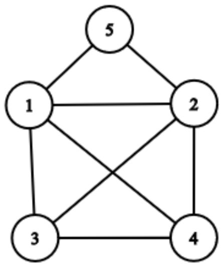
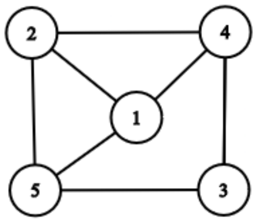
Question 1

a)

(i) 3,3,3,3,2

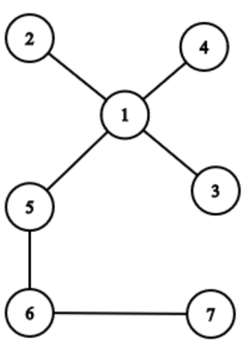


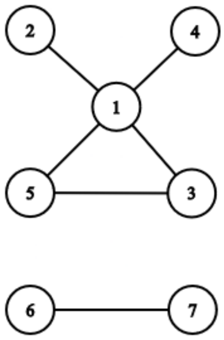
(ii) 3,3,3,2,2

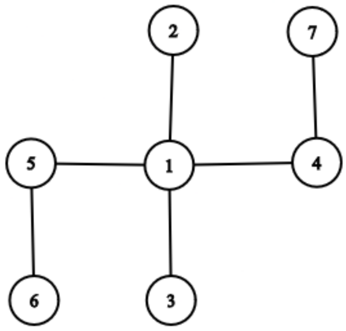


b)

1,1,1,1,2,2,4







c)

Number of Edges

$|E(G)| + |E(G)'| = \frac{n(n-1)}{2}$

$\Rightarrow 21 + |E(G)'| = \frac{10(10-1)}{2}$

$\Rightarrow |E(G)'| = \frac{10(10-1)}{2} - 21$

$\Rightarrow |E(G)'| = 45 - 21$

$\therefore\ |E(G)'| = 24$

Therefore, the number of edges of the compliment of G is 24

Question 2
a)
Let f be a bijective function from G to H
Let the correspondence between the graphs be

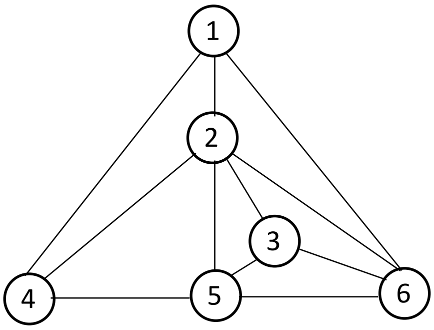
1	2	3	4	5	6
a	b	c	d	e	f

Therefore, the pair of graphs G and H are isomorphic
b)
Let f be a bijective function from G to H
Let the correspondence between the graphs be

1	2	3	4	5	6
a	d	b	c	f	e

Therefore, the pair of graphs G and H are isomorphic

Question 3



Euler's Formula for Planar Graphs
$v - e + f = 2$
$\Rightarrow 6 - 11 + f = 2$
$\therefore f = 7$
By Euler’s formula, the graph G has 7 faces which corresponds to the planar graph drawn above
Therefore, G is planar.

Question 4

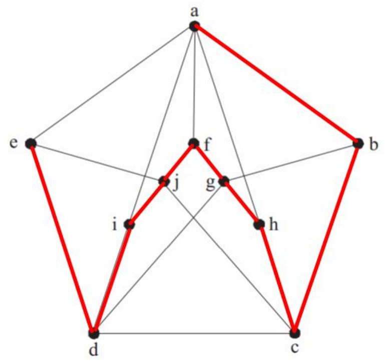
Handshake Lemma
The sum of degree of all vertices of a graph is twice the size of graph.
$\sum deg_{(v_i)} = 2|E|$

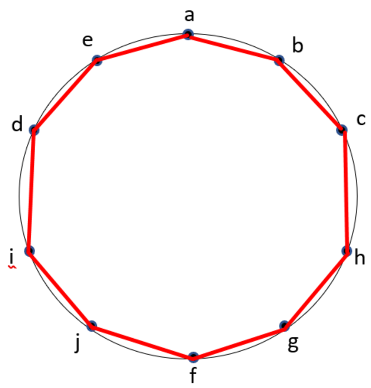
Assume that there exists a planar graph with all vertices having degree at least 6
Then:
$\sum deg_{(v_i)} = 2|E|$
$\Rightarrow 2|E| = \sum deg_{(v_i)}$
$\Rightarrow 2E \geq 6V$
$\Rightarrow E \geq 3V$

If G is planar, then we know that $E \leq 3V - 6$.
The graph G would have at least 3 vertices
Thus
$3V \leq 3V - 6$
$\Rightarrow 0 \leq -6$ which is a contradiction,
Thus, every planar graph has a vertex of degree at most s

Question 5

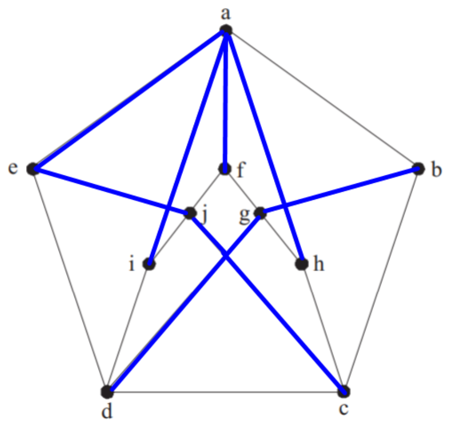
Step 1: Find a circuit that contains all the vertices of our graph
Hamiliton circuit: a-b-c-h-g-f-j-i-d-e

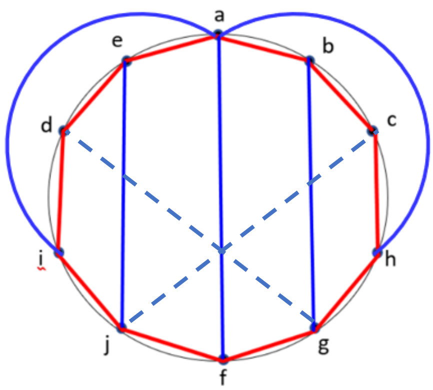




(draw it as a large circle)

Step 2: The remaining non-circuit edges, called chords, must be drawn either inside or outside the circle in a planar drawing.





Using inside-outside symmetry:
The edges af, ej and bj and ae are drawn inside
The edges ai and ah must therefore be drawn outside
The edges dg and cj are impossible to draw
Therefore, the graph is not planar

$K_{3,3}$ configuration