Semester 2
Assignment 1
LKE MNCUBE
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1.1. Find the value of the $\lim_{x \to 1^-} \left(\frac{x^2 - x}{|x-1|}\right)$

When $x \to 1-$, $x - 1$ is negative (limit from the left hand side)

$\therefore |x-1| = \begin{cases} x-1, \ x \geq 1 & RHS \\ or & \\ -(x-1), \ x < 1 & LHS \end{cases}$ ***Piecewise definition***

$|x-1|$ should be made positive, so multiply by -1

$\therefore |x-1|$. -1 = $(x-1)$.(-1)
= 1 – x

$\therefore \quad \lim_{x \to 1^-} \left(\frac{x^2 - x}{1-x}\right)$ where $\left(\frac{x^2 - x}{1-x}\right)$ = - x

$\therefore \quad \lim_{x \to 1^-} (-x)$ = -1

The correct option is:

(3) -1

1.2. Find the value of $\lim_{t \to 0} \left(\frac{\sin(5t)}{\sin 2t}\right)$

$= \lim_{t \to 0} \left(\frac{\cos(5t) \ . \ 5}{\cos(2t) \ . \ 2}\right)$ ***l'Hospital's Rule***

Substitute value of t, make t=0

$= \left(\frac{\cos(5 \, . \, 0) \ . \ 5}{\cos(2 \, . \, 0) \ . \ 2}\right)$

$= \left(\frac{\cos(\,0) \ . \ 5}{\cos(0) \ . \ 2}\right)$ where cos(0) = 1

$= \left(\frac{1 \times 5}{1\times 2}\right)$

The correct option is:

(4) $\frac{5}{2}$

1.3. Find the value of $\lim_{h\to -4}\left(\frac{\sqrt{h^2+9}-5}{h+4}\right)$

$= \lim_{h\to -4}\left(\frac{\sqrt{h^2+9}-5}{h+4}\right).\frac{(\sqrt{h^2+9}+5)}{(\sqrt{h^2+9}+5)}$ ***Multiply by conjugate*** (Rationalize denominator)

$= \lim_{h\to -4}\left(\frac{h-4}{(\sqrt{h^2+9}+5)}\right)$

Substitute value of h, make h=-4

$= \lim_{h\to -4}\left(\frac{-4-4}{\left(\sqrt{(-4)^2+9}+5\right)}\right)$ = - $\frac{4}{5}$

The correct option is:

(3) - $\frac{4}{5}$

1.4. if $y = \frac{x+4}{x+2}$, then $\frac{d^2x}{dy^2}$ is:

First order implicit differential $\left(\frac{dy}{dx}\right)$

$= \frac{\frac{d}{dx}(x+4)(x+2)-\frac{d}{dx}(x+2)(x+4)}{(x+2)^2}$

$= \frac{1.(x+2)-1.(x+4)}{(x+2)^2}$ where $\frac{d}{dx}(x+4) = 1$

And $\frac{d}{dx}(x+2) = 1$

$= \frac{(x+2)-(x+4)}{(x+2)^2}$

$= \frac{x+2-x-4}{(x+2)^2}$

$= -\frac{2}{(x+2)^2}$

Second order implicit differential $\left(\frac{d^2y}{dx^2}\right)$

$= \frac{d}{dx}\left(-\frac{2}{(x+2)^2}\right)$

$= -2.\frac{d}{dx}\left(\frac{1}{(x+2)^2}\right)$ $\quad (a.f') = a.f^{'}$: Remove constant -2

$= -2.\frac{d}{dx}\left(\frac{1}{(x+2)^2}\right)$

$= -2.\frac{d}{dx}((x+2)^{-2})$ $\quad$ where $\left(\frac{1}{(x+2)^2}\right) = ((x+2)^{-2})$

$= (-2(x+2)^{-3})$ $\quad$ ***Chain rule***

$= -2.(-2(x+2)^{-3})$

$= 4(x+2)^{-3} = \frac{4}{(x+2)^3}$

The correct option is:

(1) $\frac{4}{(x+2)^3}$

1.5. Evaluate $\int x\sqrt{1-x^2}dx$

$= \int x\sqrt{1-x^2}dx$ $\quad$ apply u substitution, where $u = 1 - x^2$

Therefore $x = dx.(1-x^2) = -\frac{1}{2x}du$

$= \int -\frac{1}{2}\sqrt{u}.du$

$= -\frac{1}{2}.\int \sqrt{u}.du$ $\quad$ $.\int a.f(x)dx = a.\int f(x)dx$:Remove constant . $-\frac{1}{2}$

$= -\frac{1}{2}.\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c$ $\quad$ ***power rule***

$= -\frac{1}{2}.\frac{2}{3}(u^{\frac{3}{2}}) + c$

$= -\frac{2}{6}u^{\frac{3}{2}} + c$

$= -\frac{2}{6}(1 - x^2)^{\frac{3}{2}} + c$ $\quad$ substitute $u = 1 - x^2$

$= -\frac{1}{3}(1 - x^2)^{\frac{3}{2}} + c$

The correct option is:

(2) $\quad -\frac{1}{3}(1 - x^2)^{\frac{3}{2}} + c$

2.1. Evaluate $\lim_{x\to-\infty}\left(\frac{\sqrt{9x^6-x}}{x^3+1}\right)$

$$= \lim_{x\to-\infty}\left(\frac{\sqrt{\frac{9x^6}{x^6}-\frac{x}{x^6}}}{\frac{x^3}{x^3}+\frac{1}{x^3}}\right)$$

divide by the highest power in denominator & numerator

$$= \lim_{x\to-\infty}\left(\frac{\sqrt{\frac{9x^6}{x^6}-\frac{x}{x^6}}}{\frac{x^3}{x^3}+\frac{1}{x^3}}\right)$$

$$= \lim_{x\to-\infty}\left(\frac{\sqrt{9-\frac{1}{x^5}}}{1+\frac{1}{x^3}}\right)$$

$$= \frac{\lim_{x\to-\infty}(\sqrt{9-\frac{1}{x^5}})}{\lim_{x\to-\infty}(1+\frac{1}{x^3})}$$

$$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a}(f(x))}{\lim_{x\to a}(g(x))}$$

$$= \frac{\underset{x\to -\infty}{lim}(\sqrt{9-\frac{1}{x^5}})}{\underset{x\to -\infty}{lim}(1+\frac{1}{x^3})}$$

$$= \frac{-3}{1} = -3$$ substitute $x = 0$

2.2. Evaluate $\lim_{x\to -3}\left(\frac{x^2+x-6}{|x+3|}\right)$

$\therefore |x+3| = \left\{\begin{matrix} x+3,\ x \geq 3 & RHS \\ or & \\ -(x+3),\ x < 3 & LHS \end{matrix}\right\}$ ***Piecewise definition***

LHS

$$= \lim_{x\to -3^-} \frac{x^2+x-6}{-(x+3)}$$

$$= \lim_{x\to -3^-} \frac{(x+3)(x-2)}{-(x+3)}$$

$$= \lim_{x\to -3^-} \frac{(x-2)}{-1}$$

$$= \frac{(-3-2)}{-1}$$ substitute $x = -3$

$$= 5$$

RHS

$$= \lim_{x\to -3^+} \frac{x^2+x-6}{(x+3)}$$

$$= \lim_{x\to -3^-} \frac{(x+3)(x-2)}{(x+3)}$$

$= \lim_{x \to -3^-} \frac{(x-2)}{1}$

$= \frac{(-3-2)}{1}$ substitute $x = -3$

$= -5$

$\therefore$ LHS limit ≠ RHS Limit

$\therefore$ Limit does not exist at $x = -3$ as the graph diverges at this point

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2.3. Evaluate $\lim_{t \to 0} \left(\frac{sin3t}{tan2t}\right)$

$= \lim_{t \to 0} \left(\frac{cos3t.3}{sec^2 2t.2}\right)$ simplify

$= \lim_{t \to 0} \left(\frac{cos3t.3.\cos^2 2t}{2}\right)$

$= \frac{cos3t.3.\cos^2 2t}{2}$

$= \frac{\cos(3)(0).3.\cos^2(2)(0)}{2}$ substitute $t = 0$

$= \frac{\cos(0).3.\cos^2(0)}{2}$

$= \frac{1 \times 3 \times 1}{2}$

$= \frac{3}{2}$

2.4. Let $f(x) = \begin{cases} x^2 + 1 \; if \; x < 1 \\ (x-2)^2 \; if \; x \geq 1 \end{cases}$

(a)

LHS

$= \lim_{x \to 1^-} x^2 + 1$

$= (1)^2 + 1$ substitute $x = 1$

$= 2$

RHS

$= \lim_{x \to 1^-} (x-2)^2$

$= ((1) - 2)^2$ substitute $x = 1$

$= 1$

(b) $\therefore$ LHS limit ≠ RHS Limit

$\therefore$ Limit does not exist at $x = 1$ as the graph diverges at this point

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2.5. Let $f(x) = \begin{cases} \frac{x^2-4}{x-2} & if \quad x < 2 \\ ax^2 - bx + 3 & if \quad 2 \leq x < 3 \\ 2x - a + b & if \quad x \geq 3 \end{cases}$

For a continuity, we need to find the limits where $x = 2$ and $x = 3$

At $x = 2$

Let this be some function where:

LHS	RHS
$= \lim_{x \to 2^-} ax^2 - bx + 3$	$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$
$= a(2)^2 - b(2) + 3$ substitute $x = 2$	$= \frac{(0)^2-4}{(0)-2}$ substitute $x = 2$
$= 4a - 2b + 3$	$= 0$

Let LHS = RHS

$\therefore 4a - 2b + 3 = 0$

At $x = 3$

Let this be some function where:

LHS	RHS
$= \lim_{x \to 3^-} ax^2 - bx + 3$	$= \lim_{x \to 3^+} 2x - a + b$
$= a(3)^2 - b(3) + 3$ substitute $x = 3$	$= 2(3) - a + b$ substitute $x = 3$
$= 9a - 3b + 3$	$= 6 - a + b$

Let LHS = RHS

$\therefore 9a - 3b + 3 = 6 - a + b$

$\therefore -8a + 2b + 3 = 0$

Solve simultaneous equation/system

$$system = \begin{Bmatrix} 4a - 2b + 3 = 0 \\ -8a + 2b + 3 = 0 \end{Bmatrix}$$

$$\begin{array}{ccc} a & b & \\ \multicolumn{3}{c}{\begin{bmatrix} 4 & -2 & 3 \\ -8 & 2 & 3 \end{bmatrix}} \end{array}$$

$$\begin{bmatrix} 4 & 0 & 6 \\ -8 & 2 & 3 \end{bmatrix} \qquad R1 + R2 \rightarrow R1$$

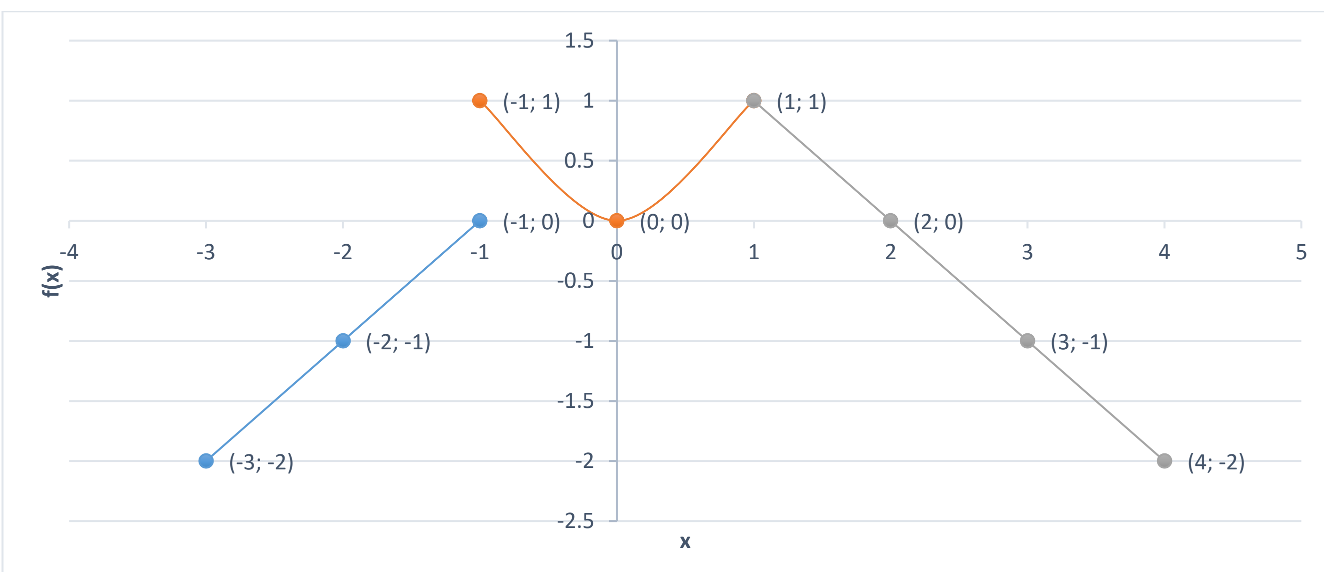
$$\begin{bmatrix} 1 & 0 & -\frac{3}{2} \\ -8 & 2 & 3 \end{bmatrix} \qquad \frac{1}{4}R1 \rightarrow R1$$

$$\begin{bmatrix} 1 & 0 & -\frac{3}{2} \\ -4 & 1 & \frac{3}{2} \end{bmatrix} \qquad \frac{1}{4}R2 \rightarrow R2$$

$$\begin{bmatrix} 1 & 0 & -\frac{3}{2} \\ 0 & 1 & -\frac{9}{2} \end{bmatrix} \qquad 4R1 + R2 \rightarrow R2$$

$a = -\frac{3}{2}$ & $b = -\frac{9}{2}$

3.1. Let $f(x) = \begin{Bmatrix} 1 + x & if & x < -1 \\ x^2 & if & -1 \leq x < 1 \\ 2 - x & if & x \geq 1 \end{Bmatrix}$



3.2.

LHS	RHS
$= \lim_{x \to -1^-} 1 + x$	$= \lim_{x \to -1^+} x^2$
$= 1 + (-1)$ substitute $x = -1$	$=(-1)^2$ substitute $x = -1$
$= 0$	$= 1$

∴ LHS limit ≠ RHS Limit

∴ Limit does not exist at $x = -1$ as the graph diverges at this point

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3.3.

LHS	RHS
$= \lim_{x \to 1^-} x^2$	$= \lim_{x \to 1^+} 2 - x$
$=(1)^2$ substitute $x = 1$	$= 2 - (1)$ substitute $x = 1$
$=1$	$= 1$

∴ LHS limit = RHS Limit

∴ Limit does exists at $x = 1$. Even though the graph changes gradient at this point is has a limit and therefore continuous

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3.4. Determine if f is differentiable at $x = 1$, by evaluating the one-sided limits

$= \lim_{h \to -1^+} \frac{f(1+h)-(f1)}{h}$ where $x = 1$

From the below calculation we see that f is differentiable at $x = 1$

LHS	RHS
$= \lim_{h\to -1^-} \frac{f(1+h)-(f1)}{h}$	$= \lim_{h\to -1^+} \frac{f(1+h)-(f1)}{h}$
$= \frac{(-1+h)^2-(-1)^2}{h}$	$= \frac{(-1+h)^2-(-1)^2}{h}$
$= \frac{h^2-h+1-1}{h}$	$= \frac{h^2-h+1-1}{h}$
$= \frac{h}{h}$	$= \frac{h}{h}$
$= 1$	$= 1$

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4. Prove $\lim_{x\to 0}(x^3+x^2)\sin\left(\frac{\pi}{x}\right) = 0$

Test the limit where:

$\lim_{x\to a} f(x) = L = \lim_{x\to a} g(x)$

$\therefore f(x \le h(x) \le g(x)$

The limit at any $\sin\left(\frac{n}{x}\right)$ does not exist, however:

$-1 \le \sin\left(\frac{n}{x}\right) \le 1$, for any number n, $x\epsilon$ positive values

$\therefore -1 \le \sin\left(\frac{\pi}{x}\right) \le 1$

$\therefore \lim_{x\to 0}\left(x^3+x^2\right)(-1) \quad \le \quad \lim_{x\to 0}\left(x^3+x^2\right)\sin\left(\frac{\pi}{x}\right) \quad \le \quad \lim_{x\to 0}(x^3+x^2)(1)$

substitute $x = 0$

$\therefore\ 0 \quad \le \quad \lim_{x\to 0}\left(x^3+x^2\right)\sin\left(\frac{\pi}{x}\right) \quad \le\ 0$

substitute $x = 0$

$\therefore\ 0 \quad \le\ 0 \quad \le\ 0$

5.1. Let $x^2 - xy + y - 3 = 0$

(a) Find $\frac{dy}{dx}$ by implicit differentiation

→ $2x - \left(1y + x\frac{dy}{dx}\right) - \frac{dy}{dx} = 0$

→ $2x - y + x\frac{dy}{dx} - \frac{dy}{dx} = 0$

→ $x\frac{dy}{dx} - \frac{dy}{dx} = y - \ 2x$

→ $\frac{dy}{dx}(x - 1) = y - \ 2x$

→ $\frac{dy}{dx} = \frac{y-2x}{(x-1)}$

(b) Find $\frac{dy}{dx}$ by rewriting y as a function of x, then using the Quotient rule

$$-xy + y = 3 - \ x^2$$
$$y \ - xy = 3 - \ x^2$$
$$y(1 \ - x) = 3 - \ x^2$$
$$y = \frac{3- x^2}{1 - x}$$

$$\therefore \frac{d}{dx}\left(\frac{u}{v}\right) = \left(\frac{v\frac{du}{x} - u\frac{dv}{dx}}{v^2}\right)$$

$$\frac{d}{dx}\left(\frac{3- x^2}{1 - x}\right) = \frac{\frac{d}{x}(3- x^2)(1 - x) - \frac{d}{x}(3- x^2)(1 - x)}{(1 - x)^2}$$

$$= \frac{(-2x)(1 - x) - (3- x^2)(-1\,)}{(1 - x)^2}$$

$$= \frac{(-2x)(1 - x) - (3- x^2)(-1\,)}{(1 - x)^2}$$

$$= \frac{x^2 - 2x + 3}{(1 - x)^2}$$

(c) Find $\frac{dy}{dx}$ by letting
$F(x,y) = \; x^2 - xy + y - 3 = 0$ and using partial differentiation by calculating

$\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$

$\frac{\partial F}{\partial x} = \; \frac{\partial}{\partial x}(x^2 - xy + y - 3)$
$= \frac{\partial}{\partial x}(x^2 - xy + y - 3)$
$= \frac{\partial}{\partial x}(x^2) - \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial x}(y) - \frac{\partial}{\partial x}(3)$ sum difference rule
$= 2x - y + 0 - 0$
$= 2x - y$

$\frac{\partial F}{\partial y} = \; \frac{\partial}{\partial y}(x^2 - xy + y - 3)$
$= \frac{\partial}{\partial y}(x^2) - \frac{\partial}{\partial y}(xy) + \frac{\partial}{\partial y}(y) - \frac{\partial}{\partial y}(3)$ sum difference rule
$= 0 - x + 1 - 0$
$= -x + 1$

5.2. Find the equation of the tangent line to $x^2 - xy + y - 3 = 0$ at the point $x = -1$

$\frac{dy}{dx} = \frac{x^2-2x+3}{(1\,-x)^2}$

Calculate m for slope formula, i.e. calculate into $f'(-1)$

$f'(-1) = \frac{(-1)^2-2(-1)+3}{(1\,-(-1))^2}$

$= \frac{(-1)^2-2(-1)+3}{(1\,-(-1))^2}$

$= \frac{6}{4}$

$= \frac{3}{2}$

Substitute $x = -1$

$$f(x) = \frac{3- x^2}{1 - x}$$

$$\therefore f(-1) = \frac{3- (-1)^2}{1 - (-1)}$$

$$= \frac{3- (-1)^2}{1 - (-1)}$$

$$= 1$$

Therefore the coordinate where $x = -1$ is (-1, $\frac{3}{2}$)

Use general slope formula to get equation:

$$y - y_1 = m(x - x_1)$$

$$\therefore y - \left(\frac{3}{2}\right) = (1)(x - (-1))$$

$$y = x - \frac{1}{2}$$

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6. Use logarithmic differentiation to find $\frac{dy}{dx}$ if $y = (sinx)^{cosx}$

$$ln.\, y = ln.\,(sinx)^{cosx}$$

$$ln.\, y = \cos(x).\, ln.\, \sin(x)$$

$$y = \cos(x).\, \sin(x)$$

$$\therefore \frac{dy}{dx} = \frac{d}{du}\sin(u)\,.\frac{d}{dx}(x^{cosx})$$

7.1. $y = \sec(3x)\,.\,sin2x$

$$= \frac{d}{dx}\sec 3x\,.\,sin2x + \frac{d}{dx}(sin2x)(sec3x)$$

$$= (\sec 3x\,.\,tan3x.\,3).\,sin2x + (cos2x.\,2)(sec3x)$$

$$= \sec 3x\,.\,tan3x.\,3\; sin2x + cos2x.\,2sec3x$$

7.3. $g(x) = \frac{3x-1}{2x+1}$

$$= \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

$$\therefore = \lim_{h\to 0} \frac{\frac{3(x+h)-1}{2(x+h)+1} - \frac{3x-1}{2x+1}}{h}$$

$$= \lim_{h\to 0} \frac{3x+3h-1}{(2(x+h)+1).h} - \frac{3x-1}{h.(2x+1)}$$

$$= \lim_{h\to 0} \frac{(3x+3h-1)\,(2x+1)}{h.(2x+1)(2(x+h)+1)} - \frac{(3x-1)(2(x+h)+1)}{h.(2x+1)(2(x+h)+1)}$$

$$= \lim_{h\to 0} \frac{5h}{h.(2x+1)(2(x+h)+1)}$$

$$= \lim_{h\to 0} \frac{5}{(2x+1)(2(x+h)+1)}$$

$$= \frac{5}{(2(0)+1)(2(x+(0))+1)}$$ substitute $x = 0$

$$= \frac{5}{(2x+1)^2}$$

8.
