**a)**

Critical point

*Substitute*

**Solve using**

**quadratic equation formula:**

and

*Substitute*

Critical Points:

Second derivatives

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Point |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  | Saddle point |
|  |  |  |  |  |  |
|  |  |  |  |  | Saddle point |

Therefore at and we have a saddle point

Therefore at and we have a local maximum

There are no global minima or maxima

**b i)**

If we have a parabola defined as , then the parametric equations are and .

For we have

The parameterisation of is

For we have

The parameterisation of is

**b ii)**

**b iii)**

Critical points are where

Therefore

Combining the above:

Therefore and

**c)**

Let be the length of the box

Let be the width of the box

Let be the height of the box

The objective function is

The restrictions for the dimensions

The volume function

Substitute restriction into

Gradients:

Determine Critical points

Language method: Optimum is at

Therefore

Combining the equations

and

Substitute back into constraint

Therefore , and

**d i)**

A picture containing dirty, small, hole, lot

Description automatically generated

**d ii)**

No. is not a type region

As a union of type regions

**d iii)**

Yes. is a type region

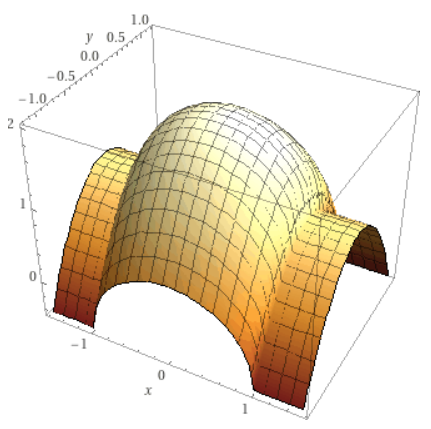
**d iv)**

**e i)**

A picture containing dirty, small, hole, lot

Description automatically generated

**e ii)**

****

**e iii)**

**f)**

Therefore

**g i)**

Therefore, the integral is path independent

**g ii)**

**g iii)**