ASSIGNMENT 03
Due date: Monday, 14 July 2025

ONLY FOR YEAR MODULE

1. Solve the boundary–value problem

$$y'' + x^2y' - 4xy = 2x^3 + 6x^2 - 2, \quad y(0) = 0, \quad y(1) = 2$$

by using the **shooting method.** Use the modified Euler method (with only one correction at each step), and take $h = 0.2$. Start with an initial slope of $y'(0) = 1.9$ as a first attempt a nd $y'(0) = 2.1$ as a second attempt. Then interpolate.

Compare the result with the analytical solution $y = x^4 - x^2 + 2x$. (7)

2. (a) Define what is meant by the eigenvalues and eigenvectors of a matrix A.

If the matrix A is

$$A = \begin{bmatrix} 2 & 0 & 1 \\ -22 & -3 & 10 \\ -12 & 0 & 9 \end{bmatrix},$$

(b) find the characteristic polynomial,

(c) find the eigenvalues and eigenvectors.

(d) Start with the approximate eigenvector (1,1,1) and use the power method to estimate the dominant eigenvalue by iterating 4 times.

(e) Use the power method to find the smallest absolute eigenvalue of A.

(f) Write a program which applies the power method to a given matrix in (d) and (e) above. (25)

3. Consider the following boundary–value problem:

$$y'' = 2x^2y' + xy + 2, \quad 1 \leq x \leq 4.$$

Taking $h = 1$, set up the set of equations required to solve the problem by the finite difference method in each of the following cases of boundary conditions:

(a) $y(1) = -1, \quad y(4) = 4;$

(b) $y'(1) = 2, \quad y'(4) = 0;$

(c) $y'(1) = y(1), \quad y'(4) = -2y(4).$

(Do not solve the equations!). (15)