Question 1

Build/design a Turing machine (TM) that determines whether a given word contains at least one instance of the substring aab. If it does, then the TM should write a T on the tape after the input word.

States (Q): $\{qStart, qCheck1, qCheck2, qCheck3, qWrite_T, qAccept\}$

Input Alphabet (Σ): $\{a, b\}$

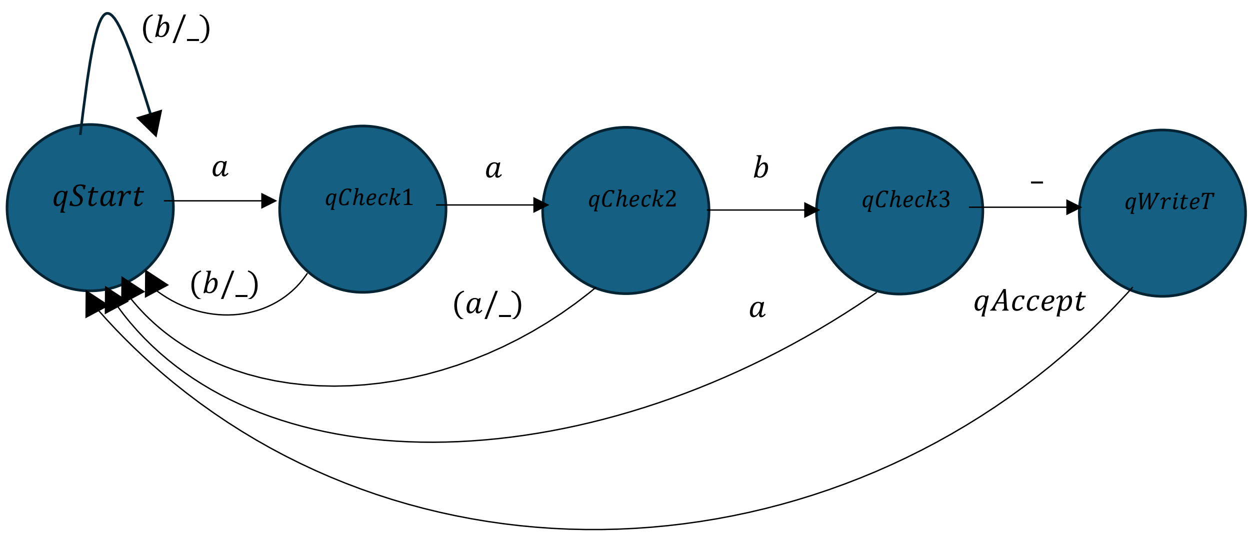
Tape Alphabet (Γ): $\{a, b, \_, T\}$

Initial State (q_0): $\{qStart\}$

Accepting State (F): $\{qAccept\}$

Transition function (δ):

$$\delta(qStart, a) = (qCheck1, a, R)$$
$$\delta(qStart, b) = (qStart, b, R)$$
$$\delta(qStart, \_) = (qAccept, \_, R)$$
$$\delta(qCheck1, a) = (qCheck2, a, R)$$
$$\delta(qCheck1, b) = (qStart, b, R)$$
$$\delta(qCheck1, \_) = (qStart, \_, R)$$
$$\delta(qCheck2, a) = (qCheck3, a, R)$$
$$\delta(qCheck2, b) = (qStart, b, R)$$
$$\delta(qCheck2, \_) = (qStart, \_, R)$$
$$\delta(qCheck3, b) = (qWrite\_T, b, R)$$
$$\delta(qCheck3, a) = (qStart, a, R)$$
$$\delta(qCheck3, \_) = (qStart, \_, R)$$
$$\delta(qWrite\_T, \_) = (qWrite\_T, T, R)$$
$$\delta(qWrite\_T, any) = (qWrite\_T, same\ symbol, R)$$



Question 2

Build/design a TM that:

- accepts all words that start with an a, and ends with a b,
- loops forever on all words that start with a b, and
- rejects all other words.

States (Q): $\{q_0, q_1, q_2, q_3, q_4, q_4, q_{loop}\}$

Input Alphabet (Σ): $\{a, b\}$

Tape Alphabet (Γ): $\{a, b, \_\}$

Initial State (q_0): $\{q_0\}$

Accepting State (F): $\{q_3\}$

Reject State: $\{q_4\}$

Transition function (δ):

$\delta(q_0, a) = (q_1, a, R)$

$\delta(q_0, b) = (q_{loop}, b, R)$

$\delta(q_0, \_) = (q_4, \_, R)$

$\delta(q_1, a) = (q_2, a, R)$

$\delta(q_1, b) = (q_2, b, R)$

$\delta(q_1, \_) = (q_2, b, R)$

$\delta(q_1, \_) = (q_4, \_, R)$

$\delta(q_2, a) = (q_2, a, R)$

$\delta(q_2, b) = (q_2, b, R)$

$\delta(q_2, \_) = (q_3, \_, L)$

$\delta(q_{loop}, a) = (q_{loop}, a, R)$

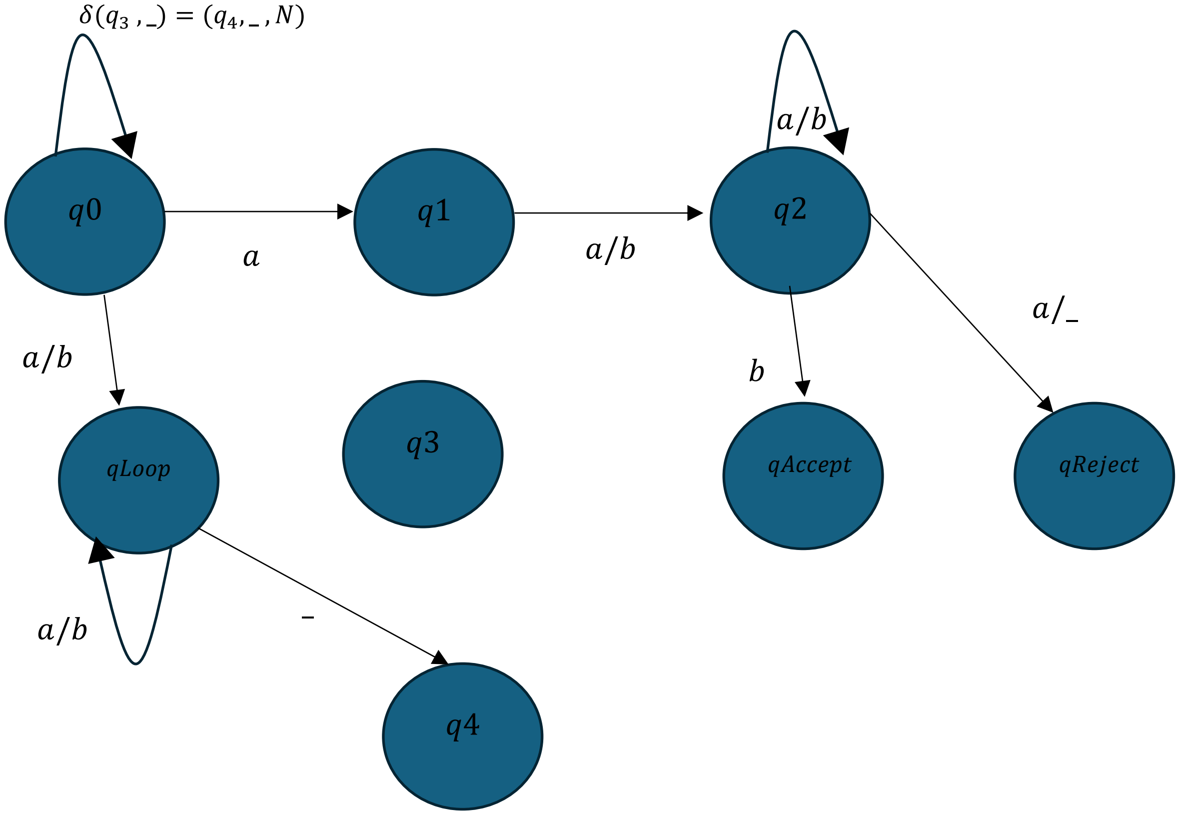
$\delta(q_{loop}, b) = (q_{loop}, b, R)$

$\delta(q_{loop}, \_) = (q_{loop}, \_, R)$

$\delta(q_3, b) = (q_{accept}, b, N)$

$\delta(q_3, a) = (q_4, a, N)$

$\delta(q_3, \_) = (q_4, \_, N)$



Question 3

Build a 2PDA that accepts the language $\{a^n b 2^n a^{n+1} b^n \mid n > 0\}$.

States (Q): $\{ q_0, q_1, q_2, q_3, q_f \}$

Alphabet (Σ): $\{ a, b \}$

Stack Alphabet(Γ): $\{ X, Y, \epsilon \}$

Initial State (q_0): $\{ q_0 \}$

Accepting State (F): $\{ q_f \}$

Transition function (δ):

q_0

$$\delta(q_0, a, \epsilon, \epsilon) = \{(q_0, X, \epsilon)\}$$
$$\delta(q_0, b, X, \epsilon) = \{(q_1, \epsilon, Y)\}$$

q_1

$$\delta(q_1, b, X, \epsilon) = \{(q_1, \epsilon, Y)\}$$
$$\delta(q_1, a, \epsilon, \epsilon) = \{(q_2, \epsilon, \epsilon)\}$$

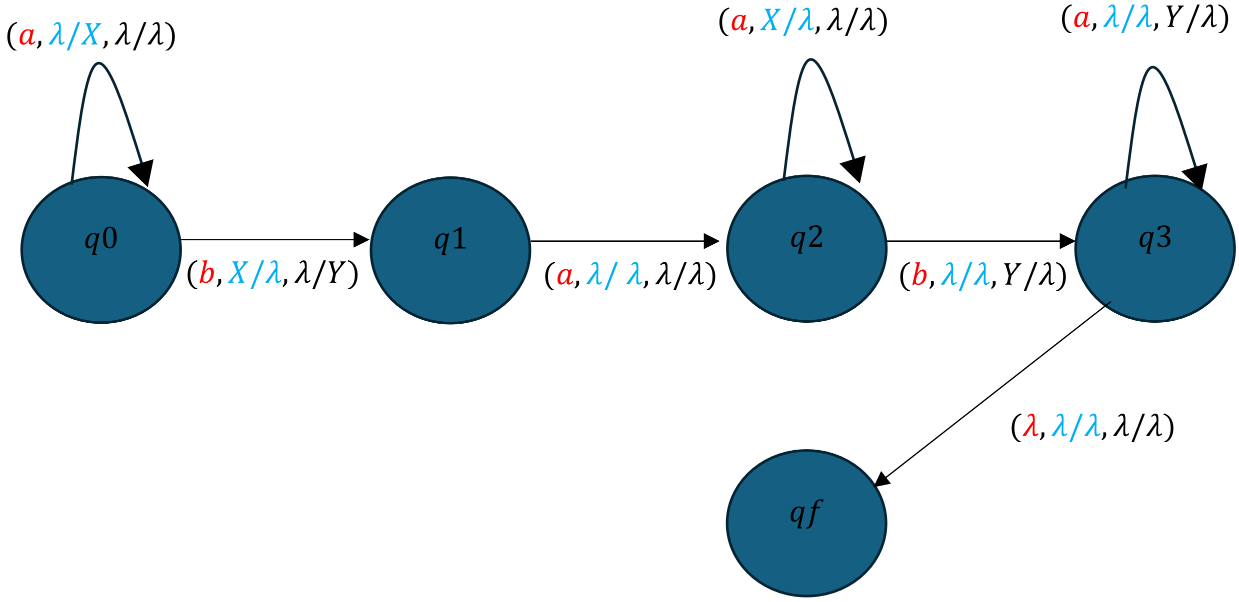
q_2

$$\delta(q_2, a, X, \epsilon) = \{(q_2, \epsilon, \epsilon)\}$$
$$\delta(q_2, b, \epsilon, Y) = \{(q_3, \epsilon, \epsilon)\}$$

q_3

$$\delta(q_3, b, \epsilon, Y) = \{(q_3, \epsilon, \epsilon)\}$$
$$\delta(q_3, \epsilon, \epsilon, \epsilon) = \{(q_f, \epsilon, \epsilon)\}$$

q_f



Question 4

Build a Turing Machine that:

- accept even number of as,
- loops forever if start with b, and
- rejects all other words.

States (Q): $\{q_0, q_{even}, q_{odd}, q_{loop}, q_{reject}, q_{accept}\}$

Input Alphabet (Σ): $\{a, b\}$

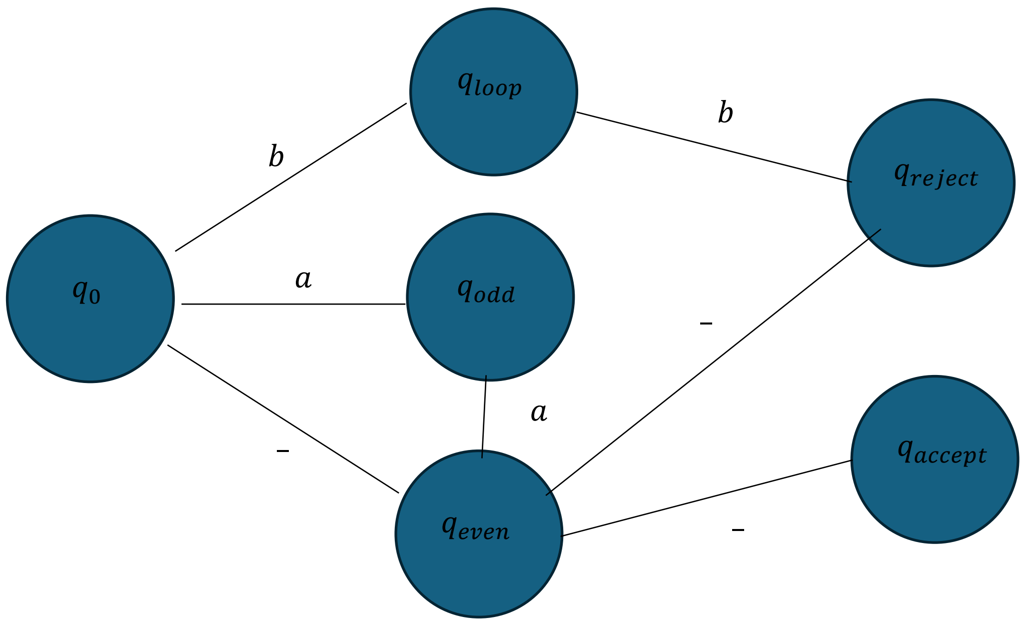
Tape Alphabet (Γ): $\{a, b, \_\}$

Start State (q_0): $\{q_0\}$

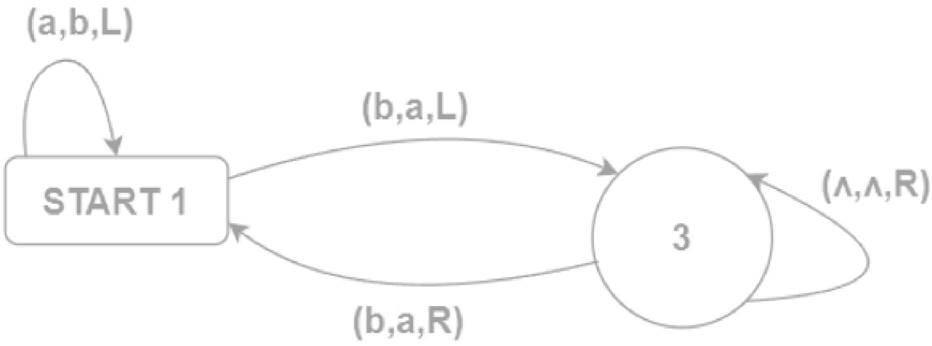
Accepting State (F): $\{qAccept\}$

Transition function (δ):

$$\delta(q_0, a) = (q_{odd}, \#, R)$$
$$\delta(q_0, b) = (q_{loop}, b, R)$$
$$\delta(q_0, \#) = (q_{even}, \#, S)$$
$$\delta(q_{odd}, a) = (q_{even}, \#, R)$$
$$\delta(q_{odd}, \#) = (q_{reject}, \#, S)$$
$$\delta(q_{even}, a) = (q_{odd}, \#, R)$$
$$\delta(q_{event}, \#) = (q_{accept}, \#, S)$$
$$\delta(q_{loop}, b) = (q_{loop}, b, R)$$



Question 5
Convert the following TM into summary table and then into their code words in CWL. What is the language accepted by this TM.



CWL:
(Current State, Input Symbol, Write Symbol, Move Direction, Next State)

1. (START1, a, b, L, START1)
2. (START1, b, a, L, 3)
3. (3, ∧, ∧, R, 3)

Language Accepted:
- Accepts strings ending in b
- Non-empty

$L = \{a^{\mathrm{n}}b | \geq 0\}$