*Problem 29.*

*Show the following:*

*(a) If A is an orthogonal matrix, then is also orthogonal matrix.*

*(b) If A and B are orthogonal matrices, then AB is also orthogonal matrix.*

**Definition**

A matrix is orthogonal if

If is an orthogonal matrix, then

**Properties**

[1] Inverse Property:

is the inverse of A,

Defined as

Where is the identity matrix

Also

[2] Inverse of Product Property:

For any two invertible matrices A and B,

the inverse of the product AB is given by:

[3] Transpose of :

is the transpose of A

[4] Transpose of Inverse

For any invertible matrix A, the transpose of its inverse is the same as the inverse of its transpose.

[1] because

[2] because

[4] because

Thus, is orthogonal

*Problem 30.*

*What is the condition on a and b for which the matrix*

*is orthogonal.*

**Definition**

A matrix is orthogonal if

If is an orthogonal matrix, then

[1] Compute

[2] Compute

*Thus*

*Problem 31.*

*Find a matrix P that orthogonally diagonalizes A; and determine ,*

*Where*

[1] Eigenvalues of A

To orthogonal diagonalized matrix

Find an orthogonal matrix such that ,

where is a diagonal matrix

Eigenvalues : OR

[2] Eigenvectors of A

Eigenvectors of

*Thus*

Eigenvectors of

*Thus ,*

*Thus,*

[3] Compute

*Problem 32. Find the spectral decomposition of matrix*

[1] Eigenvalues of A

*Thus,*

Eigenvalues : OR OR

[2] Eigenvectors of A

Eigenvectors of

*Thus ,*

Eigenvectors of

*Thus , ,*

[3] Spectral Decomposition

Given

Thus