

Question 1

1.1.

$$4z'' - 4z' - 3z = \cos 2x$$

ODE: second order linear nonhomogenous

[1] Find roots related homogenous equation

Characteristic equation

$$4\lambda^2 - 4\lambda - 3 = 0$$

$$(2\lambda - 3)(2\lambda + 1) = 0$$

$$\lambda = \frac{3}{2} \text{ or } \lambda = -\frac{1}{2}$$

[2] Solution of homogenous equation

Use the method of variation of constants, where the general solution is of the form $y_1(x) = C_1 Y_1(x) + C_2 Y_2(x)$

$$\therefore y_0(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$$

$$\therefore y_0(x) = C_1 e^{\frac{3}{2}x} + C_2 e^{-\frac{1}{2}x}$$

[2] Solution of nonhomogenous equation

Use the method of undetermined coefficients, where the integral solution is of the form $y_2(x) = A \cos x + B \sin x$

$$\therefore y_2(x) = A \cos 2x + B \sin 2x$$

We have that

$$A = -\frac{19}{425}, \quad B = -\frac{8}{425}$$

$$\therefore y_2''(x) = -\frac{19}{425} \cos 2x - \frac{8}{425} \sin 2x$$

The general solution of the nonhomogenous equation is of the form

$$z = y_0(x) + y_2(x)$$

Hence,

$$y_0(x) + y_2(x)$$

$$\therefore z = C_1 e^{\frac{3}{2}x} + C_2 e^{-\frac{1}{2}x} - \frac{19}{425} \cos 2x - \frac{8}{425} \sin 2x$$

Question 2

Given

25 grams salt

350l of water

Pumping rate 4l/min

Let

$A(t)$ be the amount of salt in the tank

R_{IN} be the rate of incoming salt

R_{OUT} be the rate of outgoing salt

$$\therefore dA = R_{IN} - R_{OUT}$$

$$\therefore R_{IN} = 4g/second$$

$$\therefore R_{OUT} = \frac{4}{350} A(t)$$

ODE: first order linear

$$dA = \left(4 - \frac{4}{350}A\right) dt$$

$$dA = \left(4 - \frac{2}{175}A\right) dt$$

$$1 = \left(4 - \frac{2}{175}A\right) \frac{dt}{dA}$$

$$1 - \left(4 - \frac{2}{175}A\right) \frac{dt}{dA} = 0$$

$$\frac{dt}{dA} \left(\frac{2}{175}A - 4\right) + 1 = 0$$

$$\frac{1}{175} \left(175 - 700 \frac{dt}{dA} A + 2A \frac{dt}{dA}\right) = 0$$

$$175 - 700 \frac{dt}{dA} A + 2A \frac{dt}{dA} = 0$$

$$\frac{dt}{dA} A(2A - 700) = 175$$

$$\frac{dt}{dA} A = \frac{175}{(2A-700)}$$

Integrate:

$$\int -\frac{175}{(2A-700)} dA$$

$$= \int -\frac{175}{2} \cdot \frac{1}{A-350} dA$$

$$= -\frac{175}{2} \int \frac{1}{A-350} dA$$

$$= -\frac{175}{2} \int \frac{1}{u} dA$$

$$= -\frac{175}{2} \log(u) + c$$

$$= -\frac{175}{2} \log(A - 350) + c$$

$$A(t) = -\frac{175}{2} \log(700 - 2A) + c$$

$$u = A - 350$$
$$du = dA$$

Question 3

Question 4

Question 5

