Question 1

1.1

|  |  |  |
| --- | --- | --- |
| Statement |  | **If** is divisible by 4, **then** is even. |
| Contrapositive |  | **If**  is odd, **then** is not divisible by 4. |

Proof:

Assume that is odd,

Then by definition of an odd integer, there exists

such that,

,

It follows that if,

Then,

Since , we have

for some

Which implies that is not divisible by 4.

Thus, the contrapositive is true

Therefore,

**If**  is odd, **then** is not divisible by 4.

1.2

|  |  |  |
| --- | --- | --- |
| Statement |  | **If** is divisible by 4, **then** is even. |
| Converse |  | **If**  is even, **then** is divisible by 4. |

Assume that is even,

Then by definition of an odd integer, there exists

such that,

,

It follows that if,

Then,

Since , we have

for some

Which implies that is divisible by 4.

Thus, the converse is true

Therefore,

is divisible by 4.

1.3

|  |  |  |
| --- | --- | --- |
| Statement |  | **If** is divisible by 4, **then** is even. |
| Inverse |  | If is not divisible by 4, then is odd, |

Assume that is not divisible by 4,

Suppose that is even,

Then there exists such that,

,

It follows that if,

Then,

Since , we have

for some

Which implies that is divisible by 4.

But this contradicts that is not divisible by 4,

Thus, our supposition that is even must be false,

Therefore,

is divisible by 4, **then** is even.

Question 2

2.1

Prove for all

**Basis Clause**

**Let** ,

LHS

RHS

LHS=RHS

Thus, the basis case holds

**Inductive Hypothesis** Assume ,

**Extremal Clause (Inductive Step)**

Prove +1 for all for ,

Prove for all

RHS

LHS

RHS=LHS

Thus, the extremal case holds

Thus, holds for all

**2.2**

Question 3

Question 4