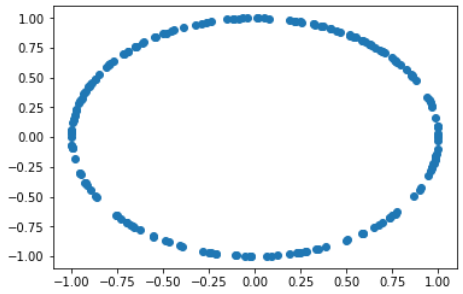
3)

```
In [114]: n=1000000
     ...: U=np.random.rand(n)
     ...: X=np.zeros(n)
     ...: for i in range(n):
     ...:     X[i]=-m.log(-m.log(U[i]))
     ...:
     ...:
     ...: mu=sum(X)/n
     ...: sigma=sum((X-mu)*(X-mu))/n
     ...: X2=sum(X*X)/n
     ...: m2=X2-mu**2
     ...: X3=sum(X*X*X)/n
     ...: sig=m.sqrt(sigma)
     ...: skew=(X3-3*mu*sigma-mu**3)/sig**3
     ...: K=sum((X-mu)*(X-mu)*(X-mu)*(X-mu))/(n*sigma**2)-3

In [115]: skew
Out[115]: 1.1414664119938298

In [116]: K
Out[116]: 2.4131072199982846


In [122]: mu
Out[122]: 0.5786255727827637

In [123]: sigma
Out[123]: 1.6454588973588518
```

4)

```
In [117]: def generate(n,d):
     ...:     BX=np.zeros((n,d))
     ...:     for k in range(n):
     ...:         X=np.random.randn(d)
     ...:         U=np.zeros(d)
     ...:         length=np.sqrt(sum(X*X))
     ...:         for j in range(d):
     ...:             U[j]=X[j]/length
     ...:             BX[k,j]=U[j]
     ...:     return(BX)
     ...:
     ...:
     ...: BX=generate(200,2)
     ...: plt.plot(BX[:,0],BX[:,1],'o')
Out[117]: [<matplotlib.lines.Line2D at 0x29618fa83c8>]
```



5)

```
In [124]: def D(X,i,j):
     ...:     s=0
     ...:     for r in range(d):
     ...:         s=s+(X[i,r]-X[j,r])**2
     ...:     return(s)
     ...:
     ...:
     ...: n=100
     ...: d=10000
     ...: p=0.5
     ...:
     ...: X=np.random.rand(n,d)
     ...: Y=np.zeros((n,d))
     ...: for i in range(n):
     ...:     for j in range(d):
     ...:         if X[i,j]<p:
     ...:             Y[i,j]=1
     ...:
     ...:
     ...: T=0
     ...: for i in range(n):
     ...:     for j in range(i):
     ...:         T=T+D(Y,i,j)
     ...:
     ...: T*2/(n*(n-1))
Out[124]: 4998.7
```

```
In [125]: def D(X,i,j):
     ...:     s=0
     ...:     for r in range(d):
     ...:         s=s+(X[i,r]-X[j,r])**2
     ...:     return(s)
     ...:
     ...:
     ...: n=100
     ...: d=10000
     ...: p=0.6
     ...:
     ...: X=np.random.rand(n,d)
     ...: Y=np.zeros((n,d))
     ...: for i in range(n):
     ...:     for j in range(d):
     ...:         if X[i,j]<p:
     ...:             Y[i,j]=1
     ...:
     ...:
     ...: T=0
     ...: for i in range(n):
     ...:     for j in range(i):
     ...:         T=T+D(Y,i,j)
     ...:
     ...: T*2/(n*(n-1))
Out[125]: 4794.072121212122
```

6)

```
In [121]: n=100
     ...: U1=np.random.rand(n)
     ...: U2=np.random.rand(n)
     ...: X=np.zeros(n)
     ...: Y=np.zeros(n)
     ...: for i in range(n):
     ...:     if U1[i]<0.5:
     ...:         X[i]=m.sqrt(2*U1[i])
     ...:         Y[i]=X[i]*U2[i]
     ...:     if U1[i]>0.5:
     ...:         X[i]=2-m.sqrt(2-2*U1[i])
     ...:         Y[i]=(2-X[i])*U2[i]
     ...:
     ...:
     ...:
     ...: plt.plot(X,Y,'o')
Out[121]: [<matplotlib.lines.Line2D at 0x296190fb4a8>]
```