Question 1

Solve the following initial value problem

,

[1] Eigenvalues of A

Thus

[2] Eigenvectors of A

R1:

R2:

Let ,

R1:

R2:

Thus

Let ,

R1:

R2:

Thus

[3] General solution

[4] Apply Initial Conditions

*Where* initial condition is

[4] Solve for , ,

*For*

[5] Solution to the initial value problem

Question 2

*We know that if is a fundamental matrix of at and is continuous in then*

*is a particular solution of*

*Use this result with to find a general solution of*

[1] Eigenvalues of A

Solve for

,

Thus ,

[2] Eigenvectors of A

For

&

For

[3] Fundamental matrix

homogeneous solution

[4] Inverse of Fundamental matrix

Therefore,

[5] Find

Given

[6] Integral

[7] Particular Solution

[8] Homogeneous and Particular Solutions:

Question 3

*If is a normalized fundamental matrix at and if F is continuous in , then*

*is a particular solution of . Use the result above to find a general solution of*

*˙*

[1] Eigenvalues of A

Solve for

,

Thus

[2] Eigenvectors of A

For

For

[3] Fundamental matrix

Given by , here A is the coefficient matrix of the homogeneous system

where P is the matrix of eigenvectors

J is the Jordan form

[4] Jordan form

[5] matrix of eigenvectors

Thus,

[6]compute

[7] compute

*Let ,*

*If , , then*

(1,1)

(1,2)

(2,1)

(2,2)

Thus,

Question 4

*(4.1) Write the companion system for the equation given below.*

*Write your final answer in terms of the appropriate trigonometric functions.*

[1] Re-write as First-order differential equation

And

Thus,

[2] Re-write in matrix form

*(4.2) Find the series solution of the blow system of first order differential equations using the*

*power series method*

*with*

*Write your final answer in terms of the appropriate trigonometric functions.*

[1] Power series

[2] Recurrence Relation

Thus

Question 5

We know that if is a normalized fundamental matrix of at ; then

*for all real numbers s and t. Use this hypothesis and*

(5.1) Show that for all real numbers s and t

[1] Initial Condition

If is the identity matrix, at we have:

[2] Fundamental matrix

Given by , here A is the coefficient matrix of the homogeneous system

[3] Matrix Exponential

Given a homogeneous system  *, the* fundamental matrix

can be expressed as , where is the matrix exponential.

(5.2) *Show that if is a normalized fundamental matrix at ; then*

for all

[1] Initial Condition

If is the identity matrix, at we have:

[2] Fundamental matrix

Given by , here A is the coefficient matrix of the homogeneous system

[3] Inverse of the Fundamental Matrix

Thus,

Question 6

*Classify the critical points of the plane autonomous system corresponding to the second order non-linear differential equation:*

[1] Re-write as First-order differential equation

Thus and

[2] Jacobian matrix

(1,1)

(1,2)

(2,1)

*derivative*

(2,2)

[3] Eigenvalues of J

Thus

[4] Solve for