[Question 1]

Which of the following statements about the function is true?

1.The function has no relative extremum.

2.The graph of the function has one point of inflection and has two relative extrema.

3.The graph of the function has two points of inflection and has one extremum.

4.The function has a singular point at

5.The function has a maximum point at

derivative:

f'(x)=0:

roots:

and

f''(x)=0:

Second derivative always positive for

Second derivative always negative

Relative extremum:

Relative minimum at

Relative maximum at

inflection points:

Second derivative always negative

inflection point at

maximum or minimum points:

(check the sign changes of the first derivative around the critical points)

Relative maximum at

[Question 2]

True or false: To have an idea on whether we should apply the bisection method to determine the root of in a given interval, we may apply the function to the endpoints of the given interval and check the sign of the corresponding outputs

[Question 3]

Suppose we wish to develop an iterative method to compute the square root of the equation for a given number .

Consider the iterative scheme where and the case where . Which of the following statements is true?

1. is locally convergent

2. would have been locally convergence if were continuous and differentiable in an interval that include

3. The interval of convergence where contains

4. is not continuous

5. None of the above is true

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represents a function of . In this case, it's a quadratic function.

represents the square of the variable .

is a constant.

It's a specific value for which we want to find the square root.

In other words, we want to find the value of such that .

So, acts as the value whose square root we're trying to compute.

When solving equations like , we're essentially finding the value(s) of where the function equals zero. In this case, it's finding the value(s) of such that , which means finding the square root of .

The iterative method proposed suggests an iterative process to approximate the square root of . In each iteration, we update the value of based on the previous value until we converge to a solution.

**[1] Locally convergent:**

To determine if is locally convergent, we typically analyze the behaviour of its derivative near the root.

derivative:

Is polynomial function, which is continuous and differentiable everywhere

evaluate at the root:

root -> value(s) of where the function equals zero

-> finding the square root of .

condition for local convergence:

(Solving this inequality gives interval where iteration is convergent around the root )

[2] evaluate specific root

evaluate :

condition for local convergence:

**[1] is locally convergent inconclusive**

**[2] derivative is continuous and differentiable everywhere.**

**does not meet confirm local convergence specifically at root**

**[3] does not meet confirm local convergence specifically at root**

**[4] derivative is continuous and differentiable everywhere.**

[Question 4]

Which of the following is true of the iterative scheme for the root equation , where is a number?

1. The convergence of is not guaranteed because the interval of convergence where does not contain .

2. is not continuous

3. The convergence of is not guaranteed because is not continuous.

4. The convergence of is not guaranteed because is not continuous.

5. None of the above is true

**[1] Locally convergent:**

To determine if is locally convergent, we typically analyze the behaviour of its derivative near the root.

derivative:

is a rational function, which is continuous and differentiable everywhere in their domain except where the denominator is zero.

is continuous and differentiable for all real numbers and for any real number .

**[1] is locally convergent inconclusive**

**[2] derivative is continuous and differentiable everywhere.**

**[3] derivative is continuous and differentiable everywhere.**

**[4] derivative is continuous and differentiable everywhere.**

[Question 5]

Let and be fixed-point iterative schemes for the root equation . The fixed-point function, 𝑔3(𝑥) below given by Newton's method is:

1.

2.

3.

4.

5. None of the above is true

[Question 6]

Consider the nonlinear equation , which has a root in the interval [0,1].

Which of the following statements is FALSE?

1. The fixed-point formula converges to the approximate solution if the initial approximation .

2. The minimum number of iterations required to approximate the root by the bisection method correct to is 26.

3. Newton's method with will converge to the approximate solution after at most 4 iterations.

4. Newton's method with will converge to the approximate solution after exactly 4 iterations

5. Both 1. and 4. are correct

derivative:

Octave

f: x\*e^x-2

f\_derivative: e^x\*(1+x)

initial\_guess: 1

tolerance: 1e-6

max\_iterations: 100

-------------------------------------------------------- iter: 0

[2] Calculate the midpoint:

x\_0 = 1

[3] Evaluate f(c) & f'(c)

f(x\_0): 0.718282

f'(x\_0): 5.43656

x\_1: 0.867879

[4] Evaluate f(c)

if f(x\_0) then x\_0 is an exact root

-------------------------------------------------------- iter: 1

[2] Calculate the midpoint:

x\_0 = 0.867879

[3] Evaluate f(c) & f'(c)

f(x\_0): 0.0671627

f'(x\_0): 4.44902

x\_1: 0.852783

[4] Evaluate f(c)

if f(x\_0) then x\_0 is an exact root

-------------------------------------------------------- iter: 2

[2] Calculate the midpoint:

x\_0 = 0.852783

[3] Evaluate f(c) & f'(c)

f(x\_0): 0.000773091

f'(x\_0): 4.34694

x\_1: 0.852606

[4] Evaluate f(c)

if f(x\_0) then x\_0 is an exact root

-------------------------------------------------------- iter: 3

[2] Calculate the midpoint:

x\_0 = 0.852606

[3] Evaluate f(c) & f'(c)

f(x\_0): 1.05842e-07

f'(x\_0): 4.34575

x\_1: 0.852606

[4] Evaluate f(c)

if f(x\_0) then x\_0 is an exact root

[5] Repeat steps 3-4 until convergence, stop

no. of iterations: 4

A solution is: 0.852606

[Question 7]

Consider again the nonlinear equation .

Applying the regula falsi method, with starting points and yields the following result:

1. 0.8525512067 after at least three iterations

2. 0.8511838582 after at most three iterations

3. 0.8526048659 after at least four iterations

4. 0.8526053068 after exactly four iterations

5. 0.8526055020 after at least 12 iterations

[Question 8]

Consider now the function which has four distinct roots. Which of the following statements is FALSE?

1. 𝑓′(𝑥) has a local minimum at 1.5408

2. has a local maximum at −0.5408

3. has no singularities and no obvious symmetries and the 𝑦 -intercept is −5

4. has a local maximum at

5. the two zeros of are −0.5408 and 1.5408

[Question 9]

Choose the appropriate one of the options below:

1. The secant method and Muller's method are similar in the sense that they both start with two points.

2. The secant method yields a complex root even when the initial approximation is a real number.

3. Muller's method determines the next approximation by considering the intersection of a parabola and the $x$-axis through three given points.

4. All of the above statements.

5. None of the above statements.

The Secant method and Muller's method both use multiple points, but their strategies for choosing the next approximation differ. The Secant method uses linear interpolation between two points, while Muller's method uses quadratic interpolation through three points.

The Secant method can yield a complex root if the initial approximations bracket a complex root. However, if the initial approximations are real and the function is real-valued, the Secant method typically converges to a real root.

Muller's method determines the next approximation by fitting a parabola through three given points, but it does not necessarily intersect the x-axis. The next approximation is found by solving the quadratic equation and selecting the root closest to one of the three given points.

Therefore, none of the statements accurately describe the characteristics of the Secant method and Muller's method.

[Question 10]

Applying Muller's method to compute the zeros of yields the following result:

1. with the starting point , and

2. with starting points ,and

3. with starting points ,𝑥0=1.5 and

4. with starting points ,and

5. None of the above is true