MAT2611

Student Number 59448873

Assignment 1

**Problem 1**

**Addendum A - Exercise 2.21**

1. Determine

2. Determine

3. Determine

Remember that:  
 *number of elements in*

*number of elements in the empty set*

*number of elements in the powerset*

*number of elements in the empty set*

1.

2.

3.

**Addendum A - Exercise 3.4**

For each of the following functions determine the image of

1. defined by .

2. defined by .

3. defined by .

*or*

1.

Therefore, the Image of is

2.

Therefore, the Image of is

3.

Therefore, the Image of is

**Problem 2**

**Addendum A - Exercise 3.11**

Consider the following two functions. Prove that both and are one-to-one correspondences.

1. defined by

1. defined by

Let be a one-to-one correspondence. Then to each there corresponds a unique such that . We define by

the unique such that

A function is said to be one-to-one correspondence if and only if is both:

Injective (one-to-one): and,

Surjective (ONTO): for all there is some such that

1. **Injectivity:**

Take and assume that

Thus

And

So

We have shown if then . Therefore is one-to-one, by definition of one-to-one.

**Surjectivity:**

We need to find an that maps to .

Suppose ;

Now we solve for in terms of .

We find

Proof:

Let be any element of .

Thus, we have found an such that

2. **Injectivity:**

Take and assume that

Thus

And

So

We have shown if then . Therefore is one-to-one, by definition of one-to-one.

**Surjectivity:**

We need to find an that maps to .

Suppose ;

Now we solve for in terms of .

We find

Proof:

Let be any element of .

Thus, we have found an such that

**Addendum A - Exercise 3.12**

Let be a one-to-one correspondence.

1. Prove that is a function.

2. Prove that is a one-to-one.

3. Prove that is onto.

4. Conclude that is a one-to-one correspondence.

1. Let and be nonempty sets.

function is said to be invertible if it has an inverse function.

Proof:

Suppose is an invertible function.

Then= a for every ;

= b for every ;

2. **Injectivity:**

A function is said to be one-to-one if

In other words, there is at most one with .

We have proven that is a function (for all there is at least one and never more than one) with

Thus is one-to-one/injective

3. **Surjectivity:**

Since , and is the inverse of .

Then

Thus is onto/surjective

4. A function is bijective if and only if is:

Injective: and,

Surjective: for all there is some such that

We have proven Injectivity in 2 and Surjectivity in 3

Thus, is a one-to-one correspondence/bijective