Question 1

1.1.

$4z'' - 4z' - 3z = \cos 2x$

ODE: second order linear nonhomogenous

[1] Find roots related homogenous equation

Characteristic equation

$4\lambda^2 - 4\lambda - 3 = 0$

$(2\lambda - 3)(2\lambda + 1) = 0$

$\lambda = \frac{3}{2}$ or $\lambda = \frac{-1}{2}$

[2] Solution of homogenous equation

Use the method of variation of constants, where the general solution is of the form $y_1(x) = C_1 Y_1(x) + C_2 Y_2(x)$

$\therefore y_0(x) = C_1 e^{\lambda_1 x} + C_1 e^{\lambda_2 x}$

$\therefore y_0(x) = C_1 e^{\frac{3}{2}x} + C_1 e^{-\frac{1}{2}x}$

[2] Solution of nonhomogenous equation

Use the method of undetermined coefficients, where the integral solution is of the form $y_2(x) = Acosx + Bsinx$

$\therefore y_2(x) = Acos2x + Bsin2x$

We have that

$A = -\frac{19}{425}$, $B = -\frac{8}{425}$

$\therefore y''_2(x) = -\frac{19}{425}cos2x - \frac{8}{425}sin2x$

The general solution of the nonhomogenous equation is of the form

$z = y_0(x) + y_2(x)$

Hence,

$y_0(x) + y_2(x)$

$\therefore z = C_1 e^{\frac{3}{2}x} + C_1 e^{-\frac{1}{2}x} - \frac{19}{425}cos2x - \frac{8}{425}sin2x$

Question 2

Given
25 grams salt
$350l$ of water
Pumping rate $4l$ /min

Let
$A(t)$ be the amount of salt in the tank
R_{IN} be the rate of incoming salt
R_{OUT} be the rate of outgoing salt

$\therefore dA = R_{IN} - R_{OUT}$
$\therefore R_{IN} = 4g/second$
$\therefore R_{OUT} = \frac{4}{350}\ A(t)$

ODE: first order linear
$dA = \left(4 - \frac{4}{350}A\right)dt$
$dA = \left(4 - \frac{2}{175}A\right)dt$
$1 = \left(4 - \frac{2}{175}A\right)\frac{dt}{dA}$
$1 - \left(4 - \frac{2}{175}A\right)\frac{dt}{dA} \ = 0$
$\frac{dt}{dA}\left(\frac{2}{175}A - 4\right) + 1 = 0$
$\frac{1}{175}\left(175 - 700\frac{dt}{dA}A + 2A\frac{dt}{dA}\right) = 0$
$175 - 700\frac{dt}{dA}A + 2A\frac{dt}{dA} = 0$
$\frac{dt}{dA}A(2A - 700) = 175$
$\frac{dt}{dA}A = \frac{175}{(2A-700)}$

Integrate:
$\int -\frac{175}{(2A-700)}dA$
$= \int -\frac{175}{2}.\frac{1}{A-350}dA$
$= -\frac{175}{2}\int \frac{1}{A-350}dA$
$= -\frac{175}{2}\int \frac{1}{u}dA$
$= -\frac{175}{2}\log(u) + c$
$= -\frac{175}{2}\log(A - 350) + c$
$A(t) = -\frac{175}{2}\log(700 - 2A) + c$

$u = A - 350$
$du = dA$

Question 3
Question 4
Question 5