A.2 Assignment 02

ASSIGNMENT 02
Due date: Monday, 12 May 2025

ONLY FOR YEAR MODULE

1. For the equation

$$y' = y\sin(\pi x)\,,\ \ y(0) = 1,$$

get starting values by the Runge-Kutta Fehlberg method for $x = 0.2$, $x = 0.4$, $x = 0.6$, and then advance the solution to $x = 1.0$ by

(a) Milne's method,

(b) the Adams-Moulton method. (10)

2. Solve the boundary–value problem

$$y'' + x^2y' - 4xy = 2x^3 + 6x^2 - 2, \quad y(0) = 0, \quad y(1) = 2$$

by using the **shooting method.** Use the modified Euler method (with only one correction at each step), and take $h = 0.2$. Start with an initial slope of $y'(0) = 1.9$ as a first attempt and $y'(0) = 2.1$ as a second attempt. Then interpolate.
Compare the result with the analytical solution $y = x^4 - x^2 + 2x$. (15)

3.

(a) The function e^x is to be approximated by a fifth-order polynomial over the interval $[-1, 1]$. Why is a Chebyshev series a better choice than a Taylor (or Maclaurin) expansion?

(b) Given the power series

$$f(x) = 1 - x - 2x^3 - 4x^4$$

and the Chebyshev polynomials

$$\begin{aligned} T_0(x) &= 1 \\ T_1(x) &= x \\ T_2(x) &= 2x^2 - 1 \\ T_3(x) &= 4x^3 - 3x \\ T_4(x) &= 8x^4 - 8x^2 + 1, \end{aligned}$$

economize the power series $f(x)$ twice.

(c) Find the Padé approximation $R_2(x)$, with numerator of degree 2 and denominator of degree 1, to the function $f(x) = x^2 + x^3$.

(20)

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