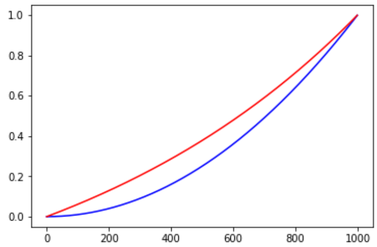
2)

```
In [319]: n=1000
     ...: X=np.random.rand(n)
     ...: Y=np.zeros(n)
     ...: for i in range(n):
     ...:     Y[i]=(m.exp(1)**X[i]-1)/(m.exp(1)-1)
     ...:
     ...: sum(Y)/n
Out[319]: 0.4225653972322869

In [320]: n=1000
     ...: X=np.random.rand(n)
     ...: Y=np.zeros(n)
     ...: for i in range(n):
     ...:     Y[i]=(m.exp(1)**X[i]-1)/(m.exp(1)-1)
     ...:
     ...: sum(Y)/n
Out[320]: 0.41938852004666005

In [321]: sum((Y-np.mean(Y))*(Y-np.mean(Y)))/(n-1)
Out[321]: 0.07978700278678036
```

b)

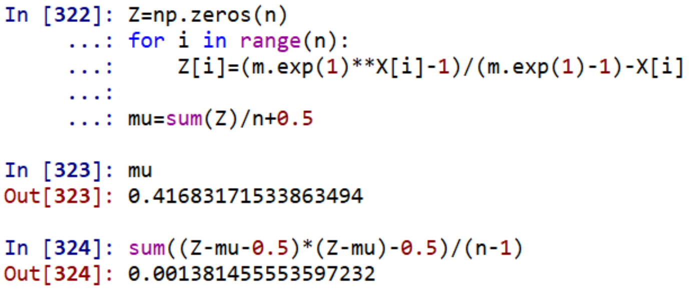
```
In [333]: G=np.zeros(1000)
     ...: T=np.zeros(1000)
     ...: for i in range(1000):
     ...:     T[i]=(m.exp(1)**(i/1000)-1)/(m.exp(1)-1)
     ...:     G[i]=(i/1000)**2
     ...:
     ...:
     ...: import matplotlib.pyplot as plt
     ...: plt.plot(G,'b',T,'r')
Out[333]:
[<matplotlib.lines.Line2D at 0x20c57cda6d8>,
 <matplotlib.lines.Line2D at 0x20c57cda828>]
```



```
In [334]: M=np.zeros(n)
     ...: r=0
     ...: for i in range(n):
     ...:     if X[i]<1/3:
     ...:         t=(X[i]*3)**(1/3)
     ...:         M[i]=((m.exp(1)**t-1)/(m.exp(1)-1))/(t**2)
     ...:         r=r+1
     ...:
     ...: sum(M)/r
Out[334]: 1.2649654462120152

In [335]: sum((M-np.mean(M))*(M-np.mean(M)))/(n-1)
Out[335]: 0.3937220264048954
```

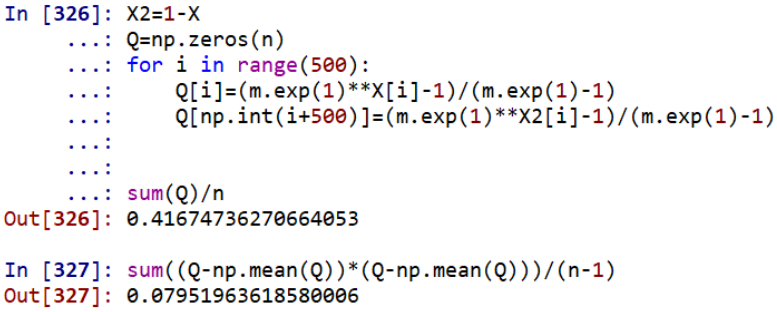
```
In [340]: Vara/Varb
Out[340]: 0.2026480548099412

In [341]: Vara/Varc
Out[341]: 57.75575086655486

In [342]: Vara/Vard
Out[342]: 1.0033622714313681

In [343]: Varb/Varc
Out[343]: 285.00520728275734

In [344]: Varb/Vard
Out[344]: 4.9512553790481615

In [345]: Varc/Vard
Out[345]: 0.017372508475383602

In [346]: Vara
Out[346]: 0.07978700278678036

In [347]: Varb
Out[347]: 0.3937220264048954

In [348]: Varc
Out[348]: 0.001381455553597232

In [349]: Vard
Out[349]: 0.07951963618580006
```

c)



```
In [322]: Z=np.zeros(n)
     ...: for i in range(n):
     ...:     Z[i]=(m.exp(1)**X[i]-1)/(m.exp(1)-1)-X[i]
     ...:
     ...: mu=sum(Z)/n+0.5

In [323]: mu
Out[323]: 0.41683171533863494

In [324]: sum((Z-mu-0.5)*(Z-mu)-0.5)/(n-1)
Out[324]: 0.001381455553597232
```

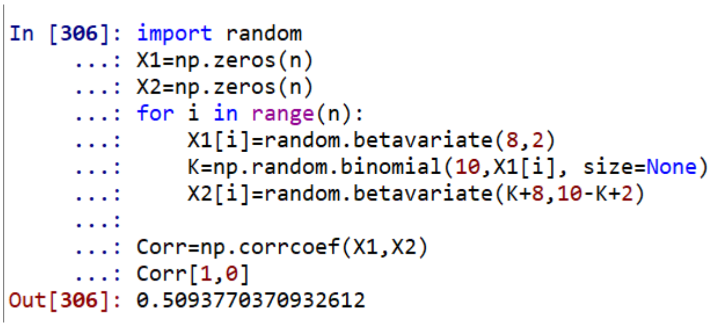
d)



```
In [326]: X2=1-X
     ...: Q=np.zeros(n)
     ...: for i in range(500):
     ...:     Q[i]=(m.exp(1)**X[i]-1)/(m.exp(1)-1)
     ...:     Q[np.int(i+500)]=(m.exp(1)**X2[i]-1)/(m.exp(1)-1)
     ...:
     ...:
     ...: sum(Q)/n
Out[326]: 0.41674736270664053

In [327]: sum((Q-np.mean(Q))*(Q-np.mean(Q)))/(n-1)
Out[327]: 0.07951963618580006
```

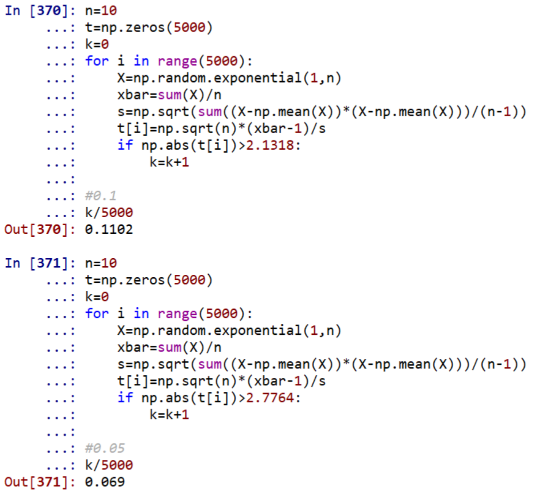
3)

n=1000

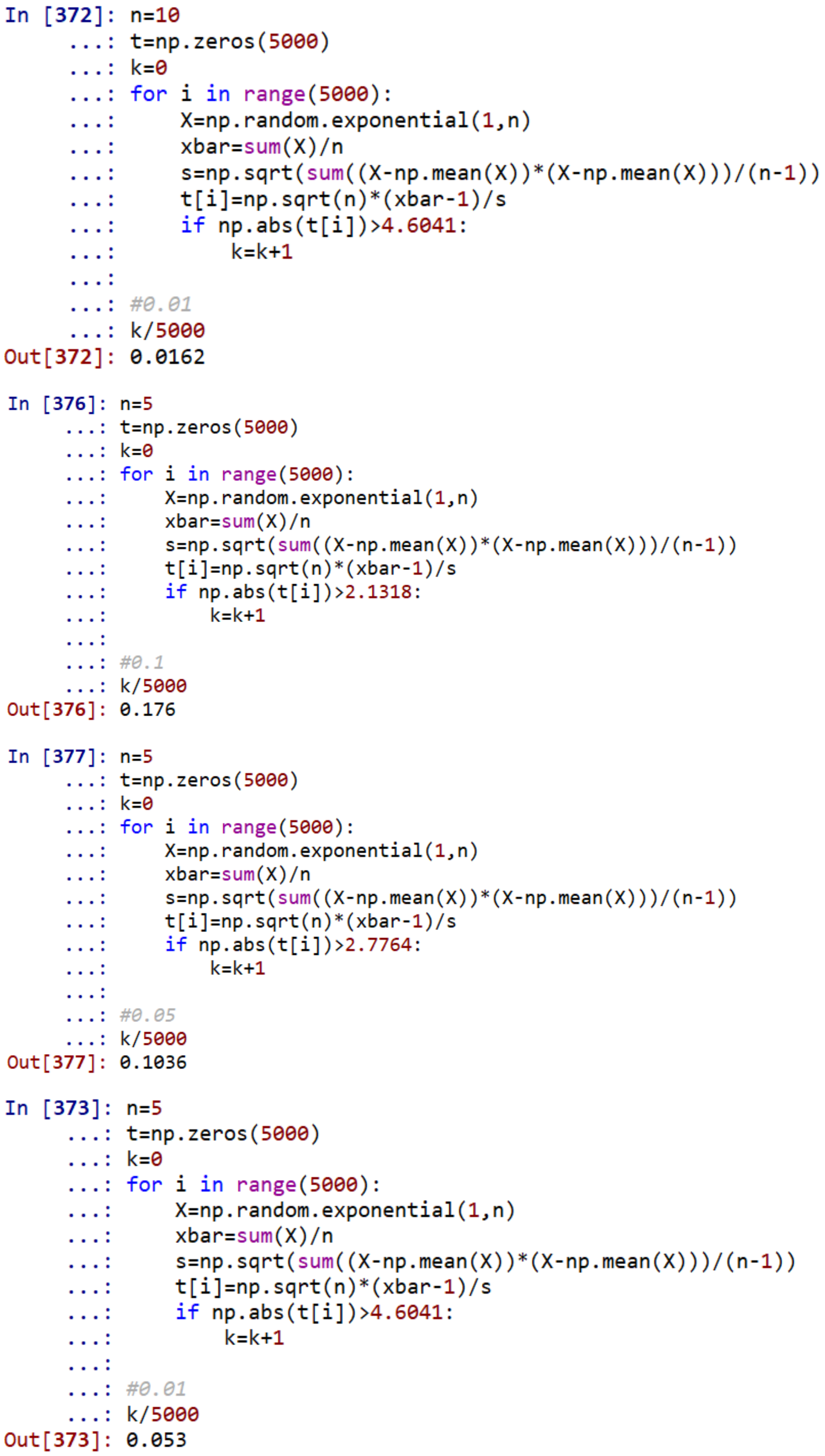


```
In [306]: import random
     ...: X1=np.zeros(n)
     ...: X2=np.zeros(n)
     ...: for i in range(n):
     ...:     X1[i]=random.betavariate(8,2)
     ...:     K=np.random.binomial(10,X1[i], size=None)
     ...:     X2[i]=random.betavariate(K+8,10-K+2)
     ...:
     ...: Corr=np.corrcoef(X1,X2)
     ...: Corr[1,0]
Out[306]: 0.5093770370932612
```

4)

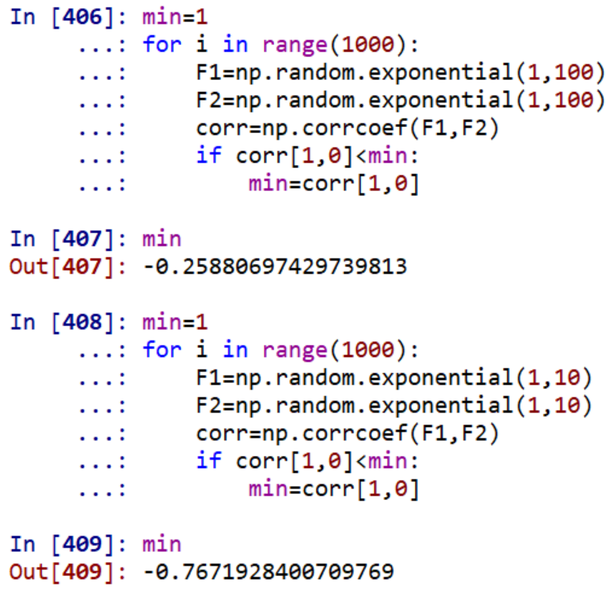


```
In [370]: n=10
     ...: t=np.zeros(5000)
     ...: k=0
     ...: for i in range(5000):
     ...:     X=np.random.exponential(1,n)
     ...:     xbar=sum(X)/n
     ...:     s=np.sqrt(sum((X-np.mean(X))*(X-np.mean(X)))/(n-1))
     ...:     t[i]=np.sqrt(n)*(xbar-1)/s
     ...:     if np.abs(t[i])>2.1318:
     ...:         k=k+1
     ...:
     ...: #0.1
     ...: k/5000
Out[370]: 0.1102

In [371]: n=10
     ...: t=np.zeros(5000)
     ...: k=0
     ...: for i in range(5000):
     ...:     X=np.random.exponential(1,n)
     ...:     xbar=sum(X)/n
     ...:     s=np.sqrt(sum((X-np.mean(X))*(X-np.mean(X)))/(n-1))
     ...:     t[i]=np.sqrt(n)*(xbar-1)/s
     ...:     if np.abs(t[i])>2.7764:
     ...:         k=k+1
     ...:
     ...: #0.05
     ...: k/5000
Out[371]: 0.069
```

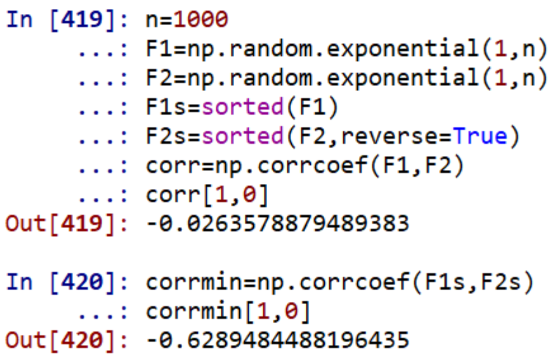


```
In [372]: n=10
     ...: t=np.zeros(5000)
     ...: k=0
     ...: for i in range(5000):
     ...:     X=np.random.exponential(1,n)
     ...:     xbar=sum(X)/n
     ...:     s=np.sqrt(sum((X-np.mean(X))*(X-np.mean(X)))/(n-1))
     ...:     t[i]=np.sqrt(n)*(xbar-1)/s
     ...:     if np.abs(t[i])>4.6041:
     ...:         k=k+1
     ...:
     ...: #0.01
     ...: k/5000
Out[372]: 0.0162

In [376]: n=5
     ...: t=np.zeros(5000)
     ...: k=0
     ...: for i in range(5000):
     ...:     X=np.random.exponential(1,n)
     ...:     xbar=sum(X)/n
     ...:     s=np.sqrt(sum((X-np.mean(X))*(X-np.mean(X)))/(n-1))
     ...:     t[i]=np.sqrt(n)*(xbar-1)/s
     ...:     if np.abs(t[i])>2.1318:
     ...:         k=k+1
     ...:
     ...: #0.1
     ...: k/5000
Out[376]: 0.176

In [377]: n=5
     ...: t=np.zeros(5000)
     ...: k=0
     ...: for i in range(5000):
     ...:     X=np.random.exponential(1,n)
     ...:     xbar=sum(X)/n
     ...:     s=np.sqrt(sum((X-np.mean(X))*(X-np.mean(X)))/(n-1))
     ...:     t[i]=np.sqrt(n)*(xbar-1)/s
     ...:     if np.abs(t[i])>2.7764:
     ...:         k=k+1
     ...:
     ...: #0.05
     ...: k/5000
Out[377]: 0.1036

In [373]: n=5
     ...: t=np.zeros(5000)
     ...: k=0
     ...: for i in range(5000):
     ...:     X=np.random.exponential(1,n)
     ...:     xbar=sum(X)/n
     ...:     s=np.sqrt(sum((X-np.mean(X))*(X-np.mean(X)))/(n-1))
     ...:     t[i]=np.sqrt(n)*(xbar-1)/s
     ...:     if np.abs(t[i])>4.6041:
     ...:         k=k+1
     ...:
     ...: #0.01
     ...: k/5000
Out[373]: 0.053
```

5)



```
In [406]: min=1
     ...: for i in range(1000):
     ...:     F1=np.random.exponential(1,100)
     ...:     F2=np.random.exponential(1,100)
     ...:     corr=np.corrcoef(F1,F2)
     ...:     if corr[1,0]<min:
     ...:         min=corr[1,0]

In [407]: min
Out[407]: -0.25880697429739813

In [408]: min=1
     ...: for i in range(1000):
     ...:     F1=np.random.exponential(1,10)
     ...:     F2=np.random.exponential(1,10)
     ...:     corr=np.corrcoef(F1,F2)
     ...:     if corr[1,0]<min:
     ...:         min=corr[1,0]

In [409]: min
Out[409]: -0.7671928400709769
```



```
In [419]: n=1000
     ...: F1=np.random.exponential(1,n)
     ...: F2=np.random.exponential(1,n)
     ...: F1s=sorted(F1)
     ...: F2s=sorted(F2,reverse=True)
     ...: corr=np.corrcoef(F1,F2)
     ...: corr[1,0]
Out[419]: -0.0263578879489383

In [420]: corrmin=np.corrcoef(F1s,F2s)
     ...: corrmin[1,0]
Out[420]: -0.6289484488196435
```