Question 1

a)

$f(x) = x^2 - \ln x^8$, where $x > 1$

$f(x) = x^2 - 8\ln x$

$f'(x) \;\; = \frac{d}{dx}\left(x^2 - 8\ln x\right)$

$f'(x) \;\; = 2x - \frac{8}{x}$

Product rule $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{d}{dx}(8\ln x)$ $\quad u = 8 \quad$ v$= \ln x$

$du = 0 \quad dv = \frac{1}{x}$

$= 8\left(\frac{1}{x}\right) + \ln x\,(0)$

$= \frac{8}{x}$

b)

Critical point

$f'(c) = 0$ or $f'(c) = undefined$

$0 = 2x - \frac{8}{x}$

$0 = 2x^2 - 8$

$0 = x^2 - 4$

$0 = (x-2)(x+2)$

Only defined for $\ln g(x)$ where $g(x) > 0$

$x = 2$

$f(1) = (1)^2 - \ln(1)^8$

$f(1) = 1 - 0$

$f(1) = 1$

The local extreme point is $(2,1)$

c)

$f''(x) \;\; = \frac{d}{dx}\left(2x - \frac{8}{x}\right)$

$f''(x) \;\; = 2 + \frac{8}{x^2}$

Concavity

$f''(c) = 0$ or $f''(c) = undefined$

Quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \left(\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right)$ $if\ v \neq 0$

$\frac{d}{dx}\left(\frac{8}{x}\right)$ $\quad v = x \quad u = 8$

$v' = 1 \quad u' = 0$

$= \frac{1(0) - 8(1)}{x^2}$

$= -\frac{8}{x^2}$

$0 = \; 2 + \frac{8}{x^2}$

$0 = \; 2x^2 + 8$

$0 = \; x^2 + 4$

$x^2 = -4$

$x = \sqrt{-4}$

$undefined\ at\ 0$. Therefore, no inflection point exists for the function

$f''(-1) \;\; = 2 + \frac{8}{(-1)^2} = 10$ $\qquad f''(1) \;\; = 2 + \frac{8}{(1)^2} = 10$

The function is positive where $x < 0$ or $x > 0$. Therefore, it is concave up where $x < 0$ or $x > 0$

Question 2

Let x be the length of the poster
Let y be the width of the poster
Therefore, the area of the poster is defined by: $A = X.Y$

$A = (x - 4 - 4)(y - 2 - 2)$
$50 = (x - 8)(y - 4)$
$y = 4 + \frac{50}{x-8}$
$A = X.Y$
$A = x.\left(4 + \frac{50}{x-8}\right)$

Product Rule
$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
$A' = x\left(-\frac{50}{(x-8)^2}\right) + (1)\left(4 + \frac{50}{x-8}\right)$
$A' = 4 + \frac{-50x}{(x-8)^2} + \frac{50}{x-8}$
$A' = 4 + \frac{-50x}{(x-8)^2} + \frac{50x-400}{(x-8)^2}$
$A' = 4 - \frac{400}{(x-8)^2}$

Quotient Rule

$\frac{d}{dx}\left(\frac{u}{v}\right) = \left(\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right)$ if $v \neq 0$

$\frac{d}{dx}\left[4 + \frac{50}{x-8}\right]$

$\frac{d}{dx}\left[4 + 50.\frac{1}{x-8}\right]$

$50.\left(\frac{1}{x-8}\right)$

$v = x - 8$ $\quad$ $u = 1$

$v' = 1$
$u' = 0$

$50.\left(\frac{(x-8)(0)-(1)(1)}{(x-8)^2}\right)$

$50.\left(\frac{-1}{(x-8)^2}\right)$

Critical point
$f'(c) = 0$ or $f'(c) = undefined$
$0 = 4 - \frac{400}{(x-8)^2}$
$4 = \frac{400}{(x-8)^2}$
$4(x-8)^2 = 400$
$(x-8)^2 = 100$
$x - 8 = 10$
$x = 18$

$A'' = \frac{800}{(x-8)^3}$

Concavity
$f''(c) = 0$ or $f''(c) = undefined$
$0 = \frac{800}{(x-8)^3}$
$0 = 800$

$undefined$

Therefore $x = 18$ is the absolute minimum

$y = 4 + \frac{50}{x-8}$

$y = 4 + \frac{50}{18-8}$

$y = 9$

Therefore, the length of the poster is $18\,cm$
Therefore, the width of the poster is $9\,cm$

Question 3

a)

$$\lim_{x \to 0} \frac{1 - \cos^3 x}{\sin^2 x}$$

L'Hôpital's Rule $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

$f'(x) = \frac{d}{x}(1 - \cos^3 x)$ $= \frac{d}{x}(1) - \frac{d}{x}(\cos^3 x)$ $= 0 - 3\cos^2 x(-\sin x)$ *chain rule* $= 3\cos^2 x \sin x$	$g'(x) = \frac{d}{x}(\sin^2 x)$ $= 2 \sin x \cos x$ *chain rule*

$$= \lim_{x \to 0} \frac{3\cos^2 x \sin x}{2 \sin x \cos x}$$

$$= \lim_{x \to 0} \frac{3}{2} \cos x$$

$$= \frac{3}{2} \cos(0)$$

$$= \frac{3}{2}$$

b)

$$\lim_{x \to \infty} \left(\cos \frac{1}{x}\right)^x$$

L'Hôpital's Rule $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

$f'(x) =$	$g'(x) =$

$=$

c) $\lim_{x\to\infty} \frac{x \ln x}{x^2-1}$

L'Hôpital's Rule $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

$f'(x) = x \ln x$ $= (x)\left(\frac{1}{x}\right) + (\ln x)(1)$ *product rule* $= 1 + \ln x$	$g'(x) = x^2 - 1$ $= 2x$

$= \lim_{x\to\infty} \frac{1+\ln x}{2x}$

L'Hôpital's Rule $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

$f'(x) = 1 + \ln x$ $= \frac{1}{x}$	$g'(x) = 2x$ $= 2$

$= \lim_{x\to\infty} \frac{\frac{1}{x}}{2}$

$= \frac{1}{2} \lim_{x\to\infty} \frac{1}{x}$

$= \frac{1}{2} . 0$

$= 0$

d) $\lim_{x\to 1} \frac{x \ln x}{x^2-1}$

$f'(x) = x \ln x$ $= (x)\left(\frac{1}{x}\right) + (\ln x)(1)$ *product rule* $= 1 + \ln x$	$g'(x) = x^2 - 1$ $= 2x$

$= \lim_{x\to 1} \frac{1+\ln x}{2x}$

L'Hôpital's Rule $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

$f'(x) = 1 + \ln x$ $= \frac{1}{x}$	$g'(x) = 2x$ $= 2$

$= \lim_{x\to 1} \frac{\frac{1}{x}}{2}$

$= \frac{1}{2} \lim_{x\to 1} \frac{1}{x}$

$= \frac{1}{2} . \frac{1}{1}$

$= \frac{1}{2}$

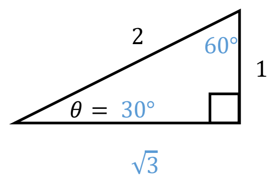
Question 4

$= tan\left(sin^{-1}\left(\frac{1}{2}\right)\right)$

Reference Triangle:

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$ $30°$

$\sin\theta = \frac{1}{2}$ *pythagoras theorem* $b = \sqrt{2^2 - 1^2} = \sqrt{3}$



Unit Circle: