LKE MNCUBE
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Unique Assignment Number: 789872

Question 1

a)

$x = 6 + 5t$

$y = 4 + 3t$

$z = 2 + t$

$\therefore t = z - t$

Substitute back into above equations

$y = 4 + 3t$

$\therefore\ y = 4 + 3(z - t)$

$\therefore\ y = 4 + 3z - 3t$

$\therefore\ y - 3z + 3t = 4$

$x = 6 + 5t$

$\therefore x = 6 + 5(z - t)$

$\therefore x = 6 + 5z - 5t)$

$\therefore x - 5z + 5t = 6$

System of equations:

$$\begin{Bmatrix} z - t = \ 2 \\ y - 3z + 3t = 4 \\ x - 5z + 5t = 6 \end{Bmatrix}$$

b)

Solve using Gaussian Elimination

$$\begin{Bmatrix} x + 4y + z = \ 0 \\ 4x + 13y - 7z = 0 \\ 7x + 22y + 13z = 1 \end{Bmatrix}$$

$\therefore$ Augmented matrix

$$\begin{bmatrix} x & 4y & z & 0 \\ 4x & 13y & 7z & 0 \\ 7x & 22y & 13z & 1 \end{bmatrix}$$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 4 & 1 & 0 \\ 4 & 13 & 7 & 0 \\ 7 & 22 & 13 & 1 \end{bmatrix} \end{array}$$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & -3 & 3 & 0 \\ 7 & 22 & 13 & 1 \end{bmatrix} \end{array}$$

R2 - 4R1 → R2

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 6 & -6 & 1 \end{bmatrix} \end{array}$$

R3 - 7R1 → R3

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 6 & -6 & 1 \end{bmatrix} \end{array}$$

$-\frac{1}{3}R2 \rightarrow R2$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 6 & -6 & 1 \end{bmatrix} \end{array}$$

R1 – 4R2 → R1

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

– 6R2+R3 → R3

$$\begin{Bmatrix} x + 5y = 0 \\ y - z = 0 \end{Bmatrix}$$

Let $z = t$

$\therefore 6y - 6(t) = 1$
$\therefore 6y = 6t$
$\therefore y = t$
Substitute $y = t$
$\therefore x + 5(t) = 0$
$\therefore x = -5t$

Solution:

$$\begin{Bmatrix} x = -5t \\ y = t \\ z = t \end{Bmatrix} t \in \mathbb{R}$$

c)

Solve lower triangular system

$$\begin{Bmatrix} x_1 - 2x_2 - x_3 + x_4 = 3 \\ x_2 + 3x_3 + 7x_4 = 3 \\ x_3 + 2x_4 = 3 \\ x_4 = 0 \end{Bmatrix}$$

∴ Augmented matrix

$$\begin{bmatrix} x_1 & 2x_2 & -x_3 & x_4 & 3 \\ & x_2 & 3x_3 & 7x_4 & 5 \\ & & x_3 & 2x_4 & 2 \\ & & & x_4 & 0 \end{bmatrix}$$

From the above: $x_4 = 0$

$x_3 + 2x_4 \ = 2$
$\therefore x_3 + 2(0) = 2$ Substitute $x_4 = 0$
$\therefore x_3 = \ 2$

$x_2 + 3x_3 + 7x_4 = 5$
$\therefore x_2 + 3(2) + 7(0) = 5$ Substitute $x_4 = 0, x_3 = \ 2$
$\therefore x_2 + 6 + 0 = 5$
$\therefore x_2 = -1$

$x_1 + 2x_2 - x_3 + x_4 = 3$ Substitute $x_4 = 0, x_3 = \ 2, x_2 = -1$
$\therefore x_1 + 2(-1) - (2) + (0) = 3$
$\therefore x_1 - 2 - 2 + 0 = 3$
$\therefore x_1 = 7$

Solution:

$$\begin{Bmatrix} x_1 = 7 \\ x_2 = -1 \\ x_3 = \ 2 \\ x_4 = 0 \end{Bmatrix}$$

d)

Solve the system:

$$\begin{bmatrix} 3x & 11y & 19z & -2 \\ 7x & 23y & 39z & 10 \\ -4x & -3y & -2z & 6 \end{bmatrix}$$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 3 & 11 & 19 & -2 \\ 7 & 23 & 39 & 10 \\ -4 & -3 & -2 & 6 \end{bmatrix} \end{array}$$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & \frac{11}{3} & \frac{19}{3} & -\frac{2}{3} \\ 7 & 23 & 39 & 10 \\ -4 & -3 & -2 & 6 \end{bmatrix} \end{array}$$

$\frac{1}{3}R1 \rightarrow \boldsymbol{R1}$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & \frac{11}{3} & \frac{19}{3} & -\frac{2}{3} \\ 0 & -\frac{8}{3} & -\frac{16}{3} & \frac{44}{3} \\ -4 & -3 & -2 & 6 \end{bmatrix} \end{array}$$

$R2 - 7R1 \rightarrow \boldsymbol{R2}$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & \frac{11}{3} & \frac{19}{3} & -\frac{2}{3} \\ 0 & -\frac{8}{3} & -\frac{16}{3} & \frac{44}{3} \\ 0 & \frac{35}{3} & \frac{70}{3} & \frac{10}{3} \end{bmatrix} \end{array}$$

$R3 - 4R1 \rightarrow \boldsymbol{R3}$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & \frac{11}{3} & \frac{19}{3} & -\frac{2}{3} \\ 0 & 1 & 2 & -\frac{11}{2} \\ 0 & \frac{35}{3} & \frac{70}{3} & \frac{10}{3} \end{bmatrix} \end{array}$$

$R2 \div -\frac{8}{3} \rightarrow \boldsymbol{R1}$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & -\frac{39}{2} \\ 0 & 1 & 2 & \frac{11}{2} \\ 0 & \frac{35}{3} & \frac{70}{3} & \frac{10}{3} \end{bmatrix} \end{array}$$

$R1 - \frac{11}{3}R2 \rightarrow \boldsymbol{R1}$

$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 & -1 & -\frac{39}{2} \\ 0 & 1 & 2 & \frac{11}{2} \\ 0 & 0 & 0 & \frac{135}{2} \end{bmatrix}$$

$\frac{35}{3}R2 - R3 \rightarrow R3$

$$\begin{Bmatrix} x - z = -\frac{39}{2} \\ y - 2z = \frac{11}{2} \end{Bmatrix}$$

Let $z = 2t$

$y - 2z = \frac{11}{2}$
$\therefore\ y - 2(2t) = \frac{11}{2}$
$\therefore\ y = \frac{11}{2} +\ 2(2t)$
$\therefore\ y = 11 + 2t$

$x - z = -\frac{39}{2}$
$\therefore x - (2t) = -\frac{39}{2}$
$\therefore x = -\frac{39}{2}+2t$
$\therefore x = -39 + t$

Question 2

a)

Solve for C such that $x, y, z\ \in\ \mathbb{Z}^+$

$$\begin{Bmatrix} 2x + y = C \\ 3y + z = C \\ x - 4z = C \end{Bmatrix}$$

$\therefore$ Augmented matrix

$$\begin{array}{ccc} x & y & z \end{array}$$
$$\begin{bmatrix} 2 & 1 & 0 & C \\ 0 & 3 & 1 & C \\ 1 & 0 & -4 & C \end{bmatrix}$$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 1 & 4 & 0 \\ 0 & 3 & 1 & C \\ 1 & 0 & -4 & C \end{bmatrix} \end{array}$$

$R1 - R3 \rightarrow \boldsymbol{R1}$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 1 & 4 & 0 \\ 0 & 3 & 1 & C \\ 0 & -1 & -8 & C \end{bmatrix} \end{array}$$

$R1 - R3 \rightarrow \boldsymbol{R3}$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 0 & -4 & C \\ 0 & 3 & 1 & C \\ 0 & -1 & -8 & C \end{bmatrix} \end{array}$$

$R1 + R3 \rightarrow \boldsymbol{R1}$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 0 & -4 & C \\ 0 & 1 & -15 & 3C \\ 0 & -1 & -8 & C \end{bmatrix} \end{array}$$

$R2 + 2R3 \rightarrow \boldsymbol{R2}$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 0 & -4 & C \\ 0 & 1 & -15 & 3C \\ 0 & 0 & -23 & 4C \end{bmatrix} \end{array}$$

$R2 + R3 \rightarrow \boldsymbol{R3}$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 0 & -4 & C \\ 0 & 1 & -15 & 3C \\ 0 & 0 & 1 & \frac{-4}{23}C \end{bmatrix} \end{array}$$

$-\frac{1}{23}R3 \rightarrow \boldsymbol{R3}$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 0 & -4 & C \\ 0 & 1 & 0 & \frac{9}{23}C \\ 0 & 0 & 1 & \frac{-4}{23}C \end{bmatrix} \end{array}$$

$\boldsymbol{R2+15}R3 \rightarrow \boldsymbol{R2}$

$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 & 0 & \frac{7}{23}C \\ 0 & 1 & 0 & \frac{9}{23}C \\ 0 & 0 & 1 & \frac{-4}{23}C \end{bmatrix}$$

R1+4R3 → R2

$$\begin{Bmatrix} x = \frac{7}{23}C \\ y = \frac{7}{23}C \\ z = \frac{7}{23}C \end{Bmatrix}$$

∴ *C would need to be 23 in order to make x,y and z be integers.*

b)

$$\begin{Bmatrix} x + \ 2y + 3z = 4 \\ x + ky + 4z = 6 \\ x + 2y + (k+2)z = 6 \end{Bmatrix}$$

∴ Augmented matrix

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & k & 4 & 6 \\ 1 & 2 & k+2 & 6 \end{bmatrix} \end{array}$$

Eliminate into Generalized row-echelon form

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & k-2 & 1 & 2 \\ 1 & 2 & k+2 & 6 \end{bmatrix} \end{array}$$

*R2- R1 → **R2***

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & k-2 & 1 & 2 \\ 0 & 0 & k-1 & 2 \end{bmatrix} \end{array}$$

*R3- R1 → **R3***

∴ Corresponding system

$$\begin{Bmatrix} x + \ 2y + 3z = 4 \\ y(k-2) + z = 6 \\ z(k-1) = 6 \end{Bmatrix}$$

i)

The system has exactly one solution where the matrix is in reduced row-echelon form.

∴ One/Unique solution exists where $(k-2)=0$ AND $(k-1)=0$

∴ One/Unique solution exists where $k=2$ AND $k=1$

ii)

No solution exists where in the last row we have $0x_1 + 0x_2 + 0x_3 = c$, where $c \neq 0$ (0,0,0,C)

∴ No solution exists where $(k-1) = \;\; 0$ AND $2 \neq 0$.

Change last equation into the form $0x_1 + 0x_2 + 0x_3 = c$, where $c \neq 0$

$\therefore (k-1) = \;\; 0$

$k=1$

Hence no solution exists if $= 1$.

iii)

Infinitely many solutions exists where in the last row we have $0x_1 + 0x_2 + 0x_3 = c$, where $c = 0$ (0,0,0,0)

∴ infinitely many solutions exist where $(k - 1\,) = 0$ AND 2 =0

Hence no solution exists if $k=1$. Although, $2 \neq 0$, so the case where we have infinitely many solutions does not exist for this system.

c)

Where:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad \overrightarrow{e_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \overrightarrow{e_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \overrightarrow{e_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

i)

$$\therefore \mathrm{A}\overrightarrow{e_1} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} . \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ d \\ g \end{bmatrix}$$

$$\therefore \mathrm{A}\overrightarrow{e_2} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} . \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ e \\ h \end{bmatrix}$$

$$\therefore \mathrm{A}\overrightarrow{e_3} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} . \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} c \\ f \\ i \end{bmatrix}$$

ii)

Let matrix B be defined as:

$$B = \begin{bmatrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \\ b_7 & b_8 & b_9 \\ \cdots & \cdots & \cdots \\ b_{n1} & b_{n2} & b_{n3} \end{bmatrix}$$

Assume that $\overrightarrow{v_3} = \overrightarrow{c_3}$, $\overrightarrow{v_2} = \overrightarrow{c_2}$, and $\overrightarrow{v_1} = \overrightarrow{c_1}$

Also assume that $\overrightarrow{c_3} = \overrightarrow{e_3}$, $\overrightarrow{c_2} = \overrightarrow{e_2}$, and $\overrightarrow{c_1} = \overrightarrow{e_1}$

So replace the first, second or third column respectively in matrix B with the vectors $\overrightarrow{e_1}$, $\overrightarrow{e_2}$, $\overrightarrow{e_3}$ respectively.

$$\therefore \mathrm{B}\overrightarrow{e_1} = \begin{bmatrix} a & b_2 & b_3 \\ d & b_5 & b_6 \\ g & b_8 & b_9 \\ \dots & \dots & \dots \\ b_{n1} & b_{n2} & b_{n3} \end{bmatrix}$$

$$\therefore \mathrm{B}\overrightarrow{e_2} = \begin{bmatrix} b_1 & b & b_3 \\ b_4 & e & b_6 \\ b_7 & h & b_9 \\ \dots & \dots & \dots \\ b_{n1} & b_{n2} & b_{n3} \end{bmatrix}$$

$$\therefore \mathrm{B}\overrightarrow{e_3} = \begin{bmatrix} a & b_2 & c \\ d & b_5 & f \\ g & b_8 & i \\ \dots & \dots & \dots \\ b_{n1} & b_{n2} & b_{n3} \end{bmatrix}$$

Question 3

$u = (0,3,0)\ v = (1,0,4)\ w = (2,4,0)$

a)

$$2\bar{u} - 2\bar{v} = 2(0,3,0) - 2(1,0,4)$$
$$= (0,6,0) - (2,0,8)$$
$$= (-2,6,-8)$$

b)

$||2\bar{u} + 3\bar{v} - \bar{w}||$

$\big||2(0,3,0) + 3(1,0,4) - (2,4,0)|\big|$
$\therefore \sqrt{2(0,3,0) + 3(1,0,4) - (2,4,0)}$
$= \sqrt{(0,6,0) + (3,0,12) - (2,4,0)}$ ***absolute value norm***
$= \sqrt{(0+3-2,\ 6+0-4,\ 0+12-0)}$
$= (1,2,12)$
$= \sqrt{1^2 + 2^2 + 12^2}$
$= \sqrt{1 + 4 + 144}$
$= \sqrt{149}$

c)

Distance between $-3\bar{u}$ and $\bar{w} - 4\bar{v}$

$\therefore -3\bar{u} = -3(0,3,0)$

$= (0,-9,0)$

And $\bar{w} - 4\bar{v} = (2,4,0) - 4(1,0,4)$

$= (2,4,0) - (4,0,16)$

$= (2-4, 4-0, 0-16)$

$= (-2,4,-16)$

Therefore the distance between $-3\bar{u}$ and $\bar{w} - 4\bar{v}$:

$= ||terminal\ point - initial\ point||$
$= \sqrt{(-2-0)^2 + (4--9)^2 + (-16-0)^2}$
$= \sqrt{(-2)^2 + (-13)^2 + (-16)^2}$
$= \sqrt{4 + 169 + 256}$
$= \sqrt{429}$

d)

$Proj_{\bar{v}}\bar{w}$
Calculate $\bar{v}$:

$\therefore \bar{v} = \sqrt{1^2 + 0^2 + 4^2}$
$\therefore \bar{v} = \sqrt{1 + 0 + 16}$
$\therefore \bar{v} = \sqrt{17}$

$Proj_{\bar{v}}\bar{w}$

$$= \frac{\bar{v}.\bar{w}}{|\bar{v}|}$$

$$= \frac{(1,0,4).(2,4,0)}{\sqrt{17}}$$

$$= \frac{(1.2+0.4+4.0)}{\sqrt{17}}$$

$$= \frac{2}{\sqrt{17}}$$

e)

The area of the parallelogram bounded by $\bar{v}$ and $\bar{w}$:

Area = $\frac{1}{2}||\bar{v} \times \bar{w}||$

$\bar{v} \times \bar{w} = \det \begin{vmatrix} 1 & 0 & 4 \\ 2 & 4 & 0 \end{vmatrix}$

$= +\begin{vmatrix} 0 & 4 \\ 4 & 0 \end{vmatrix} x - \begin{vmatrix} 1 & 4 \\ 2 & 0 \end{vmatrix} y + \begin{vmatrix} 1 & 0 \\ 2 & 4 \end{vmatrix} z$

= (0×0– 4x4) x - (1×0-4×2) y + (1x4-0x2) z
=-(16) x – (8)y + (4)z
=(-16,-8,4)

$\therefore$ Area $= \frac{1}{2}\sqrt{(-16)^2 + (-8)^2 + 4^2}$
Area = $\frac{1}{2}\sqrt{336}$

f)

The equation of the plane parallel to $\bar{v}$ and $\bar{w}$ and passing through the tip of $\bar{u}$

From 3(e) above, $(\bar{v} \times w)$ = (-16,-8,4)

As an equation**:** $(-16x - 8y + 4z)$

Let $Q(x, y, z)$ be an arbitrary point on the plane

$\therefore \bar{u}Q = terminal\ point - initial\ point$
$= (x - 0, y - 3, z - 0)$

$\bar{u}Q$ is parallel to the plane and perpendicular to the cross product $\therefore$ dot product $= 0$
$\therefore (x - 0, y - 3, z - 0)$. (-16,-8,4) = 0
$(x).(-16) + (y - 3).(4) + (z - 0).(1) = 0$
$-16x + 3y - 12 + z$ = 0
$-16x + 3y + z$ = 12

Question 4

Let $z_1 = 2 + i\sqrt{3}$ and $z_2 = 2 - i\sqrt{3}$

z_1 and z_2 are in the form $z = a + bi$

z_1 and z_2 are complex numbers

$\therefore$ Polar form of a complex number is: $Z = r(\cos\theta + isin\,\theta)$

i)

$r = |z_1|$

$= \sqrt{a^2 + b^2}$

$= \sqrt{2^2 + (\sqrt{3})^2}$

$= \sqrt{4+3}$

$= \sqrt{7}$

Where a > 0, $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\therefore \theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\therefore \theta = arctan\left(\frac{b}{a}\right)$

$\therefore \theta = arctan\left(\frac{\sqrt{3}}{2}\right)$

$\therefore$ Polar form: $Z_1 = r\left[\cos\left(arctan\left(\frac{\sqrt{3}}{2}\right)\right) + isin\left(arctan\left(\frac{\sqrt{3}}{2}\right)\right)\right].$

ii)

$r = |z_2|$

$= \sqrt{a^2 + b^2}$

$= \sqrt{2^2 + (-\sqrt{3})^2}$

$= \sqrt{4+3}$

$= \sqrt{7}$

Where a > 0, $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\therefore \theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\therefore \theta = arctan\left(\frac{b}{a}\right)$

$\theta = arctan\left(\frac{-\sqrt{3}}{2}\right)$

$\theta = -\,arctan\left(\frac{\sqrt{3}}{2}\right)$

$\therefore$ Polar form: $Z_2 = r\left[\cos\left(-arctan\left(\frac{\sqrt{3}}{2}\right)\right) + isin\left(-\,arctan\left(\frac{\sqrt{3}}{2}\right)\right)\right]$

iii)

Show that $Z_1 . Z_2 = 7$

But both Z_1 and Z_2 are complex numbers in the form $Z = r(\cos\theta + isin\,\theta)$

$Z_1.Z_2 = r\left[\cos\left(arctan\left(\frac{\sqrt{3}}{2}\right)\right) + isin\left(\ arctan\left(\frac{\sqrt{3}}{2}\right)\right)\right].r\left[(\cos\left(-arctan\left(\frac{\sqrt{3}}{2}\right)\right) + isin\left(-arctan\left(\frac{\sqrt{3}}{2}\right)\right)\right]$

Simplify, let $\theta\ =\ arctan\left(\frac{\sqrt{3}}{2}\right)$:

$\therefore Z_1.Z_2 = r[\cos(\theta) + isin\,(\theta)].r[(\cos(-\theta) + isin\,(-\theta)]$

$Z_1.Z_2 = r[\cos(\theta) + isin\,(\theta)].r[(\cos(\theta) -isin\,(\theta)]$

$Z_1.Z_2 = r^2[\cos^2(\theta) + isin\,(\theta)\ (\cos(\theta) - isin(\theta)\ cos\theta - i^2 sin^2\,(\theta)]$

$Z_1.Z_2 = r^2[\cos^2(\theta) - i^2 sin^2\,(\theta)]$ but $i^2\ =\ -1$

$Z_1.Z_2 = r^2[\cos^2(\theta) + sin^2\,(\theta)]$ ***Pythagorean identity:*** $\cos^2(\theta) + sin^2\,(\theta) = 1$

$\therefore Z_1.Z_2 = r^2$

Where $r = \sqrt{7}$

$\therefore Z_1.Z_2 = \left(\sqrt{7}\right)^2$

$\therefore Z_1.Z_2 = 7$

iv)

Determine the modulus of $\frac{Z_1}{Z_2}$

But both Z_1 and Z_2 are complex numbers in the form $Z = r(\cos\theta + isin\,\theta)$

modulus of $Z_1 = |Z_1|$
$\therefore |Z_1| =\ \sqrt{a^2 + b^2}$
$\therefore |Z_1| =\ \ \sqrt{7}$

modulus of $\frac{Z_1}{Z_2}$
$\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$, where $Z_1 \neq 0$ ***Property of conjugate***
$\therefore \frac{|Z_1|}{|Z_2|} = \frac{\sqrt{7}}{\sqrt{7}}$
$\therefore \frac{|Z_1|}{|Z_2|} = 1$

(b)

Use the Moivre’s theorem to derive a formula for the 4th roots of 8.

DeMoivre's formula

$n^{th}\ roots = r^{\frac{1}{n}}.\cos\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right) + isin\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right)$ where $0 \leq k \leq n-1$

$\therefore$

$$= \sqrt{8}^{\frac{1}{4}}.\cos\left(\frac{\pi}{4}+\frac{2\pi k}{4}\right)+isin\left(\frac{\pi}{4}+\frac{2\pi k}{4}\right)$$
$$= 8^{\frac{1}{8}}.\cos\left(\frac{\pi}{4}+\frac{2\pi k}{4}\right)+isin\left(\frac{\pi}{4}+\frac{2\pi k}{4}\right)$$

That is:

$$k=0:\ 8^{\frac{1}{8}}.\cos\left(\frac{\pi}{4}+\frac{2\pi(0)}{4}\right)+isin\left(\frac{\pi}{4}+\frac{2\pi(0)}{4}\right)$$
$$=\ 8^{\frac{1}{8}}.\left(\frac{\sqrt{2}}{2}\right).i.\left(\frac{\sqrt{2}}{2}\right)$$
$$=\frac{8^{\frac{1}{8}}}{2}\ i$$

$$k=1:\ 8^{\frac{1}{8}}.\cos\left(\frac{\pi}{4}+\frac{2\pi(1)}{4}\right)+isin\left(\frac{\pi}{4}+\frac{2\pi(1)}{4}\right)$$
$$=\ 8^{\frac{1}{8}}.\left(-\frac{\sqrt{2}}{2}\right).i.\left(-\frac{\sqrt{2}}{2}\right)$$
$$=-\frac{8^{\frac{1}{8}}}{2}\ i\ =\frac{1}{2^{\frac{5}{8}}}\ i$$

$$k=2:\ 8^{\frac{1}{8}}.\cos\left(\frac{\pi}{4}+\frac{2\pi(2)}{4}\right)+isin\left(\frac{\pi}{4}+\frac{2\pi(2)}{4}\right)$$
$$=\ 8^{\frac{1}{8}}.\left(-\frac{\sqrt{2}}{2}\right).i.\left(\frac{\sqrt{2}}{2}\right)$$
$$=-\frac{8^{\frac{1}{8}}}{2}\ i\ =-\frac{1}{2^{\frac{5}{8}}}\ i$$

$$k=3:\ 8^{\frac{1}{8}}.\cos\left(\frac{\pi}{4}+\frac{2\pi(3)}{4}\right)+isin\left(\frac{\pi}{4}+\frac{2\pi(3)}{4}\right)$$
$$=\ 8^{\frac{1}{8}}.\left(\frac{\sqrt{2}}{2}\right).i.\left(-\frac{\sqrt{2}}{2}\right)$$
$$=-\frac{8^{\frac{1}{8}}}{2}\ i\ =-\frac{1}{2^{\frac{5}{8}}}\ i$$
