Stat-6207 **Homework Set7** **Fall 2018 (Modarres)**

1) Consider a mixed congruential generator, $X_i = (\alpha x_{i-1}+c)\ mod\ m$. What is the maximum period, assuming $m = 2^{10} - 1$? Give values for α, c, and x_0 for which the maximum period can be attained.

2) Use PIT to generate an observation from each distribution:

- a. Beta$(\alpha = 2, \beta = 1)$
- b. $f(x) = \frac{e^{-(x-\alpha)/\beta}}{\beta(1+e^{-(x-\alpha)/\beta})^2}, -\infty < x < \infty$
- c. $f(x) = \frac{1}{2\beta}e^{-|x-\alpha|/\beta}, -\infty < x < \infty, -\infty < \alpha < \infty, \beta > 0$
- d. $f(x) = \frac{\beta}{\pi(\beta^2+(x-\alpha)^2)}, -\infty < x < \infty, -\infty, \alpha < \infty, \beta > 0$

3) Use Monte Carlo integration to find the first four moments of the random variable with the pdf $f(x) = e^{-e^{-x}-x}$, $-\infty < x < \infty$. Use the results to find the skewness and the kurtosis of this pdf. Attach your program and output.

4) The d-sphere is the set of all points $\mathbf{x} \in \mathbf{R^d}$ such that $\|x\| = (\mathbf{x'x})^{\mathbf{1/2}} = \mathbf{1}$. If $X_1, \ldots, X_d$ are iid $N(0,1)$, then $(U_1, \ldots, U_d)$ is uniformly distributed on the unit sphere in $\mathbf{R^d}$, where $U_j = \frac{X_j}{(X_1^2+\ldots+X_d^2)^{1/2}}$, for $j = 1, \ldots, d$. Write a function to generate random variates uniformly distributed on the unit d-sphere. Run your function to generate 200 bivariate points uniformly on the unit circle.

5) Generate n random vectors $\mathbf{X}_i$ from a d-variate Bernoulli distribution with success probability p and independent components. Compute $D_{ij} = ||\mathbf{X}_i - \mathbf{X}_j||^2 = (\mathbf{X}_i - \mathbf{X}_j)'(\mathbf{X}_i - \mathbf{X}_j)$ and $T = \frac{2}{n(n-1)}\sum_{1<=i<j<=n} D_{ij}$.

- a. What is the distribution of D_{ij}?
- b. Find the expected value of T and compare with the sample average when $(n, d, p) = (100, 10000, 0.5)$ and $(n, d, p) = (100, 10000, 0.6)$.

6) Generate and plot $n = 100$ random bivariate vectors from p.d.f. $f(X, Y) = c$ for $0 < Y < X < 1$ or $0 < Y < 2 - x < 1$.