[Problem 9]

*Determine whether the sets are are subspace of defined by:*

a*nd*

A subset or of is a subspace if it satisfies the following three properties ensure that the subset behaves like a vector space:

|  |  |  |
| --- | --- | --- |
| **A1** | **Closure under addition**  *For any vectors and in the set, is also in the set* |  |
| **A2** | **Existence of an additive identity**  *There exists a vector in the set such that for any vector in the set, .*  *Related:*  *- A3:Existence of additive inverses* |  |
| **M1** | **Closure under scalar multiplication**  *For any scalar and any vector in the set, is also in the set.*  *Implied by:*  *- A1:**Closure under addition* |  |

|  |  |
| --- | --- |
| A1 | Let  be arbitrary vectors in  where and are real numbers    Then      Therefore A1 holds. |
| A2 | Let the zero vector in be  Then      Therefore A2 holds. |
| **M1** | Let be an arbitrary scalar in    Let be an arbitrary vector in  where , and are real numbers  Then      Then    Therefore M1holds. |

*Therefore is a subspace of*

|  |  |
| --- | --- |
| A2 | Let the zero vector in be  Then    Then    *but 1*  Therefore A2 Fails. |

*Therefore is not a subspace of*

[Problem 10]

*Express the following as a linear combinations of*

*; ; and ;*

Definition: linear combination

*Let be a vector space over a field , and let , ​ be vectors in .*

*A linear combination of , ​ is any expression of the form:*

*Where are scalars from the field .*

*a)*

*Let be scalars from the field .*

*Then*

*in matrix form :*

*Initial augmented matrix:*

*Forward Elimination*

*Back Substitution:*

*: -2/-2 = 1*

*: (-2 - 0.5) /0.5 = -5*

*: (6 - -2) /2 = 4*

*Thus*

*b)*

*Let be scalars from the field .*

*Then*

*Thus*

*c)*

*Let be scalars from the field .*

*Then*

*in matrix form :*

*Initial augmented matrix:*

*Forward Elimination*

*Back Substitution:*

*: -14/-2 = 7*

*: (4.5 - 3.5) /0.5 = 2*

*: (7 - 23) /2 = -8*

*Thus*

[Problem 11]

*Which of the following sets of vectors in are linearly independent.*

Definition: Linear independence

*Let be a vector space over a field , and let , ​ be vectors in .*

*Is said to be linearly independent if the only solution to the equation:*

*is the trivial solution,*

*where 0 is the zero vector in .*

*(a) (1; 2; 2; 1) ; (3; 6; 6; 3) ; (4; 2; 4; 1)*

*in matrix form :*

*; ;*

*Reduce matrix to row-echelon form (RREF):*

*Therefore, the given set of vectors is* ***linearly dependent****.*

*(b) (2; 1; 1; -4) ; (2; -8; 9; -2) ; (0; 3; -1; 5); (0; -1; 2; 4);*

*in matrix form :*

*; ;*

*Reduce matrix to row-echelon form (RREF):*

*Therefore, the given set of vectors is* ***linearly dependent****.*

*(c) (1; 1; 0; 0) ; (0; 1; 0; 1); (0; 0; 1; 1); (1; 0; 1; 0) ; (1; 0; 0; 1)*

*in matrix form :*

*; ;*

*Reduce matrix to row-echelon form (RREF):*

*Therefore, the given set of vectors is* ***linearly dependent****.*

[Problem 12]

*Determine whether the solution space of the system Ax = 0 is a line through the origin, a plane through the origin, or the origin only for*

*Forward Elimination*

solutions in

|  |  |  |
| --- | --- | --- |
| ***geometric interpretation*** | *solution space* | *Nullity of A* |
| A line through the origin | one-dimensional. | 1 |
| A plane through the origin | two-dimensional | 2 |
| The origin only | zero-dimensional | 0 |

Thus, the solution space is The origin only