Tutorial letter A02/0/2024

NUMERICAL METHODS I

APM2613

Year module

Department of Mathematical Sciences

IMPORTANT INFORMATION:

This tutorial letter contains Questions for Assessment 2 - Systems of linear equations (Chapters 6 and 7) and numerical solution of systems of nonlinear equations (Section 10.2). Please read the relevant chapters and lessons before you attempt the assignment.

Note: This is a fully online module and therefore it is only available on *my*Unisa.

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PLEASE NOTE:
In working out the following exercises, you can use Matlab/Octave to compute but the essential steps in the computations must be written out. Unless otherwise stated, no computer code is required to be submitted. The arguments of any trigonometric functions are in radians.

Question 1 [30 marks]

Consider the linear system

$$\begin{aligned} 3.333x_1 + 15920x_2 - 10.333x_3 &= 15913 \\ 2.222x_1 + 16.710x_2 + 9.612x_3 &= 28.544 \\ 1.5611x_1 + 5.1791x_2 + 1.6852x_3 &= 8.4254 \end{aligned}$$

In answering the following questions use 4 decimal place accuracy with rounding:

(1.1) Solve the system using:

(a) Gaussian elimination without pivoting. (7)

(b) Gaussian elimination with scaled partial pivoting. (7)

(c) Basic LU decomposition. (6)

(1.2) Use the appropriate Matlab/Octave command to check what the actual solution is. (2)

(1.3) Determine the number of arithmetic operations (multiplication/ division, addition/subtraction) in (1.3)(a) above. (3)

Question 2 [25 marks]

Consider the linear system $A\mathbf{x} = \mathbf{b}$ in which

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & -3 \end{bmatrix}; \qquad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix};$$

(2.1) Use Gauss-Jordan method to solve the system $A\mathbf{x} = \mathbf{b}$. (7)

(2.2) Use Gauss-Jordan method to compute the inverse A^{-1} exactly. (7)

(2.3) Use Cramer's rule to compute the inverse A^{-1}. (7)

(2.4) Solve the linear system $A\mathbf{x} = \mathbf{b}$ using the inverse obtained above. (4)

Question 3 [25 marks]

Consider again the linear system $A\mathbf{x} = \mathbf{b}$ used in Question 2. For each of the methods mentioned below use 4 decimal place arithmetic with rounding and the initial approximation $\mathbf{x}^{(0)} = (0,\, 0,\, 0)^T$.

(3.1) By examining the diagonal dominance of the coefficient matrix, A, determine whether convergence to a solution when using iterative methods to solve the system can be guaranteed. (3)

(3.2) Obtain approximate solutions to the system using three iterations of each of the following methods:

(a) the Jacobi method. (5)

(b) the Gauss-Seidel method (6)

(c) the Successive Over-Relaxation technique with $\omega = 0.4$. (6)

(3.3) Compute the residual vectors associated with the approximate solutions obtained using each method above and compare results. (4)

Question 4 [20 marks]

Consider the following nonlinear system:

$$\begin{aligned} 5x_1^2 - x_2^2 &= 0 \\ x_2 - 0.25(\sin x_1 + \cos x_2) &= 0 \end{aligned}$$

Use Newton's method to find the approximation $\mathbf{x}^{(2)}$, starting $\mathbf{x}^{(0)} = (1,\ 0)'$.