Stat-6207 **Homework Set 6** **Fall 2018 (Modarres)**

1) Write a program that uses the continued fraction expansion to approximate the CDF $Q(z)$ of the standard normal distribution. See Abramowitz-Stegun (AS) page 932 Eq. 26.2.14. Set up a forward recurrence and compute $1 - Q(4.7)$.

2) Suppose we want to approximate the tail area of the standard normal distribution $\Psi(z) = \int_z^\infty \phi(x)dx$ for large z where $\phi(z)$ is the standard normal probability density function.

- a. Use integration by parts to show that $\Psi(z) = \frac{\phi(z)}{z}(1 - z^{-2} + 3z^{-4} - 3 \times 5z^{-6} + \ldots)$.
- b. Write a function that evaluates $\Psi(4.7)$ with 12 terms and compare to problem 1.

3) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $f(x) = 1$, $0 < x < 1$.

- a) Find the joint distribution of $M = max(X_1, X_2, \ldots, X_n)$ and $N = min(X_1, X_2, \ldots, X_n)$.
- b) Find the distribution of the range $R = M - N$ and its first 6 cumulants when $n = 10$.
- c) Find the Fisher-Cornish series expansion of the quantiles of R and compare against the distribution of R using a Q-Q plot.

4) Consider a random variable X that is uniformly distributed on $[0, 1]$.

- a) Find the first 6 cumulants of X.
- b) Find the Fisher-Cornish series expansion for the quantiles of $Y = \sum_{i=1}^n X_i^2$ where X_i are i.i.d. copies of X.
- c) Evaluate the accuracy of your approximation when $n = 12$ and compare to the normal approximation of Y using a Q-Q plot.

5) Let Z_0,Z_1, and Z_2 be independent $N(0, 1)$ random variables. For $\rho \geq 0$ and $i = 1, 2$, let $X_i = \sigma_i(\sqrt{1-\rho}Z_i + \sqrt{\rho}Z_0) + \mu_i$.

- a) Show that (X_1, X_2) has a bivariate normal distribution. Find its mean vector and covariance matrix.
- b) Show that $F(x_1, x_2) = P(X_1 \leq x_1, X_2 \leq x_2) = \int_{-\infty}^\infty \prod_{i=1}^2 \Phi(\frac{\sqrt{\rho}+a_i}{\sqrt{1-\rho}})\phi(z)dz$ where $a_i = (x_i - \mu_i)/\sigma_i$, Φ is the cdf and ϕ is the pdf of $N(0, 1)$ distribution.
- c) Write a program to find $F(1.6, 2)$ when $\rho = 0.5$ using Hermit-Gauss integration and compare to pbivnorm of R.