Question 1
Consider the following data:

x	f(x)	f'(x)
0.1	-0.62049958	3.58502082
0.2	-0.28398668	3.14033271
0.3	0.00660095	2.66668043
0.4	0.24842440	2.16529366

(1.1) Find an approximation to f(0.27) using the following forms of interpolating polynomial
(a) the Lagrange form.

[1] Calculate $L_i(x)$:
For each i:

$$L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_1)}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}$$

$$L_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

[2] Lagrange polynomials, evaluated at the point x = 0.27
For each i:

$$\begin{aligned} L_0(0.27) &= \frac{(0.27-0.2)(0.27-0.3)(0.27-0.4)}{(0.1-0.2)(0.1-0.3)(0.1-0.4)} \\ &= \frac{(0.7)(-0.03)(-0.13)}{(-0.1)(-0.2)(-0.3)} \\ &= \frac{0.000273}{-.006} \\ &= -0.0455 \end{aligned}$$

$$\begin{aligned} L_1(0.27) &= \frac{(0.27-0.1)(0.27-0.3)(0.27-0.4)}{(0.2-0.1)(0.2-0.3)(0.2-0.4)} \\ &= \frac{(0.17)(-0.03)(-0.13)}{(0.1)(-0.1)(-0.2)} \\ &= \frac{0.000663}{0.002} \\ &= 0.3315 \end{aligned}$$

$$L_2(0.27) = \frac{(0.27-x_0)(0.27-x_1)(0.27-0.4)}{(0.3-0.1)(0.3-0.2)(0.3-0.4)}$$
$$= \frac{(0.17)(0.07)(-0.13)}{(0.2)(0.1)(-0.1)}$$
$$= \frac{-0.001547}{-0.002}$$
$$= 0.7735$$

$$L_3(0.27) = \frac{(0.27-0.1)(0.27-0.2)(0.27-0.3)}{(0.4-0.1)(0.4-0.2)(0.4-0.3)}$$
$$= \frac{(0.17)(0.07)(-0.03)}{(0.3)(0.2)(0.1)}$$
$$= -\frac{0.000357}{0.006}$$
$$= -0.0595$$

[3] Interpolated polynomial, evaluated at x = 0.27

$P(0.27) = f(x_0)L_0(x) + f(x_1)L_1(x) + f(x_2)L_2(x) + f(x_3)L_3(x) +$

$\Rightarrow P(0.27) = f(0.1)L_0(0.27) + f(0.1)L_1(0.27) + f(0.1)L_2(0.27) + f(0.1)L_3(0.27)$

$\Rightarrow P(0.27) =$
$(-0.62049958)(-0.0455) +$
$(-0.28398668)(0.3315) +$
$(0.00660095)(0.7735) +$
$(0.24842440)(-0.0595) +$

$\Rightarrow P(0.27) = -0.0755234$

(b) the Newton forward divided difference form.

[1] First-order divided differences

$$f[x_0, x_1] = \frac{f(x) - f(x_0)}{x_1 - x_0}$$
$$= \frac{-0.28398668 + 0.62049958}{0.2 - 0.1}$$
$$= 3.365129$$

$$f[x_1, x_2] = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$= \frac{0.00660095 + 0.28398668}{0.3 - 0.2}$$
$$= 2.9058763$$

$$f[x_2, x_3] = \frac{f(x_3) - f(x_2)}{x_3 - x_2}$$
$$= \frac{0.24842440 + 0.00660095}{0.4 - 0.3}$$
$$= 2.9058763$$

[2] Second-order divided differences

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$
$$= \frac{2.9058763 - 3.365129}{0.3 - 0.1}$$
$$= -2.296263$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1}$$
$$= \frac{2.4182345 - 2.9058763}{0.4 - 0.2}$$
$$= -2.438209$$

[3] Third-order divided differences

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_0, x_1, x_2] - f[x_1, x_2, x_3]}{x_3 - x_0}$$
$$= \frac{-2.438209 + 2.296263}{0.3 - 0.1}$$
$$= -0.4731517$$

[4] Newton forward divided difference form of the interpolating polynomial

$$
\begin{aligned}
P(x) = {} & \\
& f(x_0) + f[x_0, x_1](x - x_1) + \\
& f[x_0, x_1, x_2](x - x_0)(x - x_1) + \\
& f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow P(0.27) = {} & \\
& -0.62049958 + 3.365129(0.27 - 0.1) + \\
& 2.9058763(0.27 - 0.1)(0.27 - 0.2) + \\
& -0.4731517(0.27 - 0.1)(0.27 - 0.2)(0.27 - 0.3)
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow P(0.27) = {} & \\
& -0.62049958 + 3.365129(0.17) + \\
& 2.9058763(0.17)(0.7) + \\
& -0.4731517(0.17)(0.07)(-0.03)
\end{aligned}
$$

$$\Rightarrow P(0.27) = -0.07409884$$

(c) the Hermite form.

x	f(x)	f'(x)
0.1	-0.62049958	
	3.58502082	
0.2	-0.28398668	
	3.14033271	
0.3	0.00660095	
	2.66668043	
0.4	0.24842440	
	2.16529366	

[1] Divided differences

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = 3.58502082$$

$$f[x_1, x_2] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = 3.58502082$$

$$f[x_2, x_3] = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = 3.14033271$$

$$f[x_3, x_4] = \frac{f(x_3) - f(x_2)}{x_3 - x_2} = -0.28398668$$

$$f[x_4, x_5] = \frac{f(x_4) - f(x_3)}{x_4 - x_3} = 3.14033271$$

$$f[x_5, x_6] = \frac{f(x_5) - f(x_4)}{x_5 - x_4} = 2.66668043$$

$$f[x_6, x_7] = \frac{f(x_6) - f(x_5)}{x_6 - x_5} = 2.16529366$$

[2] Hermite Interpolated polynomial, evaluated at x = 0.27

$$
\begin{aligned}
h_0(0.27) &= \frac{(0.27-0.2)(0.27-0.3)(0.27-0.4)}{(0.1-0.2)(0.1-0.3)(0.1-0.4)} \\
&= \frac{(0.7)(-0.03)(-0.13)}{(-0.1)(-0.2)(-0.3)} \\
&= \frac{0.000273}{-.006} \\
&= -0.0455
\end{aligned}
$$

$$
\begin{aligned}
h_1(0.27) &= \frac{(0.27-0.1)(0.27-0.3)(0.27-0.4)}{(0.2-0.1)(0.2-0.3)(0.2-0.4)} \\
&= \frac{(0.17)(-0.03)(-0.13)}{(0.1)(-0.1)(-0.2)} \\
&= \frac{0.000663}{0.002} \\
&= 0.3315
\end{aligned}
$$

$$
\begin{aligned}
h_2(0.27) &= \frac{(0.27-x_0)(0.27-x_1)(0.27-0.4)}{(0.3-0.1)(0.3-0.2)(0.3-0.4)} \\
&= \frac{(0.17)(0.07)(-0.13)}{(0.2)(0.1)(-0.1)} \\
&= \frac{-0.001547}{-0.002} \\
&= 0.7735
\end{aligned}
$$

$$
\begin{aligned}
h_3(0.27) &= \frac{(0.27-0.1)(0.27-0.2)(0.27-0.3)}{(0.4-0.1)(0.4-0.2)(0.4-0.3)} \\
&= \frac{(0.17)(0.07)(-0.03)}{(0.3)(0.2)(0.1)} \\
&= -\frac{0.000357}{0.006} \\
&= -0.0595
\end{aligned}
$$

[3] Interpolated polynomial, evaluated at x = 0.27

$$
\begin{aligned}
H(x) = \\
&f(x_0)h_0^2(x) + \\
&f'(x_0)h_0(x-x_0) + \\
&f(x_1)h_1^2(x) + \\
&f'(x_1)h_1(x)(x-x_1) + \\
&f(x_2)h_2^3(x) + \cdots
\end{aligned}
$$

$$\Rightarrow H(0.27) =$$

$$\begin{aligned}
&-0.62049958 \times 0.00207025 + \\
&3.58502082 \times -0.007735 + \\
&-0.28398668 \times 0.10989025 + \\
&3.14033271 \times 0.023205 + \\
&0.00660095 \times 0.59830325 + \\
&2.66668043 \times -0.023205 + \\
&0.24842440 \times 0.00354025 + \\
&2.16529366 \times 0.007735
\end{aligned}$$

$$\Rightarrow H(0.27) =$$

$$\begin{aligned}
&-0.00128387 \\
&-0.02773844 \\
&-0.03119547 \\
&+0.07284464 \\
&+0.00394792 \\
&-0.06187495 \\
&+0.00088018 \\
&+0.01674864
\end{aligned}$$

$$\Rightarrow H(0.27) = -0.02718085$$

Question 2

Consider the following data

x	f(x)
0.3	-1.1518
-0.4	0.7028
0.5	-1.4845
0.00	0.13534

(2.1) Use a third degree Lagrange interpolating polynomial to approximate f(0.55).

[1] Calculate $L_i(x)$:

For each i:

$$\ell_0(x) = \frac{(x+0.4)(x-0.5)(0.27-0.00)}{(0.3+0.4)(0.3-0.5)(0.3-0.00)}$$

$\Rightarrow \ell_0(x) =$

$x($

$-23.8095238095238x^2 +$

$2.38095238095238x +$

4.76190476190476

$)$

$\Rightarrow \ell_0(0.55) = 0.02728$

$$\ell_1(x) = \frac{(x-0.3)(x-0.5)(x-0.00)}{(-0.4-0.3)(-0.4-0.5)(0.4-0.00)}$$

$\Rightarrow \ell_1(x) =$

$x($

$-3.96825396825397x^2 +$

$3.17460317460317x -$

0.595238095238095

$)$

$\Rightarrow \ell_1(0.55) = -0.00223$

$$\ell_2(x) = \frac{(x-0.3)(x+0.4)(x-0.00)}{(0.5-0.3)(0.5+0.4)(0.5-0.00)}$$
$$\Rightarrow x(11.1111111111111x^2 + 1.11111111111111x - 1.33333333333333)$$

$$\ell_2(x) = \frac{(x-0.3)(x+0.4)(x-0.00)}{(0.5-0.3)(0.5+0.4)(0.5-0.00)}$$
$$\Rightarrow \ell_2(x) =$$
$$x($$
$$11.1111111111111x^2 +$$
$$1.11111111111111x -$$
$$1.33333333333333$$
$$)$$
$$\Rightarrow \ell_2(0.55) = 0.9412$$

$$\ell_3(x) = \frac{(x-0.3)(x+0.4)(x-0.5)}{(0.00-0.3)(0.00+0.4)(0.00-0.5)}$$
$$\Rightarrow \ell_3(x) =$$
$$(2.0x - 1.0) \times$$
$$(2.5x + 1.0) \times$$
$$(3.33333333333333x - 1)$$
$$\Rightarrow \ell_3(0.55) = 0.03375$$

[2] Lagrange polynomials, evaluated at the point x = 0.27

For each i:

$$\Rightarrow L(0.55) =$$
$$-1.1518 \times 0.02728 +$$
$$0.7028 \times -0.00223 +$$
$$-1.4845 \times 0.9412 +$$
$$0.13534 \times 0.03375$$
$$\Rightarrow L(0.55) = -1.4305$$

(2.2) Use a Newton's divided-difference polynomial that interpolates all the points to approximate f(0.2), using the following criteria:
(a) Without rearranging the nodes;

[1] First divided differences

$$f[x_1, x_0] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$
$$= \frac{0.7028 + 1.1518}{-0.4 - 0.3}$$
$$= -2.649428571428571$$

$$f[x_2, x_1] = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$= \frac{-1.4845 - 0.7028}{0.5 + 0.4}$$
$$= -2.430333333333333$$

$$f[x_3, x_2] = \frac{f(x_3) - f(x_2)}{x_3 - x_2}$$
$$= \frac{0.13534 + 1.4845}{0 - 0.5}$$
$$= -3.23968$$

[2] Second Divided Differences

$$f[x_2, x_1, x_0] = \frac{f[x_2, x_1] - f[x_1, x_0]}{x_2 - x_0}$$
$$= \frac{2.9058763 - 3.365129}{0.5 - 0.3}$$
$$= 1.0954761904761905$$

$$f[x_3, x_2, x_1] = \frac{f[x_3, x_2] - f[x_2, x_1]}{x_3 - x_1}$$
$$= \frac{2.4182345 - 2.9058763}{0 + 0.4}$$
$$= -2.023366666666667$$

[3] Third Divided Differences

$$f[x_3, x_2, x_1, x_0] = \frac{f[x_3, x_2, x_1] - f[x_2, x_1, x_0]}{x_3 - x_0}$$
$$= \frac{-2.023366666666667 - 1.0954761904761905}{0 - 0.3}$$
$$= 10.396142857142857$$

[4] Newton Interpolated polynomial, evaluated at x = 0.2

$$P(x) = f(x_0)L_0(x) + f(x_1)L_1(x) + f(x_2)L_2(x) + f(x_3)L_3(x) +$$

$$\Rightarrow P(x) =$$
$$f(x_0) +$$
$$f[x_1, x_0](x - x_0) +$$
$$f[x_2, x_1, x_0](x - x_0)(x - x_1) +$$
$$f[x_3, x_2, x_1](x - x_0)(x - x_1)(x - x_2)$$

$$\Rightarrow P(x) =$$
$$f(x_0) +$$
$$f[x_1, x_0](x - x_0) +$$
$$f[x_2, x_1, x_0](x - x_0)(x - x_1) +$$
$$f[x_3, x_2, x_1](x - x_0)(x - x_1)(x - x_2)$$

$$\Rightarrow P(x) =$$
$$-1.1518 +$$
$$(-2.649428571428571)(x - 0.3) +$$
$$(1.0954761904761905)(x - 0.3)(x + 0.4) +$$
$$(10.396142857142857)(x - 0.3)(x + 0.4)(x - 0.5)$$

$$\Rightarrow P(0.2) =$$
$$-1.1518 +$$
$$(-2.649428571428571)(0.2 - 0.3) +$$
$$(1.0954761904761905)(0.2 - 0.3)(0.2 + 0.4) +$$
$$(10.396142857142857)(0.2 - 0.3)(0.2 + 0.4)(0.2 - 0.5)$$

$$\Rightarrow P(0.2) = -0.7655$$

(b) Rearranging the nodes in increasing order.

(2.3) Compare the results obtained in (2.2) above.

(2.4) Use the least-squares polynomial of degree two to approximate f(0.2) and compute the error.
(Your system of normal equations must be explicit).

[1] Given the polynomial $p(x) = ax^2 + bx + c$, a, b, and c

$$\begin{cases} \sum y_i = a\sum x_i^2 + b\sum x_i + c\sum 1 \\ \sum x_i y_i = a\sum x_i^3 + b\sum x_i^2 + c\sum x_i \\ \sum x_i^2\, y_i = a\sum x_i^4 + b\sum x_i^3 + c\sum x_i^2 \end{cases}$$

[2] Calculate sums:

$$\sum x_i = 0.3 - 0.4 + 0.5 + 0.0$$
$$= 0.4$$

$$\sum x_i^2 = (0.3)^2 + (-0.4)^2 + (0.5)^2 + (0.0)^2$$
$$= 0.5$$

$$\sum x_i^3 = (0.3)^3 + (-0.4)^3 + (0.5)^3 + (0.0)^3$$
$$= 0.088$$

$$\sum x_i^4 = (0.3)^4 + (-0.4)^4 + (0.5)^4 + (0.0)^4$$
$$= 0.0962$$

$$\sum y_i = -1.15180 + 0.7028 - 1.4845 + 0.13534$$
$$= -1.79816$$

$$\sum x_i y_i = +(0.3)(-1.15180)$$
$$+(-0.4)(0.7028)$$
$$+(0.5)(-1.4845)$$
$$+(0.0)(0.13534)$$
$$= -1.36891$$

$$\sum x_i^2 y_i = \quad +(0.3)^2(-1.15180)$$

$$+(-0.4)^2(0.7028)$$
$$+(0.5)^2(-1.4845)$$

$$+(0.0)^2(0.13534)$$

$$= -0.362339$$

[3] solve system

$$\begin{cases} -1.79816 = a \times 0.5 + b \times 0.4 + c \times 4 \\ -1.36891 = a \times 0.088 + b \times 0.5 + c \times 0.4 \\ -0.362339 = a \times 0.0962 + b \times 0.088 + c \times 0.5 \end{cases}$$

[4] matrix form,

$$\begin{bmatrix} 0.5 & 0.4 & 4 \\ 0.088 & 0.5 & 0.4 \\ 0.0962 & 0.088 & 0.5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1.79816 \\ -1.36891 \\ -0.362339 \end{bmatrix}$$

Where $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = A^{-1}B$

Thus,

$a \quad = (0.5) \cdot (-1.79816) + (0.4) \cdot (-1.36891) + (4) \cdot (-0.362339)$

$\quad = -1.28725$

$b \quad = (0.088) \cdot (-1.79816) + (0.5) \cdot (-1.36891) + (0.4) \cdot (-0.362339)$

$\quad = -2.47865$

$c \quad = (0.0962) \cdot (-1.79816) + (0.088) \cdot (-1.36891) + (0.6) \cdot (-0.362339)$

$\quad = -0.04077$

[5] polynomial $p(x) = ax^2 + bx + c$

$p(x) = ax^2 + bx + c$

$\Rightarrow p(0.2) = (-1.28725)(0.2)^2 + (-2.47865)(0.2) + (-0.04077)$

$\Rightarrow p(0.2) = -0.5880$

(2.5) Use the least-squares function of the form $y = \alpha e^{\beta x}$ to approximate f(0.2) and compute the error.

(2.6) Plot the graphs of the approximating polynomials
in (2.2) -(2.4).
Your graphs must be proper computer produced graphs.

Question 3 [15 marks]
Construct the natural cubic spline for the data below and use it to approximate f(0.3):
(-0.5, 5), (0, 15), (0.5, 9)

[1] cubic polynomials for each interval

$$S_1(x) = a_1 + b_1(x+0.5) + c_1(x+0.5)^2 + d_1(x+0.5)^3 \quad | \quad 0.5 \le x \le 0$$

$$S_2(x) = a_2 + b_2(x) + c_2(x)^2 + d_2(x)^3 \quad | \quad 0 \le x \le 0.5$$

[2] natural cubic spline

	=
$S_1(-0.5)$	5
$S_1(0)$	15
$S_2(0)$	15
$S_2(0.5)$	9
$S_1'(0)$	$S_2'(0)$
$S_1''(0)$	$S_2''(0)$
$S_1''(-0.5)$	0
$S_2''(0.5)$	0

[3] Equations for cubic spline

$S_1(-0.5) = 5$

$$\Rightarrow a_1 + b_1(0) + c_1(0)^2 + d_1(0)^3 = 5$$

$$\Rightarrow a_1 = 5$$

$S_1(0) = 15$

$$\Rightarrow a_1 + b_1(0.5) + c_1(0.5)^2 + d_1(0.5)^3 = 15$$

$$\Rightarrow a_1 + 0.5b_1 + 0.25c_1 + 0.125d_1 = 15$$

$$\Rightarrow 5 + 0.5b_1 + 0.25c_1 + 0.125d_1 = 15 \qquad \text{where } a_1 = 5$$

$$\Rightarrow 0.5b_1 + 0.25c_1 + 0.125d_1 = 10$$

$S_2(0) = 15$

$\Rightarrow a_2 + b_2(0) + c_2(0)^2 + d_2(0)^3 = 15$

$\Rightarrow a_2 = 15$

$S_2(0.5) = 9$

$\Rightarrow a_2 + b_2(0.5) + c_2(0.5)^2 + d_2(0.5)^3 = 9$

$\Rightarrow a_2 + 0.5b_2 + 0.25c_2 + 0.125d_2 = 9$

$\Rightarrow 15 + 0.5b_2 + 0.25c_2 + 0.125d_2 = 9$ where $a_2 = 15$

$\Rightarrow 0.5b_2 + 0.25c_2 + 0.125d_2 = -6$

$S_1'(0) = S_2'(0)$

$S_1(x)$ in the interval [-0.5, 0]

$S_1(x) = a_1 + b_1(x + 0.5) + c_1(x + 0.5)^2 + d_1(x + 0.5)^3$

$\Rightarrow S_1'(x) = b_1 + 2c_1(x + 0.5) + 3d_1(x + 0.5)^2$

$\Rightarrow S_1'(0) = b_1 + 2c_1(0 + 0.5) + 3d_1(0 + 0.5)^2$

$\Rightarrow S_1'(0) = b_1 + 0.5c_1 + 0.75d_1$

$S_2(x)$ in the interval [0, 0.5]

$S_2(x) = a_2 + b_2(x) + c_2(x)^2 + d_2(x)^3$

$\Rightarrow S_2'(x) = b_2 + 2c_2x + 3d_2(x)^2$

$\Rightarrow S_2'(0) = b_2(0) + c_2(0)^2 + d_2(0)^3$

$\Rightarrow S_1'(0) = b_2$

Thus,

$S_1'(0) = S_2'(0)$

$\Rightarrow b_1 + 0.5c_1 + 0.75d_1 \ = b_2$

$S_1''(0) = S_2''(0)$

$S_1(x)$ in the interval [-0.5, 0]

$\Rightarrow S_1'(x) = b_1 + 2c_1(x + 0.5) + 3d_1(x + 0.5)^2$

$\Rightarrow S_1''(x) = 2c_1 + 6d_1(x + 0.5)$

$\Rightarrow S_1''(0) = 2c_1 + 6d_1(0 + 0.5)$

$\Rightarrow S_1''(0) = 2c_1 + 3d_1$

$S_2(x)$ in the interval [0, 0.5]

$S_2(x) = a_2 + b_2(x) + c_2(x)^2 + d_2(x)^3$

$\Rightarrow S_2'(x) = b_2 + 2c_2x + 3d_2(x)^2$

$\Rightarrow S_2''(x) = 2c_2 + 6d_2x$

$\Rightarrow S_2''(0) = 2c_2 + 6d_2(0)$

$\Rightarrow S_2''(0) = 2c_2$

Thus,

$S_1''(0) = S_2''(0)$

$\Rightarrow 2c_1 + 3d_1 = 2c_2$

$S_1''(-0.5) = 0$

$\Rightarrow 2c_1 = 0$

$\Rightarrow c_1 = 0$

$S_2''(0.5) = 0$

$\Rightarrow S_2''(x) = 2c_2 + 6d_2x$

$\Rightarrow S_2''(0.5) = 2c_2 + 6d_2(0.5)$

$\Rightarrow 0 = 2c_2 + 3d_2$

$\Rightarrow c_2 = -1.5d_2$

[3] Solve system of equations

where $c_1 = 0$

$0.5b_1 + 0.25c_1 + 0.125d_1 = 10$

$\Rightarrow 0.5b_1 + 0.25(0) + 0.125d_1 = 10$

$\Rightarrow 0.5b_1 + 0.125d_1 = 10$

$\Rightarrow 4b_1 + d_1 = 80$

where $c_2 = -1.5d_2$

$0.5b_2 + 0.25c_2 + 0.125d_2 = -6$

$\Rightarrow 0.5b_2 + 0.25\,(-1.5d_2) + 0.125d_2 = -6$

$\Rightarrow 0.5b_2 - 0.375d_2 + 0.125d_2 = -6$

$\Rightarrow 0.5b_2 - 0.25d_2 = -6$

$\Rightarrow 2b_2 - d_2 = -24$

where $c_1 = 0$

$b_1 + 0.5c_1 + 0.75d_1 \ \ = b_2$

$b_1 + 0.5(0) + 0.75d_1 \ \ = b_2$

$b_1 + 0.75d_1 \ \ = b_2$

where $c_1 = 0$

$2c_1 + 3d_1 = \ 2c_2$

$2(0) + 3d_1 = \ 2c_2$

$3d_1 = \ 2c_2$

where $c_2 = -1.5d_2$

$3d_1 = \ 2c_2$

$3d_1 = \ 2(-1.5d_2)$

$3d_1 = \ -3d_2$

$d_1 = \ -d_2$

Where $d_1 = \ -d_2$

$4b_1 + d_1 = 80$

$4b_1 + -d_2 = 80$

$-d_2 = -4b_1 + 80$

$d_2 = 4b_1 - 80$

Where $d_2 = 4b_1 - 80$

$d_1 = \ -(4b_1 - 80)$

$d_1 = \ -4b_1 + 80$

Where $\boldsymbol{d_2 = 4b_1 - 80}$

$2b_2 - d_2 = -24$

$2b_2 - (4b_1 - 80\ ) = -24$

$2b_2\ = -104 + 4b_1$

$b_2\ = -52 + 2b_1$

Where $\boldsymbol{d_1 = \ -4b_1 + 80}$

$b_1 + 0.75d_1\ \ = b_2$

$b_1 + 0.75(-4b_1 + 80\ )\ \ = b_2$

$b_1 - 3b_1 + 60\ \ = b_2$

$-2b_1 + 60 = b_2$

Where $\boldsymbol{b_2\ = -52 + 2b_1}$

$-2b_1 + 60 = b_2$

$-2b_1 + 60 = -52 + 2b_1$

$4b_1 = 112$

$b_1 = 28$

Where $\boldsymbol{b_1 = 28}$

$-2b_1 + 60 = b_2$

$-2(28) + 60 = b_2$

$b_2 = 4$

Where $\boldsymbol{b_1 = 28}$

$d_2 = 4b_1 - 80$

$d_2 = 4(28) - 80$

$d_2 = 32$

Where $\boldsymbol{d_2 = 32}$

$d_1 =\ -d_2$

$d_1 =\ -32$

[4] Cubic splines

$S_1(x) = a_1 + b_1(x + 0.5) + c_1(x + 0.5)^2 + d_1(x + 0.5)^3$

$\Rightarrow S_1(x) = 5 + (28)(x + 0.5) + (0)(x + 0.5)^2 + (-32)(x + 0.5)^3$

$S_2(x) = a_2 + b_2(x) + c_2(x)^2 + d_2(x)^3$

$\Rightarrow S_2(x) = 15 + 4(x) + 96(x)^2 + 32(x)^3$

[5] f(0.3)

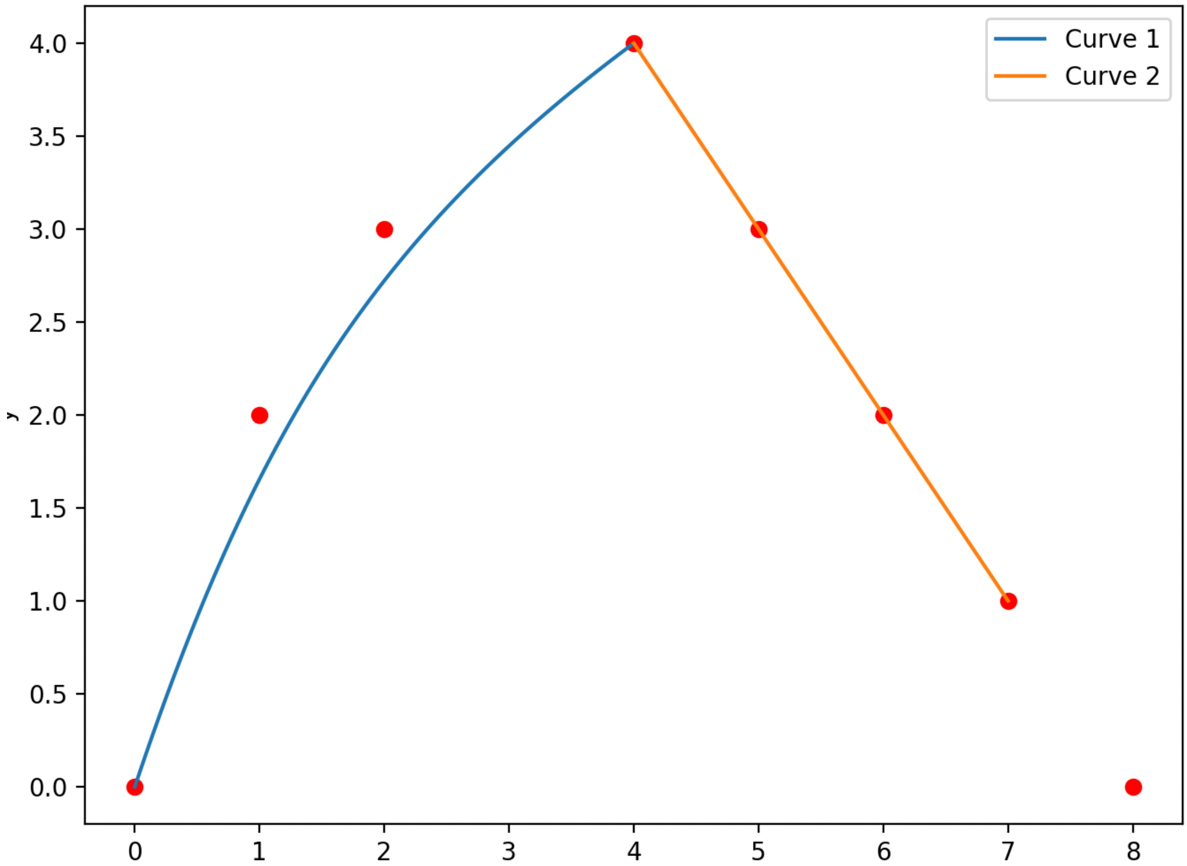
$S_2(x) = 15 + 4(x) + 96(x)^2 + 32(x)^3$

$\Rightarrow S_2(0.3) = 15 + 4(0.3) + 96(0.3)^2 + 32(0.3)^3$

$\Rightarrow S_2(0.3) = 8.42$

Question 4 [20 marks]
Consider the following set of data points in the table below:

(4.1) Using guidepoints of your choice from the data set, construct the connected Bezier curve
from the set of points.
(Hint: Divide the set of points into three parts)



(4.2) Draw the connected Bezier polynomial.

(4.3) Why is the graph smoothly connected at points 3 and 6?