[Problem 1]

(a) Give an example of a set such that there is a set with but

Let

Let

We claim that there exists a set such that but .

is an element of

- If , we have

is not a subset of

- there exists at least one element in set that is not in set ,

being or , thus

Therefore, there exists a set with but

*(b) Give an example of a set such that there is a set with but*

Let

Let

We claim that there exists a set such that but .

is a subset of

if and only if

- iff , it means every element of set is also an element of set , we have

is not an element of

- The element of is a set itself

- has the set , which is an element of ,

but itself is not an element of .

- The elements of are a sets themselves

Therefore, there exists a set with but

[Problem 2]

*Calculate the following powersets:*

The cardinality (number of elements) of the power set of a set with elements is , including the empty set and the set itself.

is the set of all subsets of

[0]

i.e. the powerset of the empty set

- has only one element

(the empty set)

- the empty set has no elements, so the only subset it can have is the empty set.

[1]

i.e. the power set of the set containing only the empty set

- has only one element

(the empty set)

- powerset is the empty set and the set itself.

Cardinality:

[2]

i.e. the power set of the set containing the empty set & the set containing the empty set

- has two elements,

(the empty set) &

(the set containing the empty set)

- all possible subsets + the empty set and the set itself.

Cardinality:

[3]

i.e. the power set of the set containing the empty set

- has only one element

(the set containing the empty set)

- powerset is the empty set and the set itself.

Cardinality:

[4]

i.e. the powerset of the powerset of the empty set

FROM [0] ABOVE

i.e. the powerset of the empty set

- has only one element

(the empty set)

- the empty set has no elements, so the only subset it can have is the empty set.

substitute into

FROM [1] ABOVE

i.e. the power set of the set containing only the empty set

- has only one element

(the empty set)

- powerset is the empty set and the set itself.

Cardinality:

[5]

i.e. the power set of the powerset of the set containing only the empty set

FROM [1] ABOVE

- has only one element

(the set containing the empty set)

- all possible subsets + the empty set and the set itself.

Cardinality:

[Problem 3]

*For each of the following functions determine the image of*

bounds of for which

*will always be positive*

*or*

*(a) defined by .*

*Interval where :*

*Interval where :*

Therefore, the Image of is

*(b) defined by .*

*Interval where :*

*Interval where :*

*will always be positive, as will always be positive*

Therefore, the Image of is

i.e. set of non-negative real numbers

[Problem 4]

*Consider the following two functions.*

*(1) defined by .*

*(2) defined by .*

*Determine whether the given functions are one-to-one correspondences.*

A function is said to be one-to-one correspondence

iff is both:

Injective (one-to-one): and,

Surjective (ONTO): for all there is some such that

*(1) defined by .*

INJECTIVE

Take and assume that:

Therefore is one-to-one, by definition of one-to-one.

SURJECTIVE

We need to find an that maps to .

s

*(2) defined by .*

INJECTIVE

Take and assume that:

The values for & could be the same, with different signs.

e.g.

Therefore is not one-to-one, by definition of one-to-one.