**Question 1**

a) Let be the statement

**Basis Clause**

Show that

is where

.

Therefore, is true

**Inductive Hypothesis**

Show that.

is where

Assume

**Inductive Step**

If is true, then must also be true

Assume

But,

Therefore, by the induction hypothesis:

Thus, is true

Hence, is true

It then follows by mathematical induction that is true.

b) Let be the statement

**Basis Clause**

Show that

is where

and

Therefore, is true

**Inductive Hypothesis**

Show that

is where

Assume

**Inductive Step**

If is true, then must also be true

Assume

But,

Therefore, by the induction hypothesis:

Re-write 4 as 3+1

Multiplying out

By regrouping

Remove from both sides

is true for all

Thus, is true

Hence, is true

It then follows by mathematical induction that is true for

**Question 2**







**Question 3**

Arrangement with unlimited repetition

**Question 4**

a)

* If no student got less than 10 out of 20, there are eleven possible marks that the students could have gotten.
* Each mark will represent a student (pigeon)
* Each container will be a pair or marks (pigeonhole)

[10,10] [11,11] [12,12] [13,13] [14,14] [15,15] [16,16] [17,17] [18,18] [19,19] [20,20]

* We note that where each container has two students, the total number of students is 22.
* We have three remaining students, that need to be assigned to one pigeonhole each.
* Each pigeonhole already contains two students.
* If we add the three remaining students to any three pigeonholes. At least three will have the same mark

b)

* Group consecutive numbers into pairs (pigeonholes):

[1,2] [3,4] [5,6]… [ ,]

Where

* If we chose integers, by the pigeonhole principle, we should get a two that are from one of the pairs mentioned above.
* The pairs are already consecutive integers so two of the numbers chosen will also be consecutive

**Question 5**

By the extended pigeonhole principle, at least one pigeonhole will contain pigeon(s).

If no student got less than 20% there are 81 possible marks that the students could have gotten.

* Each mark will represent a student (pigeon)
* Each container will be a pair or marks (pigeonhole)

….

Therefore, at least 3 students obtained the same mark

**Question 6**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | x | x | x | x | x | x |
| 2 | x | x |  |  |  |  |
| 3 | x | x | x |  |  |  |
| 4 | x | x | x | x |  |  |
| 5 | x | x | x | x | x |  |
| 6 | x | x | x | x | x | x |

a) yes. is reflexive

b) no. is not irreflexive

c) no. is not symmetric

d) no. is not asymmetric

e) yes. is antisymmetric

f) no. is not transitive

**Question 7**

a)

A picture containing necklace

Description automatically generated

b)

2: in 5, out 1

3: in 4, out 2

c)

d) 2-1-5

e)

f)

all positions non-zero

g)

* shows the possible pairs that transivity can be tested against
* In , if, for every position (a,b) and (b,c) that each have a 1, there is a 1 at (a,c), then the relation is true.
* Also, for all the positions in that are non-zero (or 1), if already has a 1 in the corresponding position, is transitive

**Question 8**

a) no. is not reflexive.

The centre (main) diagonal has all 0’s

b) yes. is irreflexive.

The centre (main) diagonal has all 0’s

c) no. is not symmetric.

For every value, the value in the transposed position is not equal.

d) yes. is asymmetric

The centre (main) diagonal has all 0’s

For every value, the value in the transposed position is not equal.

e) yes. is antisymmetric

It does not matter what values the centre (main) diagonal has

For every value and the value in the transposed position, they are both not 1

f) no. is not transitive

has 1’s in positions which does not have

**Question 9**

a)

b)

**Question 10**

a)

b)

c)

**Question 11**

**Question 12**

a)

b)