Problem 13.

Find the coordinate vectors of v relative to the basis of set

, where

(a) ; ;

(b) ; ;

To find the coordinate vectors of relative to the basis set

, we can use the formula:

(a) Thus,

Therefore,

(b) Thus,

Therefore,

Problem 14.

Let and be two subspaces of defined by

and

and

Find the bases of and

Bases of

[1] Rewrite as system of equations:

[2] Express as the span of a set of vectors:

is linearly independent

is linearly independent

Thus, both vectors form a basis for .

Bases of

[1] Rewrite as system of equations:

[2] Express as the span of a set of vectors:

is linearly independent

is linearly independent

Thus, both vectors form a basis for .

ASS4: Problem 15.

Determine whether the following form basis for :

(a) ; ;

(b) ; ;

To determine whether a set of polynomials forms a basis for , we need to check two conditions:

1. Linear Independence: The polynomials in the set must be linearly independent.
2. Spanning: The set must span , meaning that any polynomial in , can be expressed as a linear combination of the polynomials in the set.

(a) ; ;

[1] Express in terms of

Rewrite as system of equations:

Thus,

And,

And,

[2] Linearly independence

find constants , ​, and such that:

Rewrite as system of equations:

[1]

[2]

[3]

Thus,

Therefore, the polynomials are linearly independent.

(b) ; ;

[1] Express in terms of

Rewrite as system of equations:

Thus,

And,

And,

[2] Linearly independence

find constants , ​, and such that:

Rewrite as system of equations:

[1]

[2]

[3]

Thus,

Therefore, the polynomials are linearly independent.

Problem 16.

Find the basis and dimension of the solution space of given homogeneous linear system.

[1] Rewrite as system of equations:

;

Forward Elimination

-------------------------------------------------------- iter: 1

R2: R2 - 2R1

Forward Elimination

-------------------------------------------------------- iter: 2

R3: R3 - 3R1

Forward Elimination

-------------------------------------------------------- iter: 3

R2: (1/-5)R2

Forward Elimination

-------------------------------------------------------- iter: 4

R3: (5/18)R3

Forward Elimination

-------------------------------------------------------- iter: 5

R1: R1 + (8/5)R3

R2: R2 – (1/5)R3

[2] Rewrite as system of equations: