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Question 1

1.1

Vector Equation: Graph of the function

Can be defined as:

Suppose

**Then, the direction vector v**

**And, the Parametric equation of a line**  
A line with vector equation

**Therefore, the parametric equations of the line passing through the points and are given by:**

1.2

Suppose

**Then, the direction vector**

**And, the direction vector**

Where and lie in the plane.

**And, the normal vector to the plane**

The normal to the plane is given by the cross product

Since determines an arbitrary point in the plane vector v

**Which lies in the plane and is perpendicular to n.**

Thus the plane:

**Passes through and**

Question 2

2.1

Suppose and

where

We must prove that for all

For , Evaluate

For , Evaluate

Thus

For , Evaluate

For , Evaluate

Thus

Since we have shown that for both and ,

we conclude that on the set .

Thus, for all

2.2

Suppose

and

where

We must prove that for all

**Then for , we evaluate**

For , Evaluate

For , Evaluate

For , Evaluate

Thus

**Then for , we evaluate**

For , Evaluate

For , Evaluate

For , Evaluate

Thus

Since we have shown that

for

but for ,

Thus, for all

Therefore, is not true for the entire set

Question 3

Suppose V, the set of ordered pairs of real numbers, with the given operations:

: addition

is a vector space over , then it holds that V satisfies the following axioms of addition:

|  |  |  |
| --- | --- | --- |
| **A1** | **Closure under addition**  *For any vectors and in the set, is also in the set*  *Let and*  *Then the sum must be in*  *and , the sum is an ordered pair of real numbers which belong to V*  *Thus, the set is closed under addition* |  |
| **A2** | **Existence of an additive identity**  *There exists a vector in the set such that for any vector in the set, .*  *Let and*  *Then we have*  Thus, the additive identity exists and is |  |
| **A3** | **Existence of additive inverses**  *For every vector in the set, there exists a vector in the set such that*  *Let and*  *Then we have*  Thus, the additive inverse of is |  |
| **A4** | **Associativity of addition**  *For any vectors , , and in the set,*  *fundamental property of addition*  *Let , and w*  *We know that , and*  *Then because the addition of real numbers is associative,*    *Thus, addition is associative.* |  |
| **A5** | **Commutativity of addition**  *For any vectors and* *in the set,*  *fundamental property of addition*  *Let and*  *We know that and*  *Then because the addition of real numbers is commutative,*  *Thus, addition is commutative.* |  |

, : scalar multiplication

is a vector space over , then it holds that V also satisfies the following axioms and scalar multiplication:

|  |  |  |
| --- | --- | --- |
| **M1** | **Closure under scalar multiplication**  *For any scalar and any vector in the set, is also in the set.*  *Let and scalar*  *Then the scalar multiple must also be in V*  *, the scalar multiple is an ordered pair of real numbers which belong to V*  *Thus, the set is closed under scalar multiplication* |  |
| **M2** | **Distributive property - vector addition**  *For any scalar and any vectors and in the set, +cv.*  *Let , v and scalar*  Then        *Which is equivalent to*  *Thus, scalar multiplication distributes over vector addition.* |  |
| **M3** | **Distributive Property - scalar addition**  *For any scalars and and any vector in the set, +cv.*  *Let and scalars*  Then        *Which is equivalent to*  *Thus, scalar multiplication is compatible with field multiplication* |  |
| **M4** | **Associative Property**  (Compatibility of scalar multiplication with field multiplication)  *For any scalars and and any vector in the set, +cv.*  *Let , v and scalars*  Then        *Which is equivalent to*  *Thus, scalar multiplication is compatible with field multiplication.* |  |
| **M5** | **Multiplicative identity**  *For any vector in the set, , where 1 is the multiplicative identity of the underlying field*.  *Let and scalar 1*  *Then we have*  Thus, the multiplicative identity exists and is |  |

Since all the axioms of a vector space are satisfied, we can conclude that: V is a vector space over

Question 4

Let and be a square matrices.

We want to show that

**Transpose property of matrix addition**

For any two matrices, the transpose of their sum is equal to the sum of their transposes.

For each element in the matrix , its transpose will be

***Analyse***

Let be a square matrix.

We want to show that

From 1, we can apply the **Transpose property of matrix addition** to

*since*

*Thus, is symmetric.*

Question 5

is a subspace of where:

*if and only if the following three conditions hold*:

**Condition 1 : Zero vector in**

Let denote the zero function.

And

Thus, the zero vector is in  **,** where

**Condition 2: Closed under addition**

**A1**

Let .

Then

And

We now have

Therefore

Thus, is closed under addition

**Condition 3: Closed under scalar multiplication**

**M1**

Let and scalar

Then

And

And

Thus, is closed under scalar multiplication

Therefore, is a subspace of

Question 6

Given and ,

is a subspace of where:

*if and only if the following three conditions hold*:

**Condition 1: Zero vector in**

For any we can take

Let all scalars

Then

is in ,

Thus contains the zero vector

**Condition 2: Closed under addition**

The sum of the vectors

Thus, is closed under addition

**Condition 3: Closed under scalar multiplication**

If

And

For each ,

Thus, is closed under scalar multiplication

Therefore,

Question 7

**7.1**

Let and be subspaces of a vector space

Let

And

is a subspace of

*if and only if the following three conditions hold*:

**Condition 1: Zero vector in**

For any we can take

and

Also,

Then ,

Thus contains the zero vector

**Condition 2: Closed under addition**

**Let**

Then and

For some and

Then,

Which is in

Thus, is closed under addition

**Condition 3: Closed under scalar multiplication**

If

And

Then

For some and

And

Thus, is closed under scalar multiplication

Therefore, is a subspace of

**7.2**

and

*if and only if the following two conditions hold*:

**Condition 1:**

Let

Then and **,**

And also

Thus

**Condition 2:**

**Let**

Then and

For some and

Then,

Which is in

Thus, is closed under addition

Therefore, is a subspace of