Question 1

Consider the following data:

|  |  |  |
| --- | --- | --- |
| x | f(x) | f'(x) |
| 0.1 | −0.62049958 | 3.58502082 |
| 0.2 | −0.28398668 | 3.14033271 |
| 0.3 | 0.00660095 | 2.66668043 |
| 0.4 | 0.24842440 | 2.16529366 |

(1.1) Find an approximation to f(0.27) using the following forms of interpolating polynomial

(a) the Lagrange form.

[1] Calculate :

For each i:

[2] Lagrange polynomials, evaluated at the point x = 0.27

For each i:

[3] Interpolated polynomial, evaluated at x = 0.27

(b) the Newton forward divided difference form.

[1] First-order divided differences

[2] Second-order divided differences

[3] Third-order divided differences

[4] Newton forward divided difference form of the interpolating polynomial

(c) the Hermite form.

|  |  |  |
| --- | --- | --- |
| x | f(x) | f'(x) |
| 0.1 | −0.62049958 |  |
|  | 3.58502082 |  |
| 0.2 | −0.28398668 |  |
|  | 3.14033271 |  |
| 0.3 | 0.00660095 |  |
|  | 2.66668043 |  |
| 0.4 | 0.24842440 |  |
|  | 2.16529366 |  |

[1] Divided differences

[2] Hermite Interpolated polynomial, evaluated at x = 0.27

[3] Interpolated polynomial, evaluated at x = 0.27

Question 2

*Consider the following data*

|  |  |
| --- | --- |
| x | f(x) |
| 0.3 | -1.1518 |
| -0.4 | 0.7028 |
| 0.5 | -1.4845 |
| 0.00 | 0.13534 |

(2.1) Use a third degree Lagrange interpolating polynomial to approximate f(0.55).

[1] Calculate :

For each i:

[2] Lagrange polynomials, evaluated at the point x = 0.27

For each i:

(2.2) Use a Newton’s divided-difference polynomial that interpolates all the points to approximate f(0.2), using the following criteria:

(a) Without rearranging the nodes;

[1] First divided differences

[2] Second Divided Differences

[3] Third Divided Differences

[4] Newton Interpolated polynomial, evaluated at x = 0.2

(b) Rearranging the nodes in increasing order.

(2.3) Compare the results obtained in (2.2) above.

(2.4) Use the least-squares polynomial of degree two to approximate f(0.2) and compute the error.

(Your system of normal equations must be explicit).

[1] Given the polynomial , a, b, and c

[2] Calculate sums:

​

[3] solve system

[4] matrix form,

Where

Thus,

[5] polynomial

(2.5) Use the least-squares function of the form

to approximate f(0.2) and compute the error.

(2.6) Plot the graphs of the approximating polynomials

in (2.2) -(2.4).

Your graphs must be proper computer produced graphs.

Question 3 [15 marks]

Construct the natural cubic spline for the data below and use it to approximate f(0.3):

(−0.5, 5), (0, 15), (0.5, 9)

[1] cubic polynomials for each interval

|

|

[2] natural cubic spline

|  |  |
| --- | --- |
|  | = |
|  | 5 |
|  | 15 |
|  | 15 |
|  | 9 |
|  |  |
|  |  |
|  | 0 |
|  | 0 |

[3] Equations for cubic spline

where

where

**in the interval [-0.5, 0]**

**in the interval [0, 0.5]**

Thus,

**in the interval [-0.5, 0]**

**in the interval [0, 0.5]**

Thus,

[3] Solve system of equations

**where**

**where**

**where**

**where**

**where**

**Where**

**Where**

**Where**

**Where**

**Where**

**Where**

**Where**

**Where**

[4] Cubic splines

[5] f(0.3)

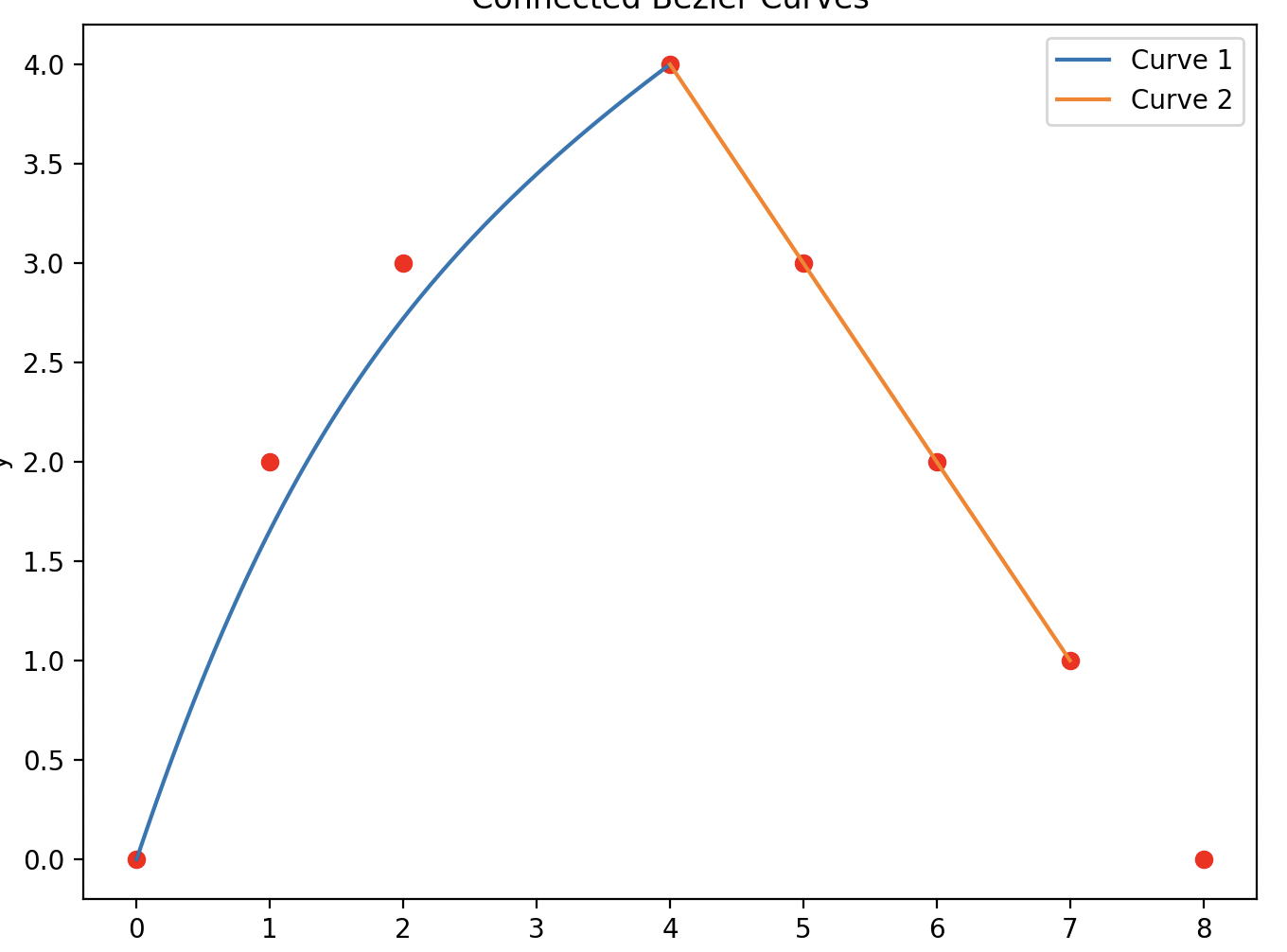
Question 4 [20 marks]

Consider the following set of data points in the table below:

(4.1) Using guidepoints of your choice from the data set, construct the connected Bezier curve

from the set of points.

(Hint: Divide the set of points into three parts)



(4.2) Draw the connected Bezier polynomial.

*(4.3) Why is the graph smoothly connected at points 3 and 6?*