

COS2601
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LKE MNCUBE

Question 1

- (i) The set $\{a b\}^*$ will be suitable because it contains, along with other words, all the words in the language ODDnotAB
- (ii) The generators are a and b
- (iii) The function CONCAT as defined in learning unit 3 will be suitable.
- (iv) Two possible recursive definitions:

First:

ODDnotAB is the smallest subset of $\{a b\}^*$

such that $a, b \in \text{ODDnotAB}$

and if $Q \in \text{ODDnotAB}$,

then also $\text{CONCAT}(bb, Q) \in \text{ODDNOTAB}$

and $\text{CONCAT}(Q, aa) \in \text{ODDNOTAB}$,

and

if $Q \in \text{ODDNOTAB}$ and Q does not end on a ,

then $\text{CONCAT}(Q, ba), \text{CONCAT}(Q, bb) \in \text{ODDNOTAB}$,

if $Q \in \text{ODDNOTAB}$ and Q does not begin with b ,

then $\text{CONCAT}(aa, Q), \text{CONCAT}(ba, Q) \in \text{ODDNOTAB}$

Second:

Rule 1: $ab \in \text{ODDNOTAB}$.

Rule 2: If $Q \in \text{AB}$ then $\text{CONCAT}(bb, Q) \in \text{AB}$.

Rule 3: If $Q \in \text{AB}$ then $\text{CONCAT}(Q, aa) \in \text{AB}$.

Rule 4: If $Q \in \text{AB}$ then $\text{CONCAT}(Q, ba) \in \text{AB}$

and $\text{CONCAT}(Q, bb) \in \text{ODDNOTAB}$

Rule 5: If $Q \in \text{AB}$ then $\text{CONCAT}(aa, Q) \in \text{AB}$

and $\text{CONCAT}(ba, Q) \in \text{ODDNOTAB}$

Rule 6: Only words generated by rules 1 to 5 are in AB.

Question 2

- (i) Rule 1: $1 \in P$.
Rule 2: If $x \in P$ then $x+1 \in P$.
Rule 3: P is the smallest set satisfying R1 and R2.
- (ii)

(iii) Let $S = \{ n \mid n \in \mathbb{Z}^+, 11 + 15 + 19 + \dots + (4n + 7) = 2n^2 + 9n \}$

Step 1:

Is $1 \in S$? Yes, since $4+7=11$ and $2+9=11$

Step 2:

Assume $k \in S$, hence $11 + 15 + 19 + \dots + (4k + 7) = 2k^2 + 9k$

Step 3:

Is $k + 1 \in S$? Yes, proof:

$$11 + 15 + 19 + \dots + (4k + 7) + (4(k+1) + 7) = 2(k+1)^2 + 9(k+1)$$

$$\begin{aligned} \text{LHS} &= 4(k+1) + 7 \\ &= 4k + 11 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= 2(k+1)^2 + 9(k+1) \\ &= 2k^2 + 4k + 2 + 9k + 9 \\ &= 2k^2 + 9k + 4k + 2 + 9 \\ &= 2k^2 + 9k + 4k + 11 \end{aligned}$$

From Step 2, we know that $11 + 15 + 19 + \dots + (4k + 7) = 2k^2 + 9k$

So LHS = RHS

Hence, $k + 1 \in S$. Thus $S = \mathbb{Z}^+$.

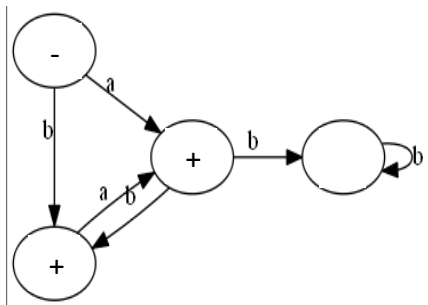
Question 3

$$M = (b^*ab^*) + (ab(ab^*))$$

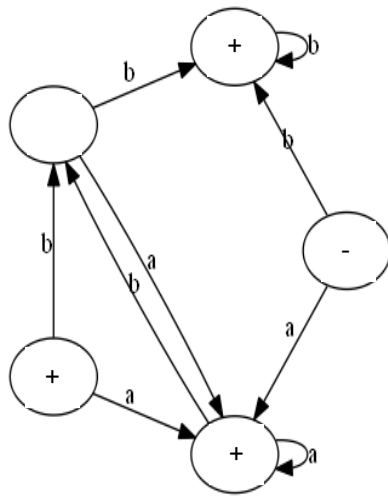
Question 4

$$L = (a+ba)baa^*$$

Question 5



Question 6



Question 7