**a)**

If and intersect, there is a point that lies on both lines. There must be such that:

*and are only distinguished for legibility*

If and intersect, there is a point that lies on both lines. There must be such that:

If and intersect, there is a point that lies on both lines. There must be such that:

Therefore, , which is a point on and

If and are in the plane they describe, the normal to the plane must be perpendicular to and

*Cross product*

, which is a point on the plane

*Dot product*

**b)**

is the vector perpendicular to the plane, where

*Convert into normal vectors*

*Cross Product*

*Find a point on the line. Choose the arbitrary point where*

*Use matrix to solve system of equations*

Therefore, a point that passes through the plane is

Therefore, the equation for the line of intersection of two planes

**c i)**

is a Cone (top portion)

is an Elliptic Paraboloid

is a Hyperbolic Hyperboloid

**c ii)**

Intersection where

The intersection of and is a circle

**d)**

Given ,

**Epsilon-Delta definition**

Multi variable

If

Then

*Because . This will always be a positive number*

*As the coordinate system approaches some random coordinate point the limit is L*

Prove from first principles that

If

Then

*Substitute =-1, ,*

If

Then

*Substitute , .*

*Add and for extra constants created by substitution*

If

Then

If

Then

Now we can start with the calculation using the function

From the Epsilon-Delta definition above, we now must find some relationship between and

or

*triangle inequality*

*Substitute ,*

From our earlier definition

If

Then

But for any if , then

Therefore

*Substitute*

If

Then

*Multiply all expressions by 6*

If

Then

If

Then

We can see that for any . Therefore

**e i)**

For the limit to exist, must approach the same value , irrespective of the curve along we approach the origin

*Curve 1: along the x-axis that approaches origin*

*Curve 2: parabola that passes through origin*

Since approaching the origin along these two different curves leads to different limits, the limit does not exist

**e ii)**

For the limit to exist, must approach the same value , irrespective of the curve along we approach the origin

*Curve 1:*

*Curve 2:*

Since approaching the origin along these two different curves leads to different limits, the limit does not exist

**e iii)**

For the limit to exist, must approach the same value , irrespective of the curve along we approach the origin

*Curve 1:*

*Curve 2:*

Since approaching the origin along these two different curves leads to different limits, the limit does not exist

**e iv)**

For the limit to exist, must approach the same value , irrespective of the curve along we approach the origin .

Since , and are continuous, we use the sum and product rules

if

if or

**f i)**

**f ii)**

**f iii)**

**f iv)**

yes.

The value of the function: .

The limit of the function:

The function value and the limit is the same at

Therefore, the function is continuous at

**f iv)**

No.

The value of the function: .

The limit of the function:

The function value and the limit are not the same at

Therefore, the function is not continuous at

**f vi)**

No.

The function is not continuous at

Therefore, is not continuous at every point of its domain

**g i)**

**Chain rule**

The gradient of a function is described by the vector function:

Therefore

To calculate in the direction of , we just need to divide by its magnitude.

Therefore, the rate of increase of at the point in the direction of the vector :

**g ii)**

Therefore, the rate of increase of at the point in the direction of the negative z-axis u:

**g iii)**

The maximum rate of change at a given point:

||

**h i)**

is a Helix (spiral)

**h ii)**

**Pythagorean Identity**

Given that

Also given:

*Substitute the parametric equations of the curve into equation*

*Apply Pythagorean identity*

Every point of the parametric curve satisfies the equation of the cylinder, and so the curve lies on the cylinder.

**h iii)**

The gradient of the tangent line is

Therefore, the gradient at :

**h iv)**

The velocity vector at instant is given by

Therefore, the speed at instant is given by

**Pythagorean Identity**

**h v)**

At the point , on the curve we have:

Resulting in the vector

At the tangent’s gradient , we have:

Resulting in the vector

The vector equation of the line :

The tangent (in terms of x, y and z):

Therefore, the re-written cartesian form:

**h vi)**

**i i)**

The composite function

**i ii)**

The gradient of a function is described by the vector function:

**i iii A)**

**i iii B)**

**j i)**

**j ii)**

**j iii)**

**j iv)**

**k i)**

**k ii)**

**k iii)**

**k iv)**

is a Conservative vector field when:

Second order derivatives of are continuous

The scalar curl of is zero *(as )*

is defined on all

Therefore, is a conservative vector field and has a potential function

**k v)**

**l)**

First, second and third order derivatives

Evaluate derivatives at the point

First Order:

**2D vector**

*General formula for1st degree Taylor polynomial*

Using matrices:

Second Order:

*General formula for 2nd degree Taylor polynomial*

Using matrices:

**2x2 symmetric matrix**

***Hessian Matrix***

*General formula for 3rd degree Taylor polynomial*

**2x2x2 symmetric tensor**

Using matrices:

Therefore or