**Question 1**

a i) is not a function since the input for 4 has more than one output

a ii) The function is not onto/surjective since every value in the domain cannot be mapped to a corresponding value in the range

a iii) The function is not one-to-one/bijective since every value in the domain cannot be mapped perfectly to a single corresponding value in the range.

a iv) is everywhere defined as value in the domain is mapped to some or other value in the range

b i) is a function since no element in the domain maps to more than one element in the range.

b ii) The function is not onto/surjective since every value in the domain cannot be mapped to a corresponding value in the range

b iii) The function is not one-to-one/bijective since every value in the domain cannot be mapped to a single corresponding value in the range.

b iv) is everywhere defined as value in the domain is mapped to some or other value in the range

**Question 2**

a)

**Therefore, is everywhere defined**

b) The function is onto/surjective if there exists an inverse function such that the composition function equals the identity function

If

Then

**Therefore is onto/surjective**

c) Suppose

Then:

**Therefore is one-to-one**

d) is invertible because is one-to-one

**Therefore is invertible as**

**Question 3**

a)

Running time is constant, it’s not affected by the input size.

b)

Running time is linear. When an algorithm accepts input size, it would perform or two times as many operations as well.

c)

Running time is quadratic. When an algorithm accepts input size, it would perform or four times as many operations as well.

d)

Running time is exponential. When an algorithm accepts input size, it would perform or the number of original operations squared as well.

**Question 4**

a) Suppose is

Then there exist constants and such that:

for all

Now

Which holds true for arbitrary values like and

Therefore, is

b) Suppose is

Then there exists and such that:

for all

Now

But we know is not a bound function (of the form )

Therefore is not asymptotically bounded by

Therefore, the inequality is false and is not

**Question 5**

a)

**Therefore, is everywhere defined**

The function is onto/surjective if there exists an inverse function such that the composition function equals the identity function

If

Then

**Therefore is onto/surjective**

Suppose

**Therefore is one-to-one**

**Therefore, is a permutation**

b)

**Therefore, is everywhere defined**

The function is onto/surjective if there exists an inverse function such that the composition function equals the identity function

If

**Solve using**

**quadratic equation formula:**

and

Then

**Therefore is not onto/surjective**

**Therefore, is not a permutation**

**Question 6**

a)

b)

**Question 7**

a)

b)

c)

d) is odd with an odd number of transpositions

e)

f) Using transpositions:

**Ordered**

Missing

**Unordered**

g)

**Question 8**

Let be a permutation on with period 5

Therefore

**Question 9**

a)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 |
| 1 | X | X |  |  |  |
| 2 | X | x | X |  |  |
| 3 |  | X | x |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  |  |  |  |

is not reflexive as

Therefore is not a partial order

b)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 |
| 1 | X | X |  |  | X |
| 2 |  | x | X |  |  |
| 3 |  | X | x |  |  |
| 4 |  |  |  | X |  |
| 5 |  | x |  |  | x |

is reflexive as

is not symmetric as for every value, the value in the transposed position is not equal

is antisymmetric as it is reflexive and not symmetric

is transitive: as and and

Therefore is a partial order

**Question 10**

a)

b) is 1. is 72

**Question 11**

**Question 12**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | a | b | c | d |
| a | X | X | x | x |
| b |  | x | X | X |
| c |  |  | x |  |
| d |  |  |  | X |

**Question 13**

**Question 14**

**Question 15**

**Question 16**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | |  | |
|  | 1 | 1 | 1 | 0 |
|  | 1 | 0 | 0 | 0 |