Homework 3

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1. Piece-wise linear mixed-effects models

First, without covariates, we consider some simple piece-wise linear mixed-effects models of post-infection CD4 as a function of time since HIV-infection. Suppose that we pick two interior knots at t = 2, 4 years after HIV infection. Give your final chosen model, and estimates and 95% Cl's for the fixed effects under the following three scenarios.

$$Y_{ij} = eta X_{ij} + b_i Z_{ij} + \epsilon_{ij}$$

 Y_{ij} : Post-infection CD4 β: The fixed effect b_i : The random effect for subject i. X_{ij} , Z_{ij} : Covariate matrices for subject i. $\epsilon_{ij} \sim N(0,\sigma 2)$ is the measurement error.

1.1 local constant fit

Using the basis functions B1(t) = 1[t<2], B2(t) = $1[2\le t<4]$ and B3(t) = $1[4\le t]$, with random-effects for each of the basis functions;

Model:

$$Y_{ij} = (eta_1 + b_{1i})B_1(t_{ij}) + (eta_2 + b_{2i})B_2(t_{ij}) + (eta_3 + b_{3i})B_3(t_{ij}) + \epsilon_{ij}$$

```
time=BMACS$Time
B1 \leftarrow function(t) I(t < 2)
B1=as.numeric(B1(time))
B2 \leftarrow function(t) I((t < 4) & (t > = 2))
B2=as.numeric(B2(time))
B3 \leftarrow function(t) I(t >= 4)
B3=as.numeric(B3(time))
B4 \leftarrow function(t) t*I(t<2)
B4=as.numeric(B4(time))
B5 \leftarrow function(t) t*I((t<4)&(t>=2))
B5=as.numeric(B5(time))
B6 \leftarrow function(t) t*I(t>=4)
B6=as.numeric(B6(time))
b1 = 1
b2=function(t) return(t)
b2=as. numeric (b2 (time))
b3=function(t) return(ifelse(t>2, t-2, 0))
b3=as.numeric(b3(time))
b4=function(t) return(ifelse(t>4, t-4, 0))
b4=as.numeric(b4(time))
data1<-cbind (BMACS, B1, B2, B3, B4, B5, B6, b1, b2, b3, b4)
model. 1. 1<-1mer (CD4 B1+B2+B3 -1 + (B1+B2+B3 -1 | ID), data= data1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00642437
## (tol = 0.002, component 1)
```

```
summary(model.1.1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: CD4 ~ B1 + B2 + B3 - 1 + (B1 + B2 + B3 - 1 | ID)
     Data: data1
## REML criterion at convergence: 12415.2
##
## Scaled residuals:
##
           1Q Median
                           30
                                    Max
     Min
## -4.0325 -0.5233 -0.0133 0.5175 3.9660
## Random effects:
## Groups Name Variance Std.Dev. Corr
## ID
            B1 69.19 8.318
##
            B2
                98. 28 9. 914 0. 83
##
            B3 145.41 12.059 0.66 0.92
## Residual
                 31.08
                        5.575
## Number of obs: 1817, groups: ID, 283
##
## Fixed effects:
##
    Estimate Std. Error t value
## B1 33, 0369
               0. 5395 61. 24
## B2 27.0514
                 0.6765 39.99
## B3 23.5989
                 0. 9351 25. 24
##
## Correlation of Fixed Effects:
##
    В1
         B2
## B2 0.678
## B3 0.469 0.751
## convergence code: 0
## Model failed to converge with max|grad| = 0.00642437 (tol = 0.002, component 1)
```

Use Bootstrap to compute 95% confidence interval.

```
set. seed(1)
unique. ID=unique (BMACS$ID)
nID=length (unique. ID)
bootsample=function() {
  resample. ID=sample (unique. ID, nID, replace=T)
  do.call("rbind", lapply(1:nID, function(i) subset(BMACS, ID==resample.ID[i])))
bootstrap. 1. 1=sapply (1:200, function(o) {
  b=bootsample()
  t=b$Time
  B1 < -function(t) I(t < 2)
  B1=as.numeric(B1(t))
  B2 \leftarrow function(t) I((t < 4) & (t > = 2))
  B2=as.numeric(B2(t))
  B3 \leftarrow function(t) I(t > = 4)
  B3=as.numeric(B3(t))
  b<-cbind(b, B1, B2, B3)
  boot. 1. 1<-1mer (CD4 B1+B2+B3 -1 + (B1+B2+B3 -1 | ID), data= b)
  return(fixef(boot. 1.1))
  })
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00220908
## (tol = 0.002, component 1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00856673
## (tol = 0.002, component 1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00321427
## (tol = 0.002, component 1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00492095
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00222261
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00224653
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0068525
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0125243
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00267139
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0121323
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00635224
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00482803
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00259056
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00709216
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00411974
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00364958
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00954143
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00543689
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0141801
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00231763
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00933153
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00256831
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00351866
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00542109
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00230842
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00305915
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0117816
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00294174
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00341783
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00290978
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00844393
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00212161
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00535532
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00518227
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0129016
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00217855
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00244716
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00444202
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00445306
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00212565
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00317359
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.018399
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00358028
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00404998
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00204446
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00307013
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00252409
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0103633
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0124394
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00690284
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00210715
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0209106
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00349202
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00970574
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.002652
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00753942
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00803499
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00655989
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00718806
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0102949
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00523073
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00202332
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00463027
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0089004
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00407611
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00269219
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00690298
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00300092
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00291619
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0107292
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00233376
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00239657
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0125484
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0218575
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00752482
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00567252
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control\converge with max|grad| = 0.0023703
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00481473
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.015146
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00547963
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00911799
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00685248
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0109838
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00417198
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00372753
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00304288
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00653601
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0391239
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0024841
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00431015
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00304222
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00953235
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00382228
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00524973
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0109404
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00517511
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00289916
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00902339
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0113726
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0047439
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0032883
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00823675
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00321383
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00221544
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00232516
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00222819
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00452097
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00444975
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0058784
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00522834
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00474734
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0076148
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0044308
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00495009
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00331432
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00687575
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00452971
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0035234
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00316736
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00617355
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00908558
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0033402
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0469821
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0152048
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00263844
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00628438
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00699319
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00570424
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0112661
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00329146
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00288976
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00380877
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00308634
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0064739
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00365006
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00266101
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00905638
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.002629
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0112703
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0046077
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0216486
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00529482
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00239339
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00561932
## (tol = 0.002, component 1)
bootstrap. 1. 1=as. matrix (bootstrap. 1. 1)
upper95=apply (bootstrap. 1.1, 1, quantile, 0.975)
lower95=apply(bootstrap.1.1,1,quantile,0.025)
estimate=fixef(model.1.1)
cbind (estimate, lower95, upper95)
```

```
## estimate lower95 upper95
## B1 33.03691 32.28070 33.90475
## B2 27.05142 25.98202 28.16932
## B3 23.59889 22.20210 25.05686
```

1.2 local linear fit without assuming continuity at the knots.

Three separate straight lines at t < 2, $2 \le t < 4$ and $4 \le t$, with random-effects for the intercepts only.

Model:

$$Y_{ij} = (\beta_1 + b_{1i})B_1(t_{ij}) + (\beta_2 + b_{2i})B_2(t_{ij}) + (\beta_3 + b_{3i})B_3(t_{ij}) + \beta_4 B_4(t_{ij}) + \beta_5 B_5(t_{ij}) + \beta_6 B_6(t_{ij}) + \epsilon_{ij}$$

The model fit 3 lines with own intercept and slop, and the knots are not forced to beconnected. Only the intercept terms have random effect.

```
model.1.2<-lmer(CD4~B1+B2+B3 -1 +B4 + B5 + B6 +(B1+B2+B3 -1 |ID), data= data1) summary(model.1.2)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: CD4 ^{\sim} B1 + B2 + B3 - 1 + B4 + B5 + B6 + (B1 + B2 + B3 - 1 | ID)
##
     Data: datal
##
## REML criterion at convergence: 12211.2
##
## Scaled residuals:
    Min
           1Q Median 3Q
## -4. 2844 -0. 5123 -0. 0176 0. 5173 4. 1792
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
##
           B1
                69.25
                        8.322
##
           B2
                103.36
                        10.166
                                0.81
##
                151.36
                        12.303
                                0.65 0.92
## Residual
                26.37
                        5. 135
## Number of obs: 1817, groups: ID, 283
##
## Fixed effects:
   Estimate Std. Error t value
## B1 36.9512 0.6196 59.642
## B2 35.1255 1.3002 27.015
## B3 30.6644 2.7162 11.289
## B4 -4.2504 0.3406 -12.478
## B5 -2.9737
                0.3920 -7.585
## B6 -1.6081
               0.5353 -3.004
## Correlation of Fixed Effects:
##
    B1 B2
                В3
## B2 0.299
## B3 0.137 0.130
## B4 -0.507 0.015 0.008
## B5 -0.001 -0.849 0.010 0.003
## B6 0.000 -0.004 -0.938 0.000 0.005
```

```
set.seed(1)
bootstrap. 1. 2=sapply (1:200, function(o) {
  b=bootsample()
  t=b$Time
  B1 \leftarrow function(t) I(t < 2)
  B1=as.numeric(B1(t))
  B2<-function(t) I((t<4)& (t>=2))
  B2=as. numeric (B2(t))
  B3 \leftarrow function(t) I(t > = 4)
  B3=as.numeric(B3(t))
  B4 \leftarrow function(t) t*I(t<2)
  B4=as.numeric(B4(t))
  B5 \leftarrow function(t) t*I((t < 4)& (t >= 2))
  B5=as.numeric(B5(t))
  B6 \leftarrow function(t) t \times I(t \ge 4)
  B6=as.numeric(B6(t))
  b<-cbind(b, B1, B2, B3, B4, B5, B6)
  fit1.2<-lmer(CD4~B1+B2+B3 -1 +B4 + B5 + B6 +(B1+B2+B3 -1 | ID), data= b)
  return(fixef(fit1.2))
})
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00867838
## (to1 = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0103108
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00851671
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00707882
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00712933
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00206851
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0183315
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00574627
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00858646
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00212266
## (tol = 0.002, component 1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00263442
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00448007
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00235939
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00923734
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00294727
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00277455
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00296463
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0128852
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00465728
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00446697
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00288329
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00340342
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.008612
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00494531
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00297096
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0207832
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0029065
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00349047
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00507791
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0036336
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0182394
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00358739
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0194179
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00795192
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00392241
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00368726
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00256654
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00200239
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00620206
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00350389
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00963585
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00564232
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0346518
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00223758
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00392579
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00782933
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00299599
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00205341
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00310093
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00468199
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00229812
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control\converge with max|grad| = 0.0102623
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0106423
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00370824
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.018832
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00324739
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0124661
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0102514
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00335906
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00255289
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00693007
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0115016
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0130126
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.017436
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0129157
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00417867
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00995554
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0052707
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00834084
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00202379
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0187494
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00327826
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0117278
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00569745
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00384368
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00878796
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00533669
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.011782
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00206759
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00605387
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0160727
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00736673
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00236131
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00517555
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00464121
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00267445
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00566353
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00359455
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00417946
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00734118
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00569488
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0144283
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00268356
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00328771
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0114044
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00319596
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00473522
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00450067
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0194618
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00448758
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00380231
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00302744
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00332929
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00621499
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0156127
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00942868
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0231709
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00540788
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00236486
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00902882
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00879823
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00352829
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00650492
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00918165
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00638041
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.013129
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00368243
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00355605
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0226767
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00407058
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00773929
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00652961
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0326263
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0144839
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00342052
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0127915
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00380593
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00302803
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00607805
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.011367
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0141681
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00944172
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0028368
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00504437
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00225272
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0104262
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00384959
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0206883
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00377185
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00904954
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00625028
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00270745
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0107735
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control$checkConv, : Model failed to converge with max \lfloor grad \rfloor = 0.0331009
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00649768
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0324134
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00563365
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00634607
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00423816
## (tol = 0.002, component 1)
bootstrap. 1. 2=as. matrix (bootstrap. 1. 2)
estimate=fixef(model.1.2)
upper95=apply (bootstrap. 1.2, 1, quantile, 0.975)
lower95=apply(bootstrap. 1.2, 1, quantile, 0.025)
cbind(estimate, lower95, upper95)
       estimate lower95
                             upper95
## B1 36.951184 35.762789 38.0564636
## B2 35.125475 33.252375 38.2445655
## B3 30.664368 25.290012 35.0678674
```

B4 -4. 250410 -5. 023388 -3. 1854724 ## B5 -2. 973743 -3. 966518 -2. 3608914 ## B6 -1. 608075 -2. 423214 -0. 4683855

1.3 local linear fit with continuity at the knots

Using B1(t) = 1, B2(t) = t, B3(t) = (t-2)+ and B4(t) = (t-4)+, with random-effects for each of the basis functions.

Model:

```
Y_{ij} = eta_1 + b_{1i} + (eta_2 + b_{2i})t_{ij} + (eta_3 + b_{3i})(t_{ij} - 2)^+ + (eta_4 + b_{4i})(t_{ij} - 4)^+ + \epsilon_{ij}
```

```
model.1.3<-lmer(CD4~b2+b3+b4+(b2+b3+b4|ID), data= data1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00562395
## (tol = 0.002, component 1)
```

```
summary (model. 1.3)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: CD4 ~ b2 + b3 + b4 + (b2 + b3 + b4 | ID)
     Data: datal
## REML criterion at convergence: 12075.3
##
## Scaled residuals:
##
             1Q Median
                               30
      Min
   -4. 4905 -0. 5299 -0. 0254 0. 4937 4. 5621
## Random effects:
## Groups Name
                        Variance Std. Dev. Corr
## ID
            (Intercept) 87.46 9.352
##
            b2
                                4,670
                                         -0.45
                        21.81
##
            b3
                        27.03
                                 5.199
                                         0.29 - 0.77
            b4
                        15.10
                                 3.886
                                         -0.04 0.12 -0.53
   Residual
                        21.58
                                 4.645
## Number of obs: 1817, groups: ID, 283
##
## Fixed effects:
##
              Estimate Std. Error t value
## (Intercept) 36.8266
                        0.6531 56.385
               -4.0310
                          0.3905 -10.324
## b3
                1.4370
                           0.5666 2.536
## b4
               1.3995
                        0. 6735 2. 078
##
## Correlation of Fixed Effects:
##
    (Intr) b2
## b2 -0.586
## b3 0.407 -0.782
## b4 -0.086 0.213 -0.556
## convergence code: 0
\#\# Model failed to converge with \max|\text{grad}| = 0.00562395 (tol = 0.002, component 1)
```

```
set.seed(1)
bootstrap.1.3=sapply(1:200, function(o) {
    b=bootsample()
    t=b$Time
    B2=function(t) return(t)
    B2=as.numeric(B2(t))
    B3=function(t) return(ifelse(t>2, t-2, 0))
    B3=as.numeric(B3(t))
    B4=function(t) return(ifelse(t>4, t-4, 0))
    B4=as.numeric(B4(t))
    cd41.3<-cbind(b, B2,B3,B4)
    fit1.3<-lmer(CD4~B2+B3+B4+(B2+B3+B4|ID), data= b)
    return(fixef(fit1.3))
})</pre>
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00384433
## (tol = 0.002, component 1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00550403
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0208836
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00594188
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00200718
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.01029 (tol
## = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00463198
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0145629
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0682363
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00480124
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00623515
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.004451
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0127015
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00353983
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00677175
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00372729
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0232877
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00949217
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00535032
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00472507
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00922235
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0085798
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00568694
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00539185
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0128462
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00325239
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.035569
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00283714
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control$checkConv, : Model failed to converge with \max|\operatorname{grad}| = 0.0114285
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00572065
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00263985
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00203326
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00413278
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00977152
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00314208
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.011041
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0187912
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0134982
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00580748
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00260219
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0056004
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0115839
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control\converge with max|grad| = 0.0106843
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

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## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00471752
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0189258
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00454542
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00937425
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00285798
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00548997
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00503191
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00320263
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0102486
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00329819
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00412478
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00769657
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0127321
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00317027
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

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## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0035155
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00838136
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0395938
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00652525
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0102937
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00507529
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0025557
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0188041
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00407003
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00297808
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00273143
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00505201
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00700015
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0028388
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

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## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00408104
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00324462
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00995 (tol
## = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0132264
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00356912
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00417925
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0121701
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00259923
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00317965
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00599315
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00918851
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00405551
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00272526
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0415357
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

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## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0123262
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control$checkConv, : Model failed to converge with max|grad| = 0.0134227
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00365007
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00343651
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00579819
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00246241
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control$checkConv, : Model failed to converge with max|grad| = 0.00689848
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.02032 (tol
## = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00325157
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00973215
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0277613
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00521767
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00716412
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00803469
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

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## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00357831
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0106967
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00309141
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00340108
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00762624
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0060754
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00737225
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00607602
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0126621
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0111093
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00579075
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00490348
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00393578
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00415862
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

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## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00425069
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0090783
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0083185
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0133835
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00552621
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0045915
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\# control$checkConv, : Model failed to converge with max|grad| = 0.0184496
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0154533
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00231783
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00544614
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00666717
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00930759
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00446868
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.002994
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

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## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00222528
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00788248
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00318781
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00356038
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00417526
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0068087
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00453019
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00754983
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00935535
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00689469
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0105421
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00478234
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00793688
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.025727
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

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## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00567073
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0175097
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00574195
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00315322
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0107042
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00537261
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00500506
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00352351
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00812539
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00592465
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00289318
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00459441
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00213241
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0049812
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

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## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0046414
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00341528
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00338308
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00265337
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00414553
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0163441
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00596458
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00880092
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00517144
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00236199
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00233471
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00773931
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0178807
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00224644
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00539758
```

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## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00319438
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00529979
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00649701
## (to1 = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00662583
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0119097
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0132367
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control$checkConv, : Model failed to converge with \max|\operatorname{grad}| = 0.00527679
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00441374
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00549592
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00547383
## (to1 = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00216565
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00264734
## (tol = 0.002, component 1)
bootstrap. 1. 3=as. matrix (bootstrap. 1. 3)
estimate=fixef(model.1.3)
upper95=apply (bootstrap. 1.3, 1, quantile, 0.975)
lower95=apply(bootstrap. 1.3, 1, quantile, 0.025)
cbind(estimate, lower95, upper95)
##
                            lower95 upper95
               estimate
## (Intercept) 36.826588 35.8096534 37.770249
               -4. 030985 -4. 5829069 -3. 355410
## b3
               1. 436992 0. 1517599 2. 058722
## b4
               1. 399518 0. 6574406 2. 857244
```

1.4 Interpretations and clinical conclusions

What are your interpretations and clinical conclusions from the results obtained in #1.1, #1.2 and #1.3?

```
anova(model.1.1, model.1.2, model.1.3, refit=F)
```

```
## Data: data1
## Models:
## model.1.1: CD4 ~ B1 + B2 + B3 - 1 + (B1 + B2 + B3 - 1 | ID)
## model.1.2: CD4 ^{\sim} B1 + B2 + B3 - 1 + B4 + B5 + B6 + (B1 + B2 + B3 - 1 | ID)
## model.1.3: CD4 ~ b2 + b3 + b4 + (b2 + b3 + b4 | ID)
           Df AIC BIC logLik deviance Chisq Chi Df Pr(>Chisq)
##
## model.1.1 10 12435 12490 -6207.6 12415
## model.1.2 13 12237 12309 -6105.6
                                     12211 203.94
                                                       3 < 2.2e-16 ***
## model.1.3 15 12105 12188 -6037.7
                                    12075 135.88
                                                       2 < 2.2e-16 ***
## --
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
```

Model.1.3 is the best model because its AIC is the smallest.

In conclusion, patients' post CD4 level decreases as time passes by since infection. The decreasing speed is the fastest in the first two years after infection, becomes slower in the 2nd-4th year, and even slower after the 4th year.

2. Extend the analysis in #1 by including smoking status as the covariate.

In this case, you will consider the varying-coeffcient mixed-effects models of the form

$$Y_{ij} = \beta_0(t) + b_{0i}(t) + \beta_1(t)X_i(t) + b_{1i}(t)X_i(t) + \epsilon_t$$

where Xi(t) is the smoking status, b0i(t) and b1i(t) are subject-specific deviation curves and ϵ_t is the independent measurement error.

Give your final chosen model, and estimates and 95% CI's for the fixed effects under the following three scenarios:

2.1 Local constant fit

Local constant fit using the basis functions in #1.1 for $\beta 0(t)$, b0i(t), $\beta 1(t)$ and b1i(t);

Model:

$$Y_{ij} = (\beta_1 + b_{1i})B_1t_{ij} + (\beta_2 + b_{2i})B_2t_{ij} + (\beta_3 + b_{3i})B_3t_{ij} + \beta_4B_1t_{ij}S_{ij} + \beta_5B_2t_{ij}S_{ij} + \beta_6B_3t_{ij}S_{ij} + \epsilon_{ij}S_{ij} + \epsilon_{i$$

#yij = β 1B1(tij)+ β 2B2(tij)+ β 3B3(tij)+ (β 4 + β 4i)B1(tij)Sij +(β 5 + β 5i)B2(tij)Sij +(β 6 + β 6i)B3(tij)Sij + β 1ij (2.1.2) #yij =(β 1 + β 1i)B1(tij)+(β 2 + β 2i)B2(tij)+(β 3 + β 3i)B3(tij)+ (β 4 + β 4i)B1(tij)Sij +(β 5 + β 5i)B2(tij)Sij +(β 6 + β 6i)B3(tij)Sij + β 1ij

```
smoke=BMACS$Smoke
S1=B1*smoke
S2=B2*smoke
S3=B3*smoke
S4=B4*smoke
S5=B5*smoke
S5=B5*smoke
S6=B6*smoke
s1=b1*smoke
s2=b2*smoke
s3=b3*smoke
s4=b4*smoke
data2=cbind (data1, S1, S2, S3, S4, S5, S6, s1, s2, s3, s4)
model. 2. 1<-1mer (CD4~1+B1+B2+B3+S1+S2+S3+ (-1+B1+B2+B3 | ID), data= data2)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00343388
## (tol = 0.002, component 1)
```

```
summary(model.2.1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: CD4 ^{\sim} -1 + B1 + B2 + B3 + S1 + S2 + S3 + (-1 + B1 + B2 + B3 |
##
     Data: data2
##
## REML criterion at convergence: 12402.3
##
## Scaled residuals:
##
     Min
           1Q Median 3Q
                                   Max
## -3.9959 -0.5247 -0.0138 0.5129 3.9590
## Random effects:
## Groups Name Variance Std.Dev. Corr
           B1 68.78 8.293
##
  TD
##
           B2
                96. 42 9. 819 0. 83
           B3 145.81 12.075 0.66 0.93
##
                 31. 09 5. 576
## Residual
## Number of obs: 1817, groups: ID, 283
##
## Fixed effects:
   Estimate Std. Error t value
## B1 32.4066 0.6700 48.370
## B2 25.9628
               0.8298 31.288
## B3 22.9001 1.1316 20.237
## S1 1.7244 1.1252 1.533
## S2 3.1522 1.4128 2.231
## S3 1.6618 2.0278 0.820
##
## Correlation of Fixed Effects:
                В3
   B1
           B2
## B2 0.690
## B3 0.490 0.769
## S1 -0.595 -0.411 -0.292
## S2 -0.405 -0.587 -0.452 0.660
## S3 -0.273 -0.429 -0.558 0.445 0.730
## convergence code: 0
## Model failed to converge with max|grad| = 0.00343388 (tol = 0.002, component 1)
```

Bootstrap

```
bootstrap. 2. 1=sapply (1:200, function(o) {
   b=bootsample()
   t=b$Time
    s=b$Smoke
   B1 \leftarrow function(t) I(t < 2)
    B1=as.numeric(B1(t))
    B2 \leftarrow function(t) I((t < 4) & (t > = 2))
    B2=as.numeric(B2(t))
    B3 \leftarrow function(t) I(t > = 4)
    B3=as.numeric(B3(t))
    S1=B1*s
    S2=B2*s
    S3=B3*s
    cd42.1<-cbind(b, B1, B2, B3, S1, S2, S3)
    \label{eq:fit2.1}  \text{fit2.1} < -1 \\ \text{mer} \left( \\ \text{CD4}^{\sim} -1 \\ + \\ \text{B1} \\ + \\ \text{B2} \\ + \\ \text{B3} \\ + \\ \text{S1} \\ + \\ \text{S2} \\ + \\ \text{S3} \\ + \left( -1 \\ + \\ \text{B1} \\ + \\ \text{B2} \\ + \\ \text{B3} \\ \mid \\ \text{ID} \right), \quad \\ \text{data=} \quad \\ \text{b)} 
    return(fixef(fit2.1))
})
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00626406
## (tol = 0.002, component 1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0121879
## (tol = 0.002, component 1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00799786
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00500773
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00367439
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00411408
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00923054
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0089244
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00229953
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00353559
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00591715
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00810924
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00809334
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0129773
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00413514
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00463014
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00310144
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0128063
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00740285
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0256185
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0112257
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00787423
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00399586
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0196126
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00212264
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0038125
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.018624
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00207939
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0111905
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0131609
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00567091
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00824391
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0077935
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00273776
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00229275
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00408391
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00294715
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00233833
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00429758
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00364677
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0250653
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00254417
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00669343
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00537635
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00901422
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00945702
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00289326
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00840554
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0029947
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00286477
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00369891
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00301397
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0123931
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00372099
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00231682
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00550318
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0189838
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0161483
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00556755
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00266372
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00275529
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0160953
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.003029
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0024335
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00837139
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00236139
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00315542
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0759306
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00873746
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00578203
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00499378
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00243631
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00896204
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00956391
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00528904
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00285088
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00831083
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00663622
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00244104
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00943303
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00743218
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00331409
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00379275
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00727321
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.004599
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00382286
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00363262
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00832509
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0214752
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00568432
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00939935
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0101802
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0158872
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00322281
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0023264
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00308113
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00563334
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00654622
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00205609
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00652632
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00357281
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00274259
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00267929
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00527013
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00401486
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.010283
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0507372
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00541955
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00946018
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00243091
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00225901
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0264062
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0030532
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0024319
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00585076
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00482877
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00499936
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00551224
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.013557
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00588862
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00462182
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00558159
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0085357
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00208697
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00383603
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00236591
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00490955
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00493105
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0103069
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00257886
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0027285
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00228569
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00794409
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00478159
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00246742
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00318504
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0193929
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00382909
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0112978
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00392507
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00259232
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00646648
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00384864
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00364685
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00360775
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00259649
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0035177
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0197822
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00649769
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0218206
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00238264
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00224831
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0109961
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00323744
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0135762
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00256307
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0062768
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00202179
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00356426
## (tol = 0.002, component 1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00504816
## (tol = 0.002, component 1)
```

```
bootstrap. 2. 1=as. matrix (bootstrap. 2. 1)
estimate=fixef (model. 2. 1)
upper95=apply (bootstrap. 2. 1, 1, quantile, 0. 975)
lower95=apply (bootstrap. 2. 1, 1, quantile, 0. 025)
cbind (estimate, lower95, upper95)
```

```
## B1 32.406614 31.4784741 33.532124

## B2 25.962782 24.4663774 27.300904

## B3 22.900084 20.9781115 24.665406

## S1 1.724428 0.1234384 3.682452

## S2 3.152200 1.1931642 5.213286

## S3 1.661812 -0.8567107 4.284879
```

###2.2 Local linear fit without assuming continuity at the knots Local linear fit without assuming continuity at the knots for $\beta 0(t)$, $\beta 1(t)$ and $\beta 1(t)$, i.e., using the basis functions as in #1.2;

Model:

$$Y_{ij} = \sum_{k=1}^{6} (eta_k + b_{ki}) B_k t_{kj} + \sum_{k=7}^{12} eta_k B_{k-6}(t_{kj}) S_{ij} + \epsilon_{ij}$$

 $\bmod e1.\ 2.\ 2 < -1 \\ model \ 3.\ 4 \\ model \ 4 \\ m$

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0758756
## (tol = 0.002, component 1)
```

```
summary (model. 2. 2)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: CD4 ^{\sim} -1 + B1 + B2 + B3 + B4 + B5 + B6 + S1 + S2 + S3 + S4 +
      S5 + S6 + (-1 + B1 + B2 + B3 + B4 + B5 + B6 | ID)
##
     Data: data2
##
## REML criterion at convergence: 12025
##
## Scaled residuals:
     Min
            10 Median
                            3Q
                                    Max
## -4.7090 -0.5044 -0.0208 0.4806 4.7382
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## TD
            B1 92.472 9.616
##
            B2 146.095 12.087
                                  0.58
##
            B3 128.970 11.356
                                 0.62 0.45
##
                 26.847
            В4
                         5. 181
                                 -0.51 0.21 0.01
##
            В5
                 11.529
                          3.395
                                 -0.23 -0.57 0.23 0.18
##
            В6
                  5. 854 2. 420
                                 -0.32 -0.07 -0.37 0.56 0.37
                 19.870 4.458
## Residual
## Number of obs: 1817, groups: ID, 283
##
## Fixed effects:
  Estimate Std. Error t value
## B1 35.96129 0.84269 42.675
## B2 34.70729 1.57023 22.103
## B3 29.02047 2.96096 9.801
## B4 -3.80888 0.56111 -6.788
## B5 -3.19370
               0. 52346 -6. 101
## B6 -1.36325
                0. 63609 -2. 143
                1. 41245 1. 980
## S1 2.79647
## S2 2.70984
                2.81916
                         0.961
## S3 4.14107
              5. 53275 0. 748
## S4 -1.25956
               0. 96835 -1. 301
## S5 -0.01128
               0.96052 -0.012
## S6 -0.65797
               1. 19457 -0. 551
##
## Correlation of Fixed Effects:
                                                   S2
## B1 B2 B3 B4 B5
                                      В6
                                            S1
                                                          S3
                                                                S4
## B2 0.277
## B3 0.155 0.064
## B4 -0.625 0.112 0.003
## B5 -0.098 -0.844 0.059 0.065
## B6 -0.089 -0.028 -0.923 0.132 0.084
## S1 -0.597 -0.165 -0.092 0.373 0.059 0.053
## S2 -0.154 -0.557 -0.035 -0.062 0.470 0.016 0.252
## S3 -0.083 -0.034 -0.535 -0.002 -0.031 0.494 0.137 0.049
## S4 0.362 -0.065 -0.002 -0.579 -0.038 -0.077 -0.623 0.106 0.005
## S5 0.054 0.460 -0.032 -0.035 -0.545 -0.046 -0.091 -0.861 0.056 0.058
## S6 0.047 0.015 0.492 -0.071 -0.045 -0.532 -0.082 -0.033 -0.930 0.117
##
## B2
## B3
## B4
## B5
## B6
## S1
## S2
## S3
## S4
## S5
## S6 0.080
## convergence code: 0
## Model failed to converge with max|grad| = 0.0758756 (tol = 0.002, component 1)
```

```
set. seed(1)
bootstrap. 2. 2=sapply (1:100, function(o) {
  b=bootsample()
  t=b$Time
  s=b$Smoke
  B1 \leftarrow function(t) I(t < 2)
  B1=as. numeric (B1(t))
  B2 \leftarrow function(t) I((t < 4) & (t > = 2))
  B2=as.numeric(B2(t))
  B3 \leftarrow function(t) I(t > = 4)
  B3=as.numeric(B3(t))
  B4 \leftarrow function(t) t*I(t<2)
  B4=as.numeric(B4(t))
  B5 \leftarrow function(t) t*I((t < 4)& (t >= 2))
  B5=as.numeric(B5(t))
  B6 \leftarrow function(t) t*I(t>=4)
  B6=as.numeric(B6(t))
  S1=B1*s
  S2=B2*s
  S3=B3*s
  S4=B4*s
  S5=B5*s
  S6=B6*s
  cd42. 2<-cbind (b, B1, B2, B3, B4, B5, B6, S1, S2, S3, S4, S5, S6)
  fit2.2<-1mer(CD4~-1+B1+B2+B3+B4+B5+B6+S1+S2+S3+S4+S5+S6+
                  (-1+B1+B2+B3+B4+B5+B6 \mid ID), data= b)
  return(fixef(fit2.2))
})
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.149315
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.310827
## (to1 = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.19428 (tol
## = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0883187
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.133764
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.149218
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0825072
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.183222
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0465642
## (tol = 0.002, component 1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.159379
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.167696
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0153243
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.047274
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0813189
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0491585
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.340067
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : unable to evaluate scaled gradient
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge: degenerate Hessian with 1
## negative eigenvalues
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0420642
## (to1 = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.172878
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.244185
## (to1 = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0364288
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control$checkConv, : Model failed to converge with \max|\operatorname{grad}| = 0.119513
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0511786
## (to1 = 0.002, component 1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0650507
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.104097
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.341237
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.34496 (tol
## = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0523937
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.378283
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.17931 (tol
## = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0365395
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.038773
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0747419
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.116405
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0887849
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0137429
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control$checkConv, : Model failed to converge with max \lceil grad \rceil = 0.248851
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.149572
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0736307
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.048332
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.100069
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0419632
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.265644
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.226445
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.098647
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0841813
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0283251
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.124693
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0694976
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.180334
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\# control$checkConv, : Model failed to converge with max|grad| = 0.0310201
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.217586
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.157742
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0323453
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0954944
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0757462
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0508772
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.070939
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.222489
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0361571
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0513087
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0523796
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.071436
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0790841
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\# control\converge with \converge with \converge with \converge of \converge with \converge of \converge
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0387749
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0741306
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.176423
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0499896
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.126854
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0479434
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.104939
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.100005
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0443719
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0548004
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0416886
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.116761
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0786234
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control$checkConv, : Model failed to converge with \max|\operatorname{grad}| = 0.0241125
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0233708
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0741281
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.166033
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.108942
## (tol = 0.002, component 1)
## boundary (singular) fit: see ?isSingular
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control\converge with max|grad| = 0.0427485
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0124643
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0627717
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0594576
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.248336
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0752689
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0748648
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.15631 (tol
## = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control$checkConv, : Model failed to converge with max|grad| = 0.0777068
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.11534 (tol
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.153169
## (tol = 0.002, component 1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0244696
## (tol = 0.002, component 1)
```

```
bootstrap. 2. 2=as. matrix (bootstrap. 2. 2)
estimate=fixef (model. 2. 2)
upper95=apply (bootstrap. 2. 2, 1, quantile, 0. 975)
lower95=apply (bootstrap. 2. 2, 1, quantile, 0. 025)
cbind (estimate, lower95, upper95)
```

```
## B1 35.96129458 34.6872007 37.4010601
## B2 34.70729341 32.7614908 37.9929169
## B3 29.02047295 25.5580445 34.0008059
## B4 -3.80887700 -4.8884098 -2.9647985
## B5 -3.19369768 -4.0601390 -2.4545433
## B6 -1.36325395 -2.3540289 -0.6475405
## S1 2.79646819 0.7843236 4.9777257
## S2 2.70984085 -1.2807460 7.8351402
## S3 4.14107049 -5.0176445 12.3485412
## S4 -1.25956211 -2.5072596 0.4835669
## S5 -0.01127876 -1.7960817 1.2931933
## S6 -0.65796835 -2.5683122 1.4367535
```

###2.3 Local linear fit with continuity at the knots Local linear fit with continuity at the knots for $\beta 0(t)$, $\beta 1(t)$ and $\beta 1(t)$, i.e., using the basis functions as in #1.3.

Model:

$$Y_{ij} = \sum_{k=1}^4 (eta_k + b_{ki}) B_k(t_{kj}) + \sum_{k=5}^8 eta_k B_{k-4}(t_{kj}) S_{ij} + \epsilon_{ij}$$

Where

$$B_1(t) = 1, B_2(t) = t, B_3(t) = (t-2)^+, B_4(t) = (t-4)^+$$

```
\verb|model.2.3<-1mer(CD4^b2+b3+b4+s1+s2+s3+s4+ (b2+b3+b4|ID), data= data2)|
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00531314
## (tol = 0.002, component 1)
```

summary (model. 2.3)

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: CD4 ~ b2 + b3 + b4 + s1 + s2 + s3 + s4 + (b2 + b3 + b4 | ID)
    Data: data2
## REML criterion at convergence: 12063.5
##
## Scaled residuals:
##
    Min 1Q Median
                         30
                                 Max
## -4.4651 -0.5208 -0.0256 0.4926 4.5480
## Random effects:
## Groups Name
                   Variance Std.Dev. Corr
## ID
           (Intercept) 87.13 9.334
                                   -0.46
##
          b2 22.05 4.696
                                   0. 29 -0. 77
##
          b3
                     27. 80 5. 272
##
           b4
                     15.71
                            3.963
                                   -0.02 0.16 -0.56
                     21.56
                            4.643
## Residual
## Number of obs: 1817, groups: ID, 283
##
## Fixed effects:
        Estimate Std. Error t value
##
## (Intercept) 36.0511 0.8120 44.395
## b2
             -3.9302
                     0. 4822 -8. 150
## b3
             1. 1842 0. 6896 1. 717
## b4
             1. 9510 0. 7987 2. 443
             2. 1685 1. 3635 1. 590
## s1
             ## s2
             0.8097
                     1. 2272 0. 660
## s3
                     1.4980 -1.348
## s4
             -2.0190
##
## Correlation of Fixed Effects:
## (Intr) b2 b3 b4 s1
                                         s3
## b2 -0.587
## b3 0.412 -0.790
## b4 -0.084 0.227 -0.567
## s1 -0.596 0.349 -0.245 0.050
## s2 0.342 -0.583 0.461 -0.132 -0.590
## s4 0.045 -0.121 0.302 -0.533 -0.079 0.220 -0.560
## convergence code: 0
## Model failed to converge with max|grad| = 0.00531314 (tol = 0.002, component 1)
```

Bootstrap

```
set. seed(1)
bootstrap. 2. 3=sapply (1:200, function(o) {
 b=bootsample()
 t=b$Time
  s=b$Smoke
  B2=function(t) return(t)
  B2=as.numeric(B2(t))
  B3=function(t) return(ifelse(t>2, t-2, 0))
  B3=as.numeric(B3(t))
  B4=function(t) return(ifelse(t>4, t-4, 0))
  B4=as.numeric(B4(t))
  S1=s
  S2=B2*s
  S3=B3*s
  cd42.3<-cbind(b, B2, B3, B4, S1, S2, S3, S4)
  fit2.3<-1mer(CD4~B2+B3+B4+S1+S2+S3+S4+
                  (B2+B3+B4 | ID), data= b)
  return(fixef(fit2.3))
})
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00840511
## (tol = 0.002, component 1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00428156
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0119241
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0461594
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0113066
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00233015
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00407779
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control\converge with max|grad| = 0.0037686
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00552758
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0049891
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00325756
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00233252
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0119261
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0156451
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control$checkConv, : Model failed to converge with \max|\operatorname{grad}| = 0.0132147
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00738808
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0151702
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00246832
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00269151
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00302506
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00647235
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00513743
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00309147
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00480925
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00371943
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00248132
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0167164
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00828814
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control\converge with max|grad| = 0.0100643
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00219429
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00514973
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00803471
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00671286
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00363518
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00778763
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00259619
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00954872
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0361168
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00300014
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00416237
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0085052
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00722409
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00563682
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

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## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00512301
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00375469
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0065597
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00605818
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0109871
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0205501
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0203599
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00420855
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00441389
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.004241
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.014756
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00528676
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00505965
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00686355
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00401119
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00370813
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00517117
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00654202
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0538421
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00956012
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.003816
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0104448
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00504334
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00585625
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00275715
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00788088
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00532663
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00220145
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0339135
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00296469
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00396968
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00977214
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00596165
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00547271
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00209155
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.035429
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00537473
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0043288
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0098875
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00567367
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00654192
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00404267
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00323466
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00604486
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00434449
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00311154
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00968102
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0175863
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00273888
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00584104
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00785083
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0141438
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00555958
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00416051
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00459319
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0026695
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00258512
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00758383
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00308144
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00751397
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00584585
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00874817
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00623804
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00570856
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00747033
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00463285
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00349791
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00791251
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00290337
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control$checkConv, : Model failed to converge with max|grad| = 0.00520842
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00938686
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00854969
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0105745
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0121012
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00402856
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00228822
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00488592
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0186342
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00418579
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0126832
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.026256
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00203689
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0044171
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00854208
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00565785
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00371826
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0036099
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00310209
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0250264
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00546777
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00364044
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00305367
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00379612
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0111652
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0162624
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.012603
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0139901
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00544736
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0269909
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.02949 (tol
## = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00762653
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00861782
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00809114
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00946077
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00839422
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00274084
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control\converge with max|grad| = 0.0102433
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00660997
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00257927
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00507825
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00822009
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00679056
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0185189
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00279631
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00307722
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0178104
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00331884
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0104224
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00521366
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0136652
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0109946
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0103638
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00938945
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00531351
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00508556
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00409838
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
```

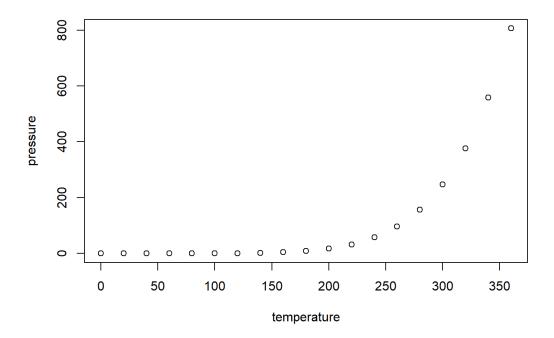
```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00508742
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control$checkConv, : Model failed to converge with max|grad| = 0.00276697
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00532242
## (to1 = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00773887
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00370049
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00577849
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
\#\# control$checkConv, : Model failed to converge with max|grad| = 0.00278924
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00232174
## (tol = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00630337
## (to1 = 0.002, component 1)
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0235799
## (to1 = 0.002, component 1)
bootstrap. 2. 3=as. matrix (bootstrap. 2. 3)
estimate=fixef(model.2.3)
upper95=apply(bootstrap. 2.3, 1, quantile, 0.975)
lower95=apply (bootstrap. 2. 3, 1, quantile, 0. 025)
cbind(estimate, lower95, upper95)
                                        upper95
                 estimate
                             lower95
## (Intercept) 36.0511410 34.6159670 37.3196434
## b2
              -3. 9302049 -4. 6689994 -3. 0488934
## b3
               1. 1842017 -0. 1162452 2. 1833758
## b4
               1. 9510086 0. 9120519 3. 5340768
## s1
               2. 1685467 0. 3467165 4. 6542024
## s2
              -0. 2945300 -1. 6647435 1. 0449123
## s3
               0.8097118 -1.4070638 2.9925775
## s4
              -2. 0190406 -4. 6836495 0. 4501607
```

###2.4 Interpretations and clinical conclusions What are your interpretations and clinical conclusions about the effects of smoking status obtained in #2.1, #2.2 and #2.3?

```
anova (model. 2. 1, model. 2. 2, model. 2. 3, refit=F)
```

```
## Data: data2
## Models:
## model.2.1: CD4 ^{\sim} -1 + B1 + B2 + B3 + S1 + S2 + S3 + (-1 + B1 + B2 + B3 |
## model.2.1:
## model.2.3: CD4 ^{\sim} b2 + b3 + b4 + s1 + s2 + s3 + s4 + (b2 + b3 + b4 | ID)
## model. 2. 2: CD4 ^{\sim} -1 + B1 + B2 + B3 + B4 + B5 + B6 + S1 + S2 + S3 + S4 +
                      S5 + S6 + (-1 + B1 + B2 + B3 + B4 + B5 + B6 \mid ID)
## model.2.2:
                \label{eq:decomposition} \mbox{Df} \quad \mbox{AIC} \quad \mbox{BIC} \quad \mbox{logLik deviance} \quad \mbox{Chisq Chi} \ \mbox{Df} \ \mbox{Pr}(\mbox{\ensuremath{\triangleright}\xsppc}\mbox{Chisq})
##
## model.2.1 13 12428 12500 -6201.2
                                                12402
## model.2.3 19 12102 12206 -6031.7
                                                 12064 338.851
                                                                        6 < 2.2e-16 ***
## model.2.2 34 12093 12280 -6012.5
                                                12025 38.492
                                                                       15 0.000763 ***
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
```

Model.2.2 is the best one because it has the smallest AIC. We can see from the model that smokers' post-CD4 level decreaseS faster then non-smokers in all time interval. The relationship between post-CD4 cell level and time is the same as the conclusion in 1.4.



Note that the echo = FALSE parameter was added to the code chunk to prevent printing of the R code that generated the plot.