Question 1

a.

i.

|  |  |  |
| --- | --- | --- |
| Converse |  | I go shopping when I take plastic bags. |
| Contrapositive |  | I do not go shopping when I do not take plastic bags |
| Negation |  | I take plastic bags and I do not go shopping. |

ii.

|  |  |  |
| --- | --- | --- |
| Converse |  | or implies |
| Contrapositive |  | or implies |
| Negation |  | implies or |

iii.

|  |  |  |
| --- | --- | --- |
| Converse |  | and then |
| Contrapositive |  | and then |
| Negation |  | and or |

b.

Indirect proof: proof of the contrapositive states that:

Let the statement be

“if the average of this set of test scores is greater than 90, then at least one of the scores is greater than 90”

Then the contrapositive, will be

“if none of the scores will be greater than 90, then the average of this set of test scores is not greater than 90”

Then we have that for every

And if the sum of these is , and

the number of elements in is ,

then we have

Therefore, the average test score will be less than 90.

Question 2

i.

hypothesis:

Let be the statement for all

**Basis clause**

is where

.

Therefore, is true

**Inductive hypothesis**

is where

Assume . Assume is true

Then also assume that

**Inductive Step**

If is true, then must also be true

But

by the induction hypothesis

Thus, is true

Hence, is true

It then follows by mathematical induction that is true.

ii.

A function is bijective if it is injective and surjective

**Injection (one-to-one)**

A function is injective where

Let ,

Therefore, is injective

**Surjection (onto)**

A function is surjective where for each there exists such that

Let , then

Thus, the function is not surjective as not every element of the codomain maps onto the domain.

**Inverse**

A function has an inverse where it is bijective (injective and surjective). The function is injective and not surjective, and is not bijective. Thus, the function has no inverse.

Question 3

a.

Thus, and

b.

Given:

and

If then

for all .

Suppose , such that , then assume

, ,

Thus,

Which is a contradiction

Therefore,

Question 4

a.

i.

conjugate:

reciprocal:

power rule:

ii.

divide by leading term

product rule:

reciprocal:

power rule:

b.

Given the sequence

;

Let as the first term in the sequence, then

first term

second term

third term

fourth term

…

for every

Lower bound (bounded below)

Let

then, , for every

Upper bound (bounded above)

Let ,

then , for every

Therefore, the given sequence is monotone decreasing

Question 5

a.

For the sequence

Let be given.

Let be given.

If , then

, then

such that

for all

Therefore,

b.

For the function

Definition: Let and .

is continuous at if for every

there exists such that , then .

Let be given.

Where ,

Where ,

Then for we have

If , then

, then

Since , and we have ,

so .

Therefore,

Thus, is continuous at

Question 6

For the function

If, then the limit from the left-hand side (LHS) will be:

LHS:

If, then the limit from the right-hand side (RHS) will be:

RHS:

LHS = RHS. Therefore, is continuous at