Question 1

1.1 $\forall x(R(x) \rightarrow \neg A(x))$

1.2 $\forall x((P(x) \wedge \neg M(b,x)) \rightarrow L(h,x))$

1.3 $\exists x(P(x) \wedge L(b,x) \wedge (M(v,x) \vee M(h,x)))$

1.4 $\forall x(L(v,x) \rightarrow M(v,x))$

1.5 $\forall x(L(b,x) \leftrightarrow L(v,x))$

1.6 $\big(A(b) \vee A(h)\big) \wedge (\neg A(b) \vee \neg A(h))$

Question 2

2.1 Every painting is painted by some artist.

2.2 There exists a rich artist who did not paint any painting.

2.3 Vincent likes all paintings painted by himself.

2.4 Every painting is liked by somebody.

2.5 There exists somebody who likes all paintings that they didn't paint themselves

Question 3

3.1 Neither. Q is missing one argument.

3.2 Neither. ∃ should have a variable.

3.3 Neither. $f(c)$ should be used as an argument.

3.4 Neither. f should have a constant, not a quantifier as an argument.

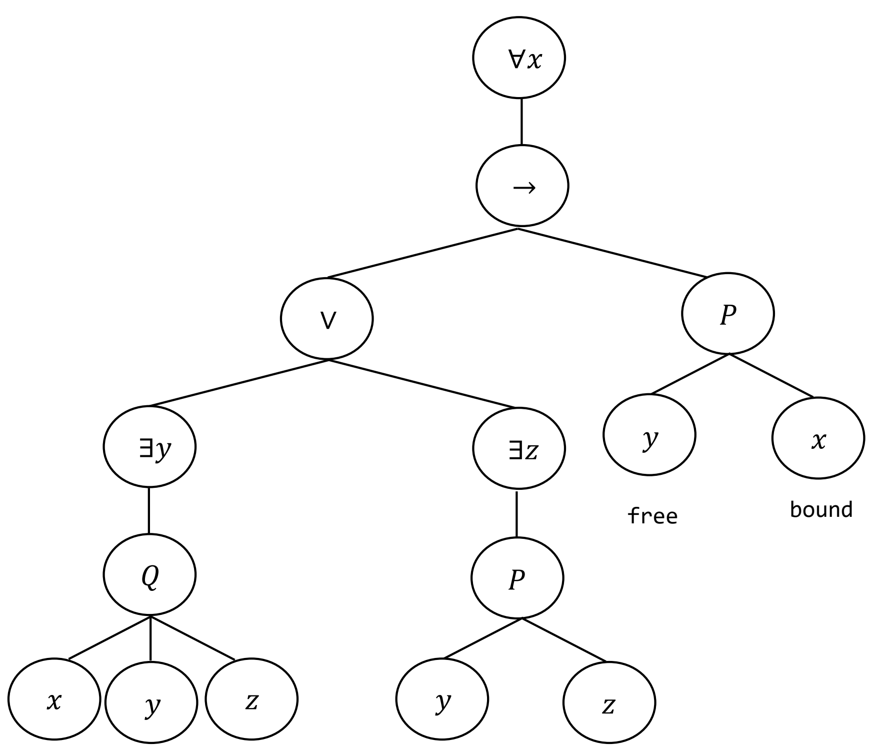
3.5 Neither. P should not have a predicate symbol as an argument

3.6 Term.

3.7 Wff.

Question 4

4.1



4.2. bound bound free free bound

4.2.2

4.2.3

Question 5

The model $\mathcal{M}$:

$A = \{1,2,3,4,5,6,7,8,9\}$
$S^M = \{2,4\}$
$Q^M = \{2,4,6,8\}$

Question 6

The model $\mathcal{M}$ where the sentence is true:

$A = \mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$
A is the set of all integers
R^M: The predicate $R(x,y)$ where x is less than y

The model $\mathcal{M}$ where the sentence is false:

$A = \{1,2,3,4,5,6,7,8,9\}$
A is the set of all integers greater than 0 and less than 10
R^M: The predicate $R(x,y)$ where x is less than y

Question 7

$\forall x \exists y\, (R(x,y) \wedge R(y,y))$
The model $\mathcal{M}$:

$A = \{a,b,c,d\}$
$R^M = \{(a,a),(b,a),(c,a),(d,b),(b,b)\}$

For the satisfaction relation $\mathcal{M} \vDash \Phi$ to be satisfied, we need to test that Φ is true for every object in the model $\mathcal{M}$.

The ordered pairs $(a,a) \in R^M$ and $(a,a) \in R^M$
The ordered pairs $(b,a) \in R^M$ and $(a,a) \in R^M$
The ordered pairs $(c,a) \in R^M$ and $(a,a) \in R^M$
The ordered pairs $(d,b) \in R^M$ and $(b,b) \in R^M$
The ordered pairs $(b,b) \in R^M$ and $(b,b) \in R^M$

Therefore, the model where satisfies the sentence $\forall x \exists y\, (R(x,y) \wedge R(y,y))$

Question 8

8.1 $\forall x\, \exists y\, S(x, y) \vdash \exists y\, \forall x\, S(x, y)$

The model $\mathcal{M}$:

$A = \mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

A is the set of all integers

R^M: The predicate $R(x, y)$ where x is divisible by y

8.2 $\exists x\, (\neg R(x) \vee \neg Q(x)) \vdash \forall x\, (R(x) \vee Q(x))$

The model $\mathcal{M}$:

$A = \mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

A is the set of all integers

R^M: The predicate $R(x)$ where x is divisible by 7

The predicate $Q(x)$ where x is divisible by 9

Question 9

9.1

9.2

1.	$\forall x(P(x) \land Q(x))$		
2.	$\boxed{c}$		
3.	$P(c) \land Q(c)$	$\forall$ Elim	1
4.	$\boxed{c}$ $P(c)$		
5.	$Q(c)$	$\land$ Elim	3
6.	$P(c) \to Q(c)$	$\to$ Intro	4-5
7.	$\forall x(P(x) \to Q(x))$	$\forall$ Intro	2-6

Goals

$\models \forall x(P(x) \to Q(x))$

9.3

1.	$\forall x \forall y(Q(y) \to F(x))$		
2.	$\exists y(Q(y))$		
3.	$\boxed{a}$ $Q(a)$		
4.	$\boxed{c}$		
5.	$\forall y(Q(y) \to F(c))$	$\forall$ Elim	1
6.	$Q(a) \to F(c)$	$\forall$ Elim	5
7.	$F(c)$	$\to$ Elim	6,3
8.	$\forall x(F(x))$	$\forall$ Intro	4-7
9.	$\forall x(F(x))$	$\exists$ Elim	2,3-8
10.	$\exists y(Q(y)) \to \forall x(F(x))$	$\to$ Intro	2-9

Goals

$\models \exists y(Q(y)) \to \forall x(F(x))$

9.4

1.	$\forall x(P(x) \vee Q(x))$		
2.	$\exists x(\neg Q(x))$		
3.	$\forall x(R(x) \rightarrow \neg P(x))$		
4.	$\boxed{c}$ $\neg Q(c)$		
5.	$P(c) \vee Q(c)$	∀ Elim	1
6.	$R(c) \rightarrow \neg P(c)$	∀ Elim	3
7.	$R(c)$		
8.	$\neg P(c)$	→ Elim	6,7
9.	$P(c)$		
10.	$\perp$	⊥ Intro	8,9
11.	$Q(c)$		
12.	$\perp$	⊥ Intro	11,4
13.	$\perp$	∨ Elim	9-10,11-12,
14.	$\neg R(c)$	¬ Intro	7-13
15.	$\exists x(\neg R(x))$	∃ Intro	14
16.	$\exists x(\neg R(x))$	∃ Elim	2,4-15

Goals

$\exists x(\neg R(x))$ ✓

9.5

1.	$\forall x(P(x) \rightarrow (Q(x) \vee R(x)))$		
2.	$\neg \exists x(P(x) \wedge R(x))$		
3.	$\boxed{c}$		
4.	$P(c)$		
5.	$P(c) \rightarrow (Q(c) \vee R(c))$	$\forall$ Elim	1
6.	$Q(c) \vee R(c)$	$\rightarrow$ Elim	4,5
7.	$Q(c)$		
8.	$Q(c)$	Reit	7
9.	$R(c)$		
10.	$P(c) \wedge R(c)$	$\wedge$ Intro	4,9
11.	$\exists x(P(x) \wedge R(x))$	$\exists$ Intro	10
12.	$\perp$	$\perp$ Intro	2,11
13.	$Q(c)$	$\perp$ Elim	12
14.	$Q(c)$	$\vee$ Elim	6,7-8,9-13
15.	$P(c) \rightarrow Q(c)$	$\rightarrow$ Intro	4-14
16.	$\forall x(P(x) \rightarrow Q(x))$	$\forall$ Intro	3-15

Goals

$\models \forall x(P(x) \rightarrow Q(x))$