**Stat-6207** **Homework Set 1** **Fall 2018 (Modarres)**

**Due: September 11**

1) Write a program to evaluate $E_n = \int_0^1 x^n \exp(x-1)dx$ for $n = 1, 2, \ldots$. Using integration by parts, show that $E_n = 1 - nE_{n-1}$ for $n \geq 2$. Use the recurrence to compute approximations to the first 9 values of $E_n$. Find an upper bound for $E_n$.

2) A computer uses an extended precision floating point number representation in hexadecimal base, with 128 bits, one for the sign, seven for the exponent in excess-64 notation, and the rest for mantissa.

- a) What is the range of numbers it can represent in normalized form?
- b) How many numbers are in this set?
- c) Give an estimate of the relative accuracy of the arithmetic.

3) What are the largest and smallest positive, finite, normalized numbers that can be represented as IEEE single precision floating point number?

4) Explain the output of the following R code

```
Mxval= .Machine$integer.max
N1= Mxval+Mxval
N2= 2*Mxval
N3= -Mxval-Mxval
N4= 2*(-Mxval);
cat("Mxval= ", Mxval,"\n")
cat("N1= ", N1,\n")
cat("N2= ", N2,\n")
cat("N3= ", N3,\n")
cat("N4= ", N4,\n")
.3 + .6 == .9
```

5) Using IEEE single precision format, for what rang of $x$ around zero, does the function $f(x) = x^{10}$ evaluates to zero?

6) Consider the computation of $trace(AB)$ using the following R code. Explain why the three ways of evaluating the $trace$ lead to different computation times. What are the floating point computation cost of each method in terms of $n$ and $m$?

n = 1000; m = 500

A = matrix(runif(n*m),n,m)

B = matrix(runif(n*m),m,n)

system.time(sum(diag(A%*%B)))

system.time(sum(diag(B%*%A)))

system.time(sum(A*t(B)))

7) Let $b \geq 2$ be an integer. Suppose $x = b^N \sum_{v=1}^{\infty} x_v b^{-v}$ and $y = b^M \sum_{v=1}^{\infty} y_v b^{-v}$ where $N$ and $M$ are signed integers, $x_v,\ y_v \in \{0, 1, \ldots, b-1\}$ and $x_1,\ y_1 \neq 0$. Prove that if $N > M$, then $x \geq y$.

8) Generate $n$ numbers uniformly distributed on the unit interval and use them to simulated a Bernoulli random variable with success probability $p = .5$. Estimate $p$ from the sample. Repeat the experiment for $n$ in $nlist = seq(100, 10000, by = 500)$ and obtain 20 estimates for $p$. Obtain a plot of $(n, \hat{p})$.

9) You want to compute $L = (1/10)^s(1 - 1/10)^{n-s}$ where $s = 10$. Suppose we use floating point numbers of the form $0.d_1d_2d_3 \times 10^E$, where $d1$, $d_2$ and $d_3$ are decimal digits and $E$ is an integer exponent in the range 100 to +100. For what values of $n$ can we compute $L$ without underflow?