Question 1

Consider the following data

|  |  |
| --- | --- |
| x | f(x) |
| 2.3 | -8.1066 |
| 2.7 | -17.7949 |
| 3.1 | -29.7652 |
| 3.5 | -40.1506 |

* 1. Use the following difference formulas to approximate f′(2.7)

a) the forward difference formula;

Where

b) the central difference formula

Where

c) the 3-point endpoint formula.

Where

(1.2) Compute f ′′(3.1) using the second derivative midpoint formula.

Where

(1.3) The above data was generated using the function . Use the Richardson’s extrapolation process to determine

, an approximation of f′(2.7) of order ,

using is approximated by the three-point midpoint formula.

**First Approximation ​:**

[1] Where

Calculate f(2.9)

3-point endpoint formula.

Thus,

[2] Where

Given:

**First-Level Richardson's Extrapolation** **​:**

[1] Richardson's extrapolation

**Second -Level Richardson's Extrapolation** **​:**

[1] Find, Where

Where

Calculate f(2.6)

Calculate f(2.8)

[2] 3-point endpoint formula.

[3] Richardson's extrapolation

[4] Find, Where

Given:

Question 2

The integral

(1.1) Use the following Newton-Cotes methods to approximate I:

a) Simpson’s rule;

[1] Calculate no of subintervals

Let

thus,

[2] Evaluate , ,

[3] Approximation

b) Simpson’s rule;

[1] Calculate no of subintervals

Let

thus,

[2] Evaluate , ,  ,

[3] Approximation

c) composite trapezoidal rule with h = 0.1

[1] Calculate no of subintervals

Let

thus,

[2] Evaluate , ,  ,

[3] Approximation

(1.2) Compute the integral analytically and determine the actual error in the approximations obtained in (1.1) above.

[1] Integration by Parts

[2] Simpson’s rule;

Error

[3] Simpson’s rule;

Error

[4] Composite Trapezoidal Rule;

Error

(1.3) Approximate using the three-point Gaussian quadrature scheme

[1] Abscissas

Legendre polynomial of degree 3

[2] Weights

[3] Change of Interval

Over Interval [1, 1.5]

Thus,

[4] Transformed Interval

[5] Three-point Gaussian quadrature rule

[6] Where

[7] Where

[8] Where

[9] Apply weights