Assignment 1

February 29, 2024

ASSIGNMENT 01
Closing Date: 8 May 2022
Total Marks:48
UNIQUE ASSIGNMENT NUMBER: **688316**

Question 1: 5 Marks

Consider the statement: If $x > 2$, then $x^2 > 4$.

(1.1) Give the contrapositive of the statement. (1)

(1.2) Give the converse of the statement, and determine whether the converse is true or not. (3)

(1.3) What is the negation of the statement? (1)

Question 2: 19 Marks

(2.1) Prove by induction that $11^n - 8^n$ is a multiple of 3 for all $n \in \mathbb{N}$. (7)

(2.2) Prove that the function given by

$$f(x) = \frac{x-3}{x+2}$$

is $1-1$ on $\mathbb{R}$ and find a formula for the inverse of f. (6)

(2.3) Consider the functions $f(x) = 2x + 1$ and $g(x) = x + 1$. Indicate the domain of definition of each of the following functions: $f, g, f \circ g, g \circ f$. (6)

Question 3: 11 Marks

Let
$$S := \left\{ \frac{n}{n+1} : n = 1, 2, 3, \ldots \right\}.$$

Find the infimum and supremum of S.

Question 4: 13 Marks

(4.1) Show that the sequence
$$a_n = \frac{n}{n+1}$$

converges to 1. (5)

(4.2) Suppose that x_n is a sequence of real numbers that converges to 1 as $n \to \infty$. Prove that the limit $1 + x_n$ converges to 2 as $n \to \infty$. (2)

(4.3) Find two convergent subsequences of the sequence $(-1)^n$ that have different limits. (2)

(4.4) Use the Monotone Convergence Theorem to prove that

$$x_n = \frac{1}{\sqrt{n}}$$

converges to 0. (4)

Johannesburg   February 29, 2024   Dr. CA Agyingi