ASSIGNMENT 04
Due date: Thursday, 11 September 2025

ONLY FOR YEAR MODULE

1. Consider the partial differential equation

$$yu - 2\nabla^2 u = 12, \quad 0 < x < 4, \quad 0 < y < 3$$

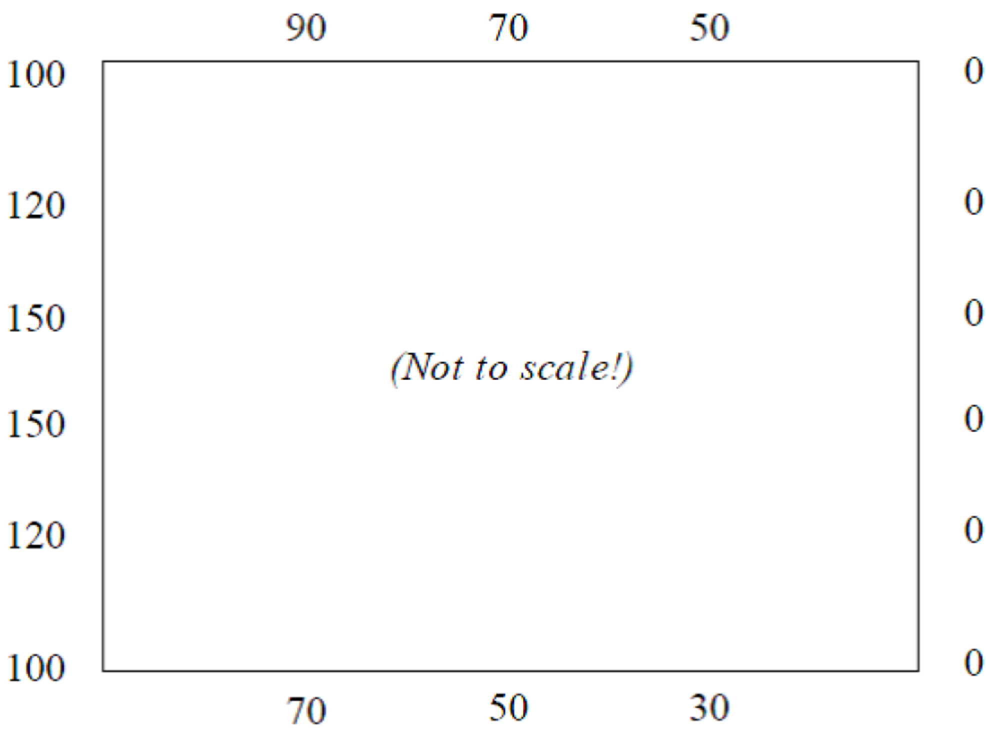
with boundary conditions

$$\begin{aligned} x &= 0 \text{ and } x = 4: \quad u = 60 \\ y &= 0 \text{ and } y = 3: \quad \frac{\partial u}{\partial y} = 5. \end{aligned}$$

   (a) Taking $h = 1$, sketch the region and the grid points. Use symmetry to minimize the number of unknowns u_i that have to be calculated and indicate the u_i in the sketch.

   (b) Use the 5-point difference formula for the Laplace operator to derive a system of equations for the u_i. (10)

2. We have a plate of 12×15 cm and the temperatures on the edges are held as shown in the sketch below. Take $\Delta x = \Delta y = 3$ cm and use the **S.O.R. method** (successive overrelaxation method) to find the temperatures at all the grid points. First calculate the optimal value of ω and then use this value in the algorithm. Start with all grid values equal to the arithmetic average of the given boundary values.



(10)

3. Solve the problem in question 2 by using the **A.D.I method** (alternating-direction-implicit method) without overrelaxation.

(15)