LKE MNCUBE
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Unique Assignment Number: 771329

Question 1

Let $\bar{u} = r$, and Let $\bar{v} = r$, where $r \in \mathbb{R}$, and represents magnitude
Therefore, $\bar{u} = P_1(0,0,r)$ and $\bar{v} = P_2(0,0,r)$, from standard position
Let standard position be A
If they both lie on a circle then $|AP_1| = |AP_2|$
But we know that $\bar{u} = r$, therefore $pu = r$
But we know that $\bar{v} = r$, therefore $pv = r$
$\therefore$ The ends of both lines u and v lie on the same circle as $|AP_1| = |AP_2|$

Question 2

$\bar{v} = terminal\ point - initial\ point$

$\therefore$ $\bar{v}$ or $\overline{P_1P_2}$ $= (P_{2x} - P_{1x};\ P_{2y} - P_{1y};\ P_{2z} - P_{1z})$

$= \{6-3; 5-(-1);\ -8-4\}$

$= \{6-3; 5-(-1);\ -8-4\}$

$= (3; 6; -12)$

Question 3

$k = \sqrt{x^2+y^2+z^2}$

$\therefore \bar{v} = \sqrt{2^2+2^2+1^2}$

$= \sqrt{4+4+1}$

$= \sqrt{9}$

$= 3$

Unit vector in direction of a : $= \frac{1}{a}\bar{a}$

$\therefore = \frac{1}{3}(2,2,1)$

$= (\frac{2}{3}, \frac{2}{3}, \frac{1}{3})$

Question 4

Let $\bar{u}.\bar{v} = \bar{u}.\bar{w}$

So then $\vec{u}.\vec{w} = |\bar{u}|.|\bar{w}|.cos\theta$ dot product definition

$\therefore$ $\vec{u} = \vec{w} .|\bar{u}|.|\bar{w}|.cos\theta$

And also $\vec{u}.\vec{v} = |\bar{u}|.|\bar{v}|.cos\theta$ dot product definition

$\therefore$ $\vec{u} = \vec{v} .|\bar{u}|.|\bar{v}|.cos\theta$

Form an equation from $\vec{u}$

$\vec{w} .|\bar{u}|.|\bar{w}|.cos\theta = \vec{v} .|\bar{u}|.|\bar{v}|.cos\theta$

$\vec{w} .|\bar{w}| = \vec{v} .|\bar{v}|$

$\therefore$ $\vec{u} = \vec{v} .|\bar{v}|$

From the above, vector u or $\vec{u}$, is equivalent to vector v times the magnitude of vector v ($\vec{u} = \vec{v} .|\bar{v}|$)

Question 5

Let u = (1; 0; 2) ; v = (2; 1; 0) and w = (0; 2; 1).

5 (i)

$3\bar{v} - 2\bar{u}$ = 3(2,1,0) – 2(1,0,2)

= (6,3,0) – (2,0,4)

= (6-2 , 3- 0, 0 - 4)

= (4,-2,-4)

5 (ii)

$||\bar{u} + \bar{v} - \bar{w}\,||\,\bar{v} \quad = \bar{v}$

$\left||(1,0,2) + (2,1,0) - (0,2,1)|\right|.(2,10) \quad = \bar{v}$

$\therefore \sqrt{(1,0,2) + (2,1,0) + (0,2,1)}.(2,10) \; = \bar{v}$ absolute value norm

$\therefore \sqrt{(1+2+0,\;\; 0+1+2,\;\; 2+0+1)}.(2,10) \quad = (2,10)$

$\therefore \sqrt{9}.(2,1,0) \; = \bar{v}$

$\therefore \left(\sqrt{9}.2, \sqrt{9}.1, \sqrt{9}.0\right) \; = \bar{v}$

$\bar{v} = \; (6,3,0)$

$\therefore \sqrt{6^2 + 3^2 + 0^2}$

= $\sqrt{45}$

5 (iii)

$(\bar{u} \; \times \bar{v}).\bar{w}$

$\therefore (\bar{u} \; \times \bar{v}) = \; det \begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \end{vmatrix}$

$= \; + \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} x - \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} y + \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} z$

= (0×0– 2x1) x - (1×0-2×2) y + (1x1-0x2) z

= (0-2) x - (0-4) y + (1-0) z

= (-2, 4, 1)

∴ (-2, 4, 1). $\bar{w}$

$= (-2,4,1).(0,2,1)$

$= (-2.0 + 4.2 + 1.1)$

$= (0 + 8 + 1)$

$= 9$

5 (iv)

$Proj_{\bar{w}}\bar{v}$

$\vec{w}.\vec{v} \; = \; |\bar{w}|.|\bar{v}|.cos\theta$ dot product definition

$\therefore Proj_{\bar{w}}\bar{v} \; = |\bar{w}|.cos\theta$

$k = \sqrt{x^2 + y^2 + z^2}$

$\therefore \bar{w} \; = \sqrt{0^2 + 2^2 + 1^2}$

$\therefore \bar{w} \; = \sqrt{0 + 4 + 1}$

$\therefore \bar{w} \; = \sqrt{5}$

$k = \sqrt{x^2 + y^2 + z^2}$
$\therefore \bar{v} = \sqrt{2^2 + 1^2 + 0^2}$
$\therefore \bar{v} = \sqrt{4 + 1 + 0}$
$\therefore \bar{v} = \sqrt{5}$

$cos\theta = \frac{adj}{hyp}$
$\therefore \theta = arccos\frac{adj}{hyp}$
$= arccos\frac{\sqrt{5}}{\sqrt{5}}$
$= \arccos(1)$
$= 0$

5(v)

$A = |\bar{u} \times \bar{v}|$
From 5(iii) above, $(\bar{u} \times \bar{v})$ = (-2, 4, 1)
$\therefore |\bar{u} \times \bar{v}| = \sqrt{(-2)^2 + 4^2 + 1^2}$
$= \sqrt{4 + 16 + 1}$
$= \sqrt{21}$
5(vi)

From 5(iii) above, $(\bar{u} \times \bar{v})$ = (-2, 4, 1)
As an equation: $(-2x + 4y + z)$
Let $\boldsymbol{Q}(x, y, z)$ be an arbitrary point on the plane
$\therefore \bar{w}Q = terminal\ point - initial\ point$
$= (x - 0, y - 2, z - 1)$
$\bar{w}Q$ is parallel to the plane and perpendicular to the cross product $\therefore$ dot product $= 0$
$\therefore (x, y - 2, z - 1).(-2,4,1) = 0$
$(x). -2 + (y - 2).4 + (z - 1).1) = 0$
$-2x + 4y - 8 + z - 1 = 0$
$-2x + 4y + z = 9$

6)
Let the plane V = ax + by + cz + d = 0
$\therefore$ $\mathrm{d} = ax + by + cz$
Let T be a point away from the plane
$\therefore T(x - x_0\ , y - y_0,\ \ z - z_0)$ or $(a - a_0\ , b - b_0,\ \ c - c_0)$

Find magnitude of T
$\therefore \mathrm{T} = \sqrt{a^2 + b^2 + c^2}$ distance equation
$\therefore \overline{\mathrm{T}} = |ax + by + cz + d|$ scalar equation of the plane
$= ax + by + cz + d$
Expression for unit vector of length 1 = $\frac{1}{a}\bar{a}$

$$\therefore\ \mathrm{q} = \frac{\mathrm{Q}}{||Q||} = \frac{(a,b,c)}{\sqrt{a^2+b^2+c^2}}\ \ or\ \ \frac{(a+b+c)}{\sqrt{a^2+b^2+c^2}}$$

Project Q onto T

$$\therefore \mathrm{Proj_Q}T = \frac{|a(x-x_{0)}\ ,b(y-y_{0\_)},c(\ z-z_0)|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|a(x-x_{0)}\ ,b(y-y_{0\_)},c(\ z-z_0)|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|a(x-x_{0)}\ ,b(y-y_{0\_)},c(\ z-z_0)|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|ax+by+cz-a_0-b_0-c_0|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{d+a_0+b_0+c_0}{\sqrt{a^2+b^2+c^2}}$$

where d = $ax + by + cz$

Let $\mathrm{Proj_Q}T = \ Unit\ Vector$

$$\therefore\ \frac{d+a_0+b_0+c_0}{\sqrt{a^2+b^2+c^2}} = \frac{(a,b,c)}{\sqrt{a^2+b^2+c^2}}$$

$\therefore d + a_0 + b_0 + c_0 = \ a + b + c$

$\therefore d + a_0 + b_0 + c_0 = \ a + b + c$

$\therefore d = \ a - a_0 + b - \ b_0 + c - \ c_0$

$\therefore d = \ \mathrm{T}$

Since T is a distance away from the plane, it is equivalent to d, which represents the constant part of the distance equation.