Tutorial letter A03/0/2024

NUMERICAL METHODS I

APM2613

Year module

Department of Mathematical Sciences

IMPORTANT INFORMATION:

This tutorial letter contains Questions for Assessment 3 - Interpolating polynomials and least-squares polynomials. Please read the relevant chapters and lessons before you attempt the assignment.

Note: This is a fully online module and therefore it is only available on *my*Unisa.

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Question 1 [25 marks]

Consider the following data:

x	$f(x)$	$f'(x)$
0.1	-0.62049958	3.58502082
0.2	-0.28398668	3.14033271
0.3	0.00660095	2.66668043
0.4	0.24842440	2.16529366

(1.1) Find an approximation to $f(0.27)$ using the following forms of interpolating polynomial

(a) the Lagrange form.

(b) the Newton forward divided difference form.

(c) the Hermite form.

(1.2) Comment on your observed comparison of the results.

Question 2 [40 marks]

Consider the following data

x	$f(x)$
0.3	-1.1518
-0.4	0.7028
0.5	-1.4845
0.00	0.13534

(2.1) Use a third degree Lagrange interpolating polynomial to approximate $f(0.55)$.

(2.2) Use a Newton's divided-difference polynomial that interpolates all the points to approximate $f(0.2)$, using the following criteria:

(a) Without rearranging the nodes;

(b) Rearranging the nodes in increasing order.

(2.3) Compare the results obtained in (2.2) above.

(2.4) Use the least-squares polynomial of degree two to approximate $f(0.2)$ and compute the error. (Your system of normal equations must be explicit).

(2.5) Use the least-squares function of the form $y = \alpha e^{\beta x}$ to approximate $f(0.2)$ and compute the error.

(2.6) Plot the graphs of the approximating polynomials in (2.2) -(2.4). Your graphs must be proper computer produced graphs.

Question 3 [15 marks]

Construct the natural cubic spline for the data below and use it to approximate $f(0.3)$:

$$(-0.5,\ 5), \quad (0,\ 15), \quad (0.5,\ 9)$$

Question 4 [20 marks]

Consider the following set of data points in the table below:

i	x	y
0	10	10
1	50	15
2	75	60
3	90	100
4	105	140
5	150	200
6	180	140
7	190	120
8	160	100
9	130	80

(4.1) Using guidepoints of your choice from the data set, construct the connected Bezier curve from the set of points.
(*Hint: Divide the set of points into three parts*)

(4.2) Draw the connected Bezier polynomial .

(4.3) Why is the graph smoothly connected at points 3 and 6?