**Question 1**

a) Let be the statement

**Basis Clause**

Show that

is where

.

Therefore, is true

**Inductive Hypothesis**

Show that.

is where

Assume

**Inductive Step**

If is true, then must also be true

Assume

But,

Therefore, by the induction hypothesis:

Thus, is true

Hence, is true

It then follows by mathematical induction that is true.

b) Let be the statement

**Basis Clause**

Show that

is where

and

Therefore, is true

**Inductive Hypothesis**

Show that

is where

Assume

**Inductive Step**

If is true, then must also be true

Assume

But,

Therefore, by the induction hypothesis:

Re-write 4 as 3+1

Multiplying out

By regrouping

Remove from both sides

is true for all

Thus, is true

Hence, is true

It then follows by mathematical induction that is true for

**Question 2**







**Question 3**

Arrangement with unlimited repetition

**Question 4**

a)

* If no student got less than 10 out of 20, there are eleven possible marks that the students could have gotten.
* Each mark will represent a student (pigeon)
* Each container will be a pair or marks (pigeonhole)

[10,10] [11,11] [12,12] [13,13] [14,14] [15,15] [16,16] [17,17] [18,18] [19,19] [20,20]

* We note that where each container has two students, the total number of students is 22.
* We have three remaining students, that need to be assigned to one pigeonhole each.
* Each pigeonhole already contains two students.
* If we add the three remaining students to any three pigeonholes. At least three will have the same mark

b)

* Group consecutive numbers into pairs (pigeonholes):

[1,2] [3,4] [5,6]… [ ,]

Where

* If we chose integers, by the pigeonhole principle, we should get a two that are from one of the pairs mentioned above.
* The pairs are already consecutive integers so two of the numbers chosen will also be consecutive

**Question 5**

By the extended pigeonhole principle, at least one pigeonhole will contain pigeon(s).

If no student got less than 20% there are 81 possible marks that the students could have gotten.

* Each mark will represent a student (pigeon)
* Each container will be a pair or marks (pigeonhole)

….

Therefore, at least 3 students obtained the same mark

**Question 6**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | x | x | x | x | x | x |
| 2 | x | x |  |  |  |  |
| 3 | x | x | x |  |  |  |
| 4 | x | x | x | x |  |  |
| 5 | x | x | x | x | x |  |
| 6 | x | x | x | x | x | x |

a) yes. is reflexive

b) no. is not irreflexive

c) no. is not symmetric

d) no. is not asymmetric

e) yes. is antisymmetric

f) no. is not transitive

**Question 7**

a)

A picture containing necklace

Description automatically generated

b)

2: in 5, out 1

3: in 4, out 2

c)

d) 2-1-5

e)

f)

all positions non-zero

g)

* shows the possible pairs that transitivity can be tested against
* In , if, for every position (a,b) and (b,c) that each have a 1, there is a 1 at (a,c), then the relation is true.
* Also, for all the positions in that are non-zero (or 1), if already has a 1 in the corresponding position, is transitive

**Question 8**

a) no. is not reflexive.

The centre (main) diagonal has all 0’s

b) yes. is irreflexive.

The centre (main) diagonal has all 0’s

c) no. is not symmetric.

For every value, the value in the transposed position is not equal.

d) yes. is asymmetric

The centre (main) diagonal has all 0’s

For every value, the value in the transposed position is not equal.

e) yes. is antisymmetric

It does not matter what values the centre (main) diagonal has

For every value and the value in the transposed position, they are both not 1

f) no. is not transitive

has 1’s in positions which does not have

**Question 9**

a)

**A picture containing sitting, hanging, necklace, pair

Description automatically generated**b)

**Question 10**

1. Let be a finite set with number of elements.

* Using the cartesian product of sets, we have
* The cartesian product contains pairs of elements from
* For each pair , we have of the elements from
* For each pair , we also have of the elements from
* Thus, there are possible ordered pairs where

Let , where is a relation on our cartesian product

* For each of the possible ordered pairs , we have two possibilities: either or
* To account for both possibilities, we have
* Thus, the number of distinct relations on is

1. Let us assume be a finite set with number of elements.

* Then again, the cartesian product of sets, we have
* The cartesian product contains pairs of elements from
* For each pair , we have of the elements from
* For each pair , we also have of the elements from
* Thus, there are possible ordered pairs where

Let , where is a reflexive relation on

* For each of the possible ordered pairs , we have two possibilities: either or
* But to be reflexive, the main (centre) diagonal of the matrix needs to be all 1’s, we can remove these from our ordered pairs
* After removing the main diagonal elements, we have ordered pairs
* To account for both possibilities, we have
* Thus, the number of distinct relations the reflexive relation on is

1. c) Let be a finite set with number of elements.

* Using the cartesian product of sets, we have
* The cartesian product contains pairs of elements from
* For each pair , we have of the elements from
* For each pair , we also have of the elements from
* Thus, there are possible ordered pairs where

Let , where is a asymmetric relation on

* For each of the possible ordered pairs , we have two possibilities: either or
* But to be asymmetric, every value and its value in the transposed position in the matrix should not be equal.
* To account for this, we have ordered pairs
* Thus, the number of distinct relations the asymmetric relation on is

**Question 11**

If is a symmetric relation on , then .

*If R is a symmetric relation on A,*

*then a related to b, and subsequently b related to a*

Using the cartesian product of sets, we can compute . This will help us identify elements to show transitivity in

By doing so, we create have the pair , where ,

*We create the element in where a is related to itself.*

*All a’s are elements of the set A*

If we suppose that , then , where , and

*assume a related to b.*

*then there exists some c that exists in ,*

*where a is related to c*

*and where c is related to b*

And if and , , then .

*a related to b (in and subsequently b related to a*

It then follows that if is a symmetric relation on , then is symmetric.

**Question 12**

To be an equivalence relation on a set, a relation or would need to be reflexive, symmetric, and transitive.

a) yes, the relation is an equivalence relation.

**Reflexivity**

The relation contains pairs in the form , where .

These pairs are .

contains all these pairs therefore it is reflexive.

**Symmetry**

The relation contains pairs in the form and where

These pairs are .

Since contains these pairs, and the only other pairs it contains are the ones explained above in its reflexivity property, is symmetrical

**Transitivity**

The relation contains pairs in the form , and where .

Examples of these pairs are , , ,

contains these pairs and can compute many others, therefore it is transitive.

b) no, the relation represented by the matrix is not an equivalence relation.

**Reflexivity**

The main centre (main) diagonal is not only 1’s, so the relation is not reflexive

**Symmetry**

For every value, it is not equal to the value in the transposed position, so the relation is not symmetric

**Transitivity**

Let the Relation , be represented by the matrix . Using cartesian product of sets, we have

has 1’s in positions which does not have. Therefore, the Relation is not transitive