ASSIGNMENT 02
Closing Date: 6 September 2024
Total Marks: 55
UNIQUE ASSIGNMENT NUMBER: 399383

Question 1: 6 Marks

Determine, using the ratio test, whether the following series converges or diverges. Carefully justify each conclusion.

(a)

$$\sum_{r=1}^{\infty} \frac{2^r}{r!}. \quad (3)$$

(b)

$$\sum_{r=1}^{\infty} \frac{r^r}{r!}. \quad (3)$$

Question 2: 10 Marks

Find the interval of convergence of the series

$$\sum_{r=1}^{\infty} \frac{1}{\sqrt[3]{r}}(x-4)^r.$$

Question 3: 13 Marks

Consider the function defined by

$$f(x) = \begin{cases} \alpha x + \beta, & x \leq -1 \\ \alpha x^3 + x + 2\beta, & x > -1, \end{cases}$$

for what value(s) of $\alpha, \beta \in \mathbb{R}$ is f differentiable at every $x \in \mathbb{R}$?

Question 4: 13 Marks

Determine if the following integral is divergent or convergent. Find the value of the integral if it converges.

$$\int_{-\infty}^{\infty} x \exp^{-x^2} dx.$$

Question 5: 13 Marks

Let $f(x) = -2x + 5$ $(-1 \leq x \leq 2)$ and let P_n be the partition

$$\left\{1, 1 + \frac{1}{n}, 1 + \frac{2}{n}, \ldots, 1 + \frac{n}{n} = 2\right\}$$

of $[1, 2]$. Calculate the lower sum $L(P_n)$ and the uppser sum $U(P_n)$ of this partition. Using the upper and lower sums computed above, show that f is Riemann integrable on $[1, 2]$ and hence compute

$$\int_{1}^{2} f(x)dx.$$

Johannesburg May 28, 2024 Dr. CA Agyingi