Stat-6207 **Homework Set 4** **Fall 2018 (Modarres)**

1) Find the 95th percentage point of the p.d.f. $f(x) = \frac{3}{8}(x^2 - x + 1)$ using a) Newton, b) bisection, and c) secant methods.

2) We have observed the function f at three values to be $f(10) = 35$, $f(15) = 10$ and $f(17) = 14$. Using the second Lagrange interpolating polynomial, find the approximate solution to $f(x) = 11$.

3) We use the third Lagrange interpolation polynomial to approximate $f(x) = \sin x$ at $x = \pi/2$ using the nodes $x_0 = \pi/2 - 2h$, $x_1 = \pi/2 - h$, $x_3 = \pi/2 + h$ and $x_3 = \pi/2 + 2h$. Suppose we want the error of this approximation to be less than 0.01. Calculate an upper bound for h.

4) Write a program to implement Halley's method to find the 99th percentile of a standard log-normal distribution $Y = \exp Z$ where $Z \sim N(0, 1)$. Start with $x_0 = 0.1$.

5) Use the Trapezoid and Simpson's rules to approximate $\int_1^2 \frac{xdx}{1+x^2}$ with $n = 4$ and $n = 40$.

6) Generate samples of size $m = 10$ and $n = 15$ from two independent bivariate normal distributions $\mathbf{X}_i \sim N_2(0, I)$ for $i = 1, \ldots, m$ and $\mathbf{Y}_j \sim N_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ for $j = 1, \ldots, n$. Write a program to build a minimal spanning tree from the combined sample and compute the multivariate runs test. Obtain summary statistics for each case below.

- Repeat the sampling experiment 100 times with $\boldsymbol{\mu} = (0, 0)', (1, 1)', (3, 3)'$ and $\boldsymbol{\Sigma} = I$.
- Repeat the sampling experiment 100 times with $\boldsymbol{\mu} = (0, 0)'$ and $\boldsymbol{\Sigma} = \begin{pmatrix} 25 & 9 \\ 9 & 4 \end{pmatrix}$.

7) Find the roots of the following system of non-linear equation using the Newton's method.

$$\begin{cases} x_1^2 - 2x_1 + x_2^2 - x3 = -1 \\ x_1 X_2^2 - x_1 - 3x_2 + x_2 x_3 = -2 \\ x_1 x_3^2 - 3x_3 + x_2 X_3^2 + x_1 X_2 \end{cases} \tag{1}$$