

Question 1

- a) 3. c isn't either medium and left of b or right of d correct Dodec(c) AND small(c)
- b) 3. Max is correct, because there is no situation in which b is small, d is large and b and d are names of the same objects.
- c) 2. (a) conjoins all the cited formulas and (b) fills in the first conjunct of the cited conjunction
- d) 2. The assumption of the subproof is a TT-contradiction.
- e) 4. Neither idempotent nor commutative
- f) 3. P must be false.
- g) 1. "John will prove a theorem only if he isn't very tired. He slept very well last night, so he'll prove a theorem."
- h) 4. Only the assumptions of every subproof in which the step-in question is embedded.
- i) 5. The set of points on the interval  $[0,1)$
- j) 4. A collection of mortal men, one aardvark and one penguin.

Question 2

- a) False
- b) True
- c) False
- d) False

Question 3

write down an informal proof, phrased in complete, well-formed English sentences

When you use proof by case or proof by contradiction

If the argument is invalid, construct a counter example

3.1) Invalid.

**Proof by cases:**

If Tapiwa or Rutendo is at home but either Themba or Tshepo is unhappy, then using proof by cases there are four cases:

In the first case, Tapiwa is home and Tshepo is unhappy.

In the second case, Tapiwa is home and Themba is unhappy.

In the third case, Rutendo is home and Tshepo is unhappy.

In the fourth case, Rutendo is home and Themba is unhappy.

Since the conclusion that "Tshepo is unhappy" is not true for all cases, we can say that the statement "Tshepo is unhappy" is invalid.

3.2) Invalid.

**Proof by cases:**

If Aggie or Cecil is not shopping, and If Cecil is shopping or Cecil and Aggie are married, then using proof by cases there are two cases:

In the first case, Cecil is shopping, or Cecil and Aggie are married.

In the second case, Angie is shopping, or Cecil and Aggie are not married.

Since the conclusion that "Cecil and Aggie are married" is not true for all cases, we can say that the statement "Cecil and Aggie are married" is invalid.

3.3) Invalid.

**Proof by cases:**

If Peter is a student or Peter is not Hungry at 2 o'clock, then using proof by cases there are four cases:

In the first case, Peter is not Hungry at 2 o'clock

In the second case, Peter is not Patient

In the third case, Peter is not Patient

In the third case, Peter is not a student

Since the conclusion that "Peter is a student" is not true for all cases, we can say that the statement "Peter is a student" is invalid.

Question 4

- a) Atomic WFF
- b) Term
- c) Neither
- d) Neither
- e) Atomic WFF

Question 5  
5.1

## 5.2

1.	$\neg S$		
2.	$P \rightarrow Q$		
3.	$Q \rightarrow (R \vee S)$		
4.	$P \vee R$		
5.	$\neg P$		
6.	$Q$	✓ $\rightarrow$ Elim	5,2
7.	$\neg R$		
8.	$(R \vee S) \vee R$	$\vee$ Intro	7
9.	$\neg P$		
10.	$Q$	✓ $\rightarrow$ Elim	9,2
11.	$R \vee S$	✓ $\rightarrow$ Elim	10,3
12.	$P \rightarrow (R \vee S)$	✓ $\rightarrow$ Intro	9-11
13.	$\neg S$		
14.	$\perp$	✓ $\perp$ Intro	13,1
15.	$\neg S$	✓ $\neg$ Intro	13-14
16.	$R$	$\neg$ Rule?	



Question 6

- 6.1)  $\exists x (\text{SisterOf}(x, \text{Bill}) \wedge \forall y (\text{SisterOf}(y, \text{Bill}) \rightarrow x = y))$
- 6.2)  $\exists x (\text{Student}(x) \wedge \text{Failed}(x, \text{Logic}) \wedge \forall y (\text{Student}(y) \wedge \text{Failed}(y, \text{Logic}) \rightarrow x = y))$
- 6.3)  $\forall x (\text{Student}(x) \rightarrow \exists y \text{Student}(y) \wedge \neg(x=y) \wedge \text{Loves}(x,y))$
- 6.4)  $\text{Takes}(\text{billy}, \text{chemistry}) \neg \Leftrightarrow \text{Takes}(\text{billy}, \text{geometry})$
- 6.5)  $\neg \exists x (\text{Student}(x) \wedge \forall y (\text{Student}(y) \wedge \neg (x = y) \Rightarrow \text{Fools}(x,y)))$
- 6.6)  $\text{Loves}(\forall x(\text{student}), \text{student})$

Question 7

- 7.1) Bill takes Logic or Geometry (or both).
- 7.2) A student loves Chemistry and Sister failed Logic
- 7.3) Bill takes either Logic or Geometry (but not both)
- 7.4) Every student loves some other student.
- 7.5) Bill only has one sister.
- 7.6) There is a student who cannot fool every other student.