Tutorial letter 101/0/2024

Linear Algebra II

MAT2611

Year Module

Department of Mathematical Sciences

TUTORIAL RESOURCE FOR MAT2611

IMPORTANT INFORMATION:
This tutorial letter contains Assignment 2 for the module MAT2611

BAR CODE

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ASSIGNMENT 02
Due date: Friday, 03 May 2024

Problem 5. Determine whether each set equipped with the given operation is a vector space. For those that are not vector space identify the vector space axioms that fail.

(1) The set $U = \{(x, 0) \in R^2\}$ with the standard operations on R^2.
(2) The set $V = \{(x, y) \in R^2 : y \geq 0\}$ with the standard operations on R^2.
(3) The set $W = \{(x, y) \in R^2 : x + y = 0\}$ with the standard operations on R^2.
(4) The set $X = \{(x, y) \in R^2\}$ with the standard vector addition but with scalar multiplication defined by $k(x, y) = \left(k^2 x, k^2 y\right)$.
(5) The set of all 2×2 matrices $Y = \left\{ \begin{bmatrix} a & b \\ c & 0 \end{bmatrix} : a, b, c \in R \right\}$ with the standard matrix addition and scalar multiplication.

[10 marks]

Problem 6. Let V be the set of all ordered pairs of real numbers, and consider the following addition and scalar multiplication operations on $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$:

$$\mathbf{u} + \mathbf{v} = (u_1 + v_1, u_2 + v_2, u_3 + v_3), \ \ k\mathbf{u} = (ku_1, ku_2, 0).$$

(1) Compute $\mathbf{u} + \mathbf{v}$ and $k\mathbf{u}$ for $\mathbf{u} = (-1, 2, -3)$, $\mathbf{v} = (2, -3, 1)$ and $k = -2$.
(2) Determine whether the Axioms 7, 8, 9 and 10 hold.

[10 marks]

Problem 7. Let V be a vector space, $\mathbf{u}$ a vector in V, and k a scalar. Then show that if $k\mathbf{u} = \mathbf{0}$, then $k = 0$ or $\mathbf{u} = \mathbf{0}$.

[10 marks]

Problem 8. Let $-\infty$ and ∞ denote two distinct objects, neither of which is in R. Define an addition and scalar multiplication on $R \cup \{\infty\} \cup \{-\infty\}$. Specifically, the sum and product of two real numbers is as usual, and for $k \in R$ define

$$k\infty = \begin{cases} -\infty & \text{if } k < 0 \\ 0 & \text{if } k = 0 \\ \infty & \text{if } k > 0, \end{cases} \qquad k(-\infty) = \begin{cases} \infty & \text{if } k < 0 \\ 0 & \text{if } k = 0 \\ -\infty & \text{if } k > 0, \end{cases}$$

$$\begin{aligned} k + \infty &= \infty + k = \infty, \quad k + (-\infty) = -\infty + k = -\infty, \\ \infty + \infty &= \infty, \quad (-\infty) + (-\infty) = -\infty, \quad \infty + (-\infty) = 0. \end{aligned}$$

Show that $R \cup \{\infty\} \cup \{-\infty\}$ is not a vector space over R.

[10 marks]

[Total: 40 marks]

– End of assignment –