Question 1

a)
$f(x) = x^2 - \ln x^8$, where $x > 1$
$f(x) = x^2 - 8\ln x$
$f'(x) \quad = \frac{d}{dx}\,(x^2 - 8\ln x)$
$f'(x) \quad = 2x - \frac{8}{x}$

Product rule $\quad \frac{d}{dx}\,(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{d}{dx}\,(8\ln x) \qquad u = 8 \qquad v = \ln x$

$du = 0 \qquad dv = \frac{1}{x}$

$= 8\left(\frac{1}{x}\right) + \ln x\,(0)$
$= \frac{8}{x}$

b)
Critical point
$f'(c) = 0 \;$ or $\; f'(c) = undefined$
$0 = 2x - \frac{8}{x}$
$0 = 2x^2 - 8$
$0 = x^2 - 4$
$0 = (x-2)(x+2)$

Only defined for $\ln g(x)$ where $g(x) > 0$
$x = 2$

$f(1) = (1)^2 - \ln(1)^8$
$f(1) = 1 - 0$
$f(1) = 1$

The local extreme point is $(2,1)$

c)

$f''(x) \quad = \frac{d}{dx}\left(2x - \frac{8}{x}\right)$
$f''(x) \quad = 2 + \frac{8}{x^2}$
Concavity
$f''(c) = 0 \;$ or $\; f''(c) = undefined$

Quotient rule $\quad \frac{d}{dx}\left(\frac{u}{v}\right) = \left(\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right)\; if\; v \neq 0$

$\frac{d}{dx}\left(\frac{8}{x}\right) \qquad v = x \qquad u = 8$

$v' = 1 \qquad u' = 0$

$= \frac{1(0) - 8(1)}{x^2}$
$= -\frac{8}{x^2}$

$0 = \; 2 + \frac{8}{x^2}$
$0 = \; 2x^2 + 8$
$0 = \; x^2 + 4$
$x^2 = -4$
$x = \sqrt{-4}$

$undefined\ at\ 0$. Therefore, no inflection point exists for the function
$f''(-1) \quad = 2 + \frac{8}{(-1)^2} = 10 \qquad\qquad f''(1) \quad = 2 + \frac{8}{(1)^2} = 10$

The function is positive where $x < 0$ or $x > 0$. Therefore, it is concave up where $x < 0$ or $x > 0$

Question 2

Let x be the length of the poster
Let y be the width of the poster
Therefore, the area of the poster is defined by: $A = X.Y$

$A = (x - 4 - 4)(y - 2 - 2)$
$50 = (x - 8)(y - 4)$
$y = 4 + \frac{50}{x-8}$
$A = X.Y$
$A = x.\left(4 + \frac{50}{x-8}\right)$

Product Rule
$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
$A' = x\left(-\frac{50}{(x-8)^2}\right) + (1)\left(4 + \frac{50}{x-8}\right)$
$A' = 4 + \frac{-50x}{(x-8)^2} + \frac{50}{x-8}$
$A' = 4 + \frac{-50x}{(x-8)^2} + \frac{50x-400}{(x-8)^2}$
$A' = 4 - \frac{400}{(x-8)^2}$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \left(\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right) \text{ if } v \neq 0$$

$\frac{d}{dx}\left[4 + \frac{50}{x-8}\right]$

$\frac{d}{dx}\left[4 + 50.\frac{1}{x-8}\right]$

$50.\left(\frac{1}{x-8}\right)$

$v = x - 8$ $u = 1$

$v' = 1$
$u' = 0$

$50.\left(\frac{(x-8)(0)-(1)(1)}{(x-8)^2}\right)$

$50.\left(\frac{-1}{(x-8)^2}\right)$

Critical point
$f'(c) = 0$ or $f'(c) = undefined$
$0 = 4 - \frac{400}{(x-8)^2}$
$4 = \frac{400}{(x-8)^2}$
$4(x - 8)^2 = 400$
$(x - 8)^2 = 100$
$x - 8 = 10$
$x = 18$

$A'' = \frac{800}{(x-8)^3}$

Concavity
$f''(c) = 0$ or $f''(c) = undefined$
$0 = \frac{800}{(x-8)^3}$
$0 = 800$

$undefined$

Therefore $x = 18$ is the absolute minimum

$y = 4 + \frac{50}{x-8}$

$y = 4 + \frac{50}{18-8}$

$y = 9$

Therefore, the length of the poster is $18\ cm$
Therefore, the width of the poster is $9\ cm$

Question 3

a)

$\lim_{x\to 0} \frac{1-\cos^3 x}{\sin^2 x}$

L'Hôpital's Rule $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

$$\begin{aligned} f'(x) &= \frac{d}{x}(1-\cos^3 x) \\ &= \frac{d}{x}(1) - \frac{d}{x}(\cos^3 x) \\ &= 0 - 3\cos^2 x(-\sin x) \quad \textit{chain rule} \\ &= 3\cos^2 x \sin x \end{aligned}$$

$$\begin{aligned} g'(x) &= \frac{d}{x}(\sin^2 x) \\ &= 2\sin x \cos x \quad \textit{chain rule} \end{aligned}$$

$= \lim_{x\to 0} \frac{3\cos^2 x \sin x}{2\sin x \cos x}$

$= \lim_{x\to 0} \frac{3}{2}\cos x$

$= \frac{3}{2}\cos(0)$

$= \frac{3}{2}$

b)

$\lim_{x\to\infty} \left(\cos\frac{1}{x}\right)^x$

L'Hôpital's Rule $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

$f'(x) =$

$g'(x) =$

$=$

c) $\lim_{x\to\infty} \frac{x \ln x}{x^2-1}$

L'Hôpital's Rule $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

$f'(x) = x\ln x$ $= (x)\left(\frac{1}{x}\right) + (\ln x)(1)$ *product rule* $= 1 + \ln x$	$g'(x) = x^2 - 1$ $= 2x$

$= \lim_{x\to\infty} \frac{1+\ln x}{2x}$

L'Hôpital's Rule $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

$f'(x) = 1 + \ln x$ $= \frac{1}{x}$	$g'(x) = 2x$ $= 2$

$= \lim_{x\to\infty} \frac{\frac{1}{x}}{2}$

$= \frac{1}{2} \lim_{x\to\infty} \frac{1}{x}$

$= \frac{1}{2} . 0$

$= 0$

d) $\lim_{x\to 1} \frac{x \ln x}{x^2-1}$

$f'(x) = x\ln x$ $= (x)\left(\frac{1}{x}\right) + (\ln x)(1)$ *product rule* $= 1 + \ln x$	$g'(x) = x^2 - 1$ $= 2x$

$= \lim_{x\to 1} \frac{1+\ln x}{2x}$

L'Hôpital's Rule $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

$f'(x) = 1 + \ln x$ $= \frac{1}{x}$	$g'(x) = 2x$ $= 2$

$= \lim_{x\to 1} \frac{\frac{1}{x}}{2}$

$= \frac{1}{2} \lim_{x\to 1} \frac{1}{x}$

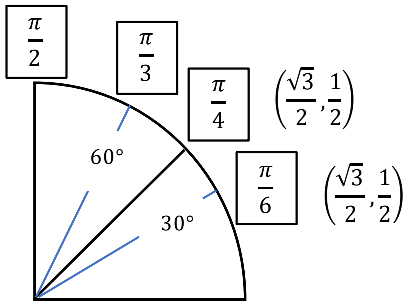
$= \frac{1}{2} . \frac{1}{1}$

$= \frac{1}{2}$

Question 4

$\tan\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$

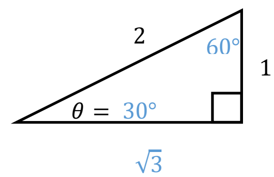
Unit Circle:



Reference Triangle:

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$ $30°$

$\sin\theta = \frac{1}{2}$ *Pythagoras theorem* $b = \sqrt{2^2 - 1^2} = \sqrt{3}$



The function $f(x) = \sin^{-1}(x)$, is only defined in the first and fourth quadrants.

If $tan\left(sin^{-1}\left(\frac{1}{2}\right)\right)$

Then $\sin\theta = \frac{1}{2}$ and θ is $30°$

Then $\tan(30°) = \frac{1}{\sqrt{3}}$

Therefore $tan\left(sin^{-1}\left(\frac{1}{2}\right)\right) = \frac{1}{\sqrt{3}}$