

**ASSIGNMENT 01**  
**Closing Date: 16 May 2025**  
Total Marks:50  
**UNIQUE ASSIGNMENT NUMBER: 695503**

**Question 1: 7 Marks**

Consider the statement A: If  $p^2$  is divisible by four, then  $p$  is even.

- (1.1) Write down the contrapositive of the statement, and determine whether the resulting statement is true or not. If it is false, find a counterexample. (2)
- (1.2) Write down the converse of the statement, and determine whether the resulting statement is true or not. If it is false, find a counterexample. (2)
- (1.3) Write down the negation of the statement, and determine whether the resulting statement is true or not. If it is false, find a counterexample. (3)
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**Question 2: 17 Marks**

- (2.1) Prove by mathematical induction that

$$\sum_{i=1}^n 5i = \frac{5n(n+1)}{2} \text{ holds for all positive numbers } n \in \mathbb{Z}. \quad (8)$$

- (2.2) Determine if the following function given by

$$f(x) = |x + 1|,$$

is 1-1 on  $\mathbb{R}$  and find a formula for the inverse of  $f$  if it is 1-1. (4)

- (2.3) Consider the functions

$$f(x) = \frac{1}{x} \text{ and } g(x) = \sqrt{x-1}.$$

Indicate the domain of definition of each of the following functions:  
 $f, g, f \circ g$ . (5)

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**Question 3: 5 Marks**

Let

$$S := \left\{ \frac{1}{x} + \frac{1}{y} : x, y \in \mathbb{N} \right\}.$$

Find the infimum and supremum of  $S$ .

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**Question 4: 21 Marks**

(4.1) Show, using the  $\epsilon - N$  method, that the sequence

$$a_n = \frac{3n^3 + 2}{n^3}$$

converges. First determine the limit. (8)

(4.2) Use the Monotone Convergence Theorem to prove that the following sequences converge.

(a)

$$a_n = \frac{n}{n+1}. \tag{5}$$

(b)

$$b_n = \frac{1}{n}. \tag{4}$$

(c)

$$c_n = 1 - \frac{1}{n}. \tag{4}$$

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