MAT2611
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Assignment 1

Problem 1

Addendum A - Exercise 2.21
1. Determine $P(\emptyset)$
2. Determine $P(\{1\})$
3. Determine $P(\{1,2,3\})$

Remember that:

$\|A\| = n$	*number of elements in A*
$\|\emptyset\| = 0$	*number of elements in the empty set*
$\|P(A)\| = 2^{\|A\|} = 2^n$	*number of elements in the powerset*
$\|P(\emptyset)\| = 2^0 = 1$	*number of elements in the empty set*

1. $P(\emptyset) = \{\emptyset\}$
2. $P(\{1\}) = \{\emptyset,\ \{1\}\}$
3. $P(\{1,2,3\}) = \{\emptyset,\ \{1\},\ \{2\},\ \{3\},\ \{1,2\},\ \{1,3\},\ \{2,3\},\ \{1,2,3\}\}$

Addendum A - Exercise 3.4
For each of the following functions determine the image of
$S = \{x \in \mathbb{R}: 9 \leq x^2\}$

1. $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = |x|$.
2. $g: \mathbb{R} \to \mathbb{R}$ defined by $g(x) = e^x$.
3. $h: \mathbb{R} \to \mathbb{R}$ defined by $h(x) = x - 9$.

$\therefore 9 \leq x^2$
$\pm\sqrt{9} \leq \sqrt{x^2}$
$\pm 3 \leq x$
$x \geq 3$ *or* $x \geq 3$

$S = \{x \in \mathbb{R}: 9 \leq x^2\}$
$S = \{x \in \mathbb{R}: x^2 \geq 9\}$
$S = \{x \in \mathbb{R}: x \leq -3, x \geq 3\}$

1. $f(x) = |x|$
   $f(-3) = |-3| = 3$
   $f(3) = |3| = 3$
   Therefore, the Image of f is $\{x \in \mathbb{R}: f(x) \geq 3\}$

2. $g(x) = e^x$
   $g(-3) = e^{-3}$
   $g(3) = e^3$
   Therefore, the Image of g is $\{x \in \mathbb{R}: g(x) \leq e^{-3}, g(x) \geq e^3\}$

3. $h(x) = x - 9$
$h(-3) = -3 - 9 = -12$
$h(3) = 3 - 9 = -6$
Therefore, the Image of h is $\{x \in \mathbb{R}: h(x) \leq -12, h(x) \geq -6\}$

Problem 2

Addendum A - Exercise 3.11
Consider the following two functions. Prove that both f and g are one-to-one correspondences.
1. $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 4x - 15$
1. $g: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 15x^3$

Let $f: A \to B$ be a one-to-one correspondence. Then to each $b \in B$ there corresponds a unique $a \in A$ such that $f(a) = b$. We define $f^{-1}: B \to A$ by

$$f^{-1}(b) = \text{the unique } a \text{ such that } f(a) = b$$

A function $f: A \to B$ is said to be one-to-one correspondence if and only if f is both:
Injective (one-to-one): $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ and,
Surjective (ONTO): for all $b \in B$ there is some $a \in A$ such that $f(a) = b$

1. **Injectivity:**
Take $x_1, x_2 \in \mathbb{R}$ and assume that $f(x_1) = f(x_1)$
Thus $4x_1 - 15 = 4x_2 - 15$
And $4x_1 = 4x_2$
So $x_1 = x_2$
We have shown if $f(x_1) = f(x2)$ then $x_1 = x_2$. Therefore f is one-to-one, by definition of one-to-one.

Surjectivity:
We need to find an x that maps to y.
Suppose $y = 5x + 11$;
Now we solve for x in terms of y .
We find $x = \frac{y+15}{4}$

Proof:
Let y be any element of $\mathbb{R}$.
$g(x) = g\left(\frac{y+15}{4}\right) = 4\left(\frac{y+15}{4}\right) - 15 = y + 15 - 15 = y$
Thus, we have found an $x \in \mathbb{R}$ such that $g(x) = y$

2. **Injectivity:**

Take $x_1, x_2 \in \mathbb{R}$ and assume that $f(x_1) = f(x_1)$

Thus $15x^3 = 15x^3$

And $(x_1)^3 = (x_2)^3$

So $x_1 = x_2$

We have shown if $f(x_1) = f(x2)$ then $x_1 = x_2$. Therefore f is one-to-one, by definition of one-to-one.

Surjectivity:

We need to find an x that maps to y.

Suppose $y = 5x + 11$;

Now we solve for x in terms of y .

We find Type equation here.

Proof:

Let y be any element of $\mathbb{R}$.

$g(x) =$

Thus, we have found an $x \in \mathbb{R}$ such that $g(x) = y$

Addendum A - Exercise 3.12
Let $f: A \to B$ be a one-to-one correspondence.
1. Prove that f^{-1} is a function.
2. Prove that f^{-1} is a one-to-one.
3. Prove that f^{-1} is onto.
4. Conclude that $f^{-1}: B \to A$ is a one-to-one correspondence.

1. Let A and B be nonempty sets.
   A function $f: A \to B$ is said to be invertible if it has an inverse function.
   Proof:
   Suppose $f: A \to B$ is an invertible function.
   Then $f^{-1}(f(a))$ = a for every $a \in A$;
   $f(f^{-1}(b))$ = b for every $b \in B$;
   $f \circ f - 1 = IB\, and\, f - 1 \circ f = IA.$

2. **Injectivity:**
   A function $f^{-1}: A \to B$ is said to be one-to-one if
   $f^{-1}(x_1) = f^{-1}(x_2) \Rightarrow x_1 = x_2$ $\qquad x_1, x_2 \in A$
   In other words, there is at most one $b \in B$ with $f(b) = a$.
   We have proven that f^{-1} is a function (for all $a \in A$ there is at least one and never more than one) $b \in B$ with $f^{-1}(b) = a$
   Thus f^{-1} is one-to-one/injective

3. **Surjectivity:**
   Since $f^{-1}(x): B \to A$, and f^{-1} is the inverse of f.
   Then $domain(f) = range(f^{-1}) = A$
   Thus f^{-1} is onto/surjective

4. A function f^{-1} is bijective if and only if f^{-1} is:
   Injective: $f^{-1}(x) = f^{-1}(y) \Rightarrow x = y$ and,
   Surjective: for all $b \in B$ there is some $a \in A$ such that $f^{-1}(a) = b$

   We have proven Injectivity in 2 and Surjectivity in 3
   Thus, $f^{-1}: B \to A$ is a one-to-one correspondence/bijective