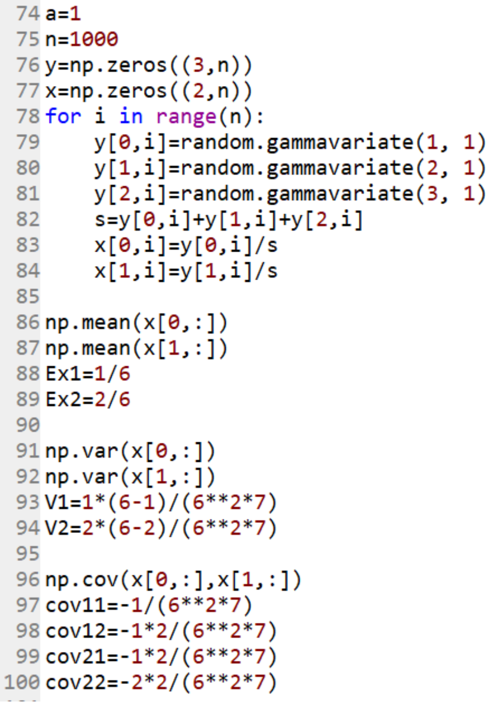
3)



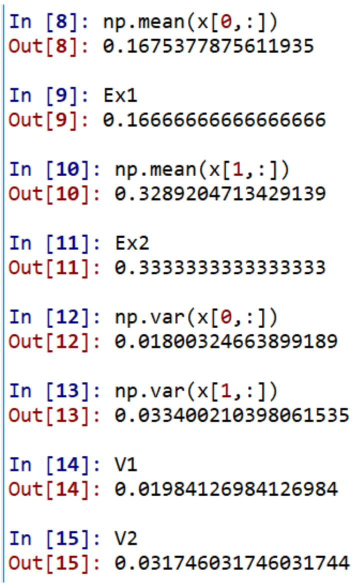
```
> n=5000
> X1<-rgamma(n,3,1)
> X2<-rgamma(n,3,1/2)
> X3<-rgamma(n,3,1/3)
> X4<-rgamma(n,3,1/4)
> X5<-rgamma(n,3,1/5)
>
> X=1/15*X1+2/15*X2+3/15*X3+4/15*X4+5/15*X5
>
> plot(density(X1))
> lines(density(X))
> lines(density(X2))
> lines(density(X3))
> lines(density(X4))
> lines(density(X5))
> |
```



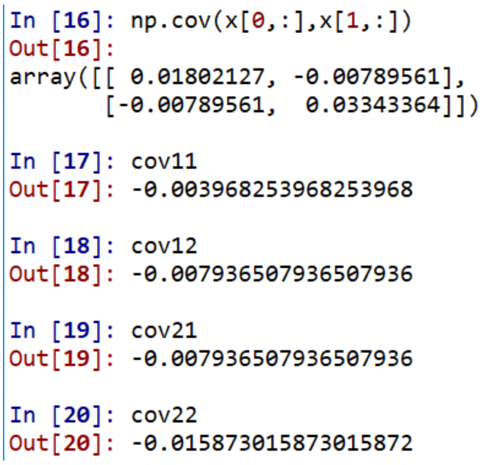
4)



```
a=1
n=1000
y=np.zeros((3,n))
x=np.zeros((2,n))
for i in range(n):
    y[0,i]=random.gammavariate(1, 1)
    y[1,i]=random.gammavariate(2, 1)
    y[2,i]=random.gammavariate(3, 1)
    s=y[0,i]+y[1,i]+y[2,i]
    x[0,i]=y[0,i]/s
    x[1,i]=y[1,i]/s

np.mean(x[0,:])
np.mean(x[1,:])
Ex1=1/6
Ex2=2/6

np.var(x[0,:])
np.var(x[1,:])
V1=1*(6-1)/(6**2*7)
V2=2*(6-2)/(6**2*7)

np.cov(x[0,:],x[1,:])
cov11=-1/(6**2*7)
cov12=-1*2/(6**2*7)
cov21=-1*2/(6**2*7)
cov22=-2*2/(6**2*7)
```

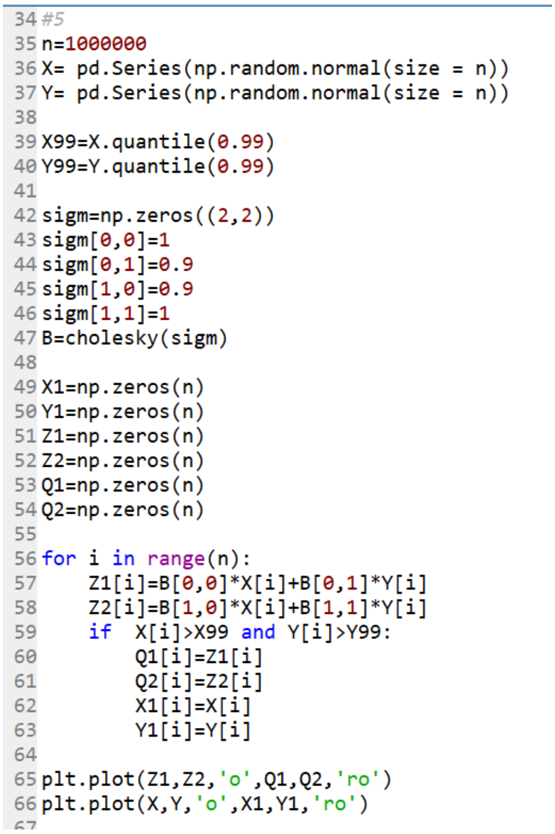


```
In [8]: np.mean(x[0,:])
Out[8]: 0.1675377875611935

In [9]: Ex1
Out[9]: 0.16666666666666666

In [10]: np.mean(x[1,:])
Out[10]: 0.3289204713429139

In [11]: Ex2
Out[11]: 0.3333333333333333

In [12]: np.var(x[0,:])
Out[12]: 0.01800324663899189

In [13]: np.var(x[1,:])
Out[13]: 0.03340021039806l535

In [14]: V1
Out[14]: 0.01984126984126984

In [15]: V2
Out[15]: 0.031746031746031744
```

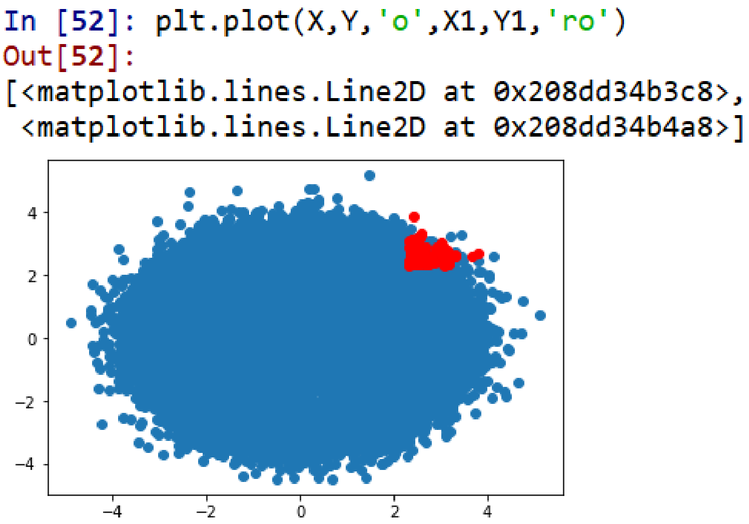


```
In [16]: np.cov(x[0,:],x[1,:])
Out[16]:
array([[ 0.01802127, -0.00789561],
       [-0.00789561,  0.03343364]])

In [17]: cov11
Out[17]: -0.003968253968253968

In [18]: cov12
Out[18]: -0.007936507936507936

In [19]: cov21
Out[19]: -0.007936507936507936

In [20]: cov22
Out[20]: -0.015873015873015872
```

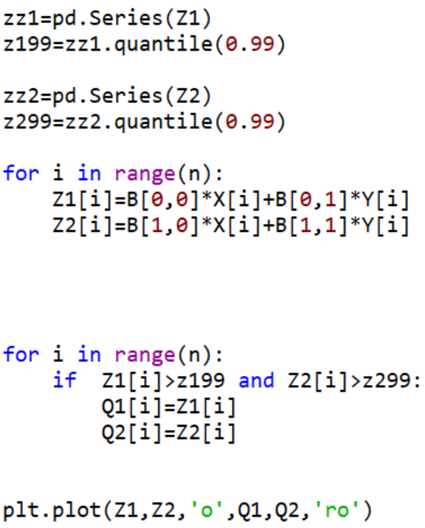
5)



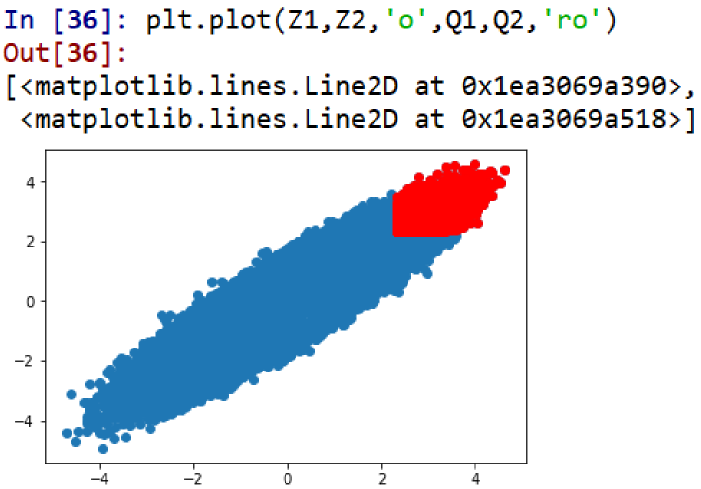
```
#5
n=1000000
X= pd.Series(np.random.normal(size = n))
Y= pd.Series(np.random.normal(size = n))

X99=X.quantile(0.99)
Y99=Y.quantile(0.99)

sigm=np.zeros((2,2))
sigm[0,0]=1
sigm[0,1]=0.9
sigm[1,0]=0.9
sigm[1,1]=1
B=cholesky(sigm)

X1=np.zeros(n)
Y1=np.zeros(n)
Z1=np.zeros(n)
Z2=np.zeros(n)
Q1=np.zeros(n)
Q2=np.zeros(n)

for i in range(n):
    Z1[i]=B[0,0]*X[i]+B[0,1]*Y[i]
    Z2[i]=B[1,0]*X[i]+B[1,1]*Y[i]
    if  X[i]>X99 and Y[i]>Y99:
        Q1[i]=Z1[i]
        Q2[i]=Z2[i]
        X1[i]=X[i]
        Y1[i]=Y[i]

plt.plot(Z1,Z2,'o',Q1,Q2,'ro')
plt.plot(X,Y,'o',X1,Y1,'ro')
```



```
In [52]: plt.plot(X,Y,'o',X1,Y1,'ro')
Out[52]:
[<matplotlib.lines.Line2D at 0x208dd34b3c8>,
 <matplotlib.lines.Line2D at 0x208dd34b4a8>]
```



```
zz1=pd.Series(Z1)
z199=zz1.quantile(0.99)

zz2=pd.Series(Z2)
z299=zz2.quantile(0.99)

for i in range(n):
    Z1[i]=B[0,0]*X[i]+B[0,1]*Y[i]
    Z2[i]=B[1,0]*X[i]+B[1,1]*Y[i]



for i in range(n):
    if  Z1[i]>z199 and Z2[i]>z299:
        Q1[i]=Z1[i]
        Q2[i]=Z2[i]


plt.plot(Z1,Z2,'o',Q1,Q2,'ro')
```



```
In [36]: plt.plot(Z1,Z2,'o',Q1,Q2,'ro')
Out[36]:
[<matplotlib.lines.Line2D at 0x1ea3069a390>,
 <matplotlib.lines.Line2D at 0x1ea3069a518>]
```